Isochronous Machines with Sector Magnets
Fields and Sectors Edges,
Conditions for Similidude

A. Riche, CERN

Summary

Isochronous cyclotron with circular closed orbit

Isochronous machine with NS sectors and straight sections

→ $B(p)$ given → straight section ?

ex : $B(p) = Cte$

$B(p) = \alpha p$

→ $SS(p)$ given → $B(p)$?

ex $SS(p) = Cte$

Focusing

FFAG triplet type (KEK)

Thomas focusing + effects of alternation of fields and gradients in sectors

Spiral ?

Many variables problem : discriminate good parameters on graphs representing Q_H Q_V β_H β_V obtained analytically as functions of at least 2 variables.

An FFAG with isochronicity + similitude ? CONCLUSION

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ISOCHRONOUS MACHINE (CYCLOTRON)
CIRCULAR CLOSED ORBITS

circular machine without sectors

Isochronicity : ω = constant

$$\rho_c = v_c / \omega_c$$

The field on the particle with momentum p_c at radius ρ_c is

$$B_c = p_c / (e\,\rho_c) = m_0\,\gamma_c\,v_c / (e\,\rho_c) = m_0\,\omega_c\,\gamma_c / e$$

$$B_c[\rho] = [E_0 / e\,c]\,(\omega_c / c) \Big/ \sqrt{1 - (\omega_c\,\rho_c / c)^2}$$

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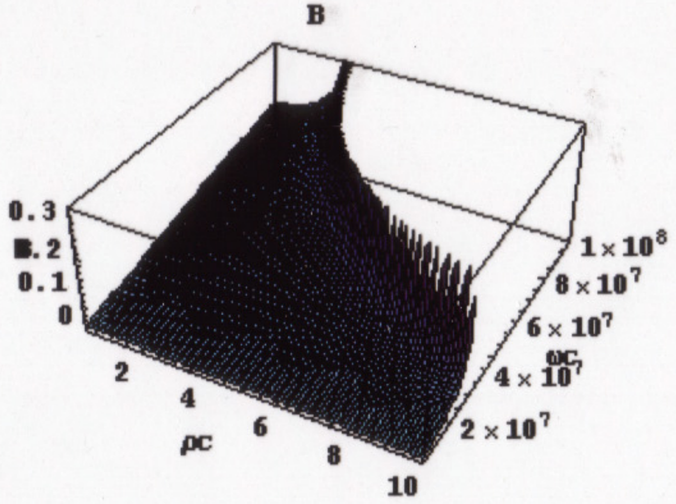
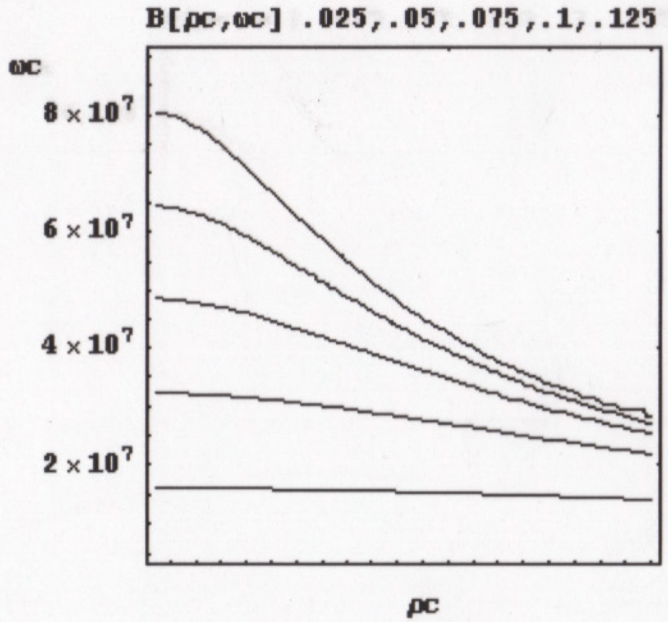
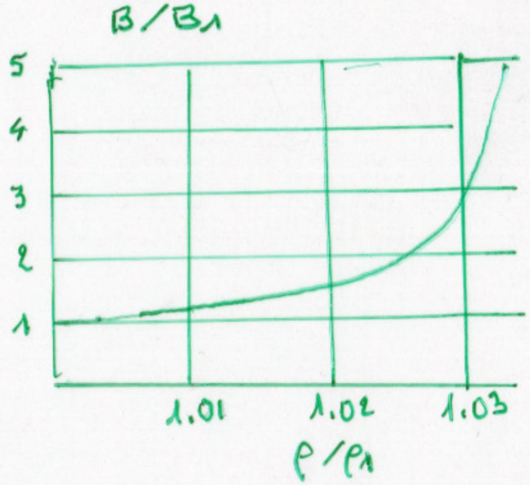
B

Isochronous Cyclotron

```
Bc[ρc_, ωc_] := Bρ0 (ωc/c) / (e √(1 - (ωc.ρc/c)^2) );
Plot[B[ρc], {ρc, 1, 10}, AxesLabel -> {"r", "B"}, Ticks -> Automatic,
  GridLines -> Automatic];
```







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ISOCHRONOUS MACHINE (CYCLOTRON) WITH NS SECTORS AND STRAIGHT SECTIONS;

Closed orbits made of arcs of circles in the sectors (with a total length of $2\pi\rho$), and straight lines (with a total length of Ns s) in between.

By comparison with the synchronous cyclotron, for a closed orbit with the same length

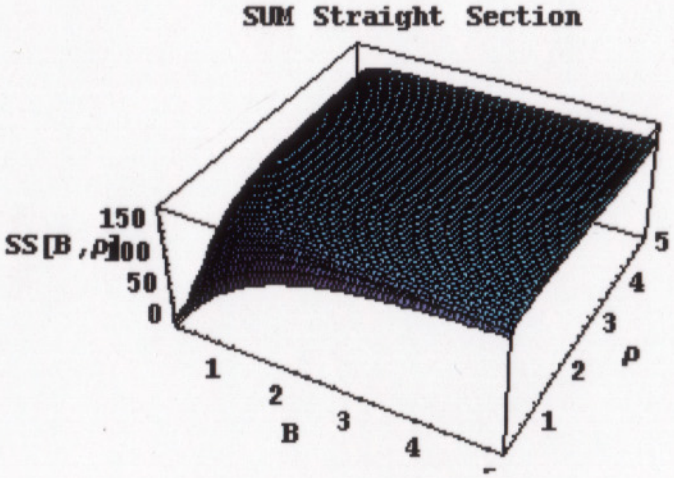
$$\text{Length} = \pi\rho + Ns\,s = Lc = 2\pi\rho c \quad (1)$$

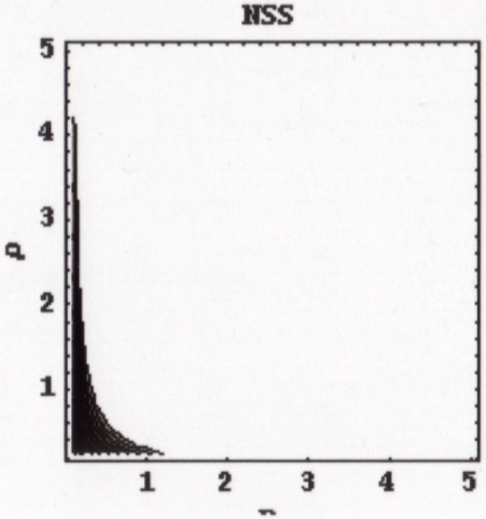
$$Bc[\rho c] = (E0/c)\,(\omega c/c) \Big/ \sqrt{1-(\rho c\,\omega c/c)^2} \quad (2)$$

and the same momentum (same speed) $\quad B[\rho]\,\rho = Bc\ \rho c \quad (3)$

$$B[\rho,\omega] = (E0/c) \Big/ \sqrt{(c/\omega)^2/(\rho + NS\,s/(2\pi))^2 - 1}$$

$$NS\,s\,[\rho,\omega] = 2\pi\left(1\Big/\left((\omega c/c)\sqrt{(E0/c)^2/(B\rho)^2+1}\right) - 1\right)$$





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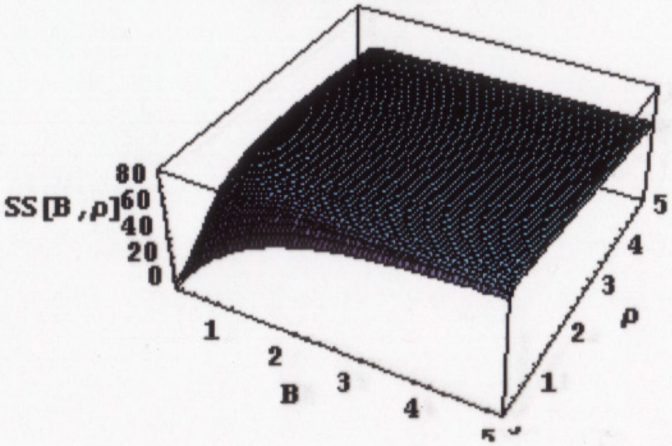
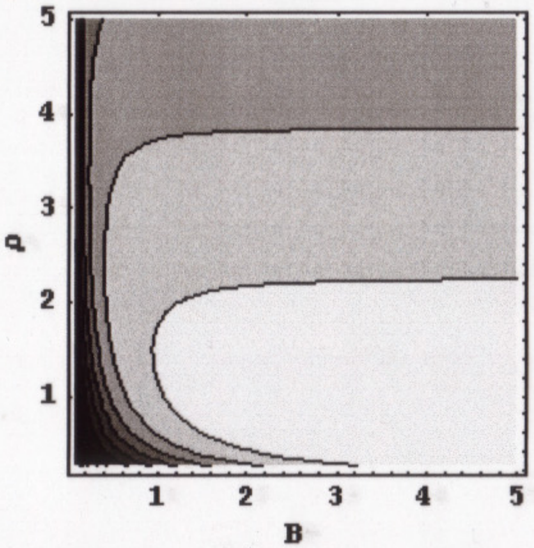
ω = 20 10^6

SUM Straight Section



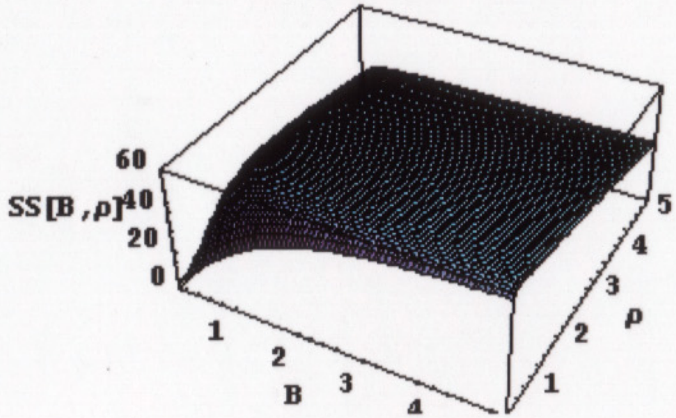
NSS



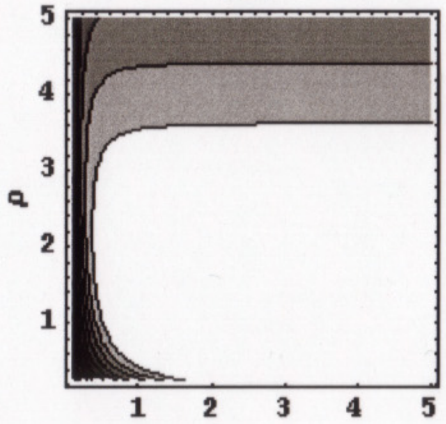
NSS{5, 7.5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90, 100}

ω = 30 10^6

SUM Straight Section

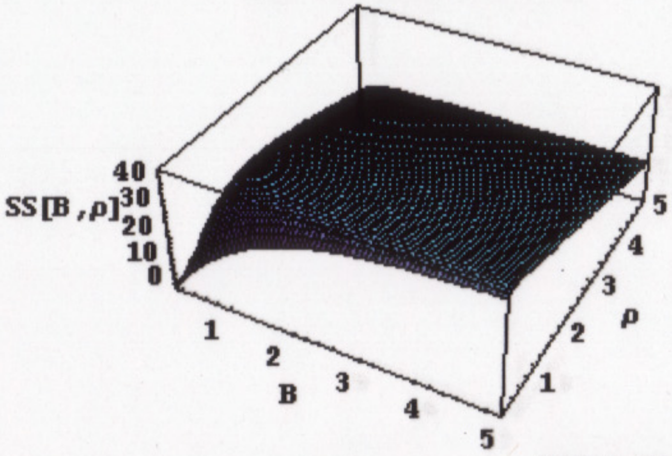


NSS

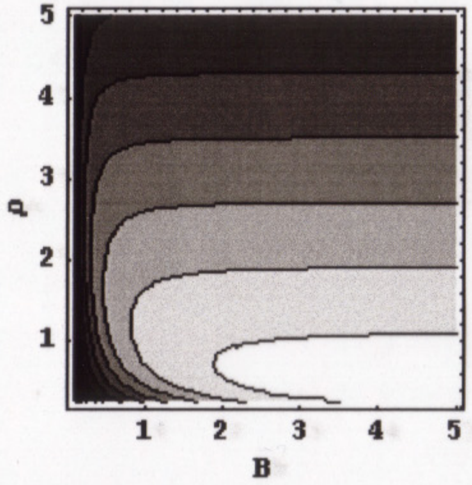


ω = 40 10^6

SUM Straight Section



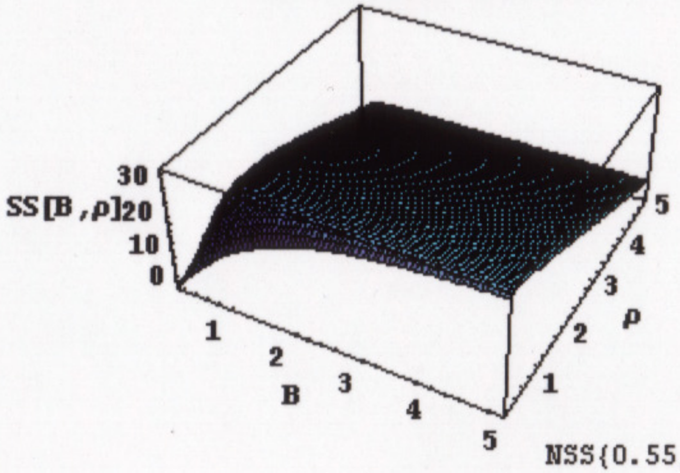
NSS



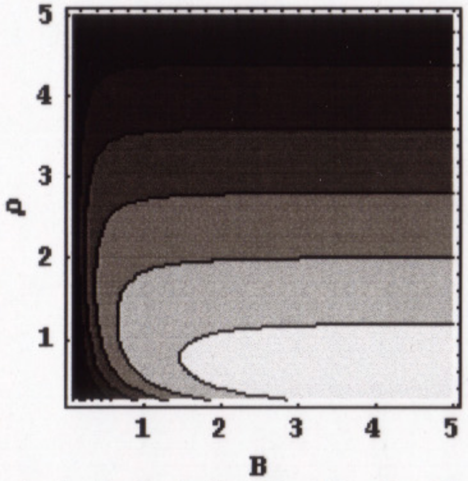
NSS{0.55, 2, 2.5, 3, 3.5, 5, 7.5, 10, 15, 20, 25, 30, 35, 40

ω = 50 10^6

SUM Straight Section



NSS



NSS{0.55, 2, 2.5, 3, 3.5, 5, 7.5, 10, 15, 20, 25, 30, 35, 40}

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ISOCHRONOUS MACHINE (CYCLOTRON) WITH NS SECTORS ARCS, AND STRAIGHT SECTIONS;

$$B[r] = Cte$$

$$NSs = 2\pi \left(1 / \left((\omega c / c) \sqrt{(E0 / c)^2 / (B\rho)^2 + 1} \right) - 1 \right)$$

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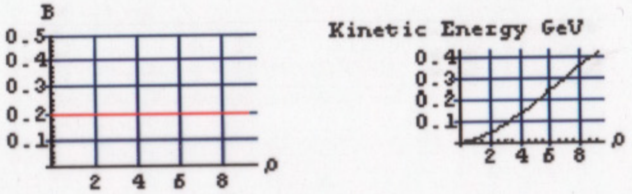
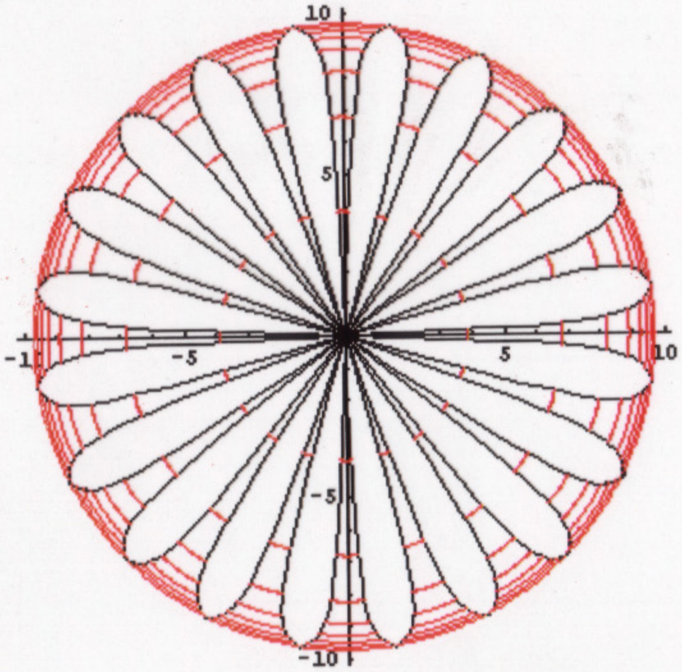
NS= 20, B= 0.2, ω= 30000000

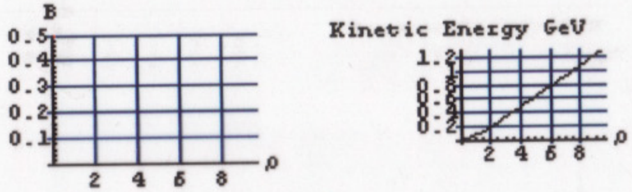
W kinetic= 0.45967 10^9 eV, ρmax= 9.71847

