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FFAG WORKSHOP

TRACKING STUDY for FFAG

Masamitsu Aiba (KEK FFAG Gr.)

Contents

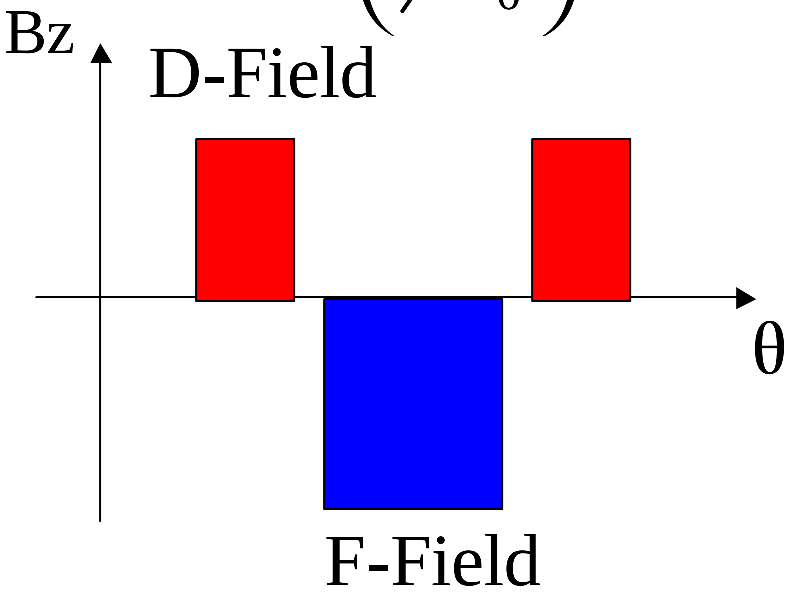
Acceptance of FFAG

The motivation of the study is
to clear the behavior of the acceptance of FFAG
and to establish the strategy of the design
for FFAG with the large acceptance .

Tracking Simulation

○Hard Edge Field
on Median Plane

$$B = B_0 \left(r / r_0 \right)^k$$



Easy to change the parameters

○Numerical Integration
with fourth order Runge-Kutta

Equations of motion

$$m(\ddot{r} - r\dot{\theta}^2) = e(r\dot{\theta}B_z - \dot{z}B_\theta)$$

$$m\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt} = e(\dot{z}B_r - \dot{r}B_z)$$

$$m\ddot{z} = e(\dot{r}B_\theta - r\dot{\theta}B_r)$$

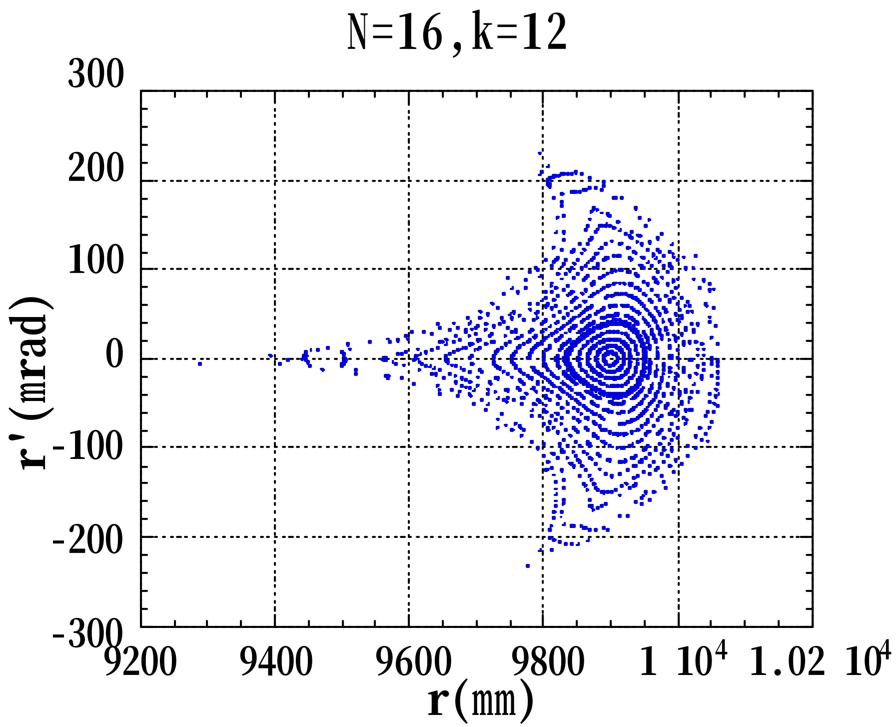
Basic Parameters of Two Rings

These parameters are obtained with linearized model.
Momentum range is 0.3GeV/c to 1GeV/c.

	ring1	ring2
magnet type	triplet(DFD)	triplet(DFD)
num. of cell	32	16
k value	50	15
orbit excursion	0.5m	0.7m
average radius	21m	10m
B @ F/D	1.8T/1.8T	2.8T/2.8T
F/2 opening angle	0.026rad.	0.052rad.
D angle	0.018rad.	0.036rad.
phase advance (H/V)	120deg./61deg.	131deg./103deg.

