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FLEXIBLE COMPUTATIONAL MODEL OF PENCIL BEAM DOSE DISTRIBUTION FOR SPOT-SCANNING*

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The developed model of 'pencil beam' dose distribution is based on analytical approximation of the Bragg curve for proton 'broad' beams and precise calculation of the multiple Coulomb scattering effect by using the Monte Carlo technique.

Using this model one can define a dose at any elementary volume in 3D-space, delivered by a 'pencil' proton beam with arbitrary initial beam parameters including beam size, particle distribution, beam energy and energy spread.

GOAL OF HADRON THERAPY

Create the geometrical distribution of the dose in cancer tumor such that it conforms exactly the 3-dimensional shape of the target volume (3D-conformal therapy)

ADVANTAGES OF PROTON BEAM

- With a single proton beam it is possible to **localize the dose** not only **in the lateral direction** but also as **a function of the depth** in the patient's body.
- Compared to photons one can achieve with proton beams a **general reduction of the integral dose outside of the target volume** by a factor of 2 or 3.
- **Intensity-modulated proton beams** are expected to produce superior results in the case of treatment of **tumours with complex 3D-shape**.

DOSE DELIVERY SYSTEMS

- Accurate dose control in both treatment volume and normal tissue of patient.
- Specified accuracy of the delivered dose is ±2%, placing great demands on beam delivery systems.

3D-DOSE DELIVERY SYSTEMS

Main purpose - minimize damage of normal tissue by forming the treatment field into an arbitrary 3D-shape.

- 'Pencil beam' scanning systems (PSI, GSI, HIMAC)

  A small (~5mm) diameter beam is controlled in the transverse direction by scanning magnets. Variation of the beam energy provides changing dose in depth.

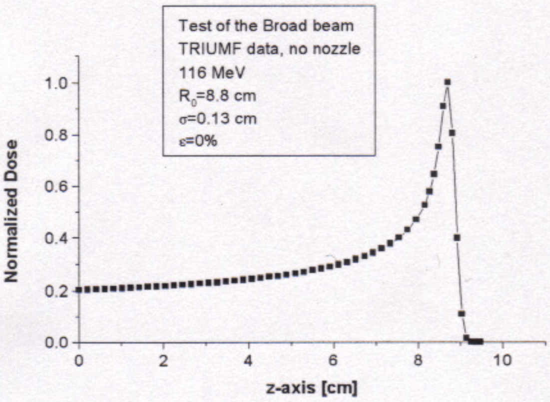
  This method should be adapted to motion of target (cancer tumor) during scanning.

- Intensity Modulated Hadron Therapy

  Changing at the same time the beam intensity and beam energy during treatment on demand of the control system with high precision.

EXPERIMENTAL RESULTS:

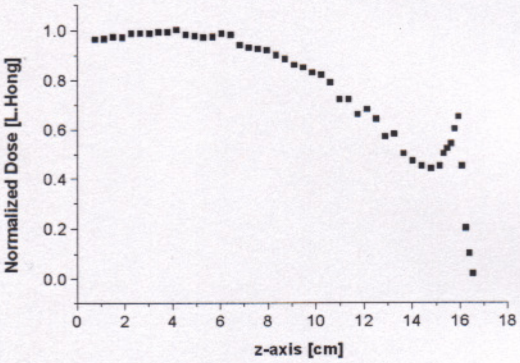
Broad proton beam - 116MeV



Pencil proton beam - 160 MeV [L.Hong]

Beam radius = 2.5 mm

Diode size = 1mm × 1mm



L.Hong et al., Physics in Medical and Biology, 41 (1996), pp. 1033-1053.

MODELS OF 'PENCIL' BEAM DOSE DISTRIBUTION

All models of the proton 'pencil' beam in the homogeneous water phantom developed before are based on the tabulated experimental results of the energy loss for proton beam [ICRU Report 49] or on numerical calculations of the Bragg curve.

Th.Bortfeld [Medical Physics, 24(12), 1997, pp.2024-2033]

Analytical model of the Bragg curve for proton beam that has a physical basis...

MODEL OF PROTON PENCIL BEAM IN HOMOGENEOUS MEDIA

- Following the model of the electron pencil beam, the dose distribution of the pencil beam $D_p(x,y,z)$ in some arbitrary point in the 3D-coordinate system can be separated into a central-axis term $G(z)$ and an off-axis term $F(x,y,z)$

$$D_p(x, y, z) = F(x, y, z) \cdot G(z) \quad .$$

- The central-axis term $G(z)$ is the dose distribution of the broad beam. The off-axis term $F(x, y, z)$ is assumed equal to the lateral flux distribution due to multiple Coulomb scattering (MSC) effect defined at some z-coordinate along the beam axis:

$$F(x, y, z) = \frac{1}{2\pi \cdot \sigma_{MCS}^2} \exp[-\frac{x^2 + y^2}{2\sigma_{MCS}^2}]$$

where $\sigma_{MCS}(z)$ is the RMS-size of the lateral distribution of the beam including the scattering (MCS) effect.

'STOPPING' POINT

- The stopping point is equal to the depth at which the dose has dropped to about *80%* of its maximum value beyond the Bragg peak. If the proton beam passes through the water from $z=0$ till $z=R_0$ (in this case R_0 is the coordinate of the stopping point), the beam deposits energy. The remained energy $E(z)$ at the arbitrary depth $z \leq R_0$ should meet the range-energy relationship [Th.Bortfeld]

$$R_0 - z = \alpha E^p(z) \quad ,$$

where p and α are approximation coefficients.

- For the energies between *10* and *250MeV* the parameters p and α should be equal approximately to *1.77* and $2.2 \cdot 10^{-3}$. Using these values one can get high coincidence between the range-energy table and the analytical fit.

RANGE (depth) STRAGGLING

Nature of the interaction of radiation with matter is the statistical one. Then the actual ranges are distributed about the mean range, that is a function of particle energy including the energy spread.