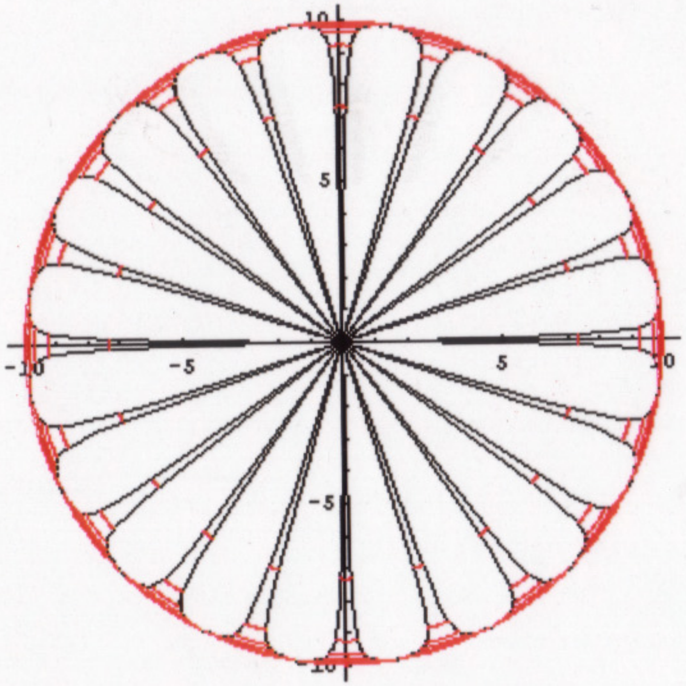




NS= 20, B= 0.5, ω= 30000000,

W kinetic= 1.35843 10^9 eV, ρmax= 9.94965





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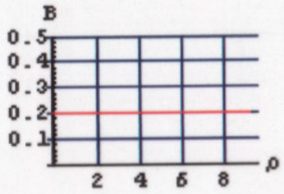
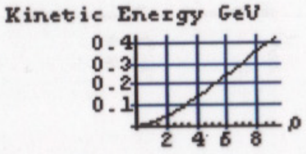
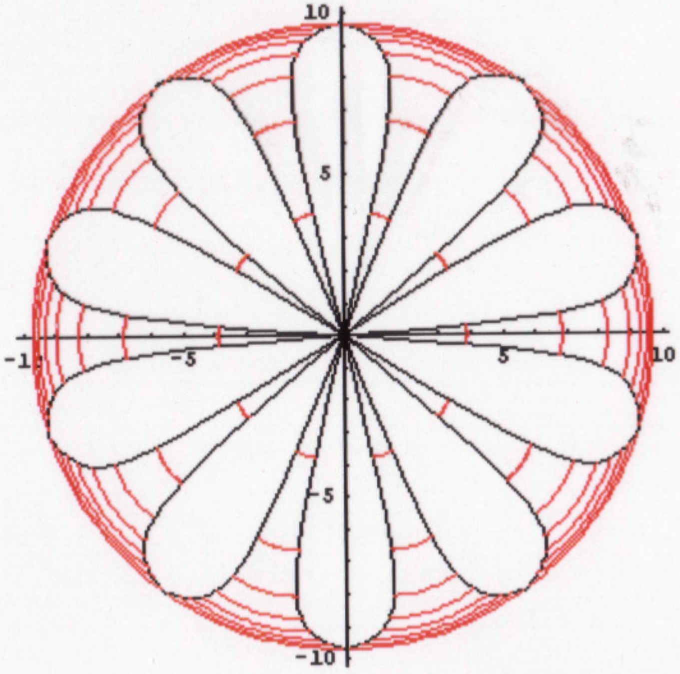
NS= 10, B= 0.2, ω= 30000000

, W kinetic= 0.45967 10^9 eV, ρmax= 9.71847

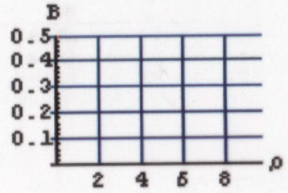


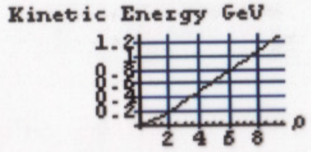


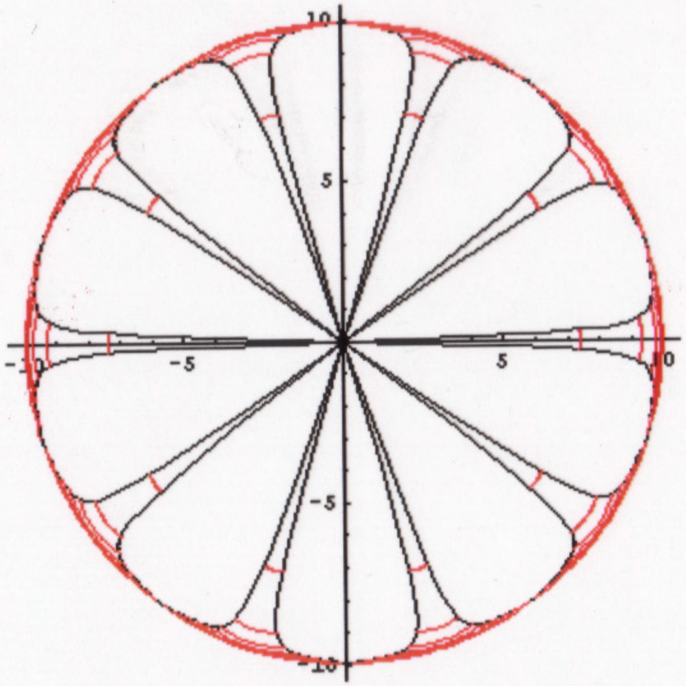


NS= 10, B= 0.5, ω= 30000000

, W kinetic= 1.35843 10^9 eV, ρmax= 9.94965







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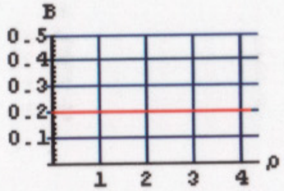
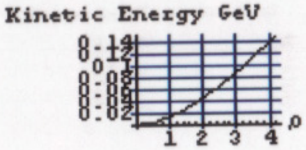
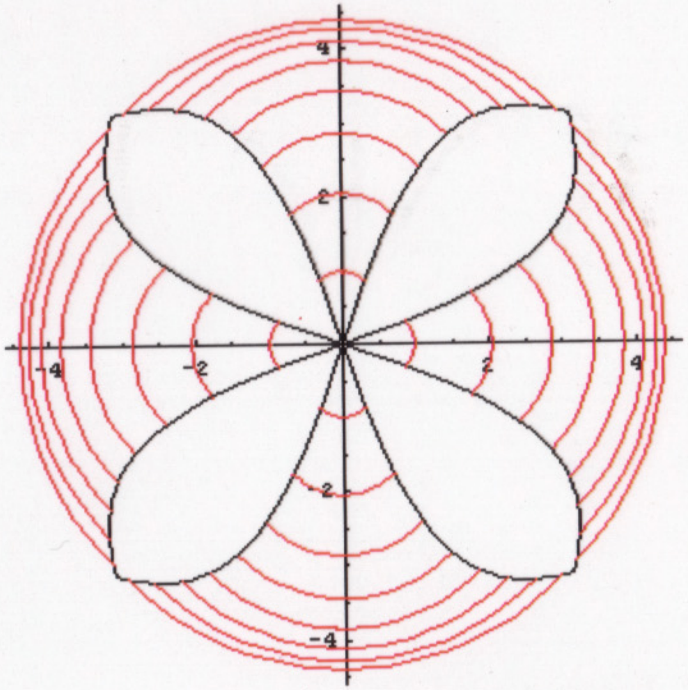
NS= 4, B= 0.2, ω= 60000000

, W kinetic= 0.160085 10^9 eV, ρmax= 4.4218

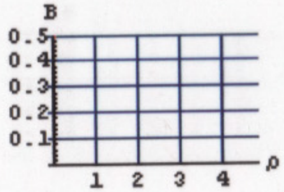


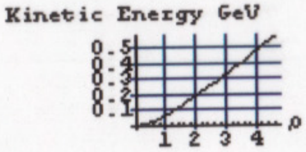


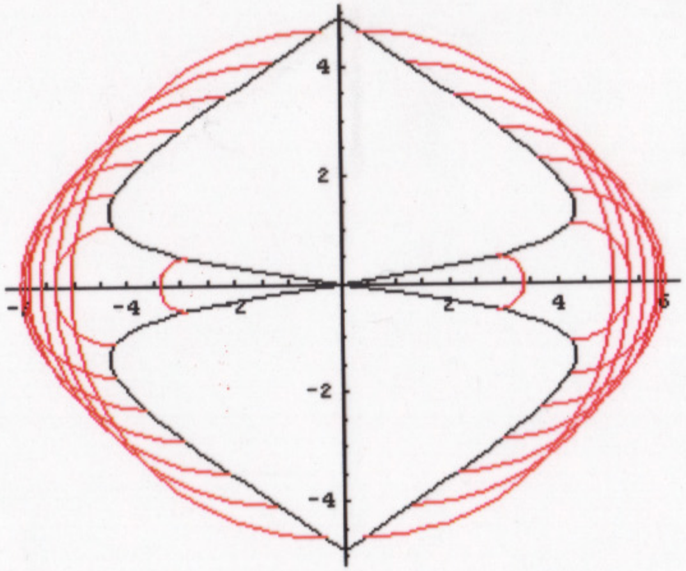


NS= 2, B= 0.5, ω= 60000000

, W kinetic= 0.609463 10^9 eV, ρmax= 4.90911







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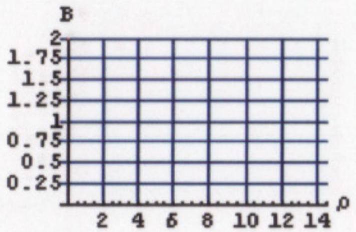
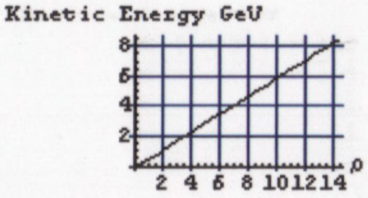
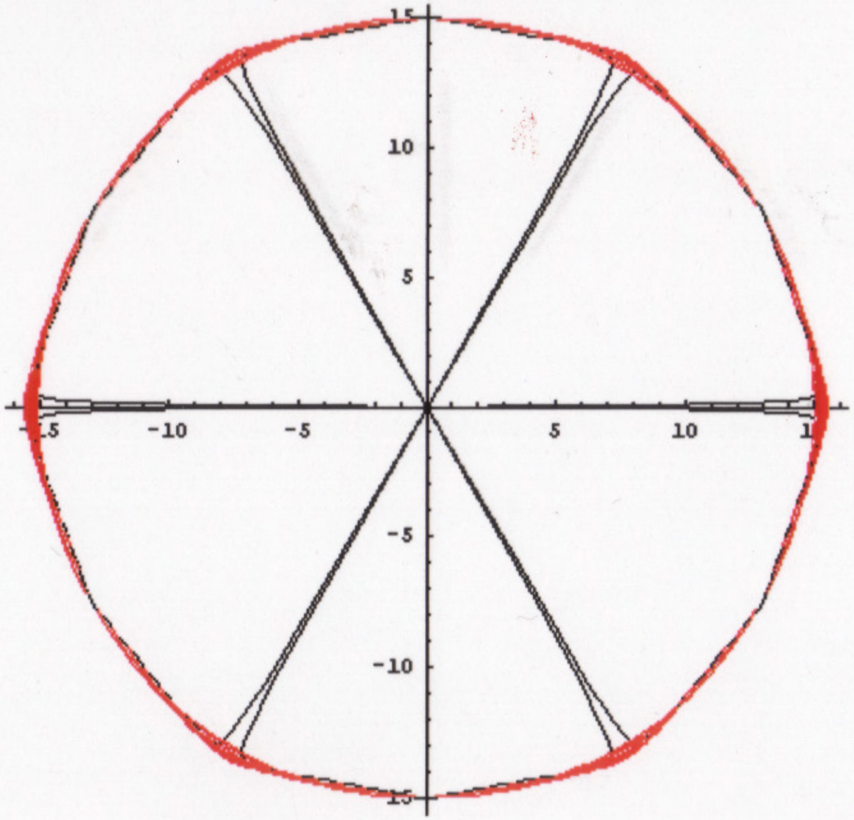
NS= 6, B= 2, ω= 20000000

, W kinetic= 8.84805 10^9 eV, ρmax= 14.9878







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ISOCHRONOUS MACHINE (CYCLOTRON) WITH NS SECTORS ARCS, AND STRAIGHT SECTIONS;

SS[r] = Cte

$$B = (E0 / c) \Big/ \sqrt{(c / \omega)^{2} / (\rho + NS\, s / (2\pi))^{2} - 1}$$

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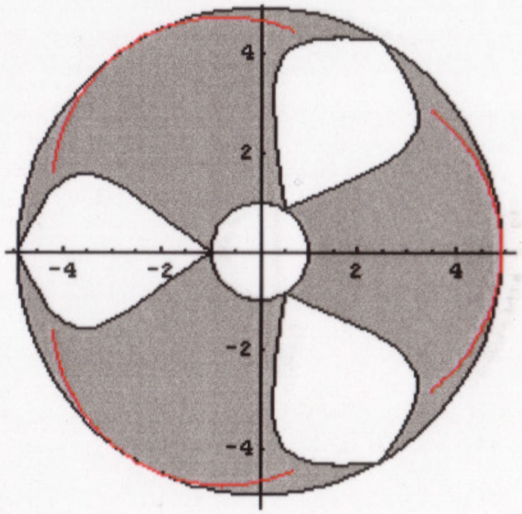
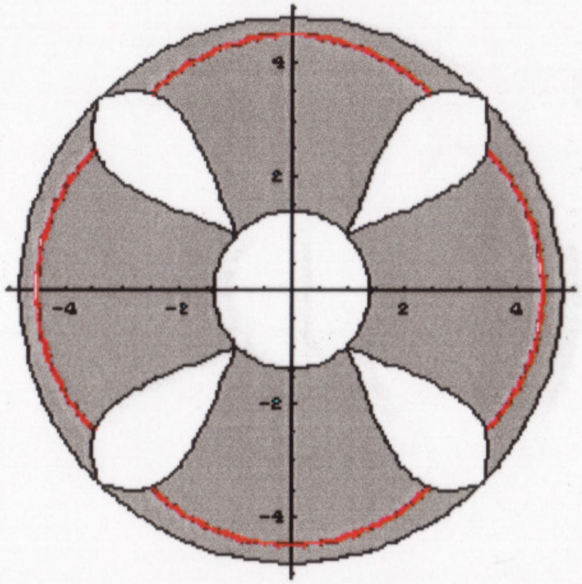
$$B[\rho] = \alpha\ \ \rho$$





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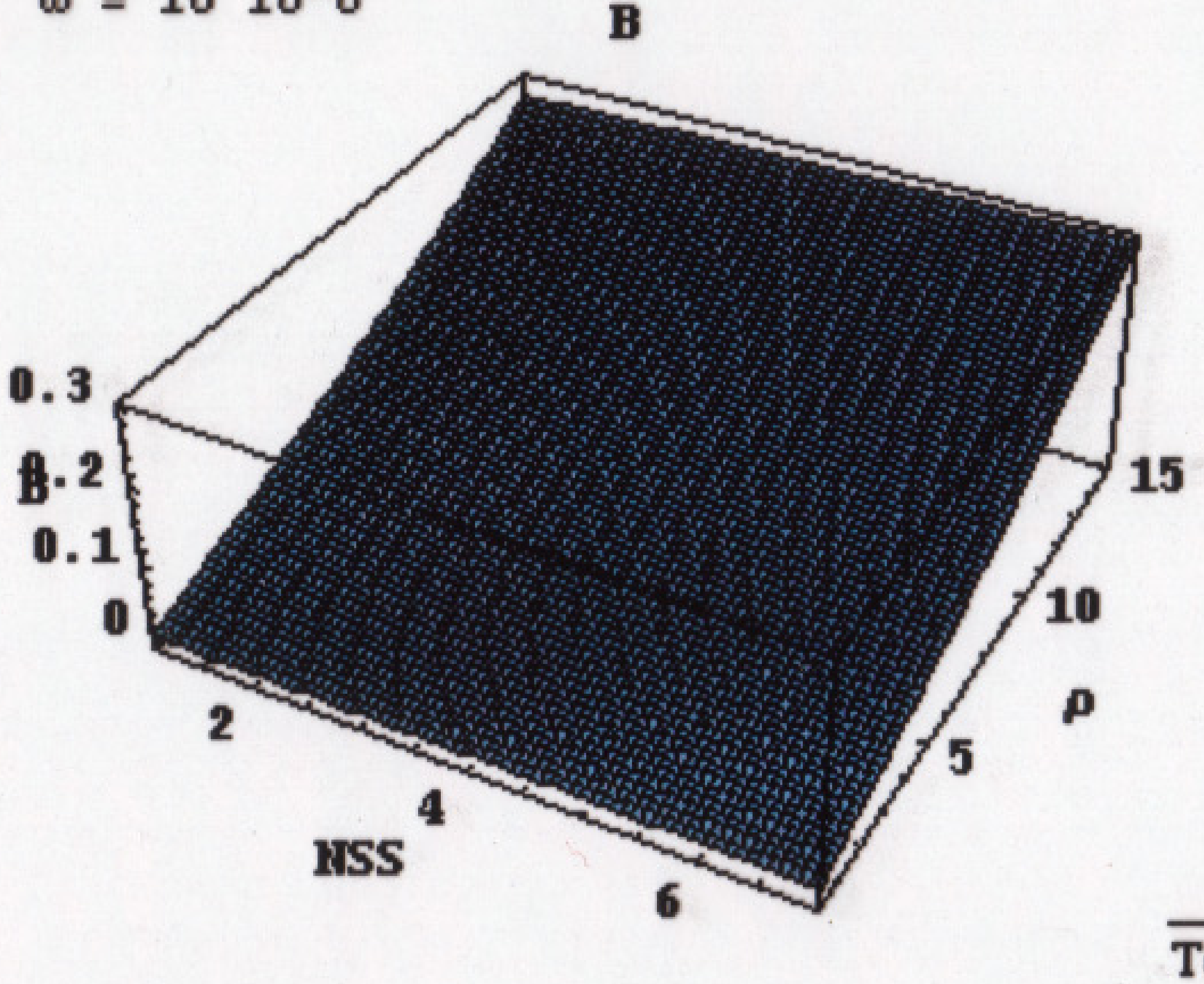
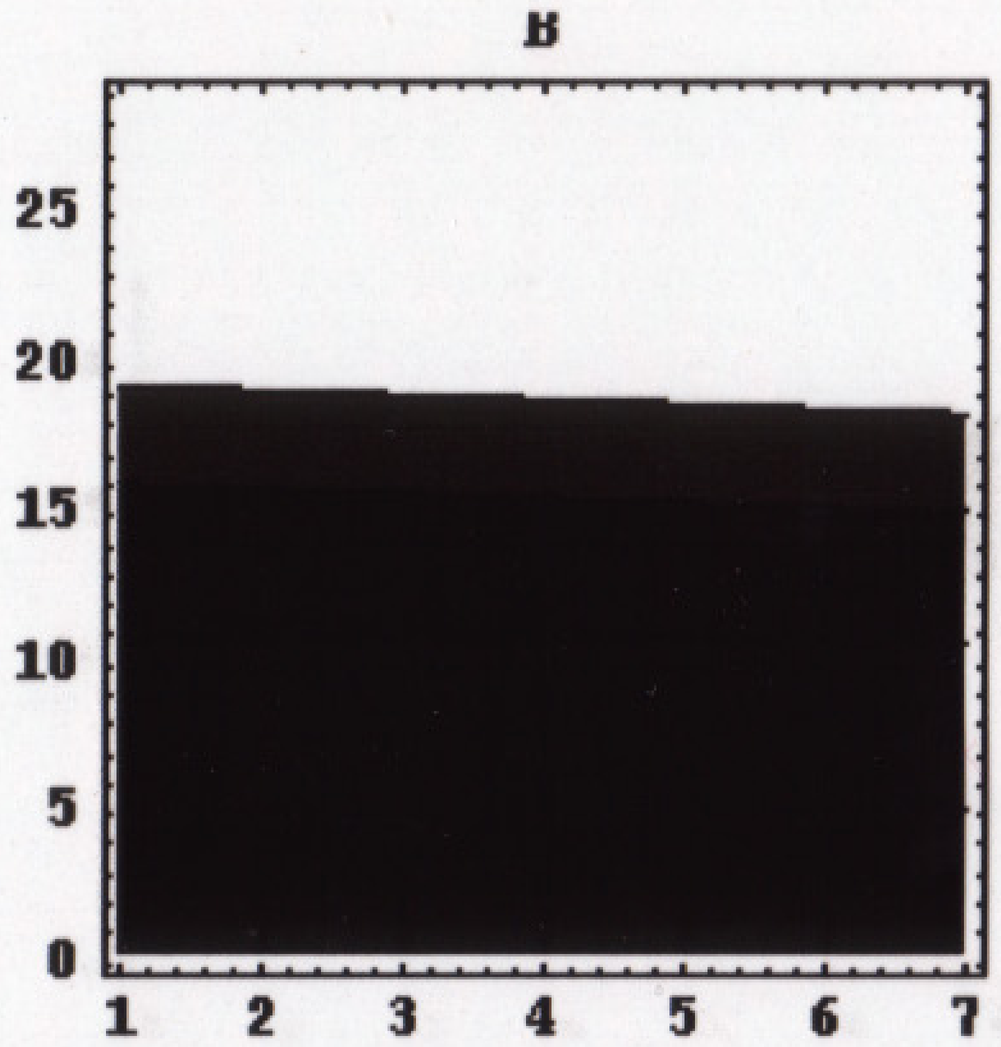
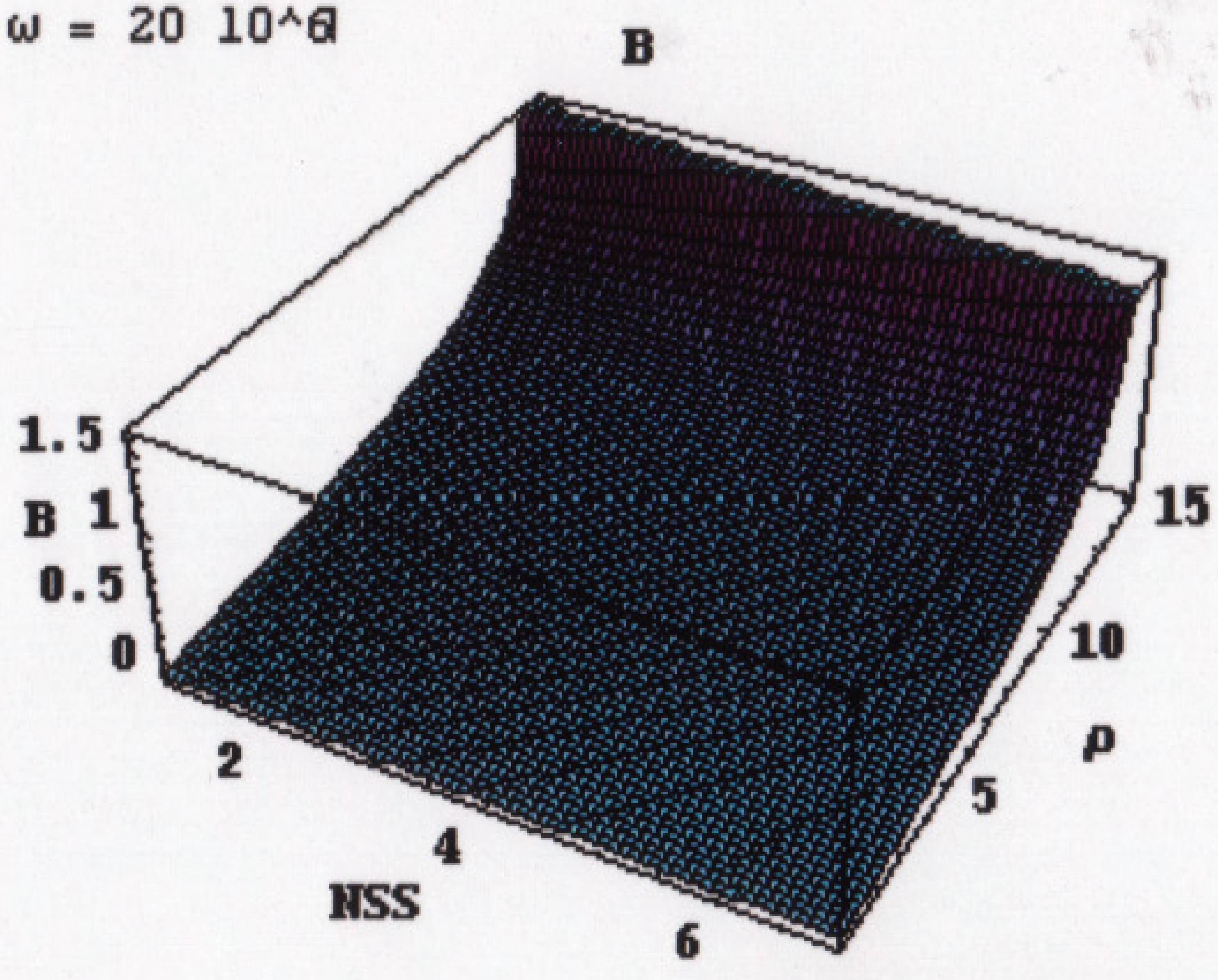
ω = 10 10^6

