BAND GAP THEME [1]

SEMICONDUCTOR [SCHRODINGERS EQ.]
    Energy gap
PHOTONIC CRYSTAL [MAXWELL'S EQ.]
    Wavelength gap
ACCELERATORS [LORENTZ FORCE]
    Strong focussing and FFAG
MHD STAILITY [MHD EQ.]
    Average magnetic well
TOROIDA ALFVEN MODE [MHD]
    Mode in the gap is unstable

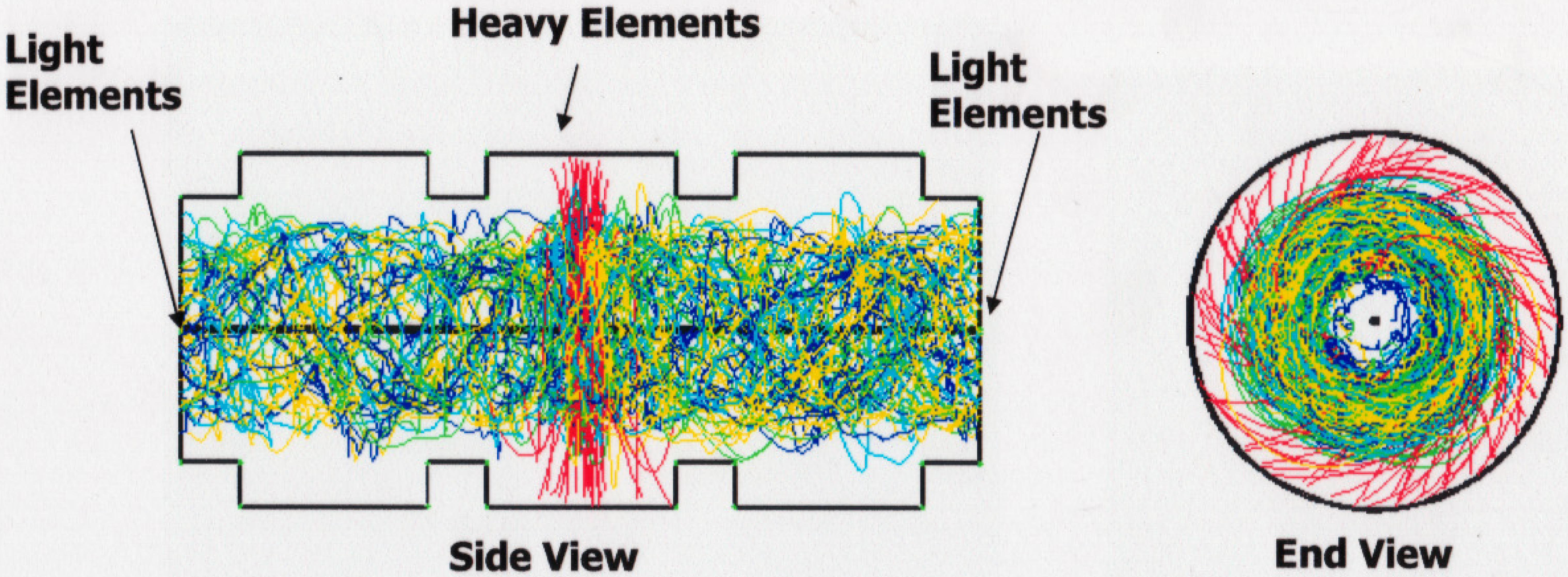
PERIODIC [SPACE OR TIME] POTENTIAL

HILL'S EQUATION SOLUTION

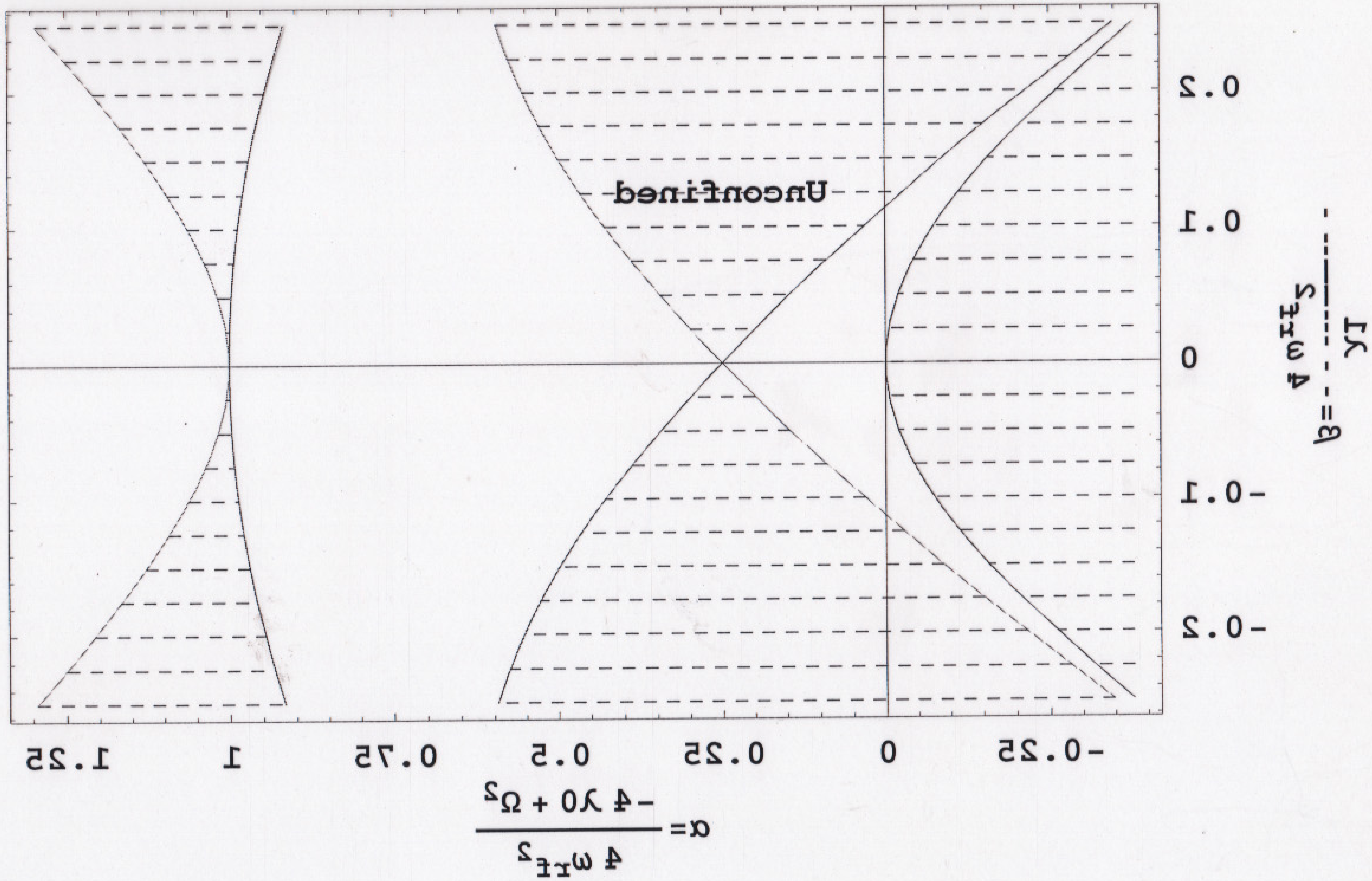
PROPAGATING	NONPROPAGATING
OR	
STABLE	UNSTABLE

DEPENDING ON MAGNITUDES OF
AVERAGE AND OSCILLATING POTENTIAL

Monte Carlo simulations demonstrate separation & throughput when effects of collisions included



O	blue	Cr	cyan
Na	green	Sr Cs Pu	red
Al	yellow		



Unconfined
0.2
0.1
0
-0.1
-0.2
$\beta = \frac{\gamma_1}{4\,\omega_{rf}^2}$
-0.25
0
0.25
0.5
0.75
1
1.25
$\alpha = \frac{-4\lambda_0 + \Omega^2}{4\,\omega_{rf}^2}$

Ion orbits in plasma

Difference from particle orbits in accelerators

$\nabla^2 \Phi \neq 0$ Φ : Electrostatic potential

$\Phi = \Phi [\Psi]$ Ψ : magnetic flux function

Plasma electrons move easily along magnetic flux lines and eliminate $E_{||}$

$\nabla \times \nabla \times \mathbf{A} \cong 0$

$2 \mu_0 p / B^2 << 1$ p : plasma pressure

For $A_\theta = A_\theta [r,t]$ and $\Phi = \Phi [r,t] = \Phi [r A_\theta]$

$$H = [1 / 2m]\{[p_r^2+p_z^2] + r^{-2}[p_\theta - e r A_\theta]^2\}$$

$$+ e \Phi$$

Band gap mass Filter

When the electrode voltage contains d.c. and a.c. components, the characteristics of the ion orbits display the band gap structure like the electrons in semiconductors or the photons in photonic crystal. The gaps represent the unconfined orbits. The standard Filter is a special case without the a.c. component. If the d.c. component is absent, there is similarity with the cyclotron resonance method [J.Dawson's] in that the frequency at the gap is the cyclotron frequency. However the radius of the orbit grows exponentially in the band gap as compare to the linear growth in the cyclotron method. The ions in the gap are collected in the same fashion as the heavy ions in the standard Filter rather than the vanes intercepting the high energy ions.