- **Non-nuclear interactions** of protons in water $\Rightarrow$ Gaussian approximation:

  The distribution of the depth, projected on the z-axis, at which protons have lost the part (E_0-E) of their energy, is approximately a Gaussian distribution about the mean depth $z(E,E_0)$ with the standard deviation $\sigma(z)$

Near the STOPPING point ($R_0-10\sigma \leq z \leq R_0+5\sigma$): $\sigma \approx 0.012 R_0^{0.935}$

- **Nuclear interactions** of protons in water is less critical:

  The contribution of these particles into overall dose is significantly smaller and smoother.

DEPTH-DOSE FUNCTION

in the case of Gaussian approximation including the range straggling and the beam energy spread [Th.Bortfeld]:

$$G(z) = G_{elast}(z) + G_{non-elast}(z) =$$

$$\Phi_0 \frac{e^{-\varsigma^2/4}\sigma^{1/p}\Gamma(1/p)}{\sqrt{2\pi}\rho p \alpha^{1/p}(1+\beta R_0)}[\frac{1}{\sigma}D_{-1/p}(-\varsigma) + (\frac{\beta}{p} + \gamma\beta + \frac{\varepsilon}{R_0})D_{-1/p-1}(-\varsigma)]$$

where $\varsigma = \frac{R_0 - z}{\sigma}$, $D_n(y)$ is the parabolic cylinder function, $\Gamma(x)$ is the gamma function. Φ_0 is the initial flux, β is the slope parameter of the relation for the flux reduction (β=0.012 cm^{-1}). ρ is the density of the homogeneous media, γ is the parameter describing a fraction of locally absorbed energy released in non-elastic nuclear interactions (γ = 0.6). ε is the fraction of primary flux contributing to the 'tail' of the energy spectrum ($\varepsilon \approx 0 \div 0.2$).

Beam has small energy spread ($\sigma_{E,0} << E_0$), then the Gaussian energy spectrum translates into the Gaussian range spectrum and one can use the following equation to define the range spectrum of the 'real' beam (σ):

$$\sigma^2 = \sigma^2_{mono} + \sigma^2_{E,0}\left(\frac{dR_0}{dE_0}\right)^2 = \sigma^2_{mono} + \sigma^2_{E,0}\alpha^2 p^2 E_0^{2p-2} \quad ,$$

where $\sigma_{mono} \approx 0.012\, R_0^{0.935}$.

ENERGY STRAGGLING

- Equation for (dE/dx) describes only the average energy loss of a particle, but does not describe the energy loss distribution.

- For thick absorber, where the number of collisions is large, the energy loss distribution can be shown to be GAUSSIAN. The width of this distribution was calculated by Bohr in the case of non-relativistic particles for an absorber of thickness L [cm].

- σ_E-energy straggling of the relativistic particles (β=v/c) in homogeneous water phantom (with density ρ [g/cm^3]) can be calculated as

$$\sigma_E^2 = \frac{1-\frac{1}{2}\beta^2}{1-\beta^2}\cdot 0.1569\rho\cdot\frac{Z}{A}L \quad [\mathrm{MeV}^2] \quad .$$

- After definition of σ_E-energy straggling, it is necessary to randomize the energy loss by adding the product of $\langle\Delta E\rangle_{LOSS}\times g\times\sigma_E$, where g is a Gaussian random number.

MULTIPLE COULOMB SCATTERING EFFECT

- Protons passing through the homogeneous water media lose energy primarily through multiple Coulomb interactions.

- MCS effect in a thick scatterer can be simulated by using a modified Highland's equation, suggested by Gottschalk [NIM,B74, 1993, pp.467-490]:

$$\theta_0(L) = 14.1 \cdot z \cdot \left(1 + \frac{1}{9}\log_{10}\frac{L}{L_R}\right) \cdot \sqrt{\int_0^L \left(\frac{1}{p\beta c}\right)^2 \cdot \frac{dL'}{L_R}} ,$$

where z is the number of elementary charges of the incoming particles, L_R is the radiation length for the homogeneous media, L is the depth, p is the momentum of the particle inside of the scatterer, $\beta = v/c$ is the particle velocity.

- The modified Highland equation gives the RMS value of the scattering angle when the three-dimensional scattered distribution is projected onto a plane.

- The theory breaks down when particles have lost on average 97% of their energy near the stopping point.

MONTE CARLO TECHNIQUE

- After passing through a scatterer *L*, the particle will suffer a scattering angle θ_0 and a displacement δx, which are connected by the following relation [C.Caso et al., "Review of particle properties", The European Physical Journal, C3:1, 1998]

$$\left\langle \delta x^2 \right\rangle = \left(\frac{L}{\sqrt{3}} \theta_0 \right)^2 .$$

- $\{x_{k0}, x'_{k0}\}$, k=1…$N_{particles}$ INITIAL particle distribution

$$x_{ki} = x_{k0} + x'_{k0} \cdot z_i + \delta x_{ki} \quad ,$$

$$x'_{ki} = x'_{ko} + \theta_{ki} \quad ,$$

$$\delta x_{ki} = \frac{g_{k1} z_i \ \theta_{ki} \sqrt{1-\rho^2}}{\sqrt{3}} + \frac{g_{k2} \rho \ z_i \theta_{ki}}{\sqrt{3}} \quad ,$$

$$\theta_{ki} = g_{k2} \theta_0 \ , \quad \rho = \sqrt{3}/2$$

with $\{g_{1k}, g_{2k}\}$ as the Gaussian random numbers.

$\{x_{ki}, x'_{ki}\}$ lateral particle distribution in any point (z_i) of the thick scatterer along the axis

We can construct the dose distribution of the pencil beam!