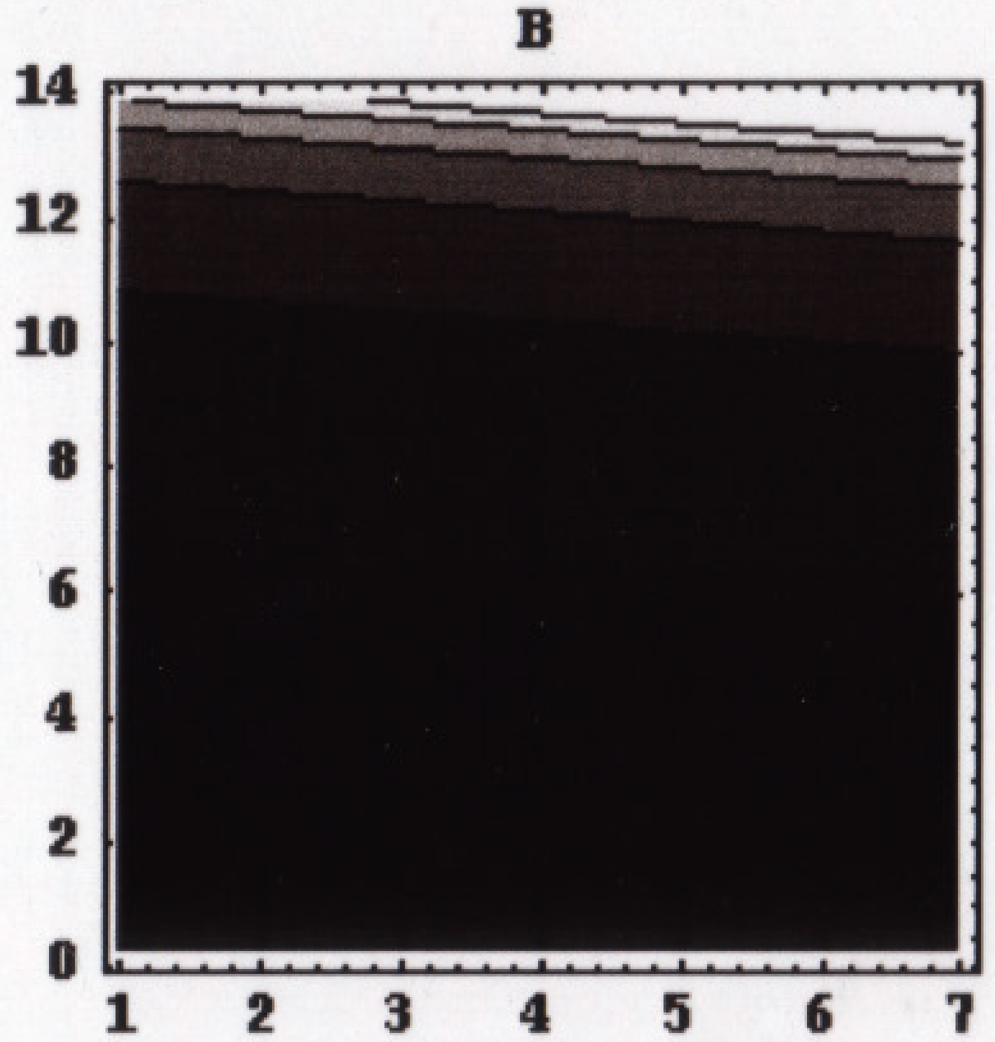




$\frac{B}{Tesla}$ {0.55, 2, 2.5, 3, 3.5, 5, 7.5, 10, 15, 20, 25, 30, 35, 40}

ω = 20 10^6





- {0.55. 2. 2.5. 3. 3.5. 5. 7.5. 10. 15. 20. 25. 30. 35. 40}

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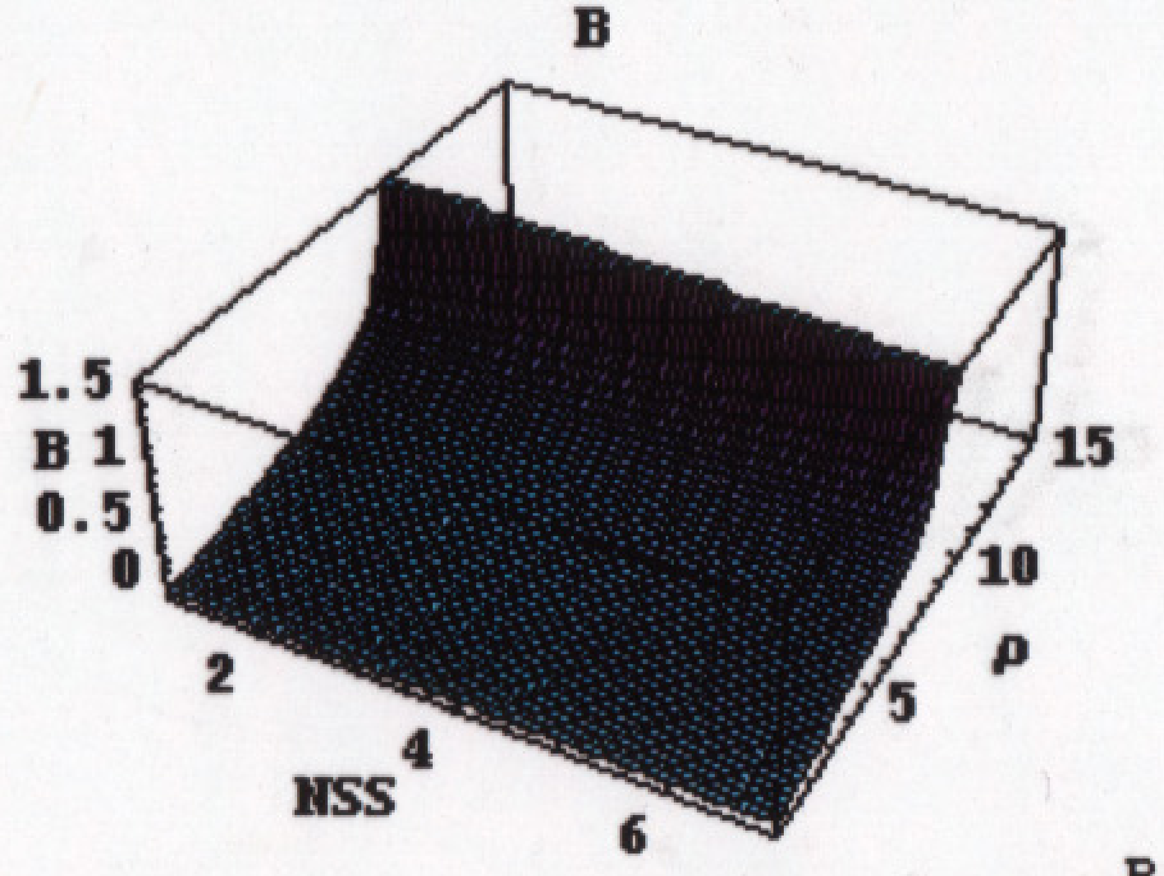
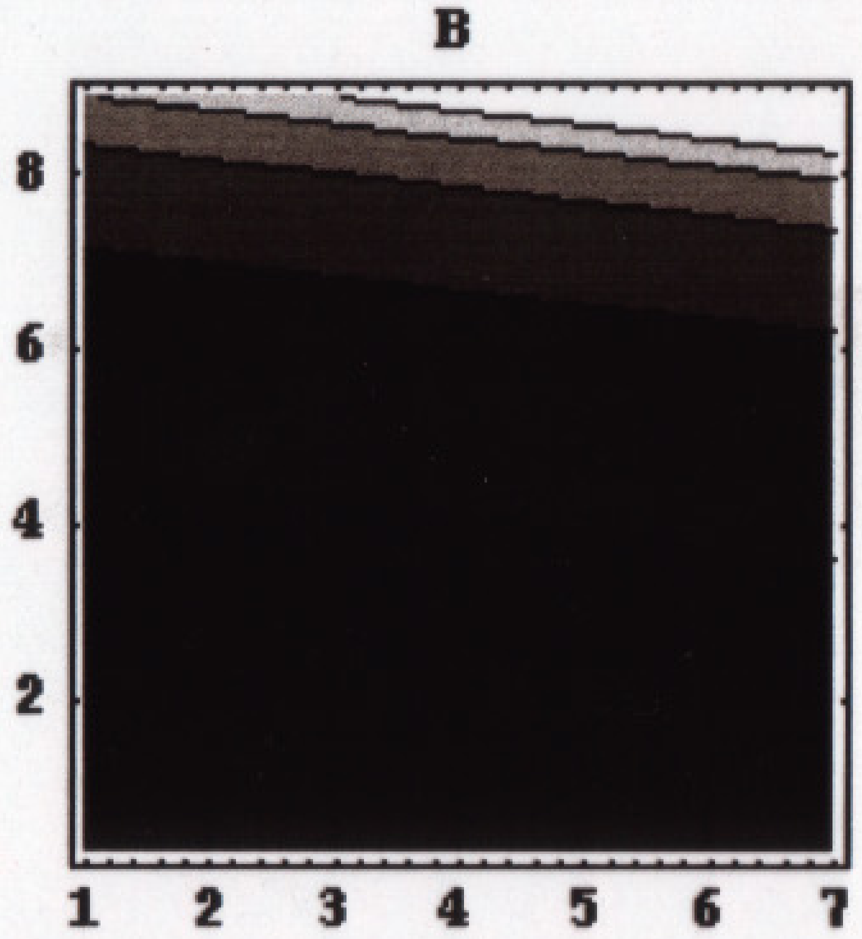
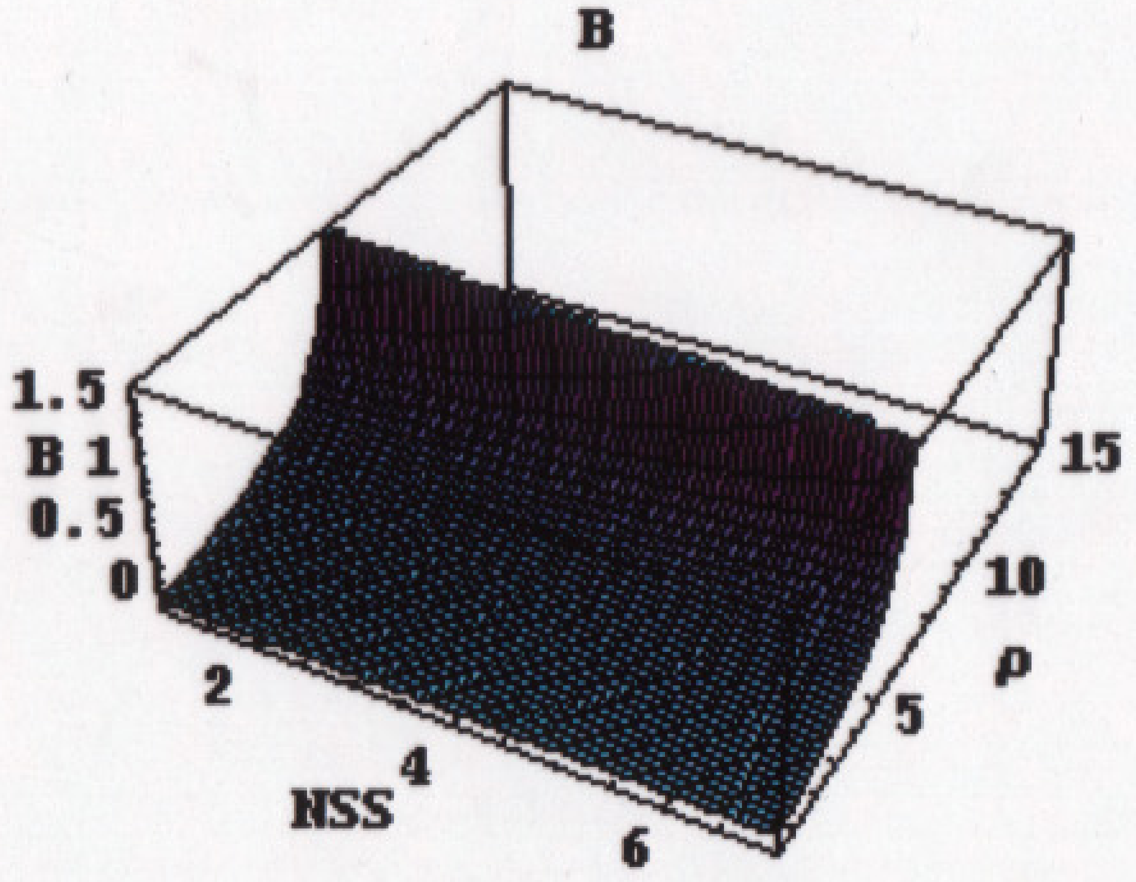
ω = 30 10^6

