Particle dynamics in electron FFAG

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Abstract. To simulate non-scaling muon FFAG, an electron FFAG from 10 to 20 MeV is proposed. There are mainly two subjects to be studied by an electron FFAG. One is acceleration without and RF bucket and the other is fast crossing of integer and half-integer resonances. In this paper, we will show tracking results of those subjects in 6D phase space. Some criteria of resonance strength and crossing speed are obtained.

Keywords: Neutrino Factory, FFAG, Resonance
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INTRODUCTION

As a muon accelerator, FFAG (Fixed Field Alternating Gradient) machine is seriously studied. Unlike conventional FFAG, that satisfies scaling law and therefore the transverse tune is constant, FFAG for muon acceleration is so called non-scaling type consisting of dipole and quadrupole magnets. Magnets may be simpler than ones for scaling FFAG. Transverse tune, however, is not constant during acceleration. Not only higher order resonances such as sextupole and octupole, integer and half-integer resonances have to be crossed several times. Effects on particle amplitude is not clear although crossing speed should be fast and that helps in general.

Another major concern is acceleration. To accelerate muons, which have very short life, fast acceleration has to be utilized. It is not practical to modulate RF frequency to synchronize muon velocity. Instead, fixed frequency RF system is installed and acceleration out of longitudinal buckets is planned. In this paper, we will show tracking results both on acceleration and resonance crossing.

LATTICE MODEL

As a model lattice, we pick up a lattice designed by Trbojevic and Courant. It was presented at FFAG workshop at TRIUMF in 2004 [1]. The number of cell is 45 and the total circumference is 15 m. Figure 1 shows lattice functions.

Although lattice design itself is one of major study items and modeling by existing codes such as SYNCH, MAD, SAD, PTC has to be done carefully, we do not discuss the details. In order to incorporate necessary optical structure, each magnet is split into 10 thin lens elements for Simpsons [2]. Tune, lattice functions, and path length as a function of momentum are reasonably reproduced compared with original design.

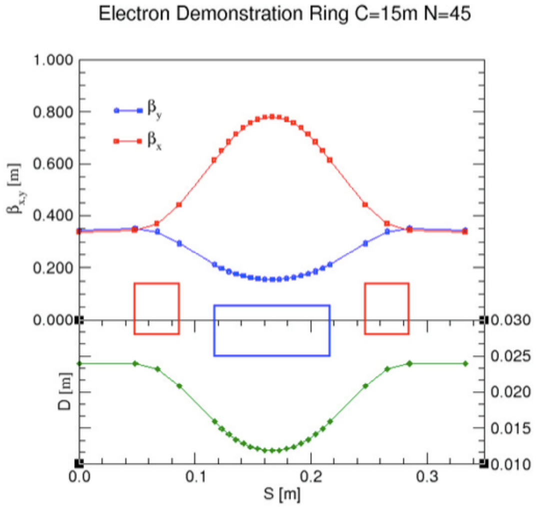


FIGURE 1. Lattice functions of electron FFAG designed by Trbojevic and Courant (from reference [1]).

ACCELERATION WITHOUT AN RF BUCKET

First we look at trajectory in longitudinal phase space. RF frequency is 1.5 GHz and total voltage is 2.52 MV (=0.056 MV x 45 cell). Among particles aligned in phase direction, ones with initial phase of 0 to 150 degree (or 0 to –0.08 m) are accelerated. Momentum spread of those particles becomes, however, quite large after 5 turns.

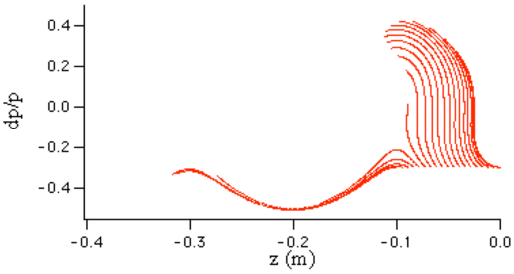
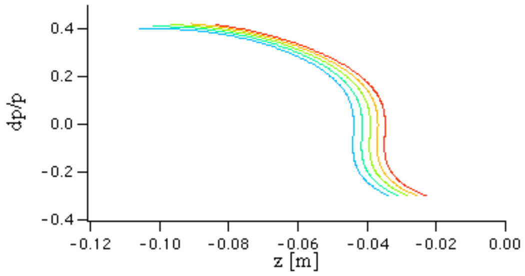


FIGURE 2. Trajectory in longitudinal phase space.