COMPARISON WITH EXPERIMENTAL DATA

- 'Broad beam' dose distribution / data set from TRIUMF (Canada) /

- 'Pencil beam' dose distribution along the beam axis

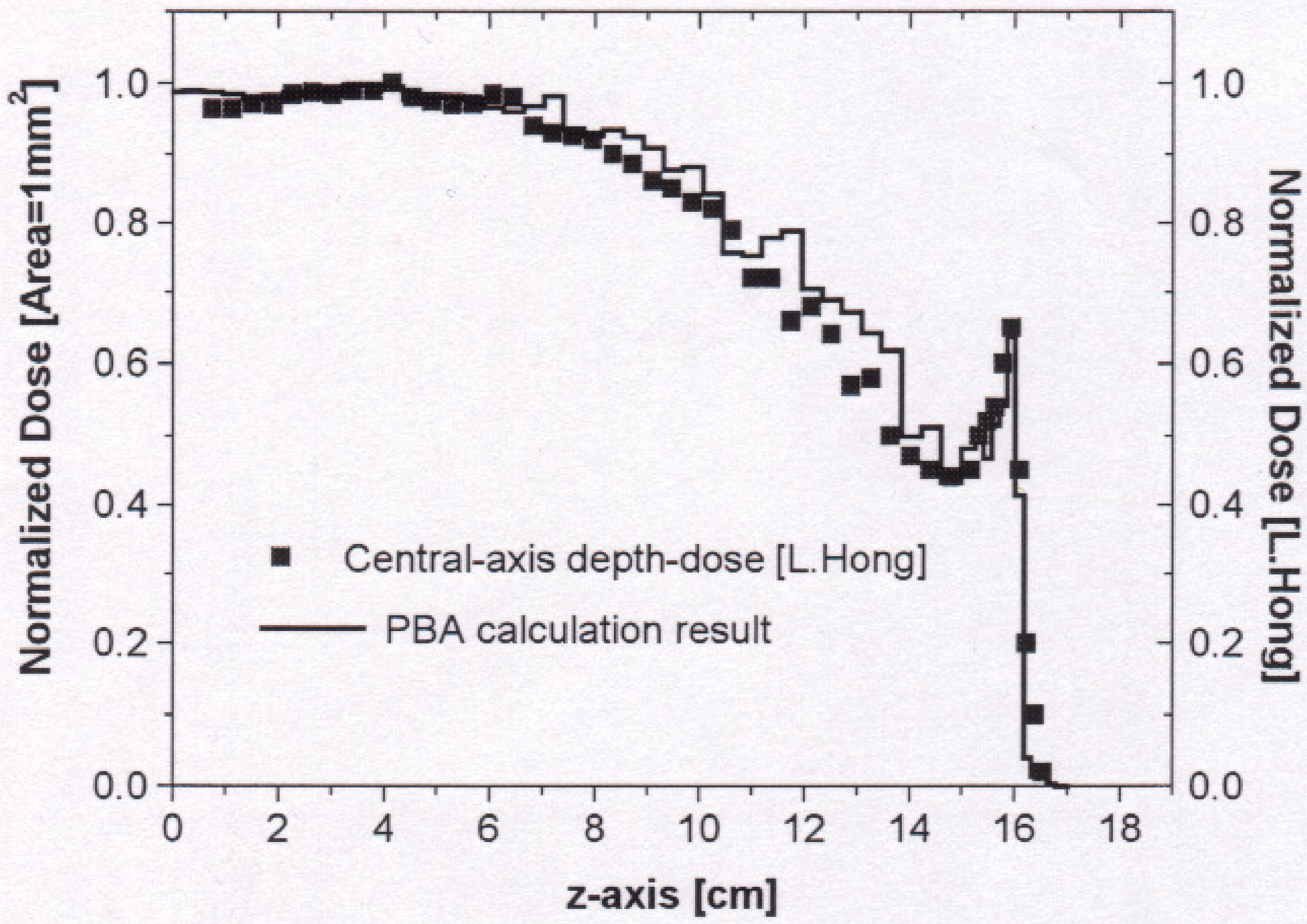
  160MeV with the energy spread $\sigma_W = \pm$ 500keV

  Beam radius = 2.4 mm

  Silicon diode with size 1mm$\times$1mm

  ...data-set corresponding to 6 cm air gap between the collimator and the water phantom

PBA - Monte Carlo simulation with 10'000 particles



FLEXIBILITY OF THE SIMULATION MODEL

- Prediction of the dose at ANY ELEMENTARY VOLUME of the 3D-space delivered by a 'pencil' proton beam