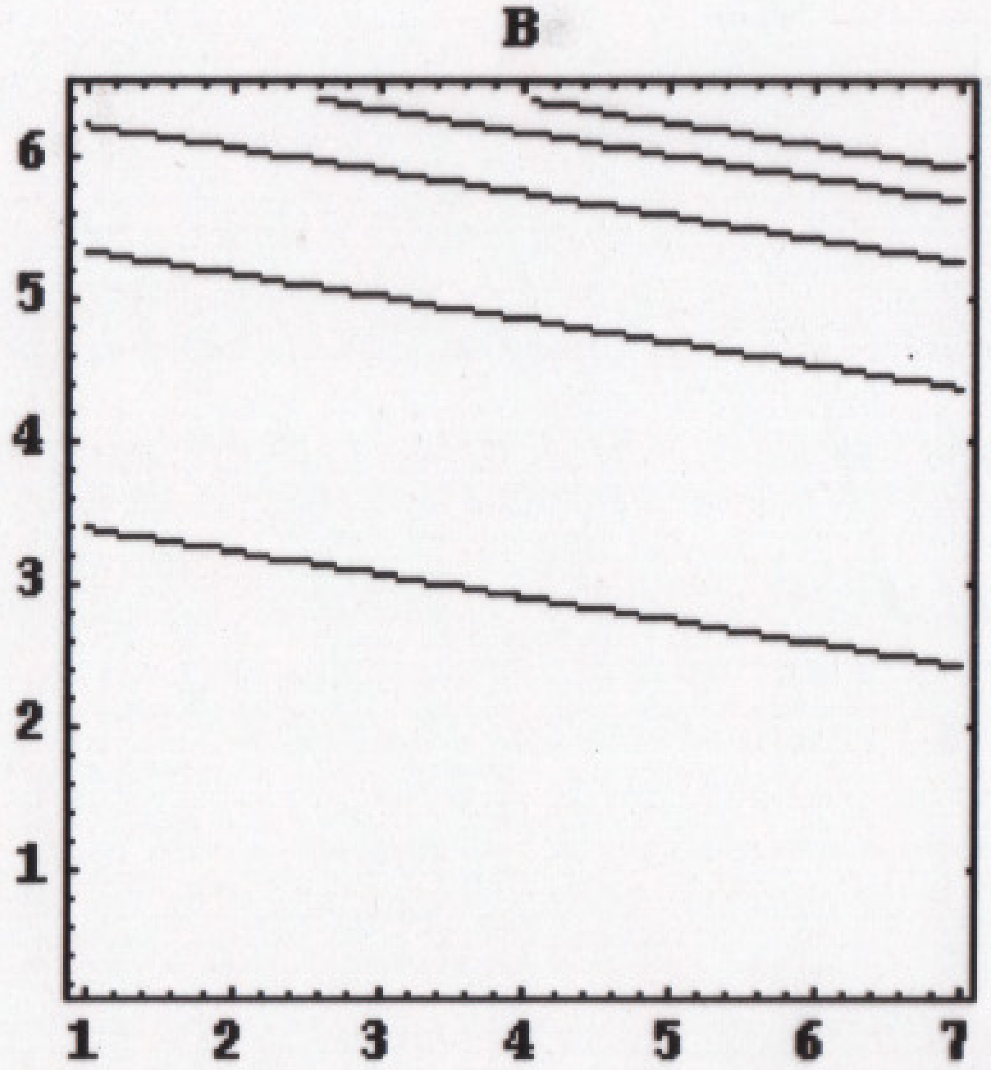




B {0.55, 2, 2.5, 3, 3.5, 5, 7.5, 10, 15, 20, 25, 30, 35, 40

ω = 40 10^6





}

B {0.55, 2, 2.5, 3, 3.5, 5, 7.5, 10, 15, 20, 25, 30, 35, 40}

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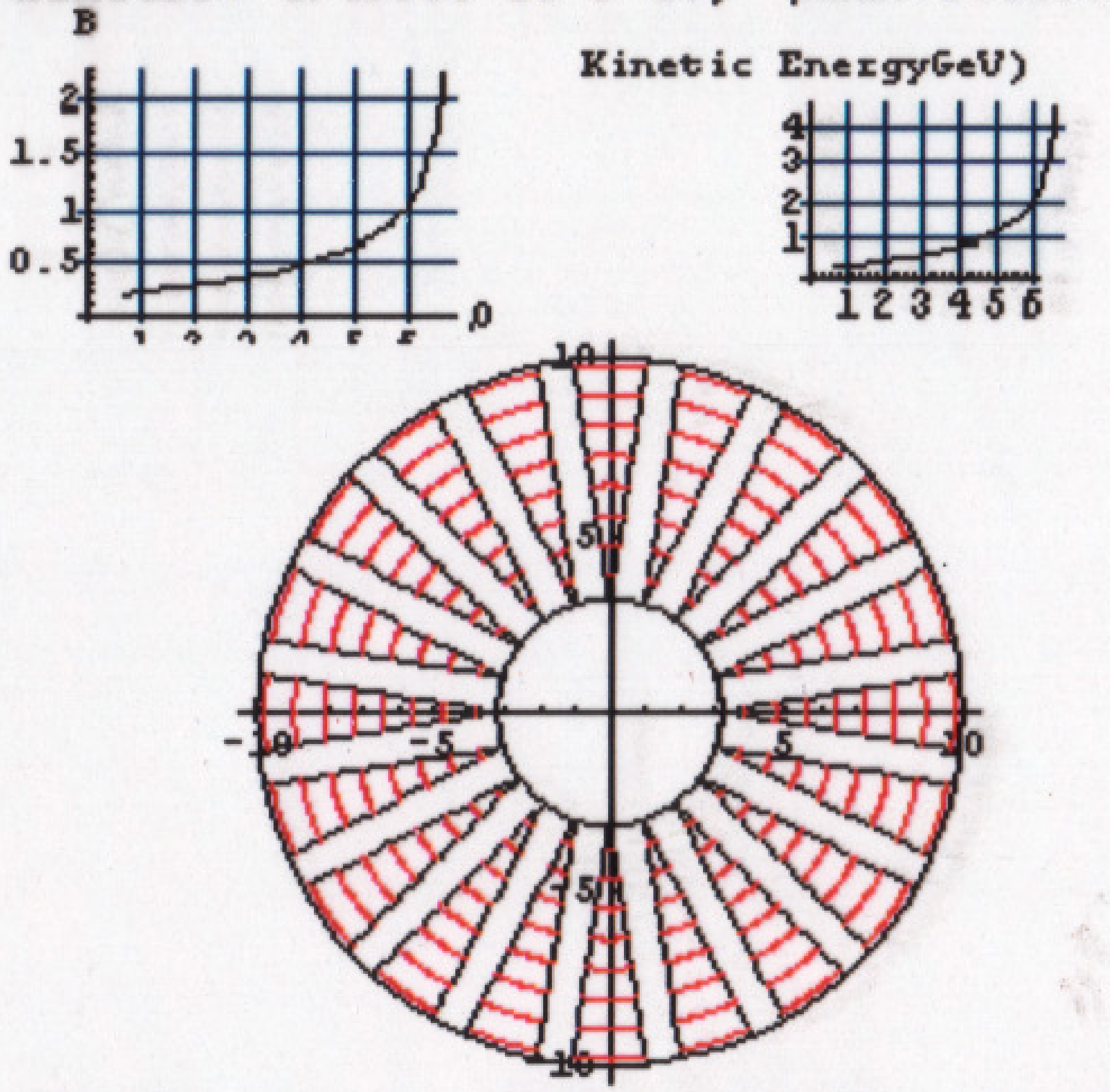
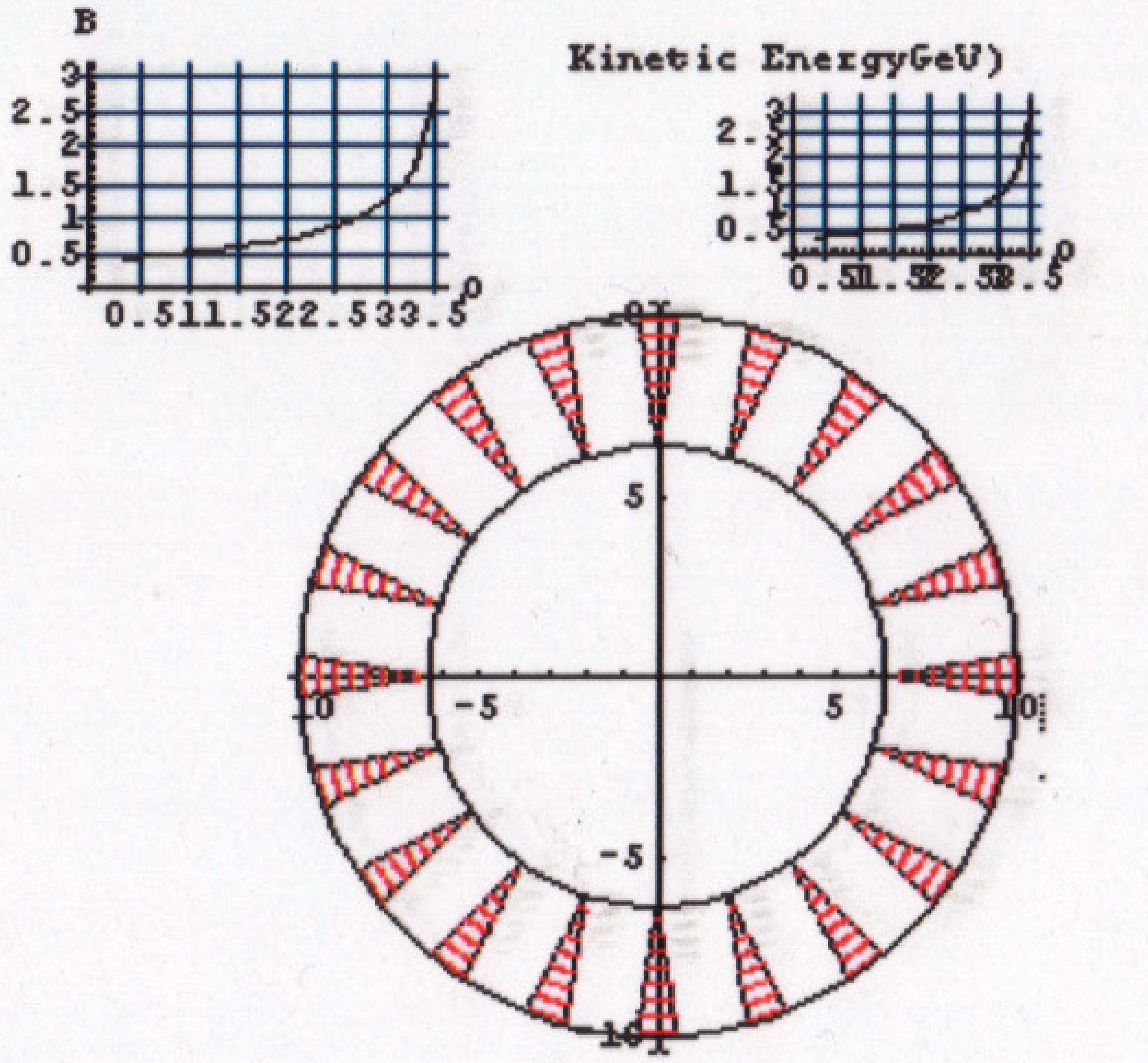
NS= 20, s= 1, Bmax= 2.26576, ω= 130

W kinetic= 4.48835 10^9 eV, ρmax=6.80998



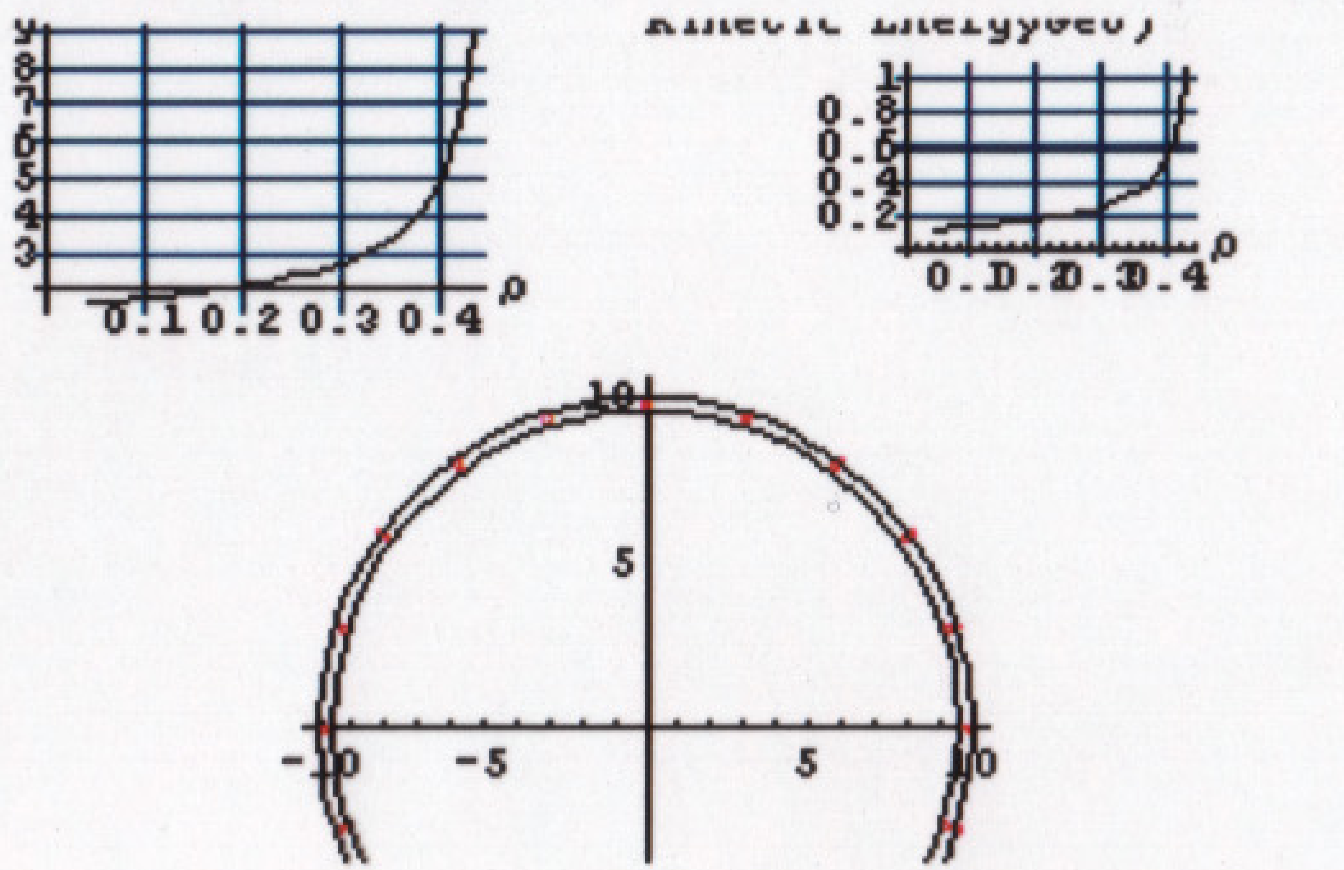
NS= 20, s= 2, Bmax= 3.12746, ω=30

W kinetic= 3.26389 10^9 eV, ρmax=3.62689



NS= 20, s= 3, Bmax= 9.00546, ω= 30

W kinetic= 1.06672 10^9 eV, ρmax=0.443787



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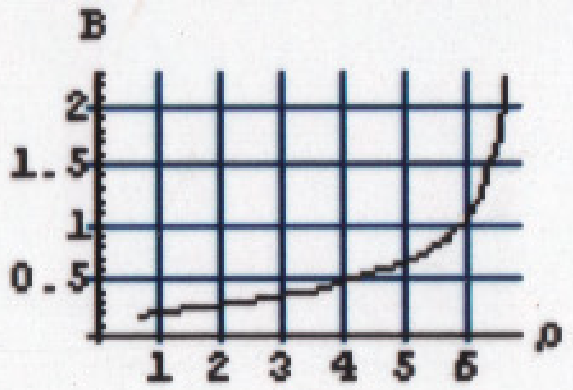
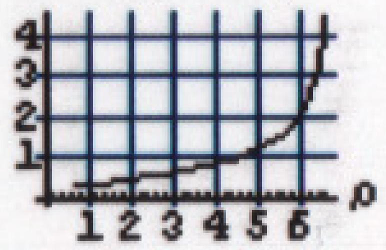
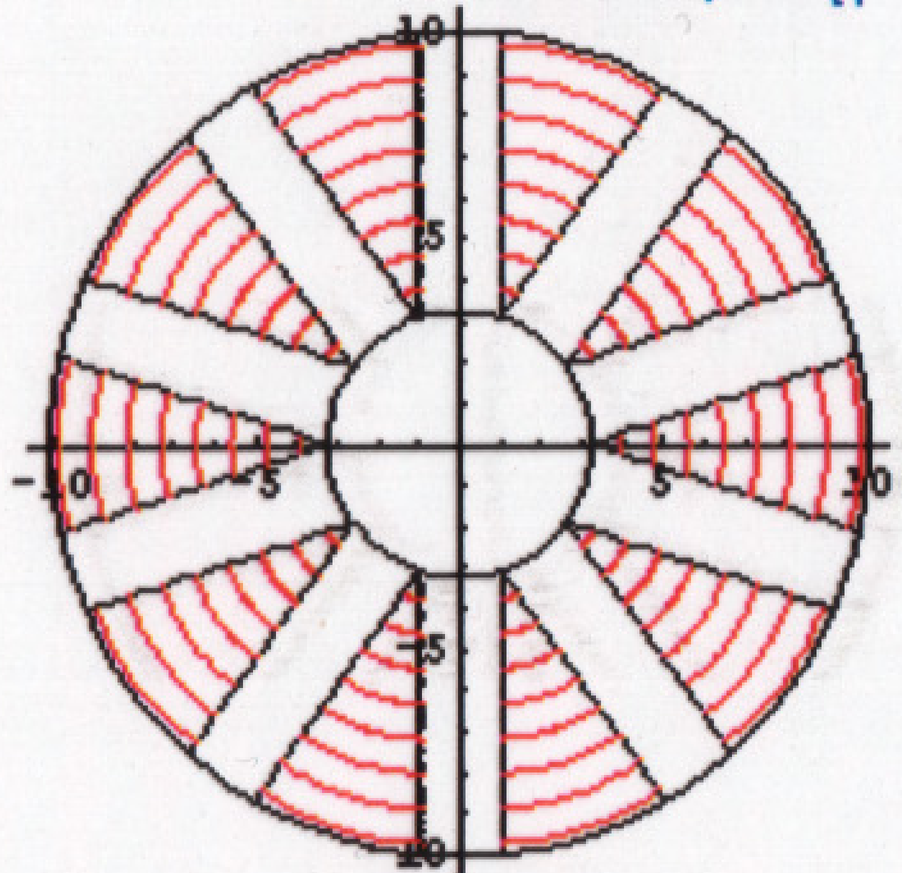
NS= 10, s= 2, Bmax= 2.26576, ω= 30000000,
W kinetic= 4.48835 10^9 eV, ρmax=6.80998

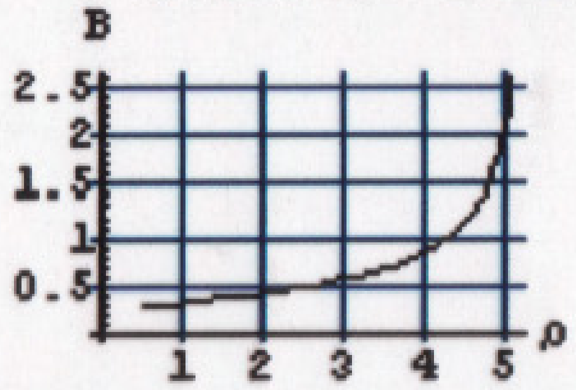


Kinetic EnergyGeV)

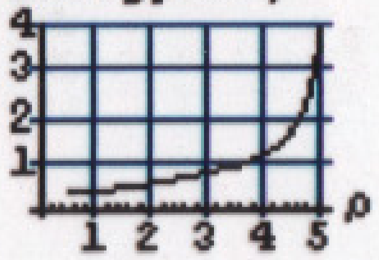


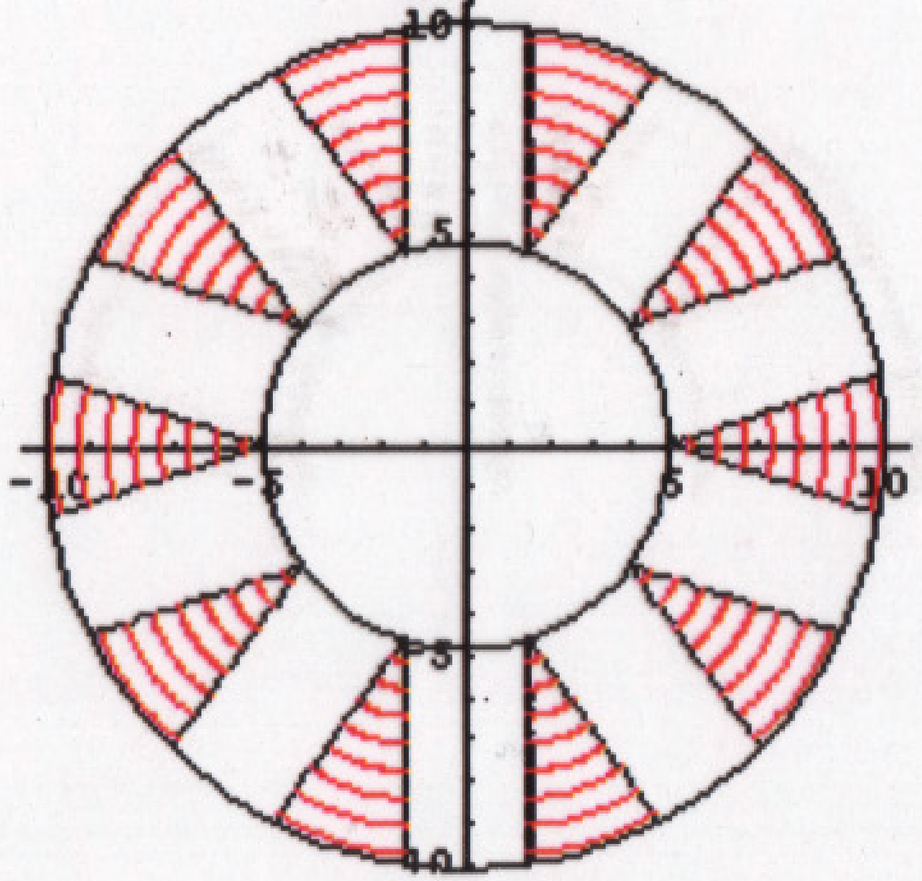


NS= 10, s= 3, Bmax= 2.59781, ω= 30000000,
W kinetic= 3.92703 10^9 eV, ρmax=5.21844