Horizontal acceptances of two rings are surveyed with various k values.

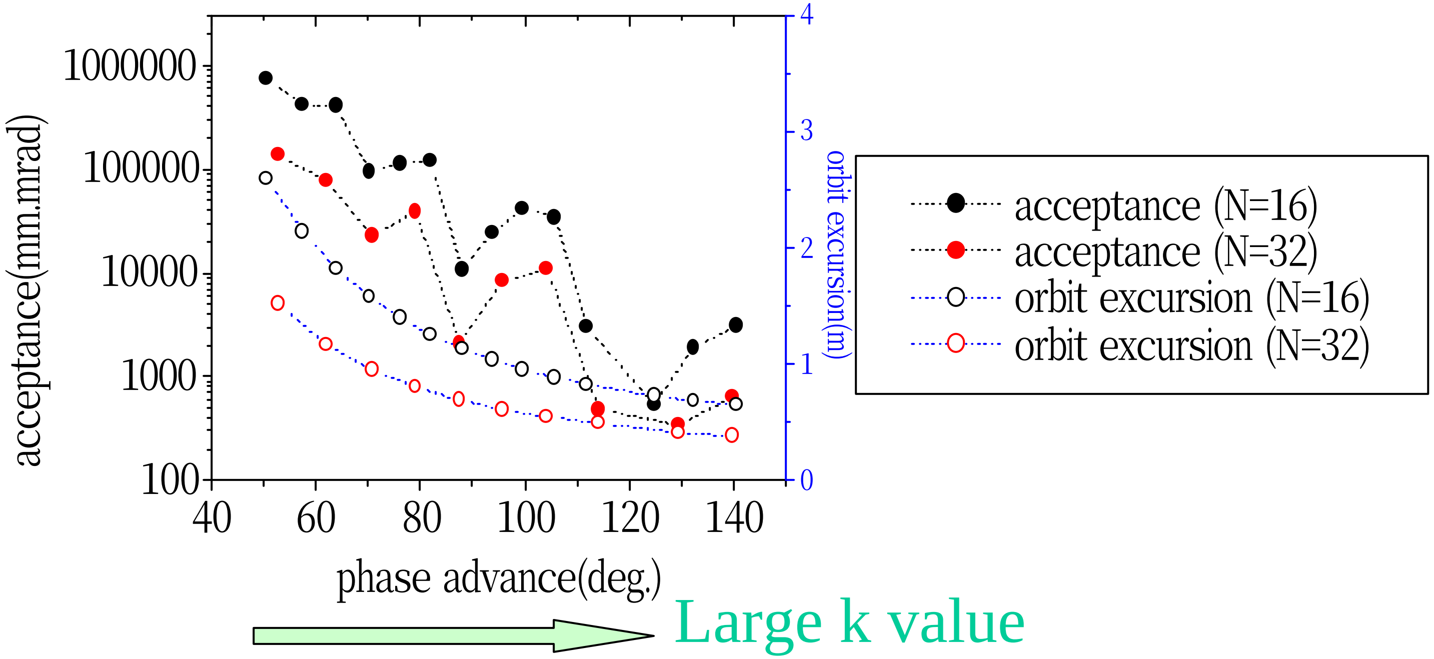
Example of Phase Space Plot



The Shape of phase space plot is distorted from ellipse.
In this case, the acceptance can be defined as
the total area of phase space divided by π.

Acceptance vs. Phase Advance

Number of turn = 128 turn , physical aperture is set almost infinite



Tendency between the acceptance

and the phase advance is the same in two rings.

-- Acceptance becomes smaller as the phase advance increases.

-- Acceptance becomes small rapidly around the structure resonance lines.

Normalization(1)

$$B = B_0\left(\frac{r}{r_0}\right)^k = B_0\left(1+\frac{k}{r_0}x+\frac{k(k-1)}{2!r_0^{\ 2}}x^2+\cdots\right) \qquad (r = r_0 + x)$$

$$\cong B_0\left(1+(\frac{k}{r_0}x)+\frac{1}{2!}(\frac{k}{r_0}x)^2+\cdots\right) \qquad (\because k >> 1)$$