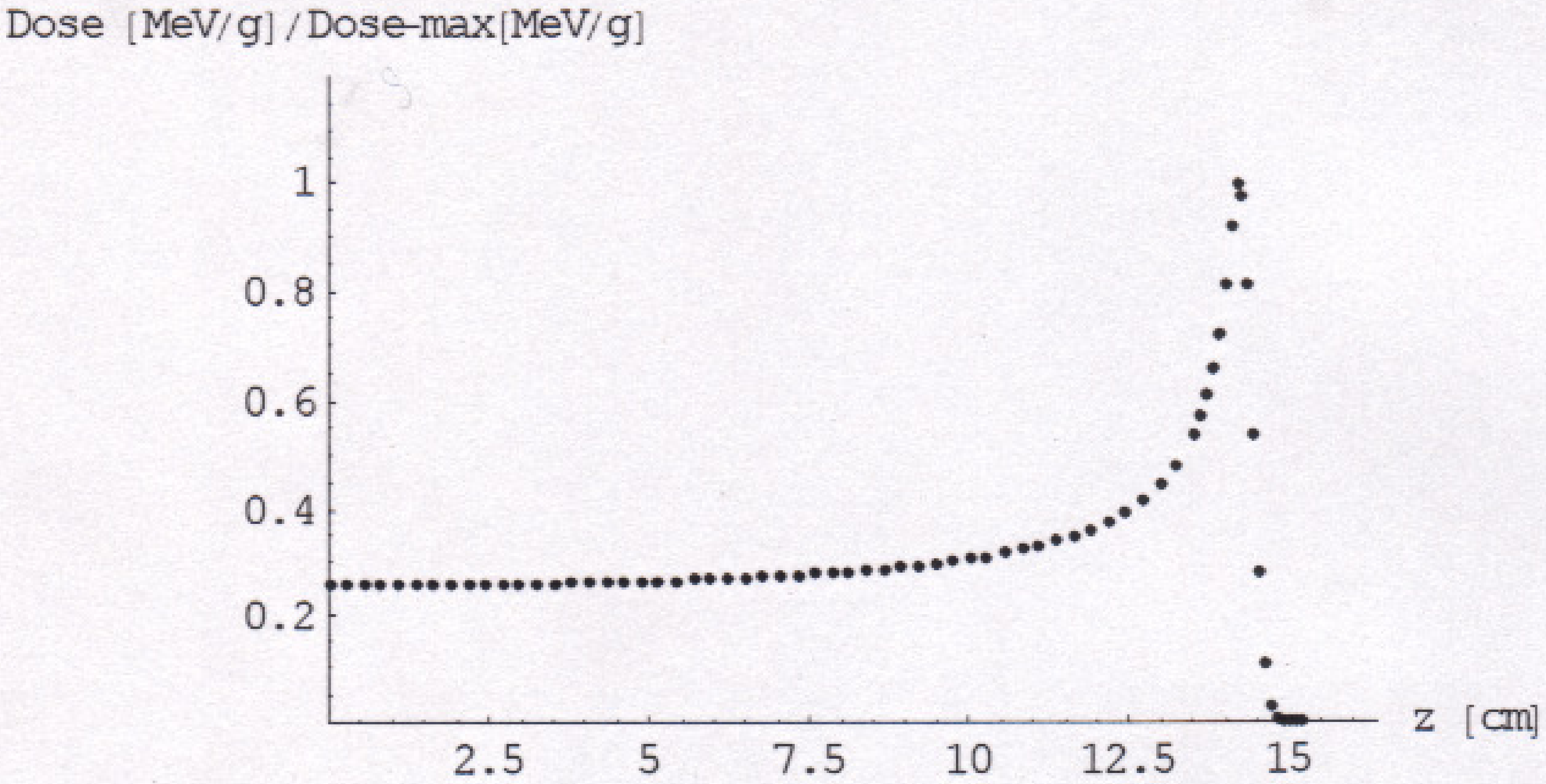
- Proton beam:
  - different initial beam size
  - different initial energy spread
  - different kind of particle distribution of the beam (Gaussian, Uniform ...)

The model is built by using the 'MATHEMATICA' software (version 4.0.1.0).

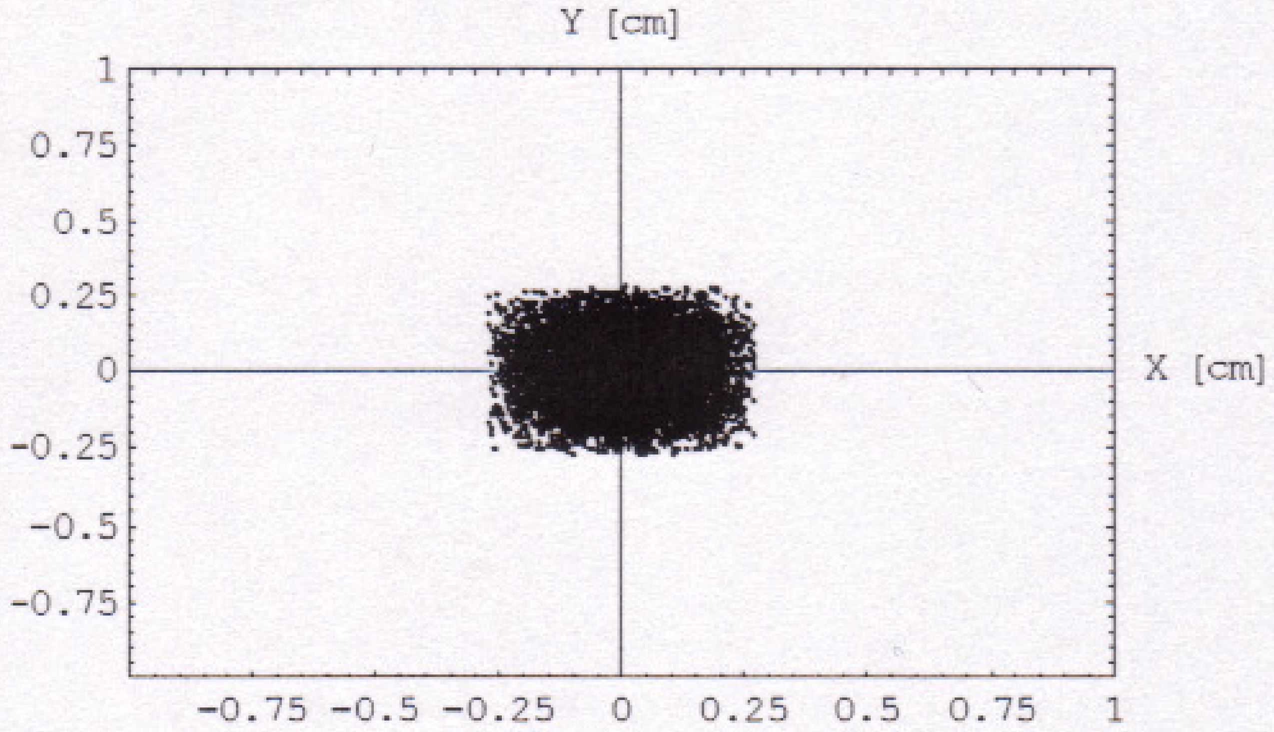
Simulation time of 3D-dose distribution of a 'pencil' proton beam (160MeV / 5'000p) without any optimizations is ~ 10min (Pentium III, 600MHz).