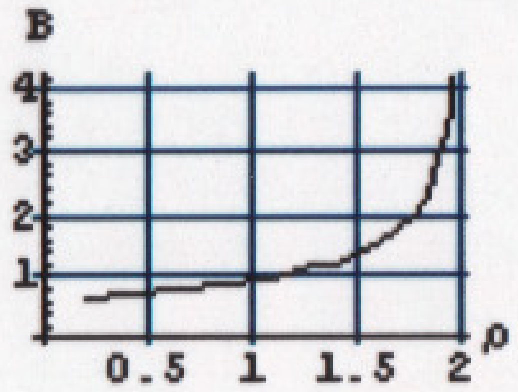


Kinetic EnergyGeV)

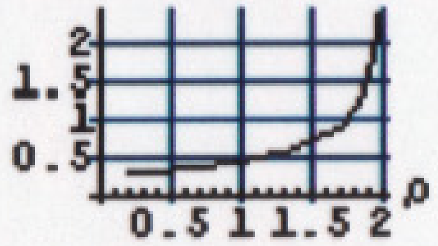


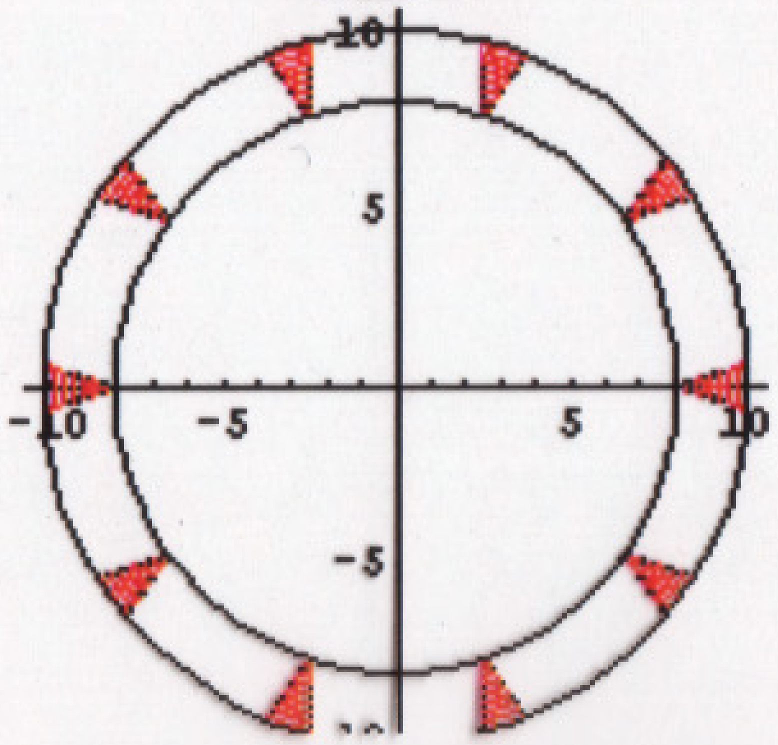


NS= 10, s= 5, Bmax= 4.18998, ω= 30000000,
W kinetic= 2.42094 10^9 eV, ρmax=2.03534



Kinetic EnergyGeV)





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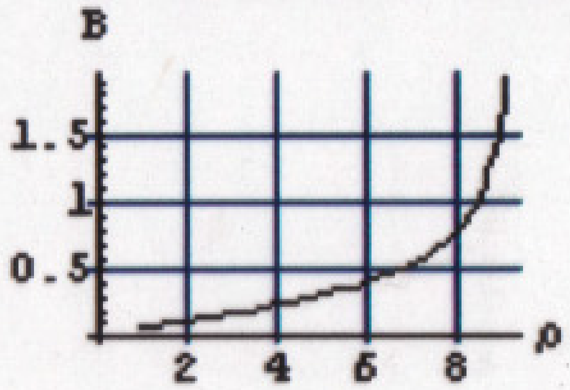
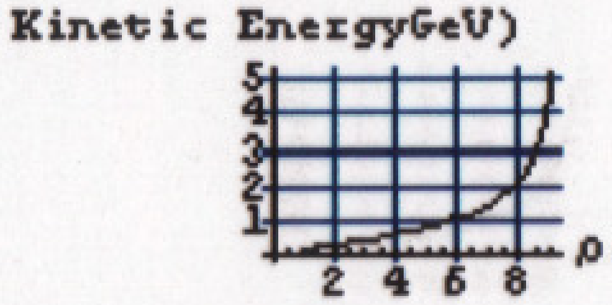
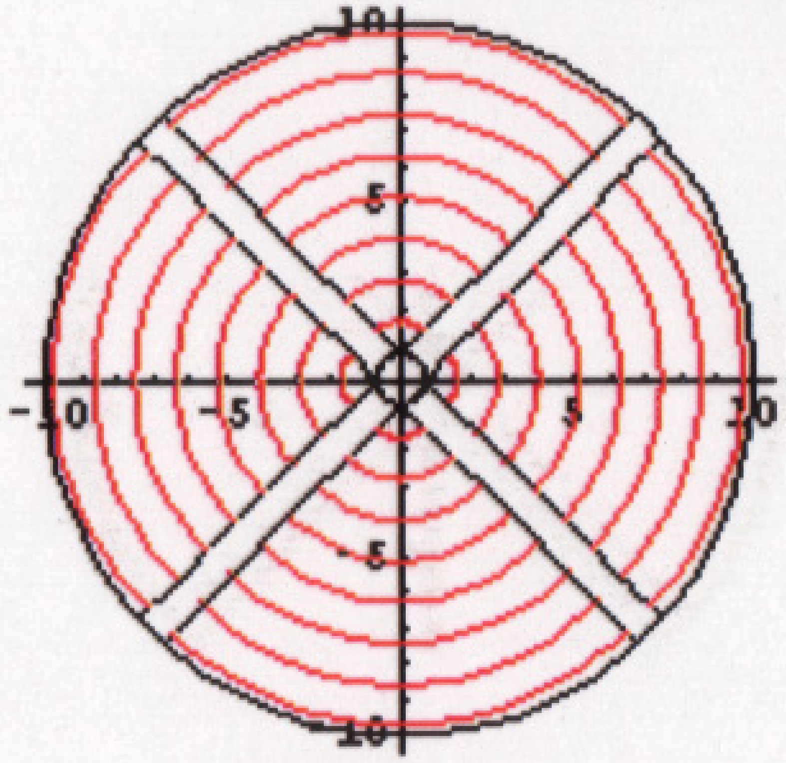
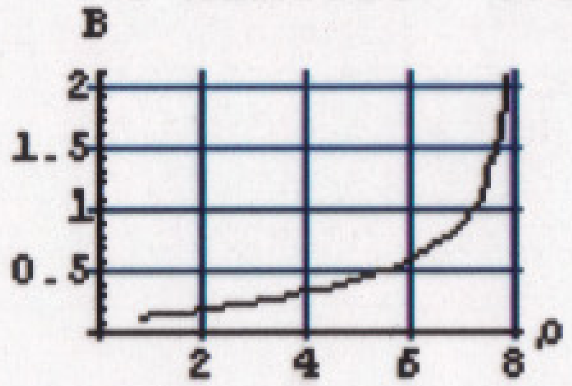
NS= 4, s= 1, Bmax= 1.92163, ω= 30000000,
W kinetic= 5.25247 10^9 eV, ρmax=9.35646

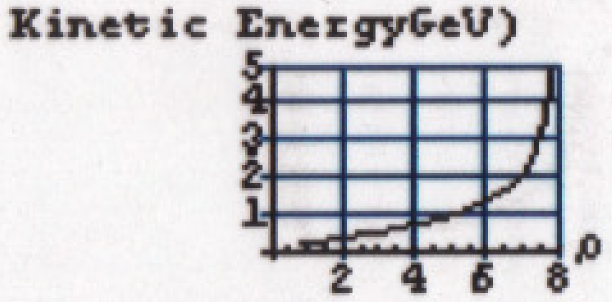


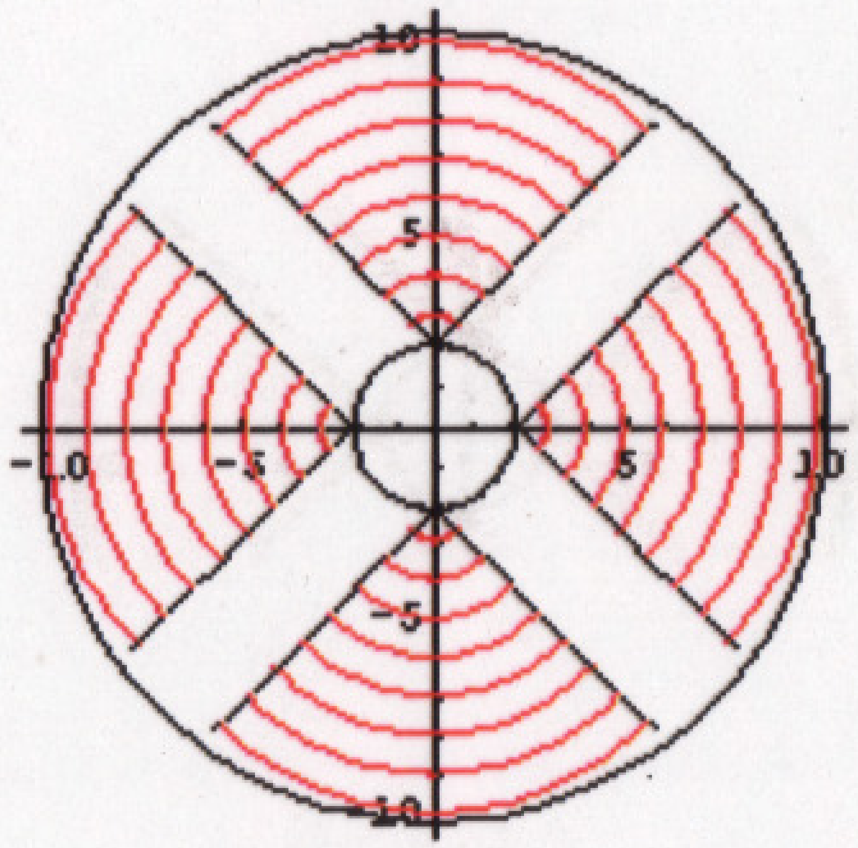




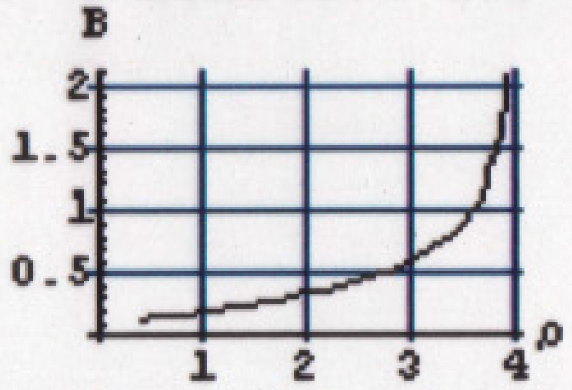
NS= 4, s= 3, Bmax= 2.07356, ω= 30000000,
W kinetic= 4.88728 10^9 eV, ρmax=8.08322

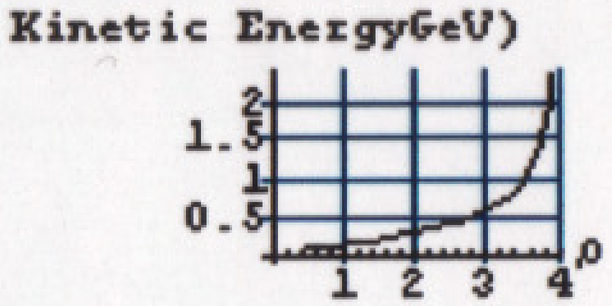


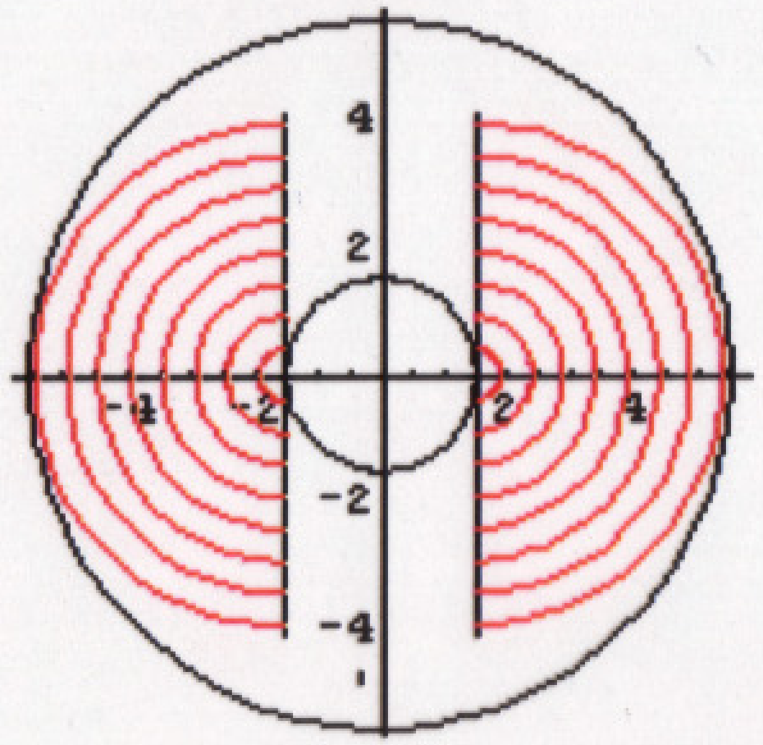




NS= 2, s= 3, Bmax= 2.07356, ω= 60000000,
W kinetic= 2.37679 10^9 eV, ρmax=4.04161







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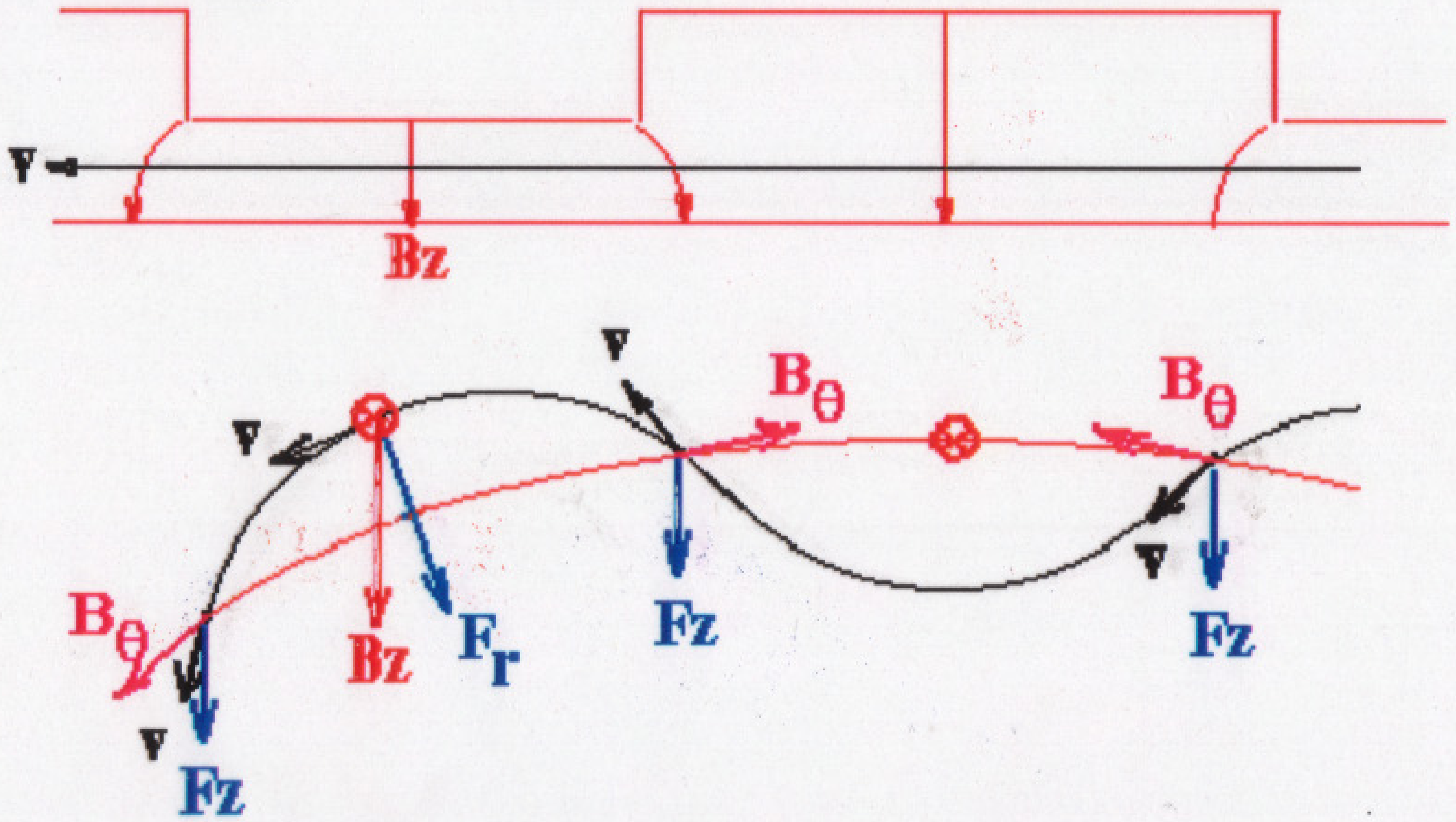
1 - An azimutal modulation of the intensity of the field (hills an valleys)

The resulting azimutal component Bθ gives a force which is alway axially focusing

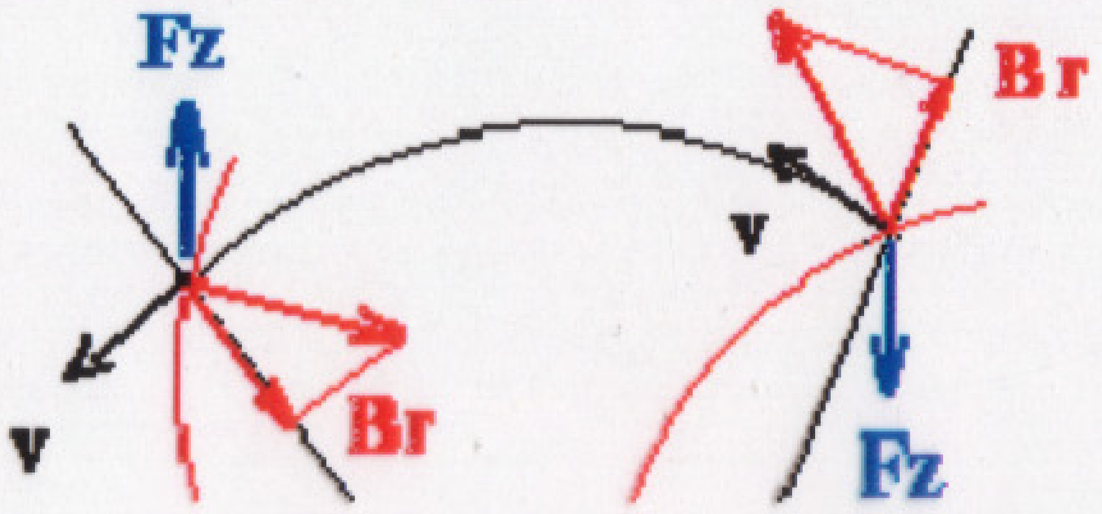
(Thomas Focusing)



2 - If the hills and the valley follow a spiral,

The radial field is alternatively positive and negative,

resulting in a focusing by the alternate gradient principle.