Non-dimensional quantity of amplitude is $(\frac{k}{r_0}x)$

$$W = \frac{x^2}{b}$$

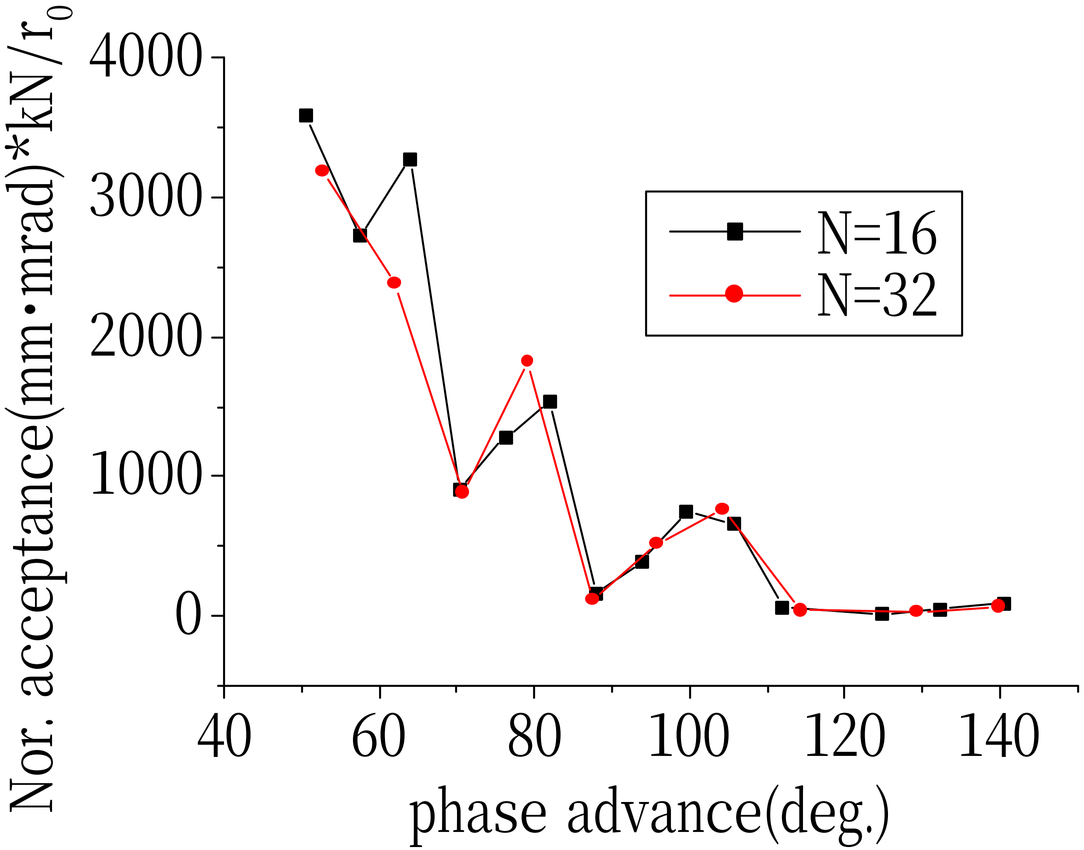
b : *beta function*

n : *tune*

$$\cong x^2\left(\frac{k}{r_0 N}\right) = \frac{r_0}{kN}(\frac{k}{r_0}x)^2 \qquad \because \boldsymbol{b} \cong \frac{r_0}{\boldsymbol{n}} \qquad \because \boldsymbol{n} \cong \frac{k}{N^2}\times N$$

Normalize factor is $\frac{r_0}{kN}$

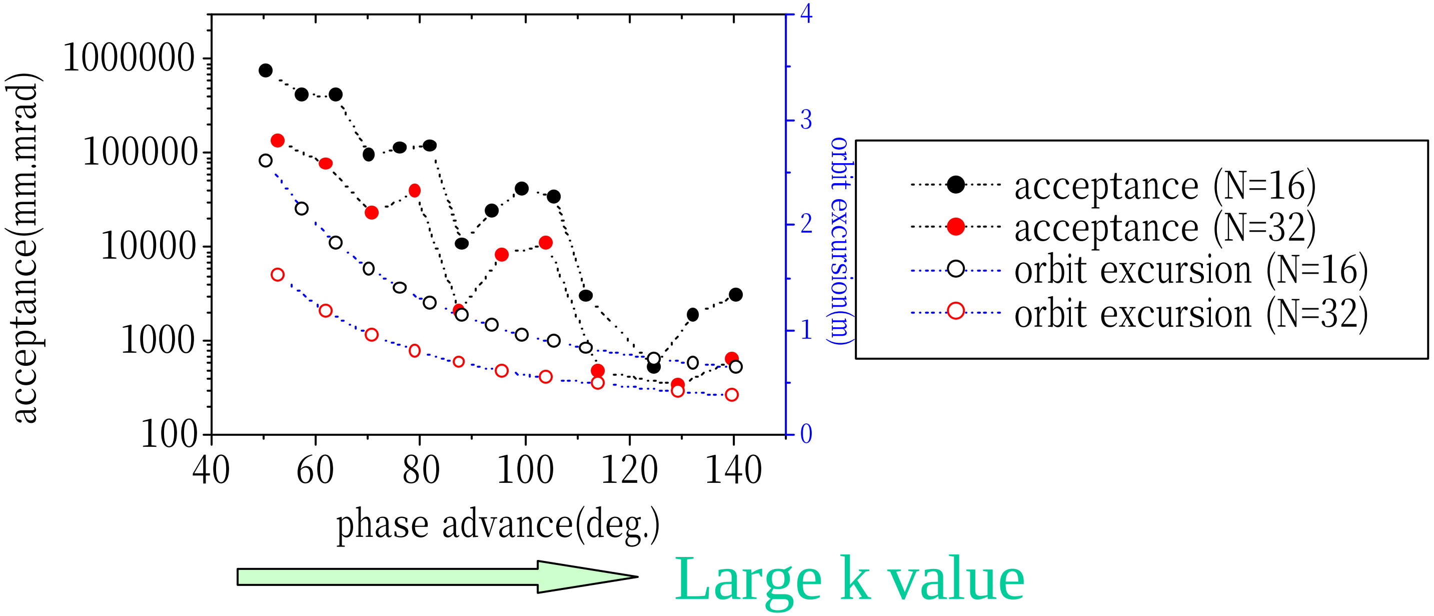
Normalization (2)



Phase advance, k, N and r0 are fixed,
the horizontal acceptance is estimated from this figure.