150MeV proton beam / $dW/W_{kin} = \pm 0.5$MeV, $dp/p = \pm 2 \cdot 10^{-3}$ /

- Dose distribution of BROAD beam



- Multiple Coulomb scattering



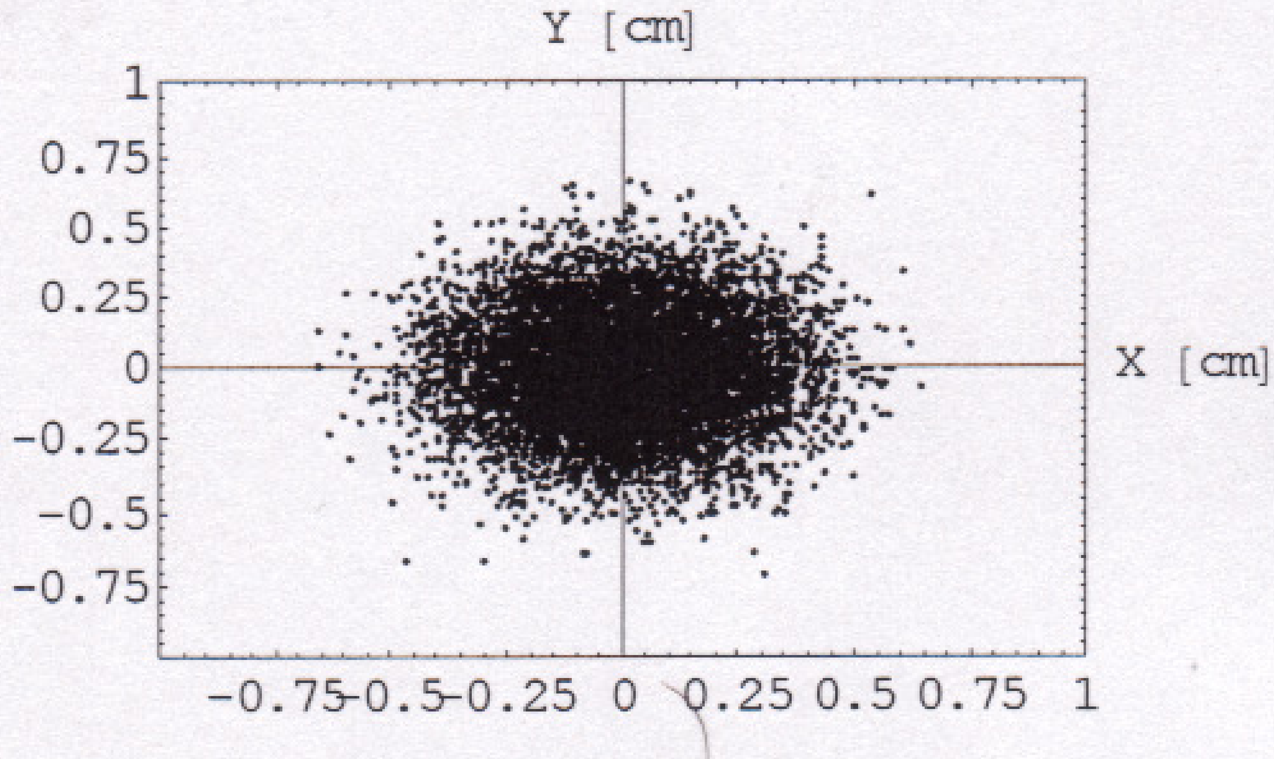
Initial beam size = ± 2.5 mm

Gaussian distribution

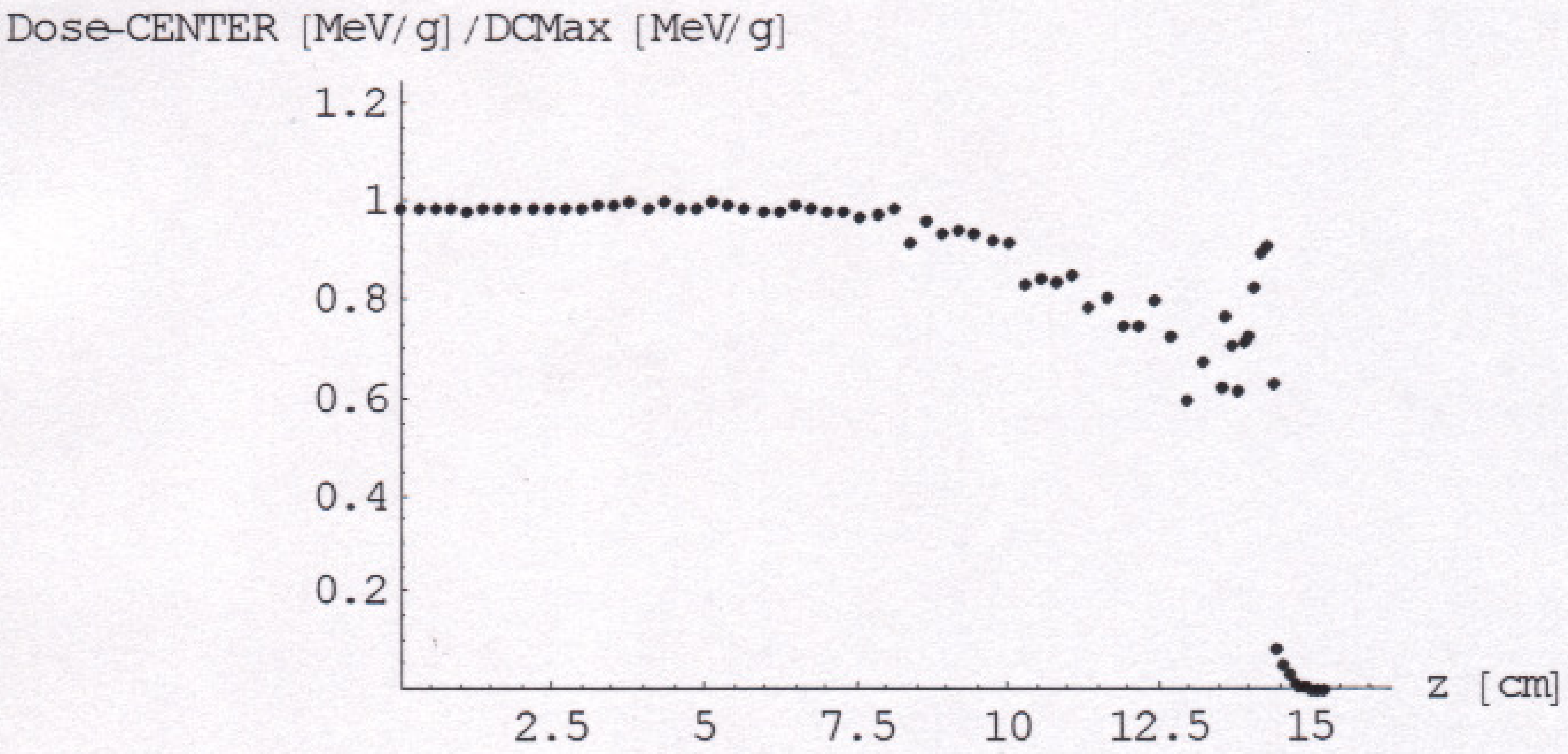
σ_{init} ~ 1mm