The single particle orbits have already been discussed ["Orbits in Filter" in z\ ohkawa]. The equation of motion without collisions is given by

$$d^2 / dt^2 \, \mathbf{q} = -\Omega \, \mathbf{J} \, d / dt \, \mathbf{q} + \lambda \, \mathbf{q} \qquad [1]$$

where

$$\mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Omega = e B / m$$

$$\lambda = 2 e V[t] / [m a^2]$$

We go to the rotating frame by putting

$$\mathbf{q} = \mathbf{M}\,\mathbf{s}$$

$$\mathbf{M} = \{ \cos[-\Omega t/2] + \sin[-\Omega t/2]\,\mathbf{J} \}$$

and obtain

$$d^2 / dt^2\ \mathbf{s} + [\Omega^2 / 4 - \lambda]\,\mathbf{s} = 0 \qquad [2]$$

Since λ is a periodic function of time, the above equation is a Hill's equation. The solution is given by

$$\mathbf{s} = f \exp[\sigma t] + f^* \exp[-\sigma t] \qquad [3]$$

where f and f* are periodic functions and σ is the characteristic exponent. If σ is imaginary the solution does not grow and if σ is real the solution grows. The boundary of the gaps occurs at $\sigma=0$. The ions with imaginary σ is confined but the ions with real σ are filtered out.

If λ is sinusoidal, namely

$$\lambda = \lambda_0 + \lambda_1 \cos \omega t \qquad [4]$$

the equation is Mathieu equation.

$$[\tfrac{1}{4}] d^2 / d\tau^2\,\mathbf{s} + [\alpha - 4\beta \cos 2\tau]\,\mathbf{s} = 0 \qquad [5]$$

where

$$\tau = \omega t / 2$$

$$\alpha = [\Omega^2 / 4 - \lambda_0] / \omega^2$$

$$\beta = \lambda_1 / [4\omega^2]$$

For small values of β, the gap boundaries are given by

$$4\,\alpha_0 = -2^5\,\beta^2 + 2^5\,7\,\beta^4 \text{ ------}$$

$$4\,\alpha_1 = 1 \pm 8\,\beta - 8\,\beta^2 \text{ -------}$$

$$4\,\alpha_2 = 4 + 80/3\ \beta^2 \text{ ------}$$

The regions of real values of σ, namely no confinement of the ion orbits, are given by

$$4\,\alpha < -32\,\beta^2 \qquad [6]$$

$$1 + 8\,\beta > 4\,\alpha > 1 - 8\,\beta \qquad [7]$$

For the voltage given by

$$V = V_0 + V_1 \cos \omega\, t \qquad [8]$$

The above conditions become

$$m > [\, e\,B^2\,a^2 / 8\,V_0]\ \{ 1 + 8\,V_1^{\,2} / B^2\,\omega^2\,a^4 \} \qquad [9]$$

and

$$4\,eV_1/a^2 > m\ \{ e^2B^2/\,m^2 - 8\,eV_0/\,ma^2 - \omega^2\} > -4eV_1/a^2 \qquad [10]$$

We examine the above conditions near the points of interest.
1] $\alpha \sim 0$ and $\beta \sim 0$. The standard Filter corresponds to the case $\beta = 0$. The condition [9] gives the cut-off mass. The r-f voltage V_1 tends to confine the orbits and the cut-off mass is slightly increased.
2] $\alpha \sim 0$ and finite β. This case is similar to the alternate gradient focusing for accelerators. The condition is given by

$$m^{-1} > [4V_0+2V_1]/ eB^2a^2 + \{ [4V_0+2V_1]^2/ [eB^2a^2]^2 +\omega^2\}^{1/2} \quad [11]$$

In this case the *lighter* ions are cut off.

3] $4\alpha \sim 1$ and small β. As shown by eq [7], the cut-off occurs even with infinitesimal V_1. The r-f frequency is in resonance with the oscillation frequency of the orbit, namely

$$\omega^2 = \Omega^2 - 8 e V_0 / ma^2 \quad [12]$$

or

$$\omega = \Omega [1 - m / m_0]^{1/2}$$

where m_0 is the cut-off mass of the standard Filter.

We discussed the method of getting rid of the doubly ionized ion species using this effect before. For example Sr^{++} 90 has the equivalent mass number of 45. If the cut-off mass is 75, an r-f voltage at the frequency $\omega = 0.63\ \Omega$ superposed on the d.c. voltage will take the doubly ionized Sr out.