Acceptance vs. Phase Advance

Number of turn = 128 turn , physical aperture is set almost infinite



Large k value

Tendency between the acceptance and the phase advance is the same in two rings.

-- Acceptance becomes smaller as the phase advance increases.

★-- Acceptance becomes small rapidly around the structure resonance lines.

What Determines Acceptance in FFAG?

- Scaled FFAG has no tune-spread caused by dP/P.

→The tune-shift caused by the higher order components of the guide field determines the acceptance, especially around the strong resonance.

Direction of Tune Shift around the phase advance of 120deg. (1)

Sextupole is the dominant source in this case.

$$H = J/\beta(s) + \frac{S(s)}{6}x^3$$

$$\equiv J/\beta(s) + V(\phi, J, s)$$

$J-\phi$: *action – angle*

V : *perturbation term*

$S(s)$: *coffeiciant of sextupole*

Average over ϕ and over s

$$<V_{J_1}G_\phi> = -\frac{J_1^{\,2}}{64C}\int_0^C ds\,\beta(s)^{3/2}S(s)\int_S^{S+C}\beta(s')^{3/2}S(s')ds'$$

$$\times\left\{\frac{3\cos(\psi(s')-\psi(s)-\pi\nu)}{\sin\pi\nu}+\frac{\cos 3(\psi(s')-\psi(s)-\pi\nu)}{\sin 3\pi\nu}\right\}$$

G : *genarating function*

$$\nu_1 = \frac{1}{2\pi}\int_0^C\left(\frac{1}{\beta(s)}+\frac{\partial<V_{J_1}G_\phi>}{\partial J_1}\right)ds = \nu + \frac{C}{2\pi}\frac{\partial<V_{J_1}G_\phi>}{\partial J_1}$$

Reference:R.D.Ruth AIP-153 p193

Direction of Tune Shift around the phase advance of 120deg. (2)

Consideration for single cell and only dominant term

$$\Delta n = \frac{\partial < V_{J_1} G_f >}{\partial J_1} \cong -N \frac{J_1}{32C} \int_0^{C/N} ds\, b(s)^{3/2} S(s) \int_S^{S+C/N} ds'\, b(s')^{3/2} S(s') \frac{\cos 3(y(s') - y(s))}{\sin 3pm}$$

N : *number of cell*
m : *tune per cell*

Sextupole coefficient S(s) of FFAG

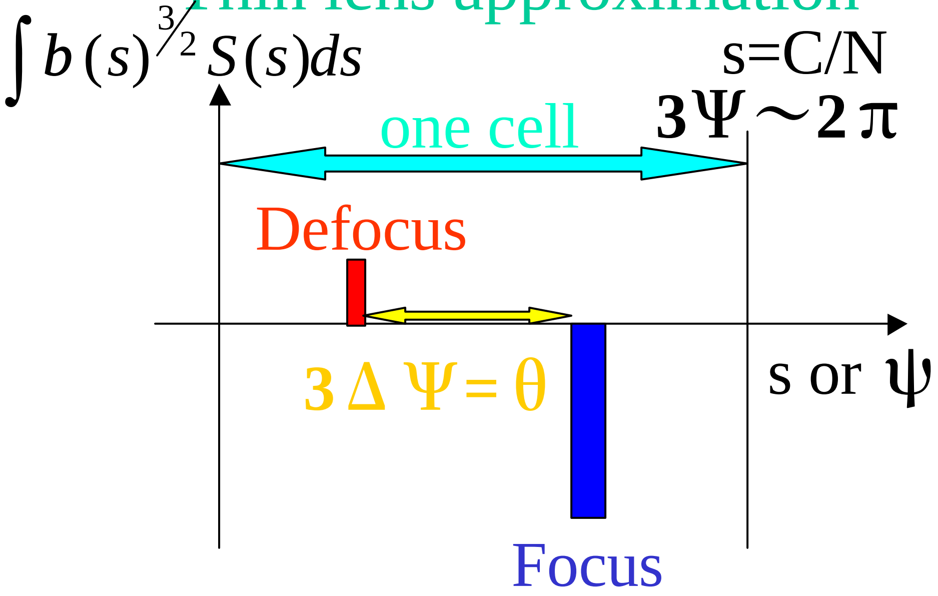
$$B = B_0 \left(\frac{r}{r_0} \right)^k = B_0 \left(1 + \frac{k}{r_0} x + \frac{k(k-1)}{2! r_0^2} x^2 + \cdots \right)$$

(Tayler expansion, $r = r_0 + x$)