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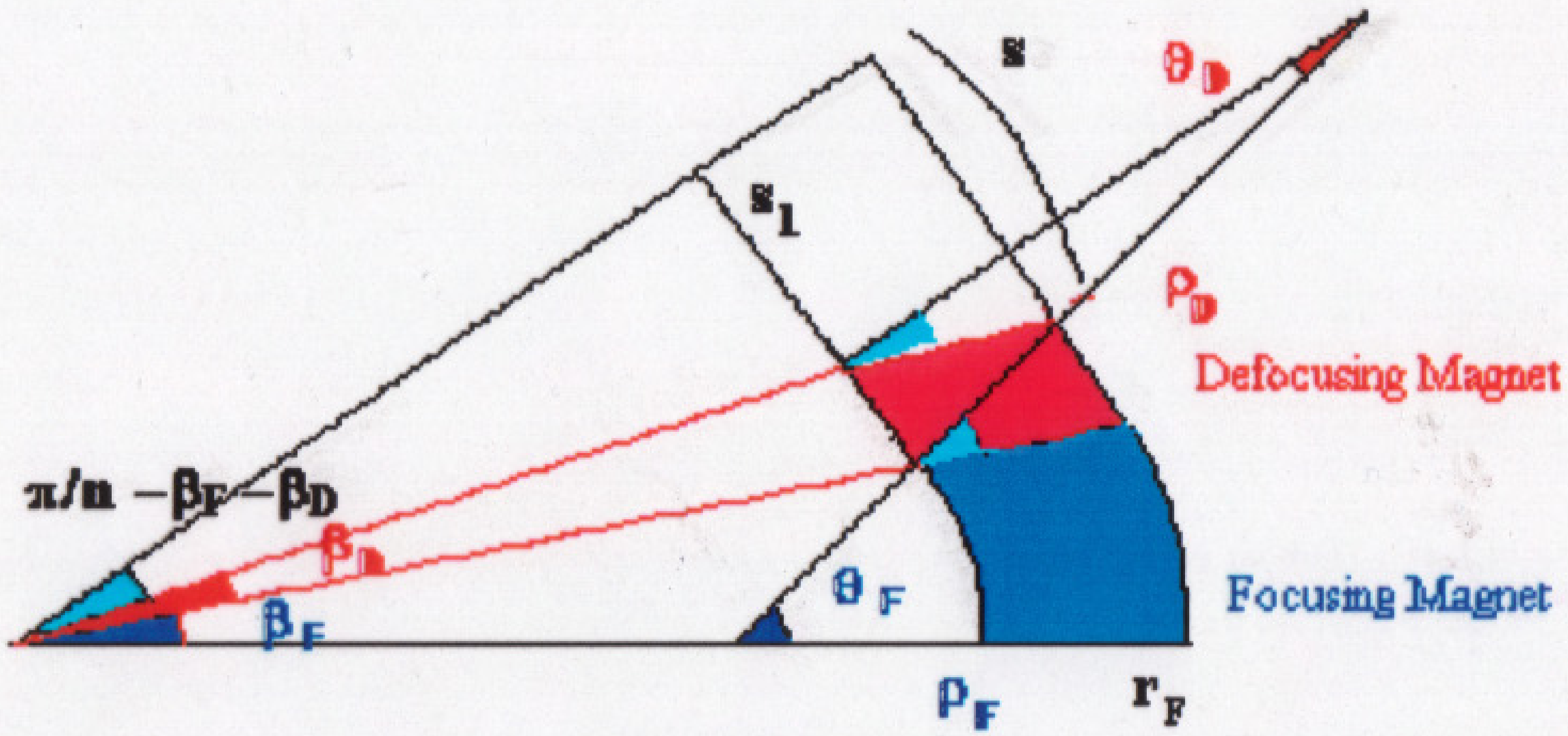

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1,478	β_f/β_D
16	N or ang = π/N
26 d.	θ_f
0,052 d	β_f
0.035 d	$\beta_d = \beta_f/(\beta_f/\beta_D)$
14.75 d	$\theta_d = \theta_f - \pi/N$
0,1185	ρ_f/r_f $[\theta_f, \beta_f]$
0.141	ρ_D/r_f $[\theta_f, \beta_f]$
0,108	S/r_f $[\theta_f, \beta_f, \beta_f$
1.011	r_0/r_f $[\theta_f, \beta_f]$
	r_d/r_f

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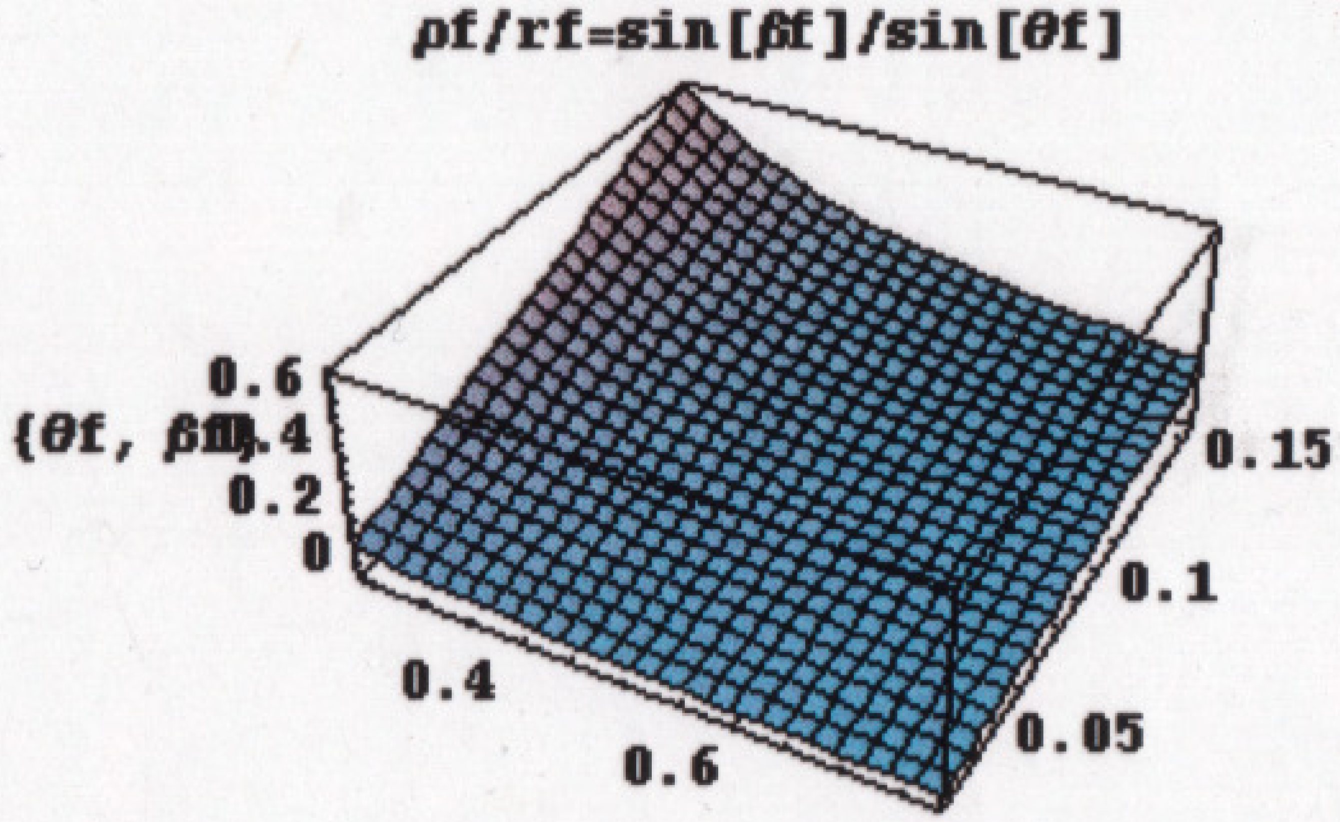