If V_0 is absent, the resonance frequency is the cyclotron frequency. The resonant ions will be filtered out spatially. It is different from the cyclotron heating method by J.Dawson where the discrimination is in the velocity space and a special collector to intercept the large ion orbits is required.

Since the mass differences are small for the isotope separation, we estimate the resonance width. We write the resonance width given by eq [10] in terms of the mass m and the mass $m \pm \Delta m$. We obtain

$$\Delta m / m = [\omega^2 + \Omega^2]^{-1} [4 eV_1/ a^2 m] \quad [13]$$

The high resolution of mass requires small r-f voltage and the growth rate of the orbit is smaller accordingly. The growth rate $\Gamma$is given by

$$\Gamma = 2 \beta \omega = e V_1 / [ma^2 \omega] \quad [14]$$

The resolution in terms of the growth rate becomes

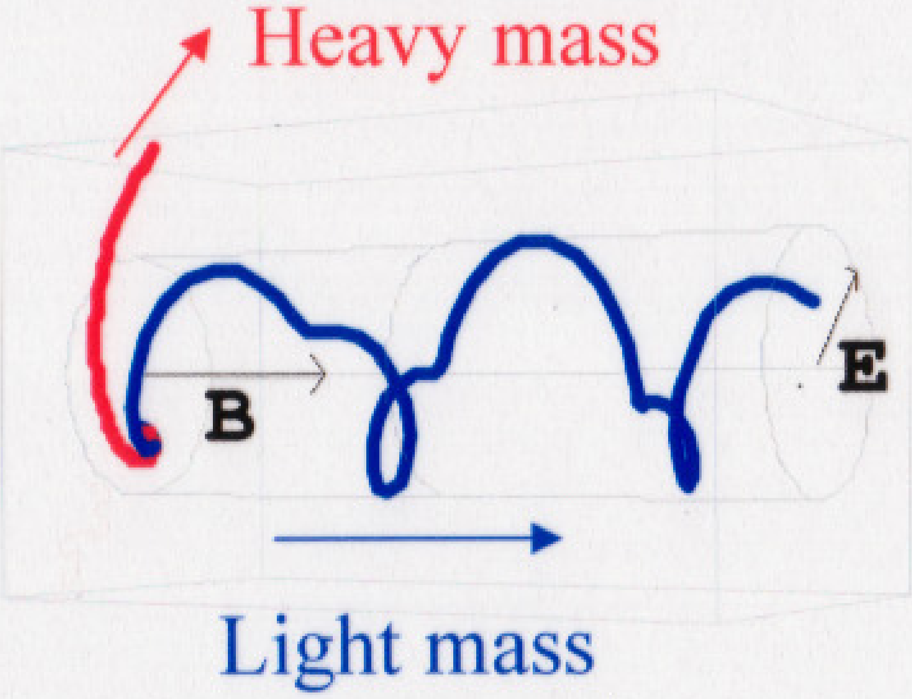
$$\Delta m / m = \Gamma \omega / [\omega^2 + \Omega^2] \qquad [15]$$

When the ion collisions are present, the collisional broadening of the resonance must be taken into account. The resolution with the collisions is given by replacing Γ of the above equation with the collision frequency.

The above analysis shows that the standard Filter and the parametric cyclotron Filter are the special cases of the band gap mass filter. The choice of the operating point on the band gap map depends of the specific applications.

Separation

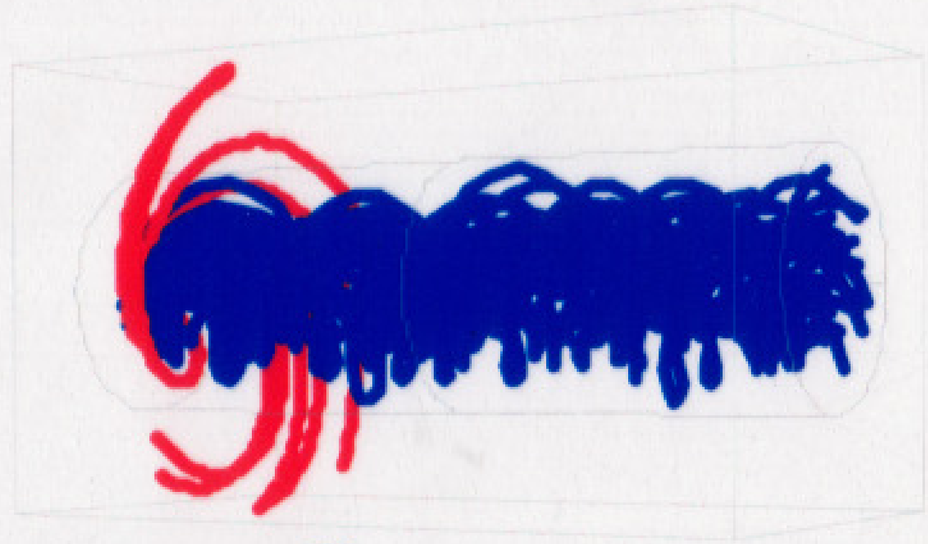
Single ions [a, B, E]