To make momentum spread small, initial phase spread has to be more strictly limited. For example, when the initial phase is between 40 and 60 degrees, final momentum spread is significantly reduced as shown in Fig. 2 (a). Phase and momentum spread as a function of time are also depicted in Fig. 2 (b) and (c), respectively.



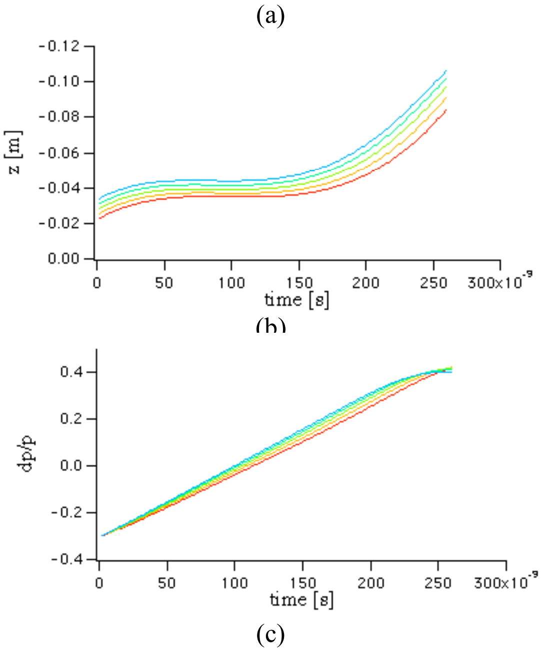


FIGURE 2. Trajectory in longitudinal phase space with initial phase spread of 40 to 60 degrees. (a) longitudinal phase space, (b) phase vs. time, (c) dp/p vs. time.

So far, no transverse amplitude is assumed in the tracking. Now we introduce transverse motion. Figure 3 shows longitudinal motion as a function of horizontal and vertical initial amplitude. A particle with larger transverse amplitude gains less momentum. When the transverse amplitude is 10 mm, the particle cannot be accelerated to the final momentum.

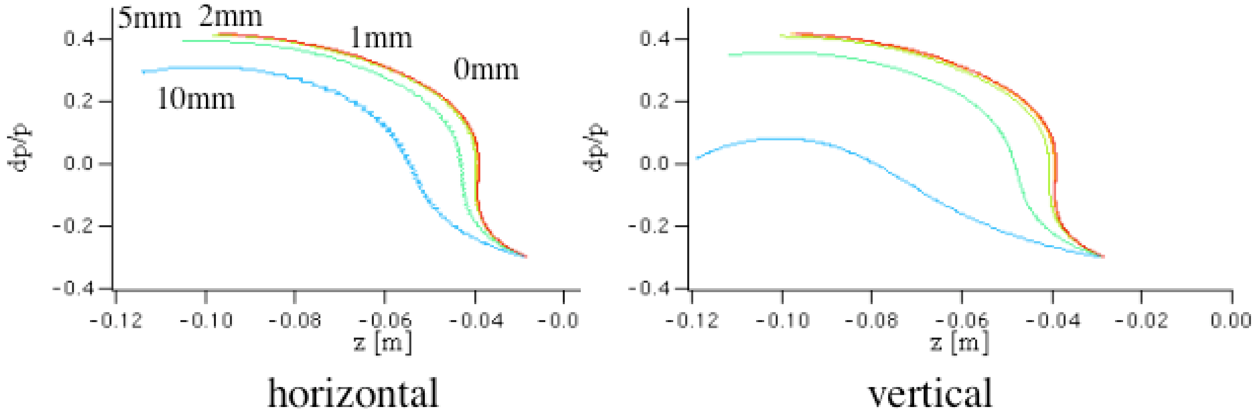


FIGURE 3. Longitudinal motion with finite transverse initial amplitude. When the transverse amplitude is 10 mm, its particle cannot be accelerated to the final momentum.

RESONANCE CROSSING

Integer Resonance Crossing

We observed transverse amplitude growth as a function of alignment errors and crossing speed. As alignment errors, we tested 7 cases. In each case, magnets are randomly misaligned and its maximum amplitude (in both positive and negative) is 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, and 1mm, respectively. Figure 4 shows horizontal and vertical amplitude as a function of time at left and center, respectively, and longitudinal phase space at right in each case. Because of reference orbit shift due to acceleration, the oscillation center of betatron motion in horizontal place increase monotonically. Amplitude growth without alignment errors in vertical place indicates that beta functions are actually increasing as a result of tune decrease.