Particle distribution near the Stopping point

- Maximum beam size ~ 0.75 cm
- σ-beam size ~ 1.9 mm

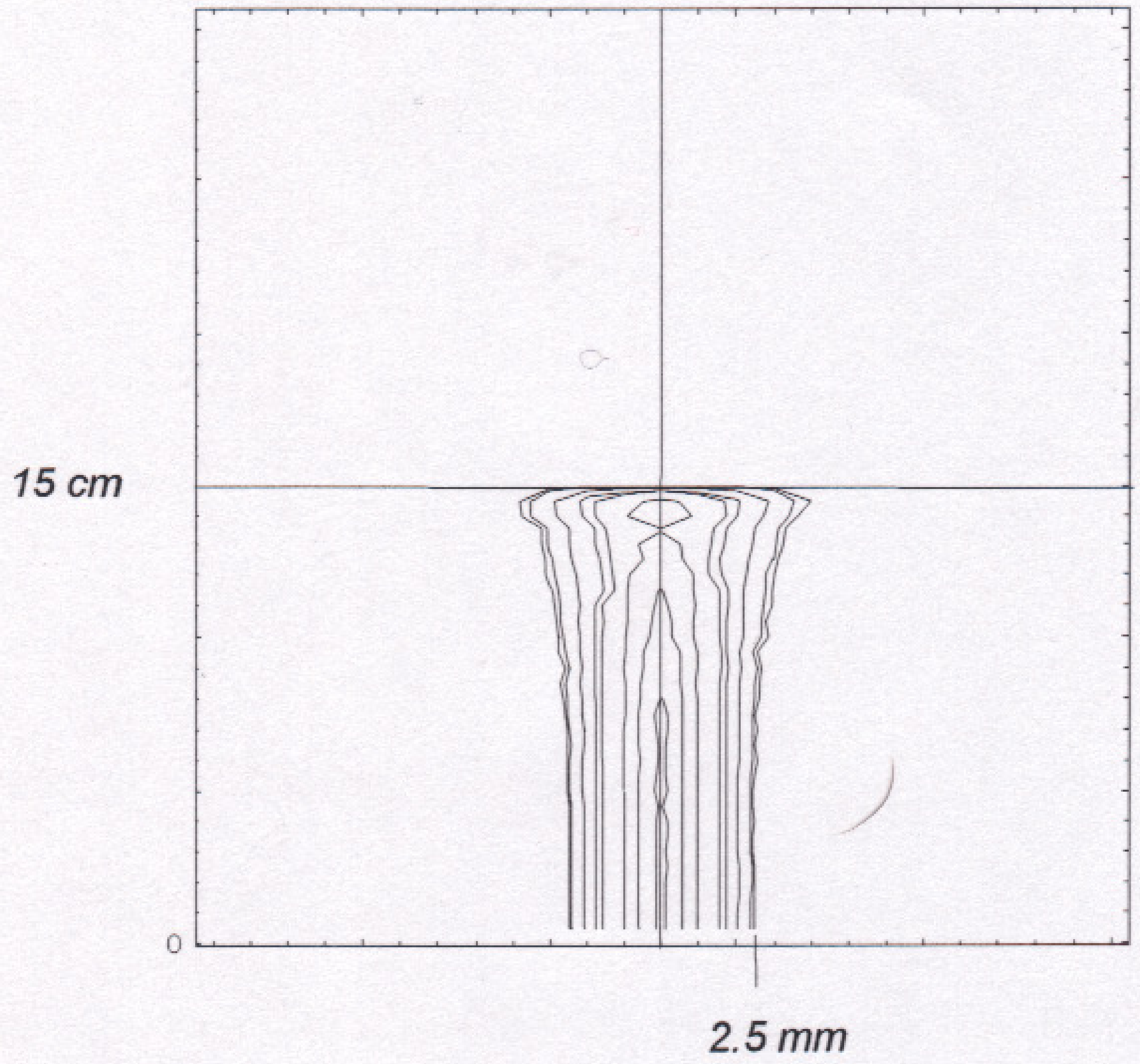


- Axial dose distribution of 150MeV pencil beam
  - initial beam size = ± 2.5 mm (σ_{init} ~ 1 mm)
  - energy spread = ± 500 keV
  - mesh-size = 1 mm × 1mm