Throughput

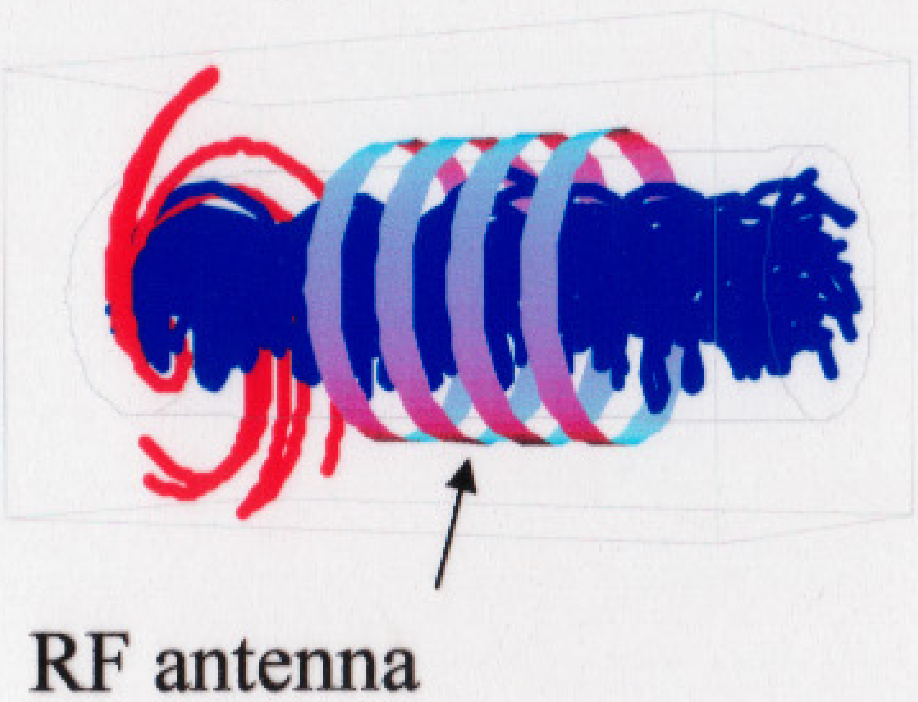
Plasma [n] collisions

Increase fields [B, E] fewer collisions