$B_0 < 0 \rightarrow S(s) < 0$ @Focus
$B_0 > 0 \rightarrow S(s) > 0$ @Defocus

Direction of Tune Shift around the phase advance of 120deg. (3)

Thin lens approximation



$$b_F(s) > b_D(s)$$

$$|S_F(s)| \geq |S_D(s)|$$

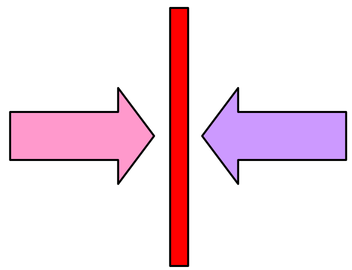
$R_F \equiv$ (blue bar)

$R_D \equiv$ (red bar)

$$\Delta n = \frac{\partial < V_{J_1} G_f >}{\partial J_1} \cong -N \frac{J_1}{32C} [-R_F(-R_F - R_D \cos q) + R_D(R_F \cos q + R_D)] \frac{1}{\sin 3pm}$$

$$= -N \frac{J_1}{32C} (R_F{}^2 + 2R_F R_D \cos q + R_D{}^2) \frac{1}{\sin 3pe}$$

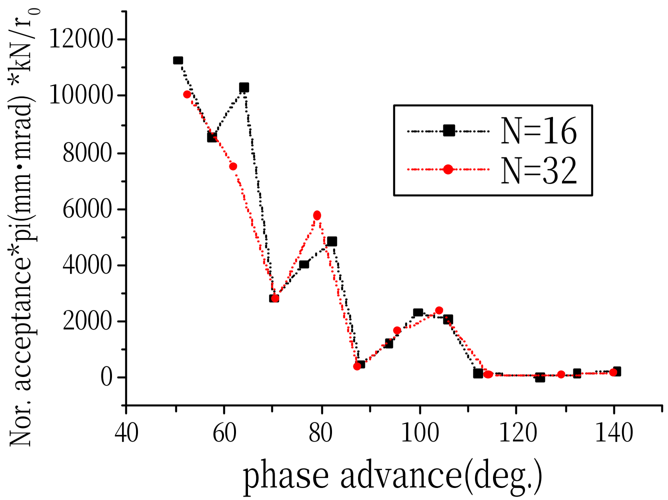
$$(\because m = \frac{1}{3} + e)$$



120deg.

Summary for Acceptance of FFAG

The normalized relation between the acceptance and the phase advance is obtained



The tendency of the relation is analyzed with the formalisms of tune shift.

Only the phase advance determines the horizontal acceptance in FFAG.

Resonance Crossing

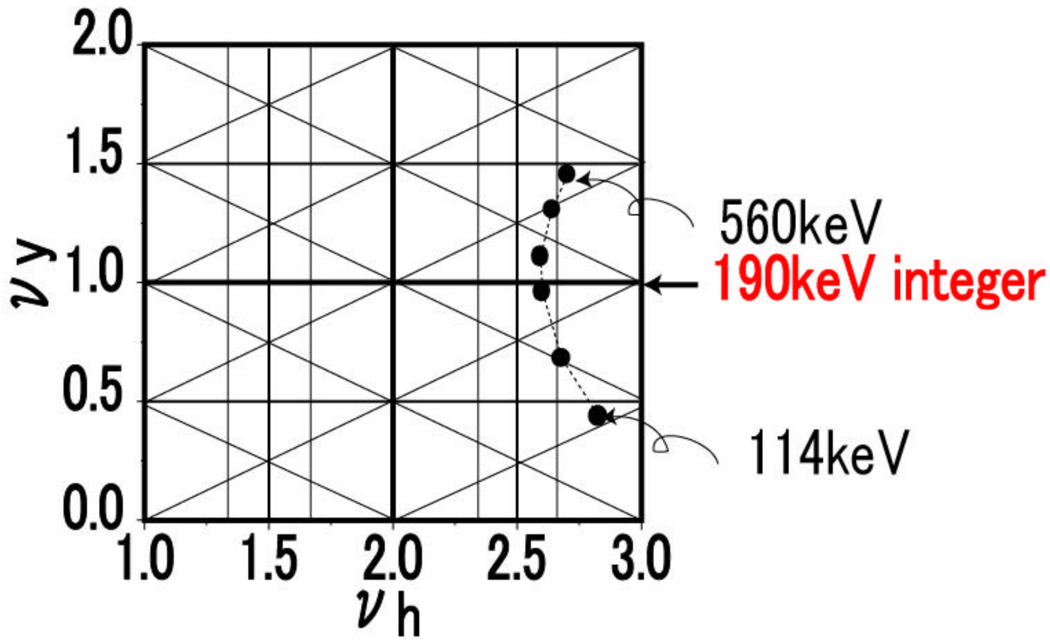
We want to know whether the resonance crossing is possible or not,
for the design of non-scaling FFAG
or the design of FFAG under the condition in which to keep scaling is difficult.