Iso-Dose lines of the proton pencil beam

D/Dmax:
0.005
0.01
0.05
0.15
0.22
0.50
0.75
0.95



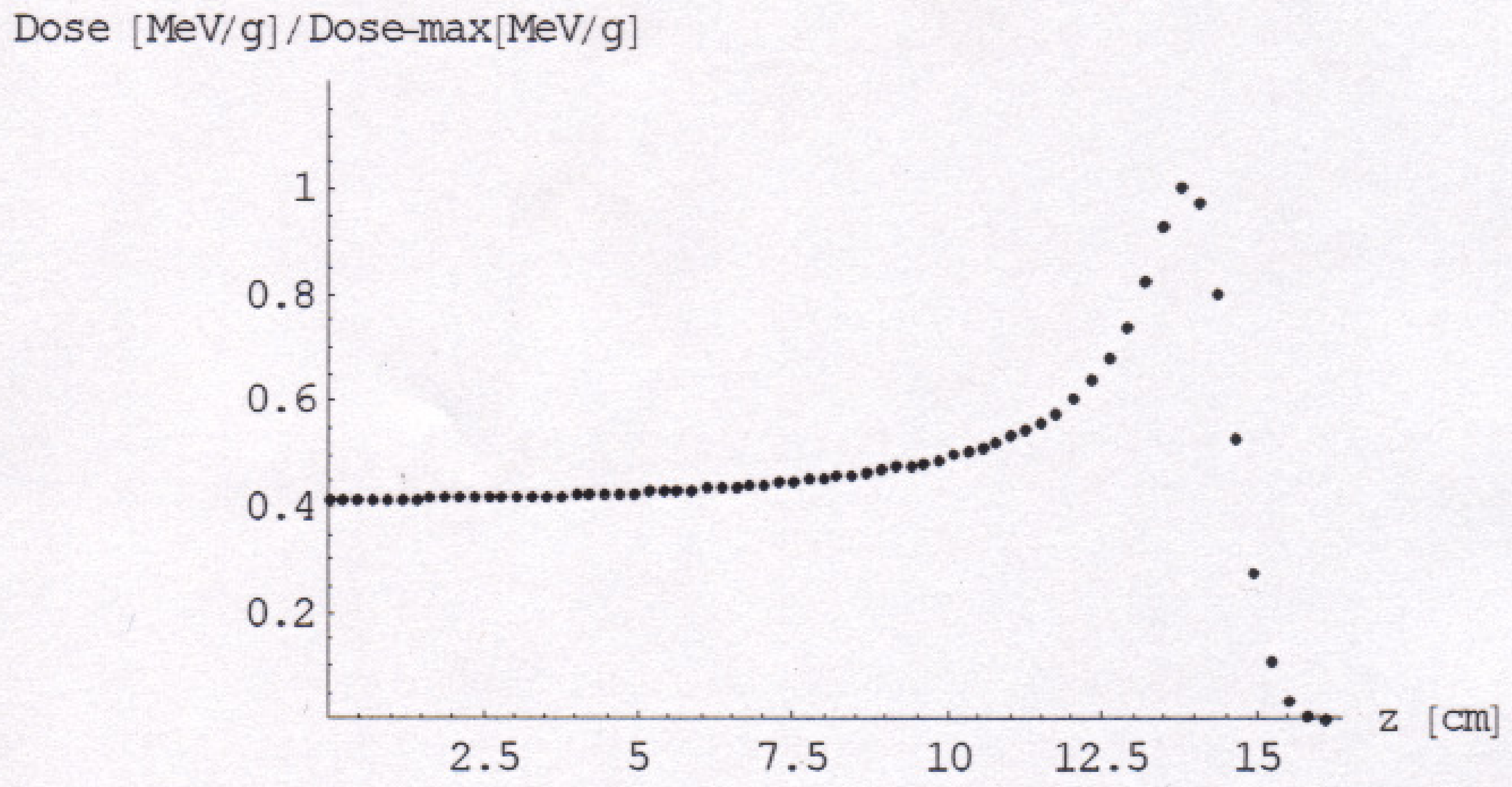
LIST OF MAIN BEAM PARAMETERS FOR SPOT SCANNING

Energy range	W_{kin}	MeV	60 … 150 (FFAG)
Stopping point	R_0	cm	~ 2.8 …14.4 (FFAG)
Energy spread	$\Delta W/W_{kin}$		$< 5 \times 10^{-3}$
Energy variability	ΔW_{step}	MeV	< 1
Energy variability accuracy		MeV	$\pm$ 0.1
Beam intensity on target	<I>	nA	~ 10
RMS beam size		mm	~ 2 mm

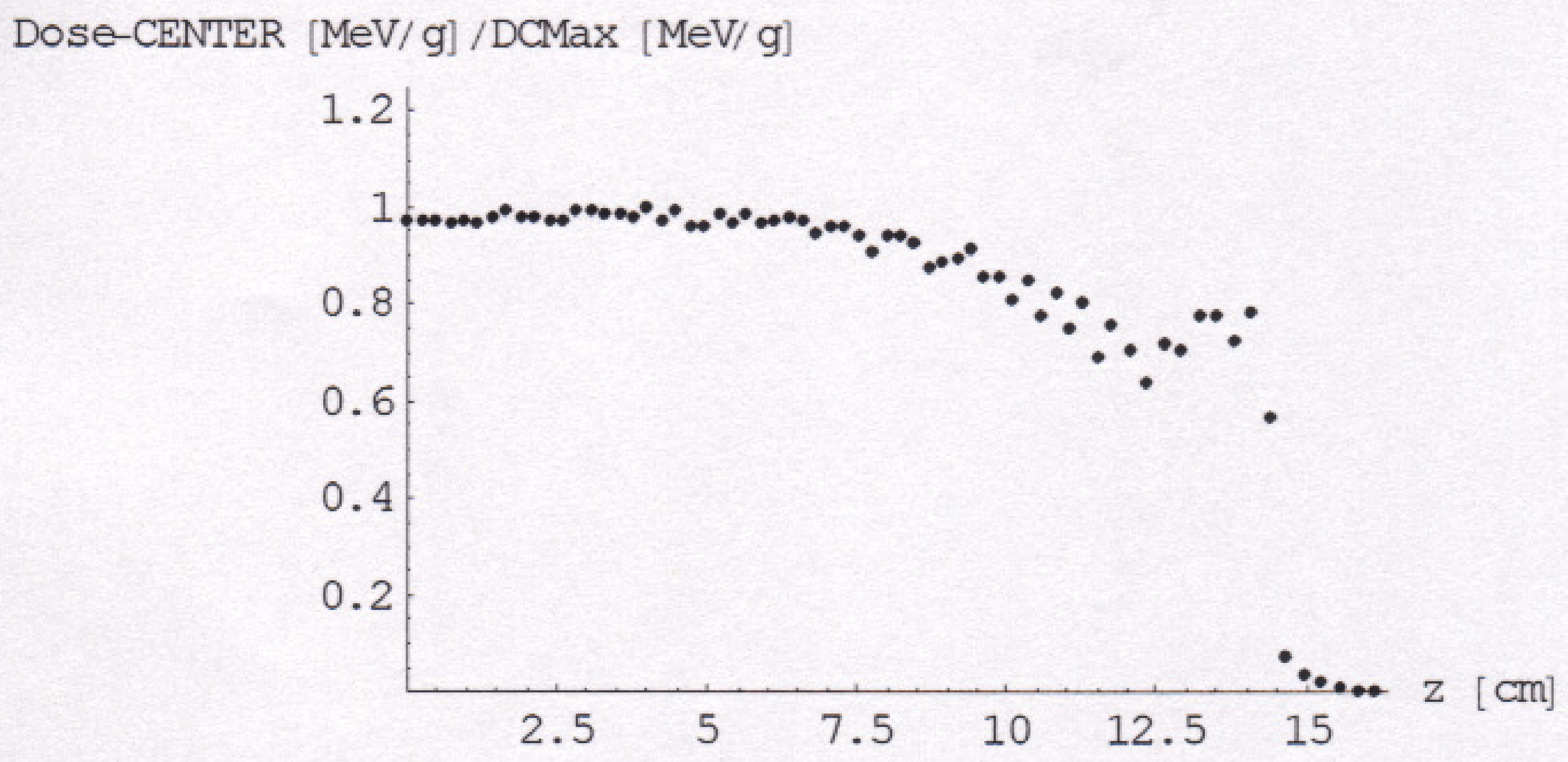
- A GAUSSIAN beam profile is desired in order to obtain a smooth lateral dose distribution by overlapping the neighboring spot of the 'spot-scanning' pattern.