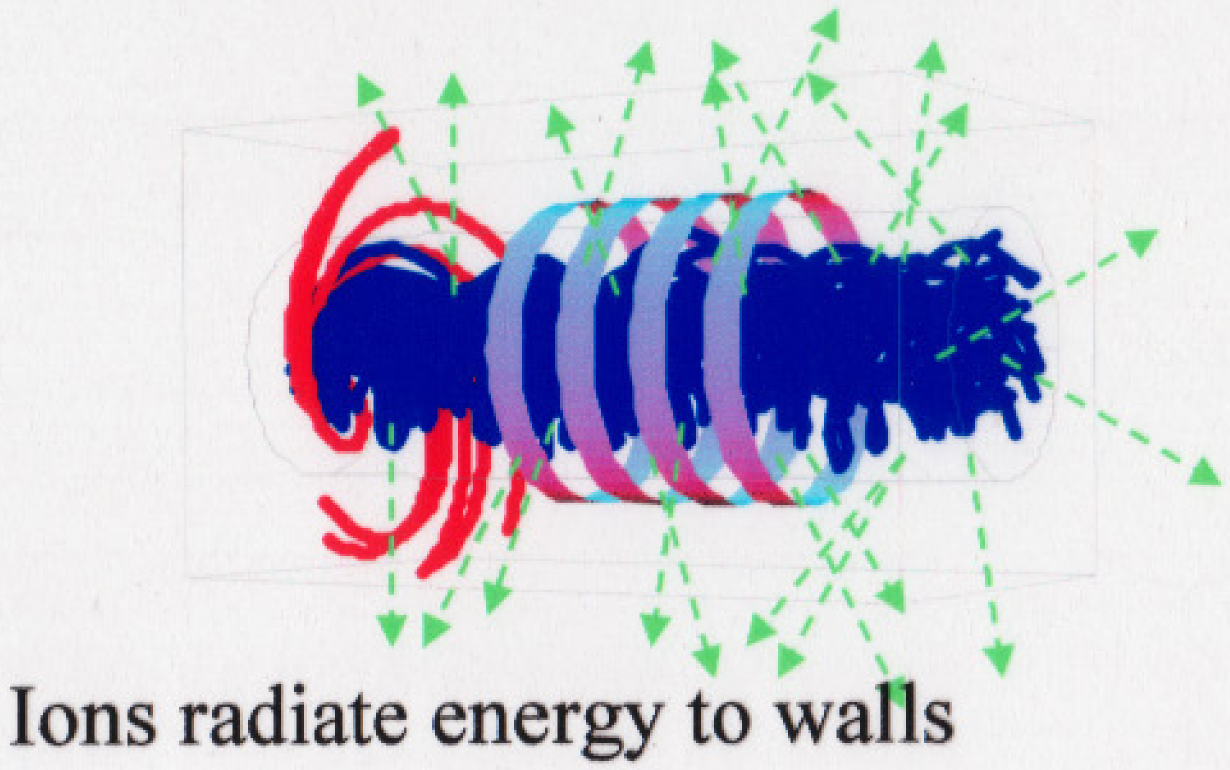


Energy Cost

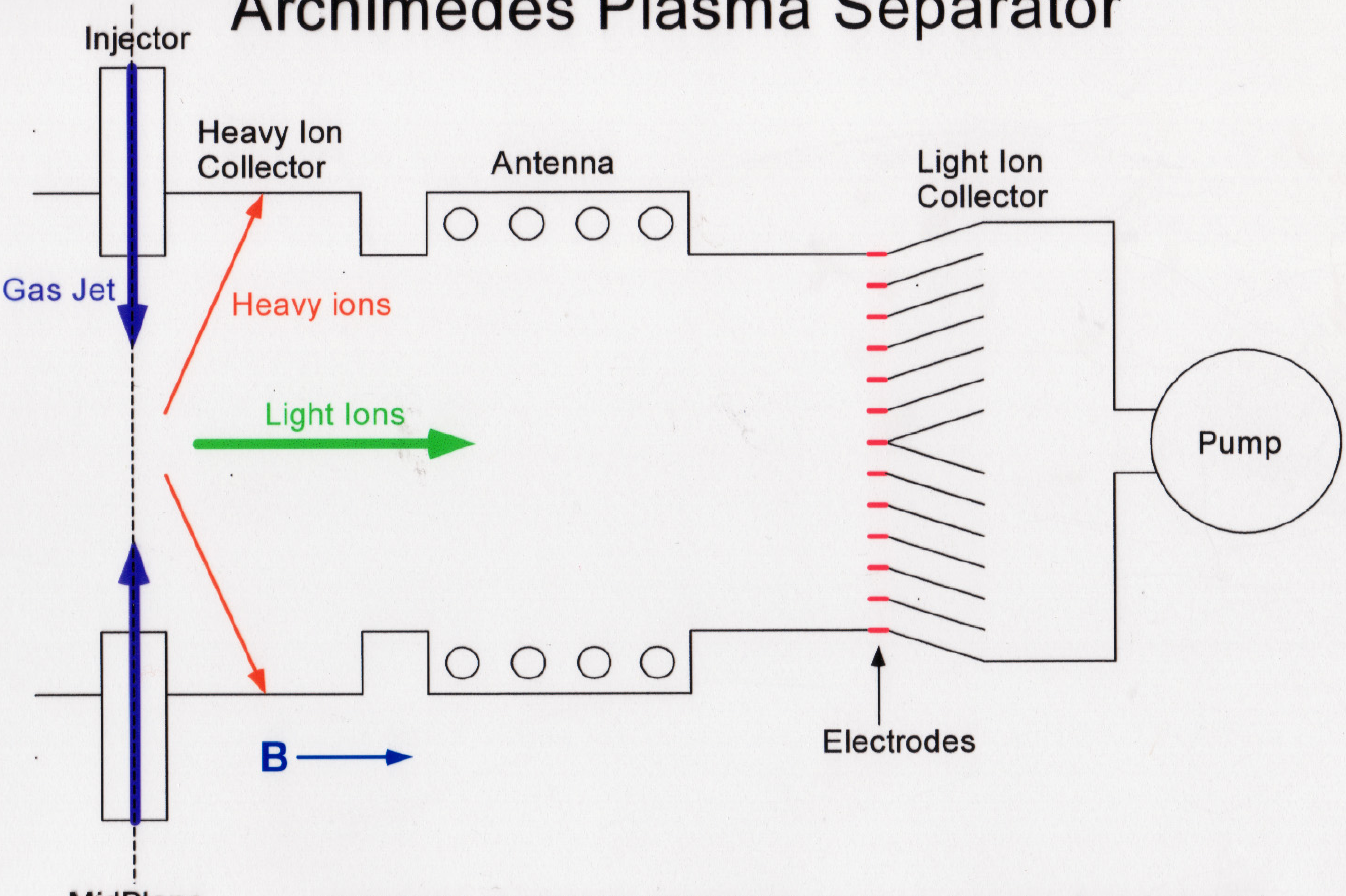
RF Energy needed for ionization [P_{RF}, T_e]