Test Field

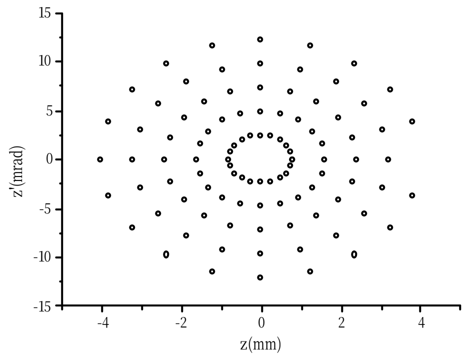
The test field which has large tune shift dare to be generated with 3-dimentional field calculation(TOSCA).

Num of Cell=6 (periodicity)
Test Particle=electron
Mean Radius～0.2m
k value = 2.1

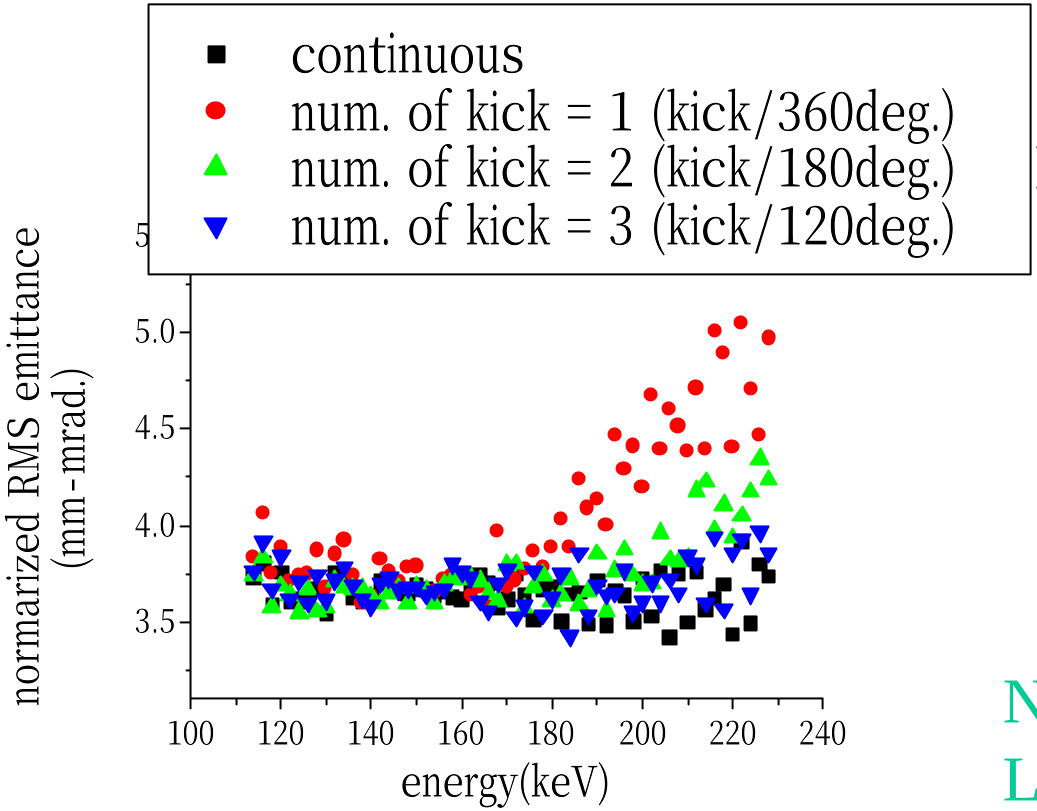
Tune Diagram of Test Field

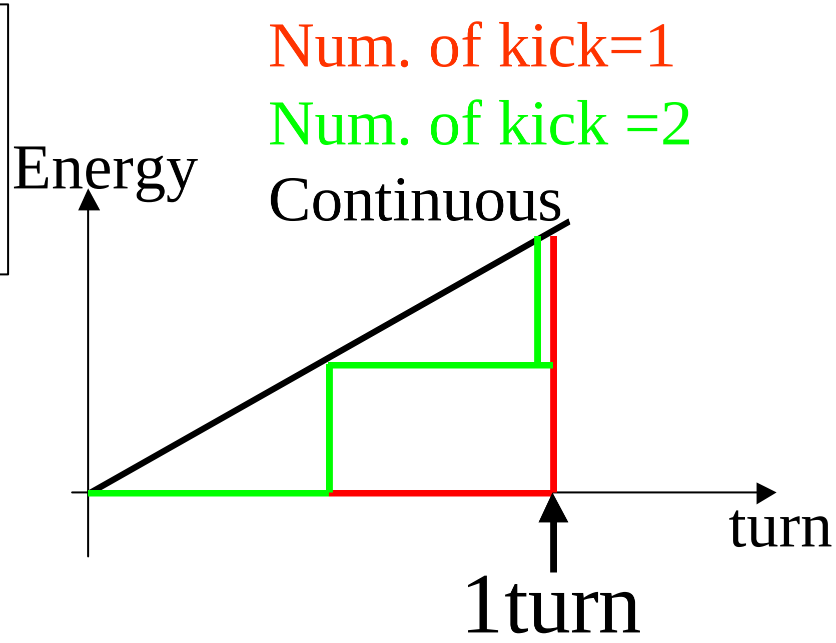


Injection beam 100particles (vertical phase space)



Vertical Emittance Growth Caused by Acceleration Field





Net Energy Gain= 2kV/turn

Longitudinal Emittance = 0

The quantity of the emittance growth decreases as the number of kick increases.