- Axial dose distribution of 150MeV pencil beam
    - initial beam size = ± 2.5 mm (σ_{init} ~ 1 mm)
    - energy spread = ± 3000 keV
    - mesh-size = 1 mm × 1mm

BROAD beam

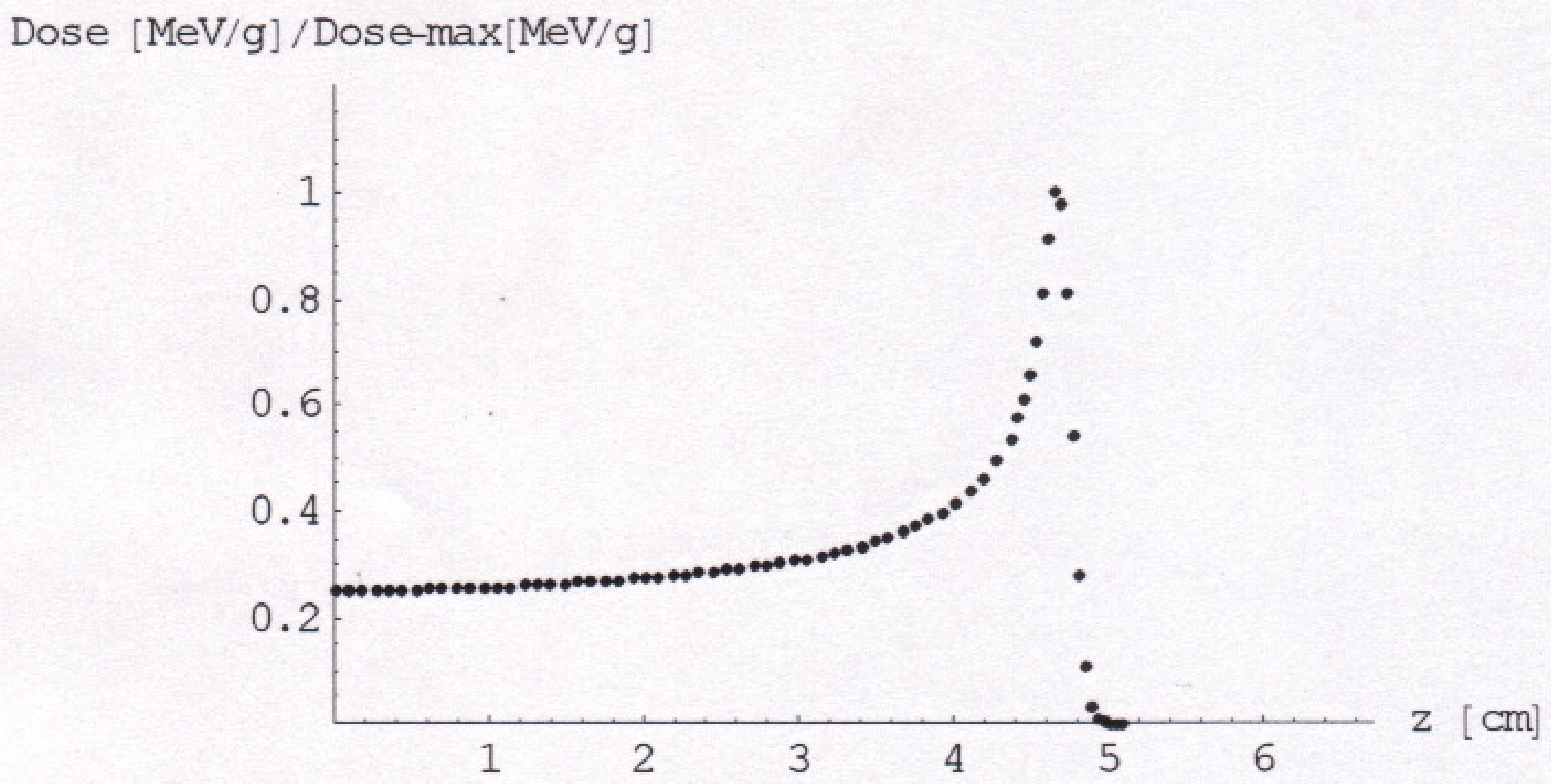


PENCIL beam



- Axial dose distribution of 80MeV pencil beam
  - initial beam size = ± 2.5 mm (σ_{init} ~ 1 mm)
  - energy spread = ± 500 keV
  - mesh-size = 1 mm × 1mm

BROAD beam



PENCIL beam