RF Energy needed to offset radiation loss [n, T_e]

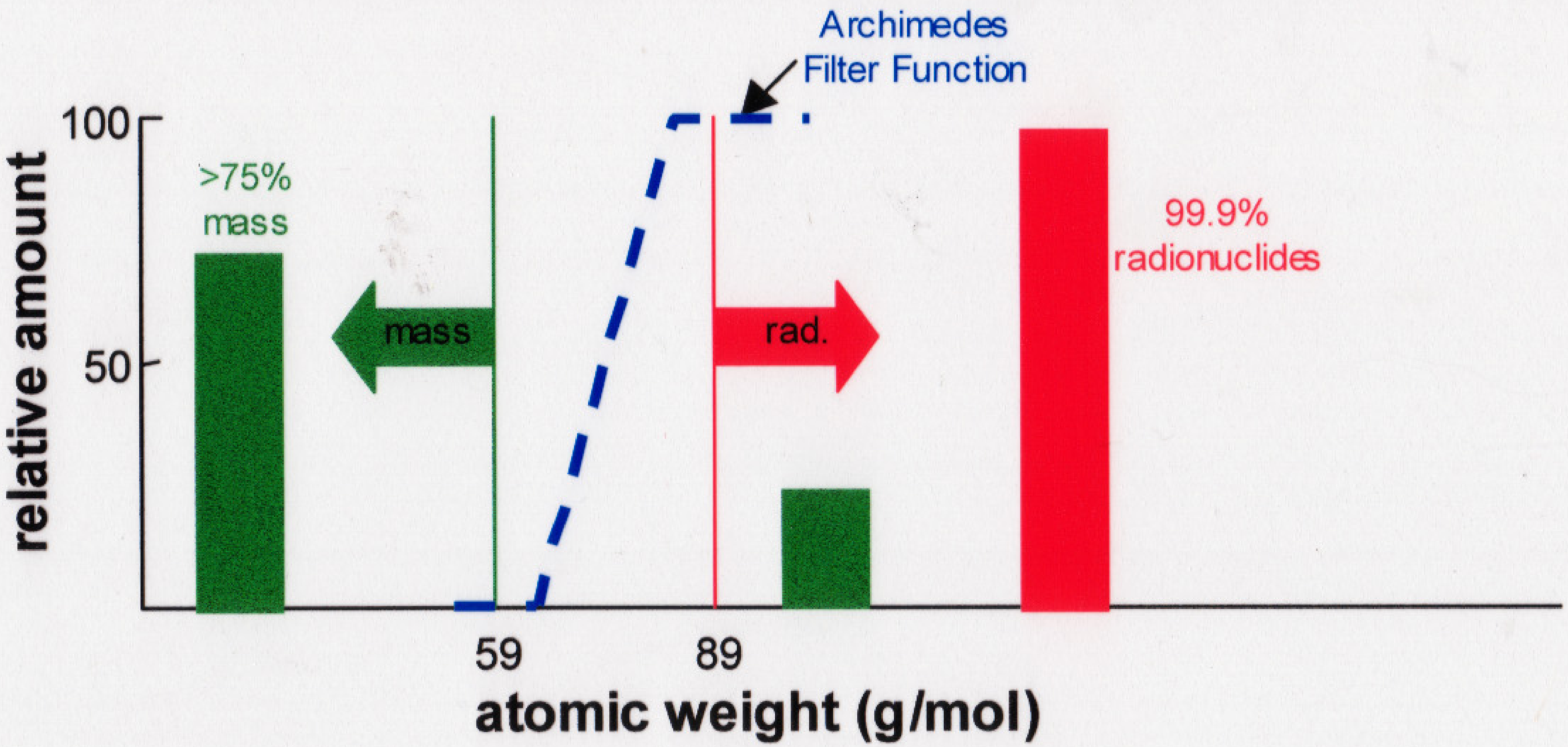


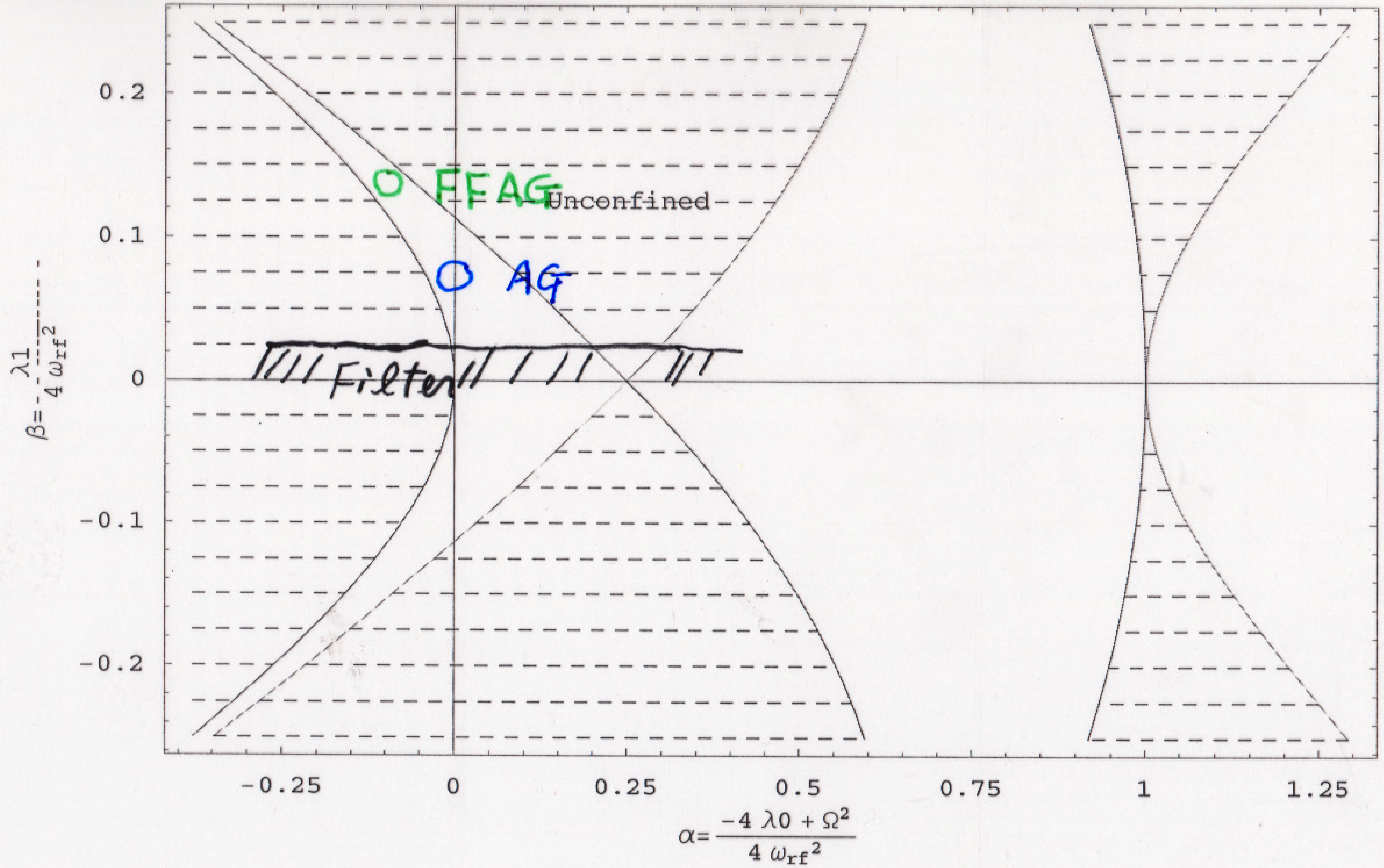
Archimedes Plasma Separator



Separations Efficiency

- **Separation of radioactivity from mass below 89 results in reduction of high level waste glass by factor of four...**





FFAG
Unconfined
AG
Filter
0.2
0.1
0
-0.1
-0.2
-0.25
0
0.25
0.5
0.75
1
1.25
$\beta = -\frac{\lambda 1}{4\,\omega_{rf}^2}$
$\alpha = \frac{-4\,\lambda 0 + \Omega^2}{4\,\omega_{rf}^2}$

ARCHIMEDES FILTER Commercial Separator

Simplified Cross Sectional View Showing Internal Components For Operation At a 0.36 Ionized Fraction

For Hanford Waste, operation with reduced ionization fraction allows:

- Highest effective throughput unit
- Smallest number of separators
- Minimum power requirements