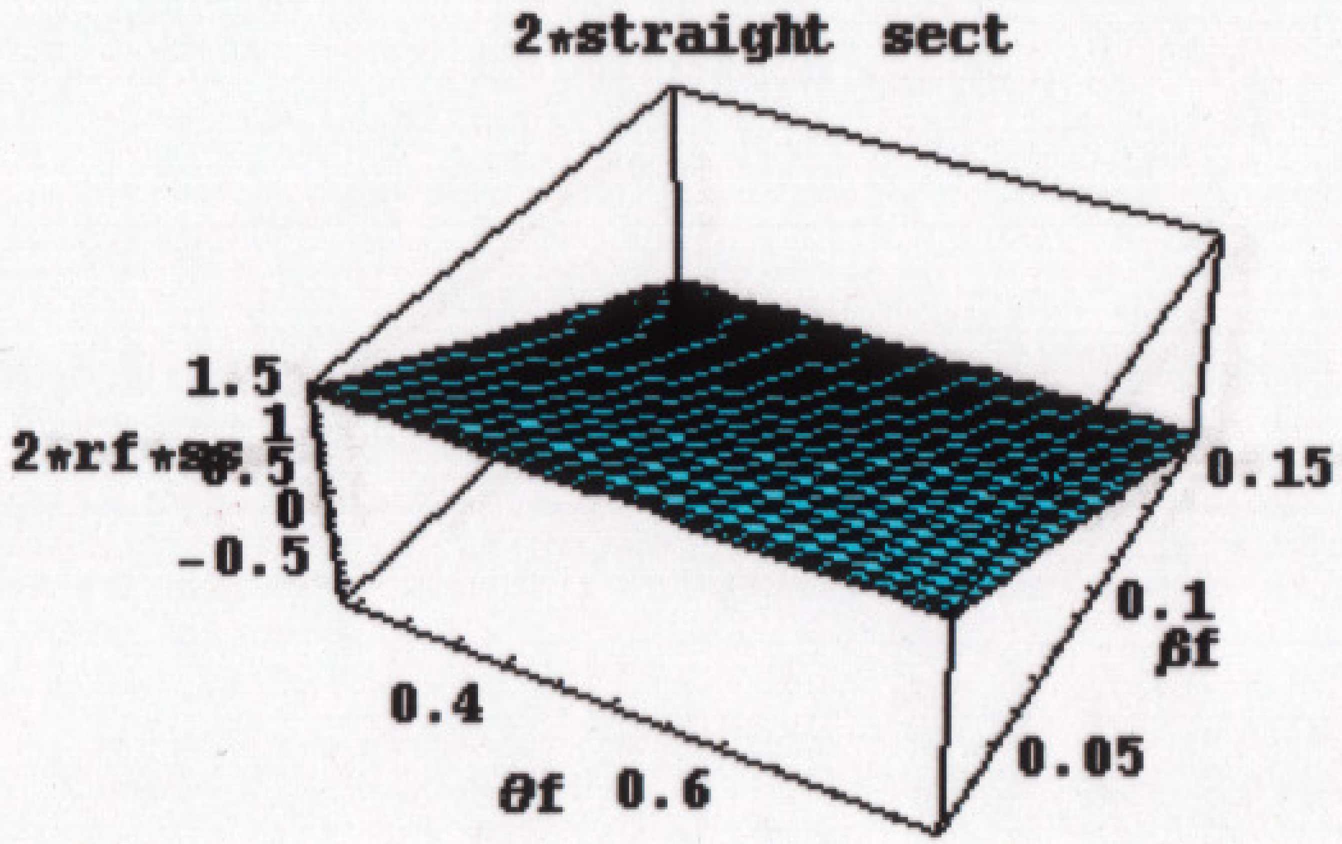

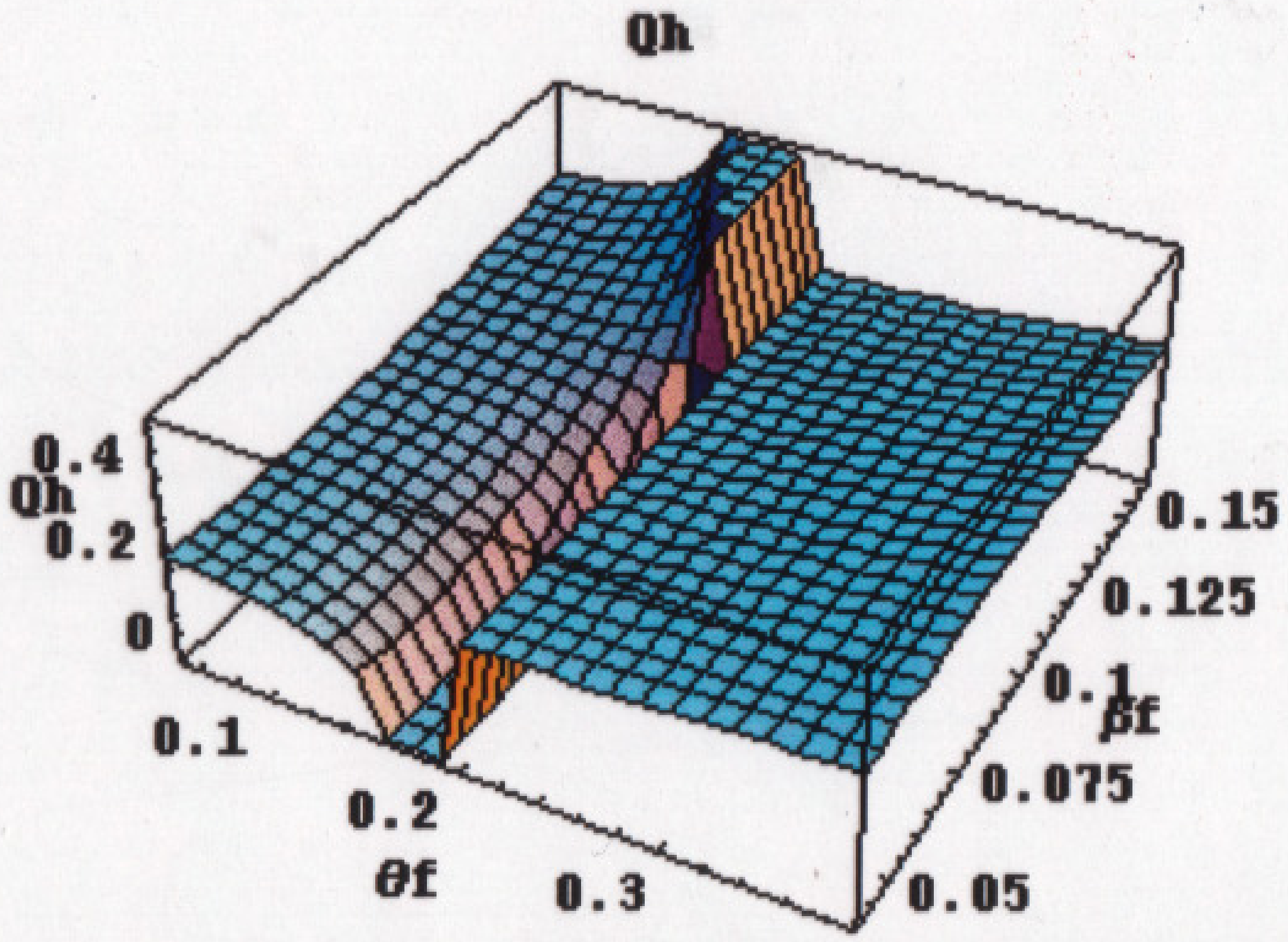

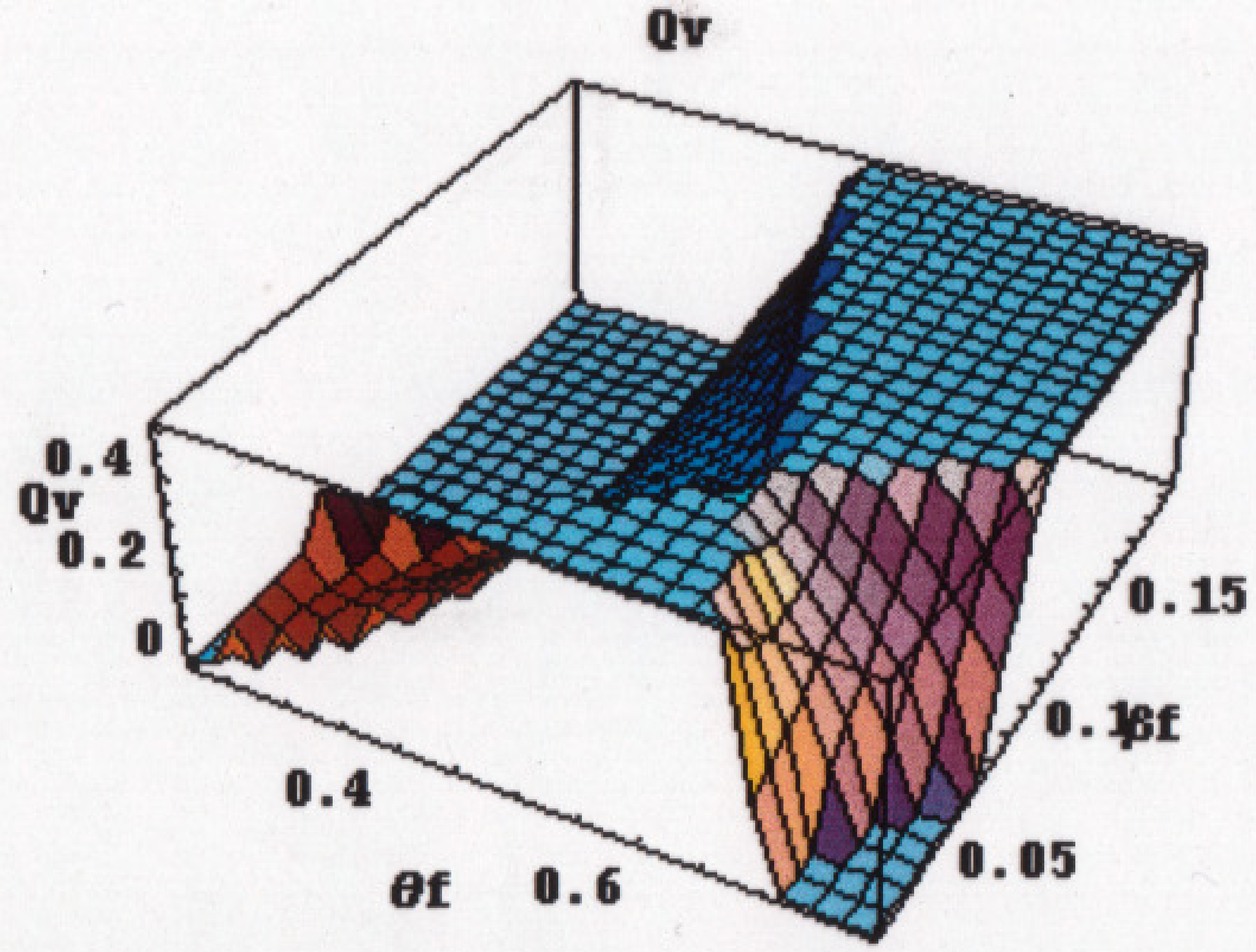

16 modules S D F D S, rf = 10 m, kρ = 1.5, βf / βd = 1.47908,

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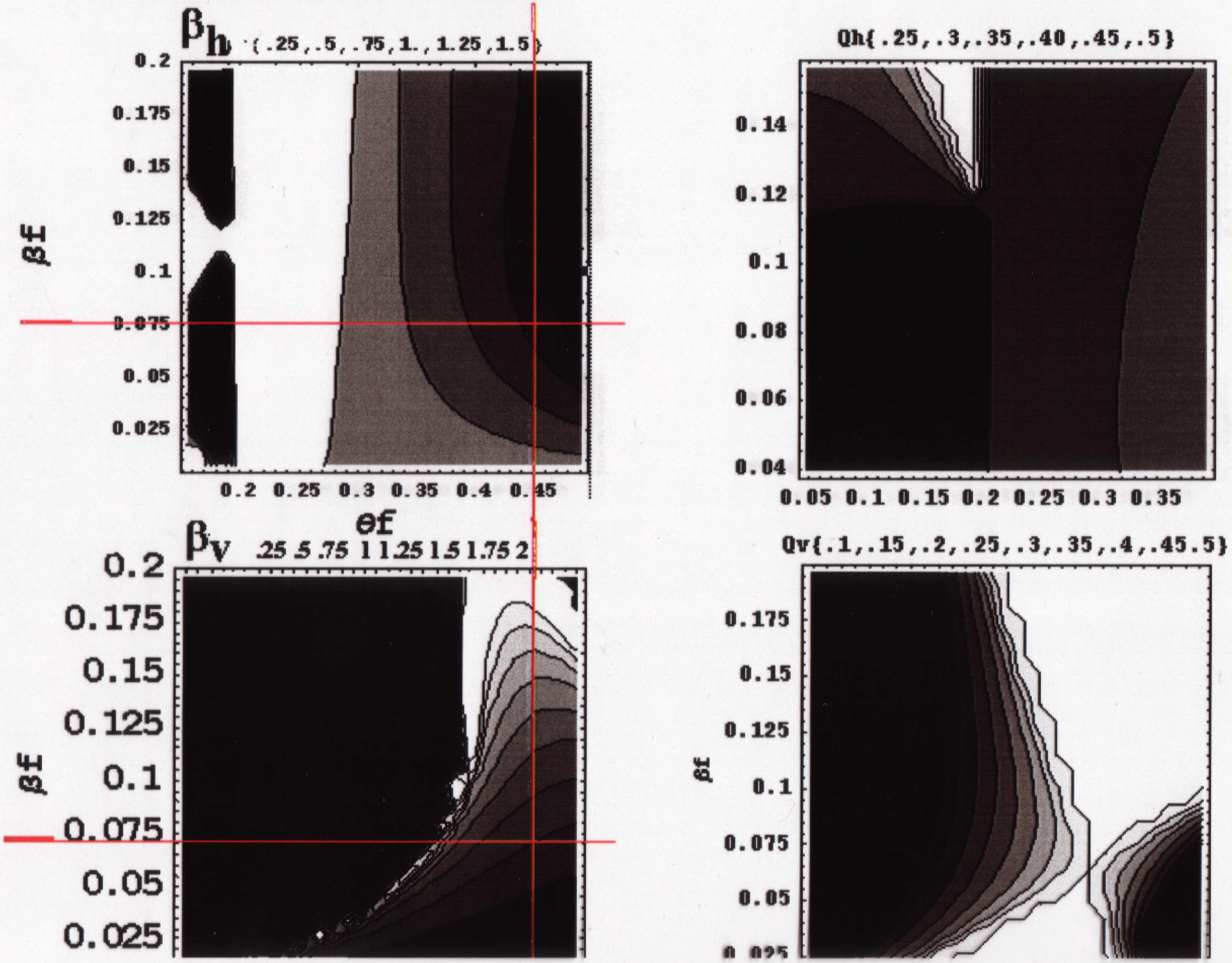
βh {.25,.5,.75,1.,1.25,1.5}
βf
θf
Qh{.25,.3,.35,.40,.45,.5}
βv .25 .5 .75 1 1.25 1.5 1.75 2
Qv{.1,.15,.2,.25,.3,.35,.4,.45.5}
βf

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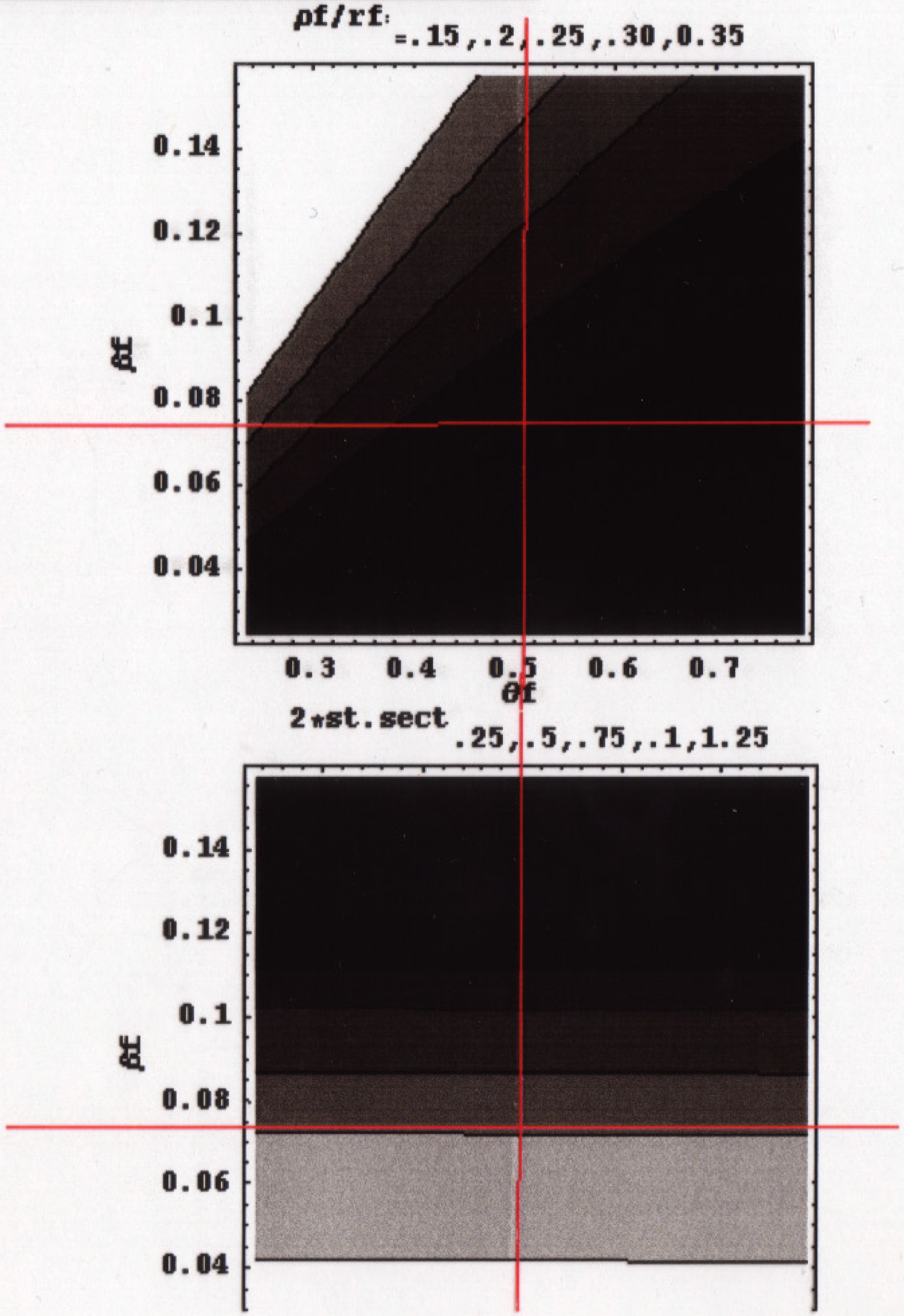
ρf/rf: =.15,.2,.25,.30,0.35
0.14
0.12
0.1
0.08
0.06
0.04
βf
0.3 0.4 0.5 0.6 0.7
θf
2*st.sect .25,.5,.75,.1,1.25
0.14
0.12
0.1
0.08
0.06
0.04
βf

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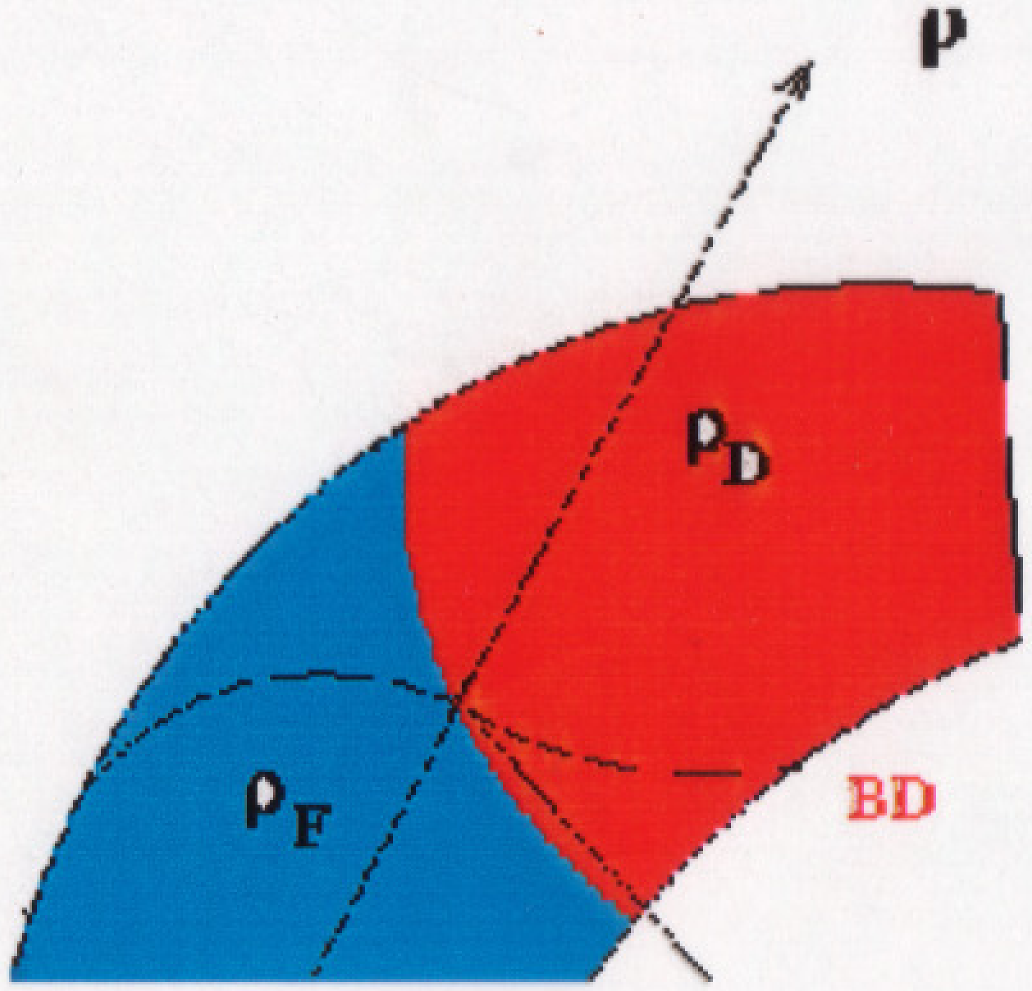
RELATION BETWEEN THE FIELDS OF ADJACENT MAGNETS MULTIPOLES

M, on the trajectory at the limit between BF and BD

$$BF\ \rho F = BD\ \rho D$$

$$\rho F = BF_M + \rho\, \partial BF_M / \partial\rho + \ldots + \rho^n\, \partial^n BF_M / \partial \rho^n$$



$$BD = BD_M + \rho\, \partial BD_M / \partial\rho + \ldots + \rho^n\, \partial^n BD_M / \partial \rho^n$$

for any point M. Then : $\rho D / \rho F$ =

$$BF / BD = BF_M / BD_M = \partial^n BF_M / \partial \rho^n / \partial^n BD_M / \partial \rho^n$$

Multipoles are defined by : $K_n = (1 / B\rho)\ \partial^n B / \partial \rho^n$

So $K_n F * \rho F = K_n D\ \rho D$

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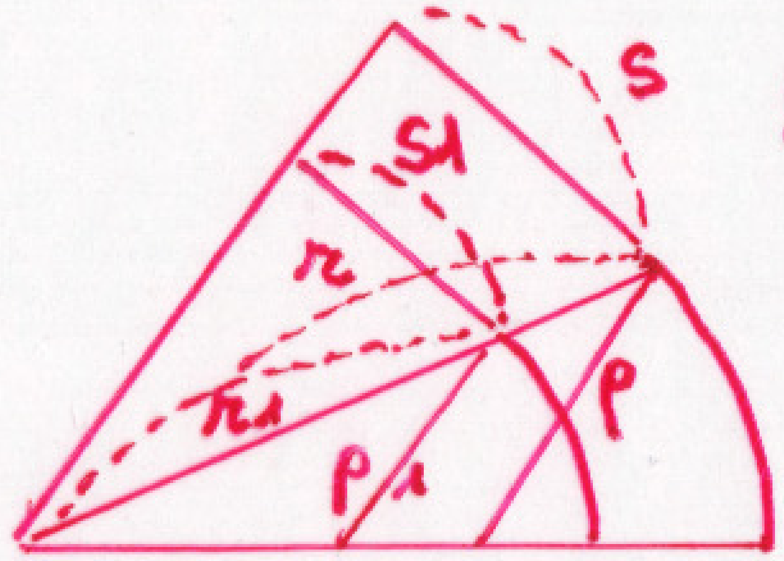
Achieving Isochronicity and Similitude

Similitude :



$$r/r_1 = \rho/\rho_1 = S/S_1 = v/v_1$$

Isochronicity :

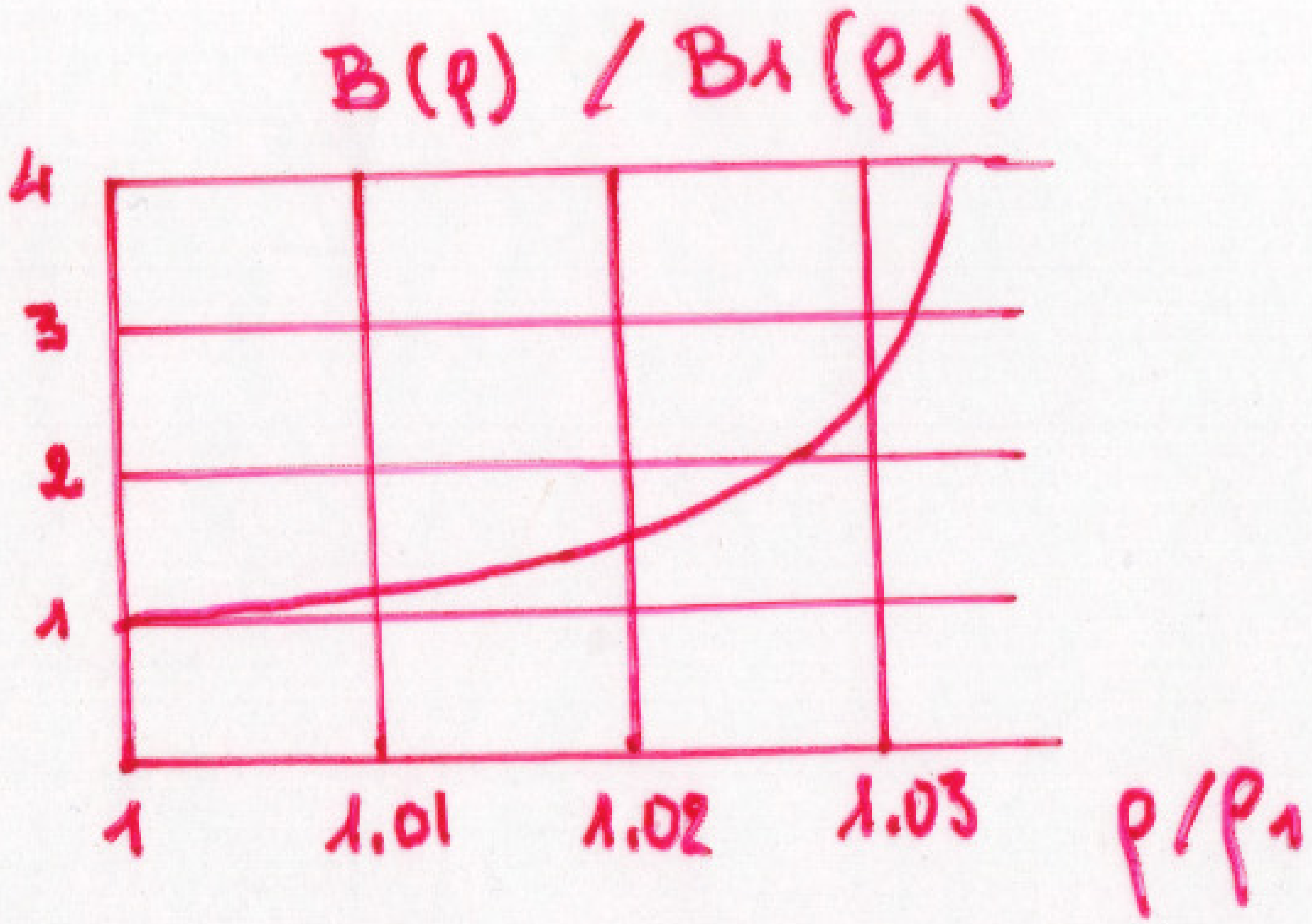
$$v/v_1 = \frac{B\rho}{B_1\rho_1}\,\frac{B_1}{B} = \frac{mv}{m_1 v_1}\,\frac{B_1}{B}$$