Process of Emittance Growth Caused by Acceleration Field

$$(z_0, z_0') \xrightarrow{\text{acceleration}} (z_1, z_1')$$

$$z_0 = \frac{P_z}{P_q} \quad , \quad z_1 = \frac{P_z}{P_q + \Delta P_q}$$

$$z_1' - z_0' = \frac{P_z}{P_q} - \frac{P_z}{P_q + \Delta P_q} = \frac{-\Delta P_q P_z}{P_q (P_q + \Delta P_q)}$$

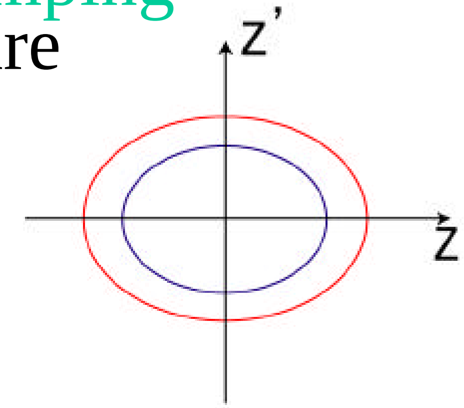
$$= \frac{-\Delta P_q}{(P_q + \Delta P_q)} z_0' \cong \frac{-\Delta P_q}{P_q} z_0'$$

Acceleration Matrix M_{acc}

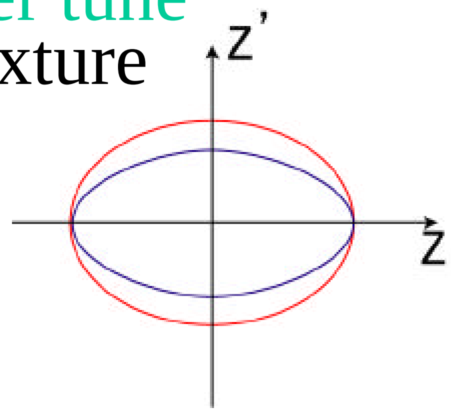
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 - \dfrac{\Delta P_q}{P_q} \end{pmatrix}$$

($\frac{\Delta P_q}{P_q} << 1$:adiabatic damping)