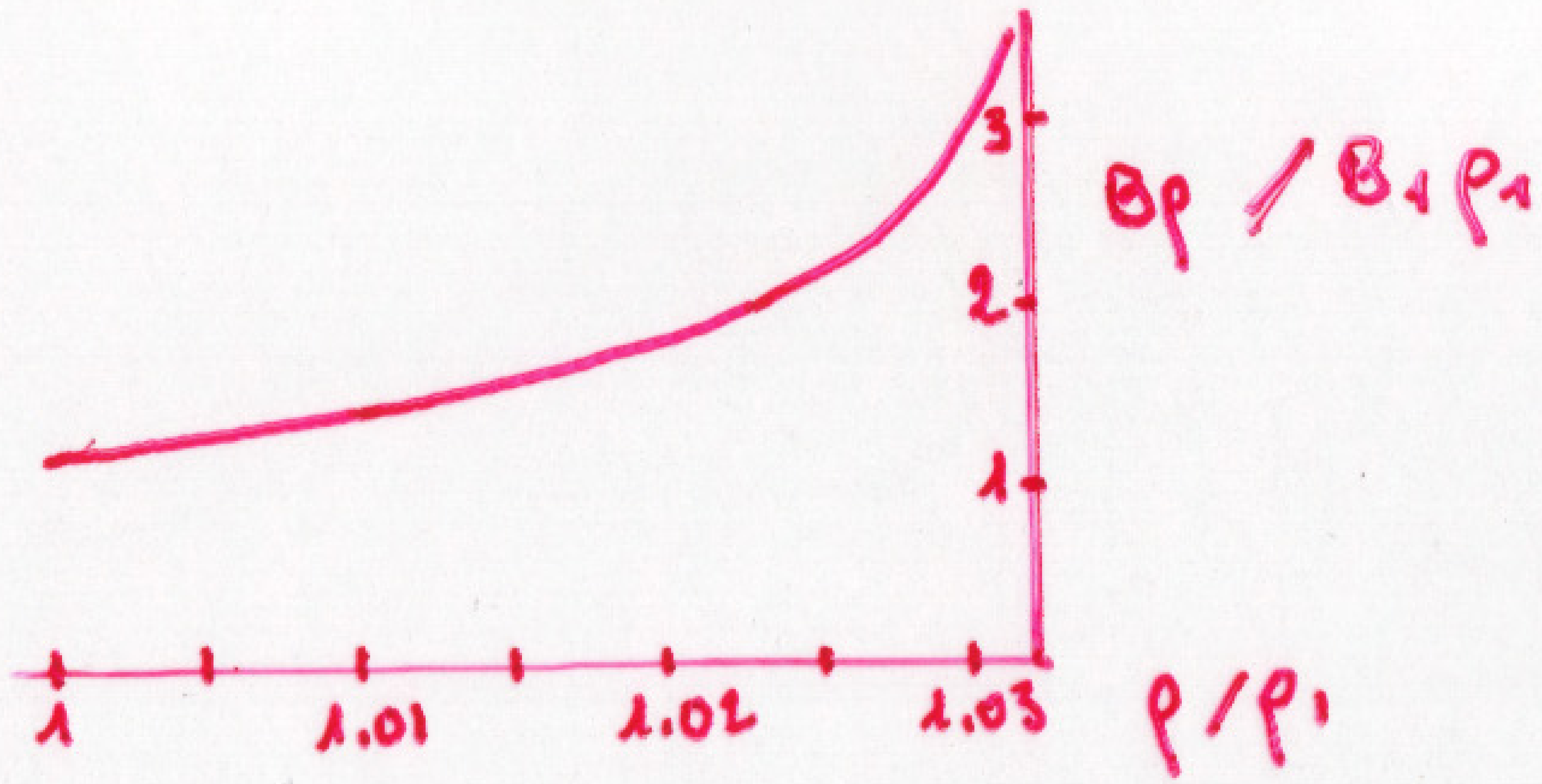
$$\rightarrow \frac{m}{m_1} = \frac{B}{B_1}$$

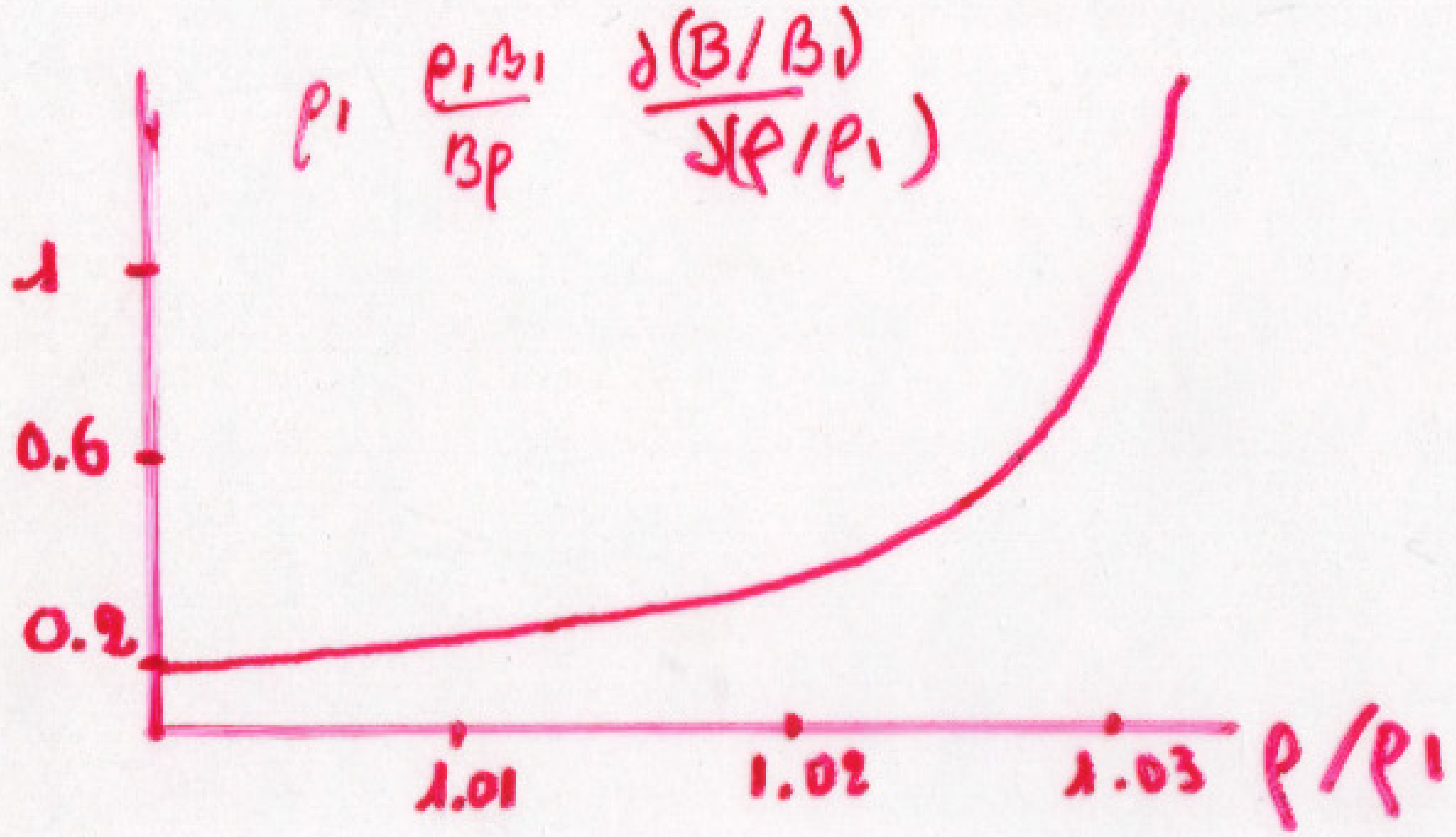
and $$\frac{\rho}{\rho_1} = \left[\frac{1-\frac{1}{\gamma^2}}{1-\frac{1}{\gamma_1^2}}\right]^{1/2}$$

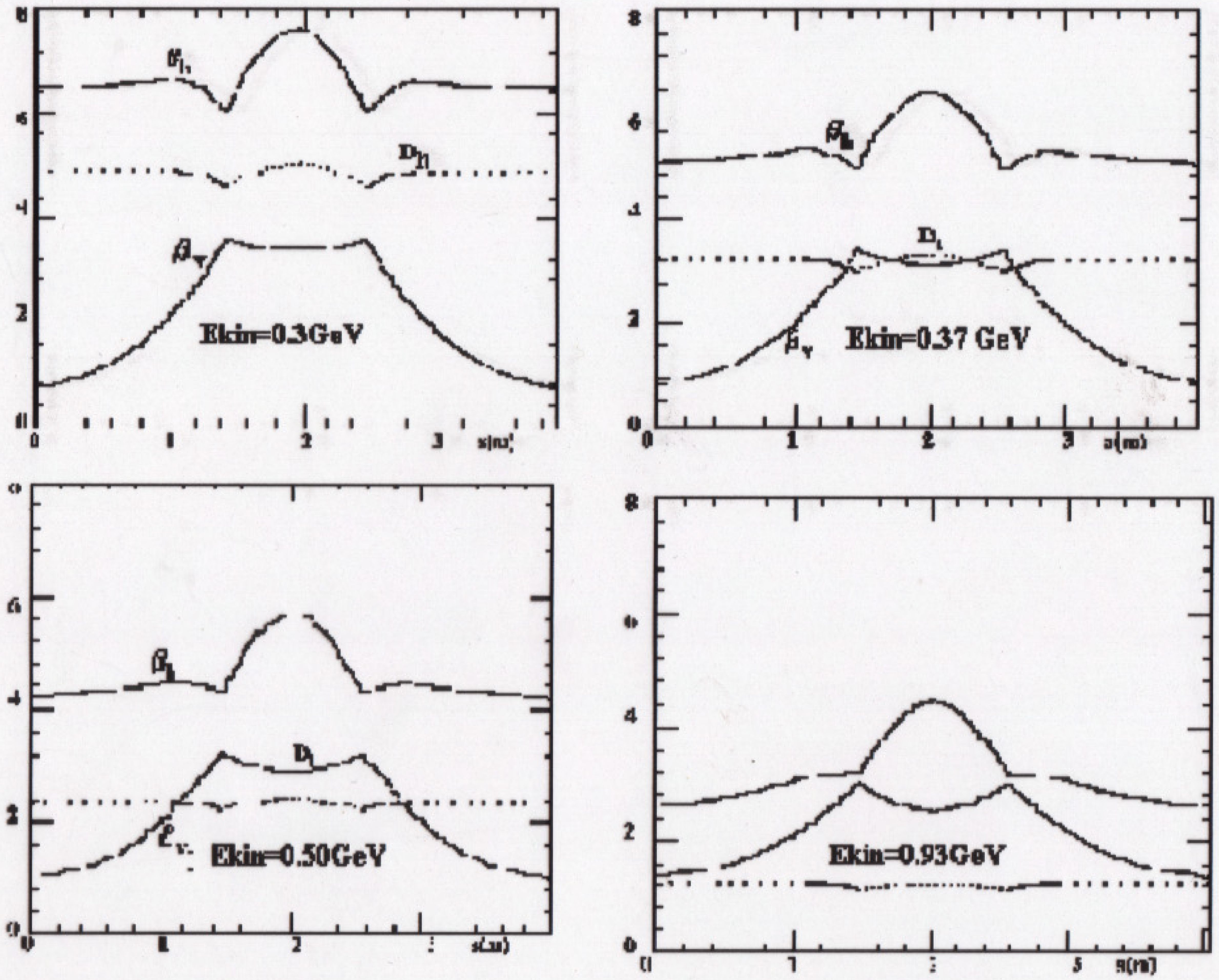
$$\rightarrow B = B_1\sqrt{\gamma_1^2-1}\Big/\sqrt{\gamma_1^2/(\gamma_1^2-1)-(\rho/\rho_1)^2}$$

$B(\rho)$ for isochronisme + similitude


B(ρ) / B1(ρ1)
4
3
2
1
1
1.01
1.02
1.03
ρ/ρ1


3
2
1
Bρ / B1ρ1
1
1.01
1.02
1.03
ρ/ρ1


ρ1
ρ1B1 / Bρ
∂(B/B1) / ∂(ρ/ρ1)
1
0.6
0.2
1.01
1.02
1.03
ρ/ρ1


Ekin=0.3GeV
Ekin=0.37 GeV
Ekin=0.50GeV
Ekin=0.93GeV

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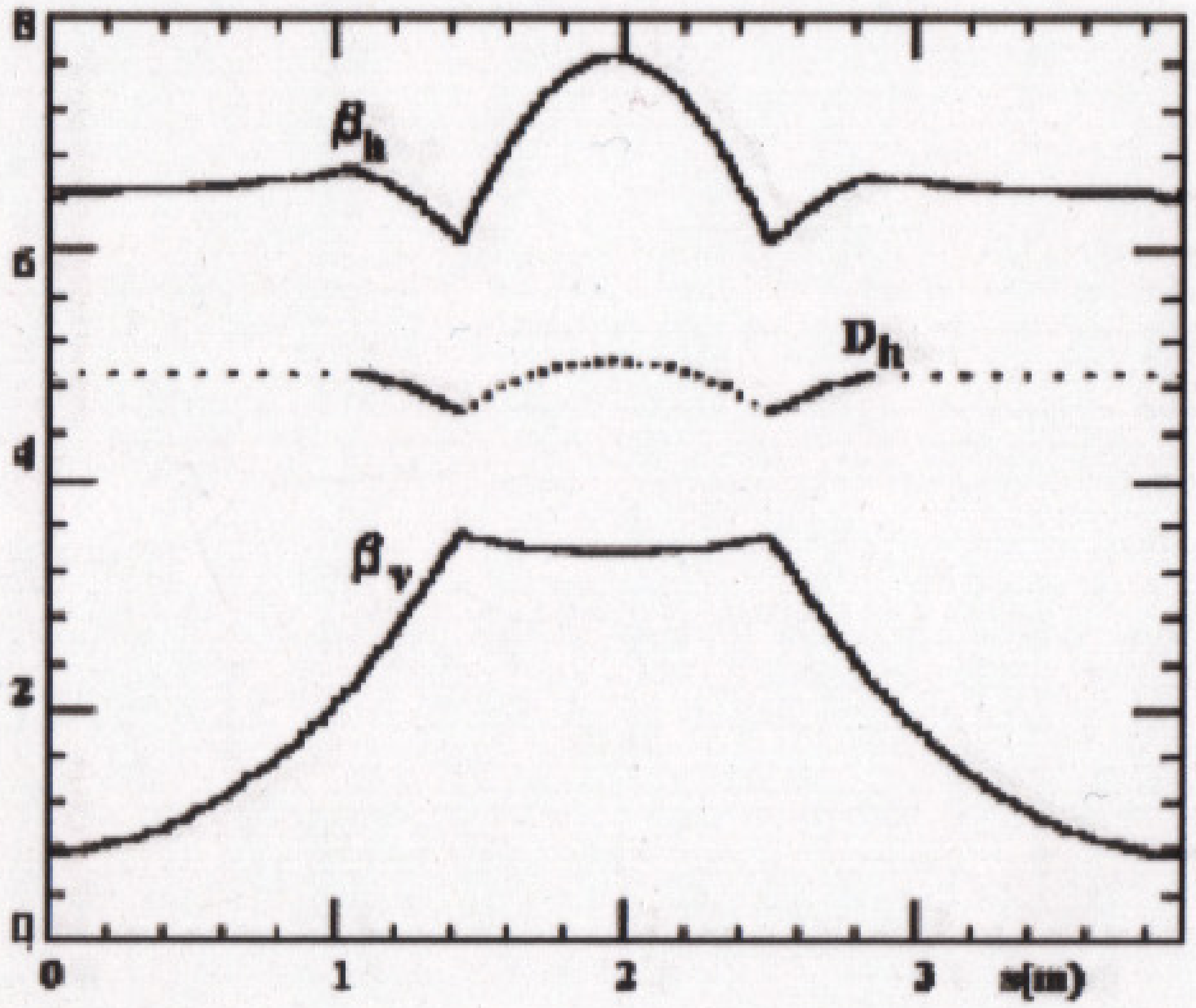
Calculated with MAD **(result to be verified)**

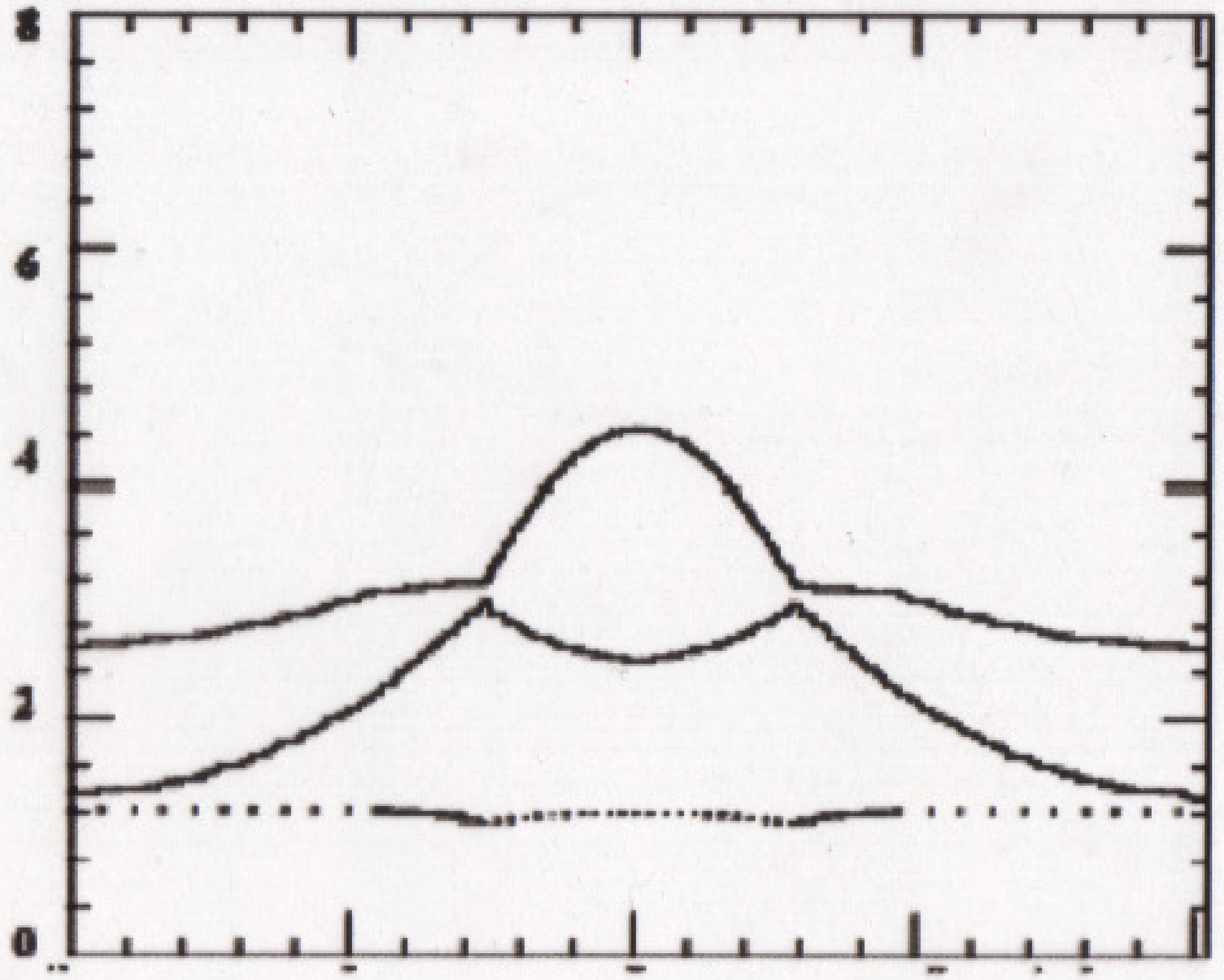


Ekinetic 0.3 GeV

$K_F = (1/B\rho)\ \delta B/\delta\rho = 0.0727$

K_D -0.05147

Qx =	0.094 [illegible]	Qy =	0.39
BetaX(max) =	7.651232	BetaY(max) =	3.53
x(max) =	0.000000	y(max) =	0.00
x(rms) =	0.000000	y(rms) =	0.00
Dx(max) =	5.034124	Dy(max) =	0.00
Dx(rms) =	4.829884	Dy(rms) =	0.00



Ekinetic 0.93 GeV

$K_F = (1/B\rho)\ \delta B/\delta\rho = 0.4197$

K_D -0.4018

Qx =	0.206716	Qy =	0.325073
BetaX(max) =	4.446613	BetaY(max) =	2.998812
x(max) =	0.000000	y(max) =	0.000000
x(rms) =	0.000000	y(rms) =	0.000000
Dx(max) =	1.232627	Dy(max) =	0.000000
Dx(rms) =	1.195128	Dy(rms) =	0.000000

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Conclusion:

There is a large number of parameters to explore, even if one choose only the solution with triplets.

To make the exploration easier, we have tried :

For the isochronicity,

to look at fields giving arcs of circles for the trajectories in the sector bending magnets and having a variation with the distance to the machine centre such that this variation, combined with the length of the trajectory in the magnet for the momentum leads to the isochronicity

For the focusing,

to develop an analytical representation of the parameters Qh, Qv, βh, βv, in function of the geometry such that a selection can be made by looking at graphs and not by a root mean square optimisation.

We have not used the method enough to have yet the solutions it may give.