ordinary damping
phase mixture



around integer tune
non phase mixture

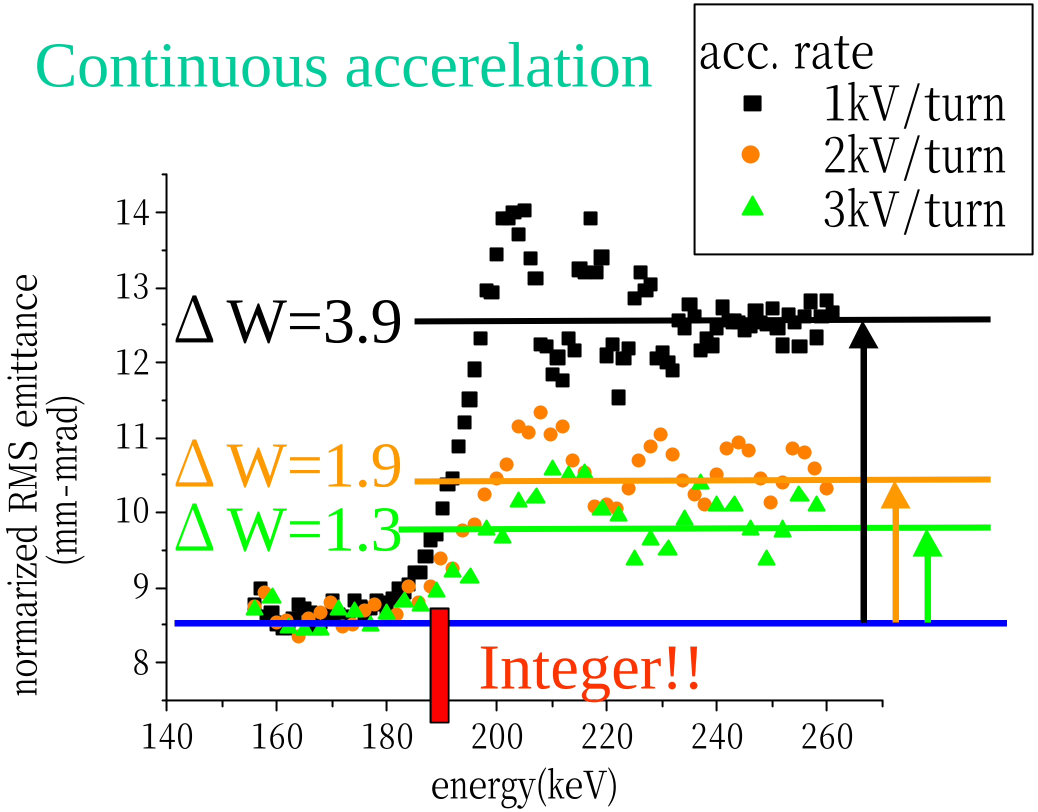


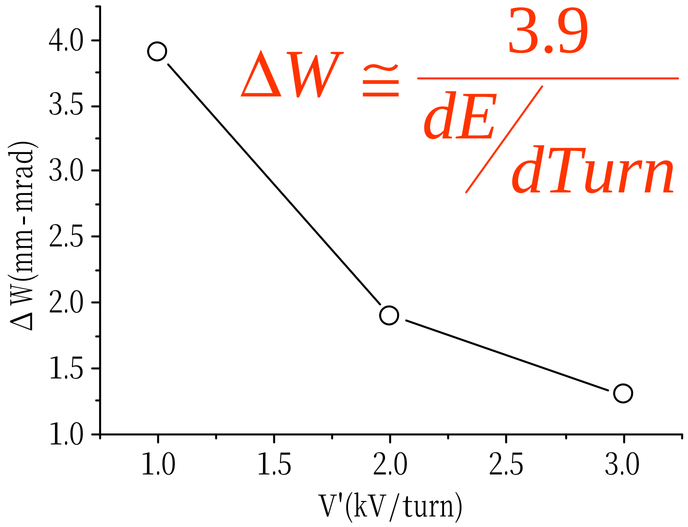
Mismatching of beta-function

Vertical Emittance Growth Caused by Alignment Error

One of the magnet is displaced 0.1mm vertically.

Continuous acceleration





$$\Delta W \cong \frac{3.9}{dE/dTurn}$$

The emittance growth is in inverse proportion to the acceleration rate.

Quadrupole Error in Misalignment of FFAG Magnet

Br of main body(B_θ=0): $B_r = \frac{kB_0}{r_0}\left(\frac{r}{r_0}\right)^{k-1} z - \frac{k^2(k-2)B_0}{3!\,r_0^{\,3}}\left(\frac{r}{r_0}\right)^{k-3} z^3 \cdots$

Feed Down Caused by Misalignment $z \rightarrow z - z_0$

$$-\frac{k^2(k-2)B_0}{3!\,r_0^{\,3}}\left(\frac{r}{r_0}\right)^{k-3}(z-z_0)^3 = -\frac{k^2(k-2)B_0}{3!\,r_0^{\,3}}\left(\frac{r}{r_0}\right)^{k-3}(z^3 - 3z_0 z^2 + 3z_0^{\,2} z - z^3)$$

Term of Quadrupole Like: $Q_{error} z = -\frac{k^2(k-2)B_0 z_0^{\,2}}{2r_0^{\,3}} z$

In this case $-\frac{k^2(k-2)B_0 z_0^{\,2}}{2r_0^{\,3}} \cong 4\times 10^{-13}\ (T/m) \Leftarrow z_0 = 0.1mm$

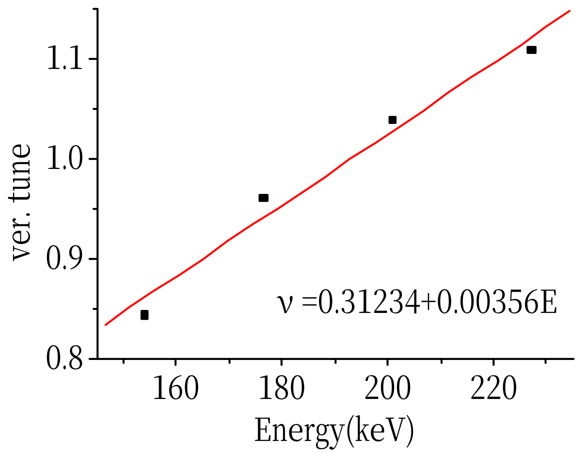
$$dn = \frac{1}{2p}\int_0^C b\Delta K ds = \frac{1}{2p}\int_0^C b\,\frac{Q_{error}}{B_0 r} ds \cong 3\times 10^{-14}$$

Stop band is negligible.

Criterion for Rate of Tune Change

Magnetic field determines $\frac{dn}{dE}$ $\frac{dE}{dTurn}$: *acceleration rate*

Rate of tune change $\frac{dn}{dE}\frac{dE}{dTurn} \equiv \frac{dn}{dTurn}$



$$\frac{dn}{dE} = 0.00365\ (keV^{-1})$$

$$\frac{dn}{dTurn} = 0.00365\ (keV^{-1}) * 3(keV / turn)$$

$$\approx 0.01(turn^{-1})$$

If the COD is the same order as that of this example,
The vertical integer resonance crossing will be achieved
with the rate of tune change an order of 0.01($turn^{-1}$)

Summary

Acceptance of FFAG

- ★ The normalized relation between the acceptance and the phase advance is obtained
- ★ The tendency of the relation is analyzed with the formalisms of tune shift.

  Only the phase advance determines the horizontal acceptance in FFAG

Resonance Crossing

- ★ The Acceleration field can be the source of the resonance and the quantity of the emittance growth decreases as the number of kick increases.
- ★ The vertical integer resonance crossing will be achieved with the rate of the tune change an order of 0.01(turn^{-1})