When alignment errors are more than +-0.05 or +-0.1 mm, amplitude growth, especially in vertical plane, becomes significant.

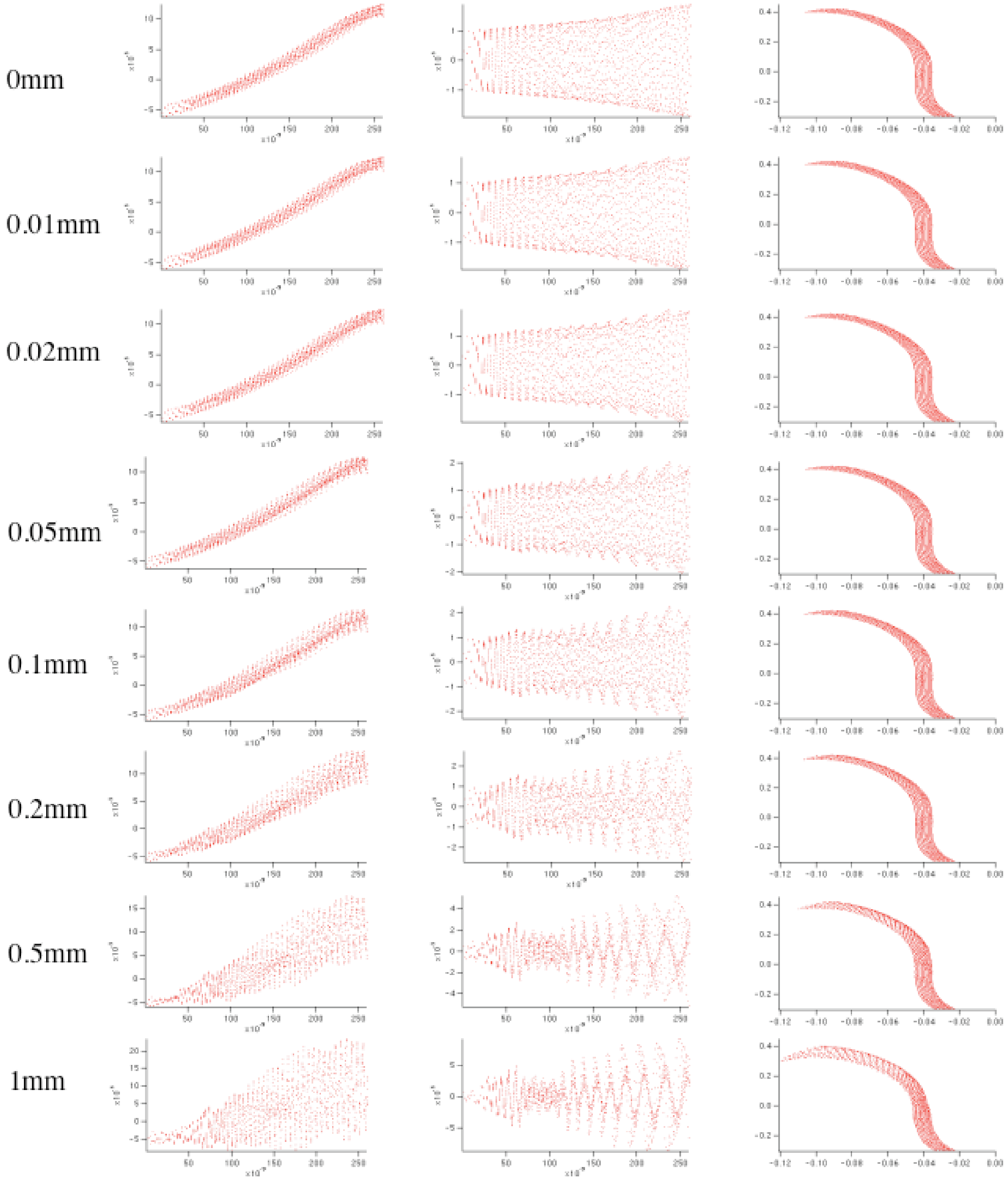
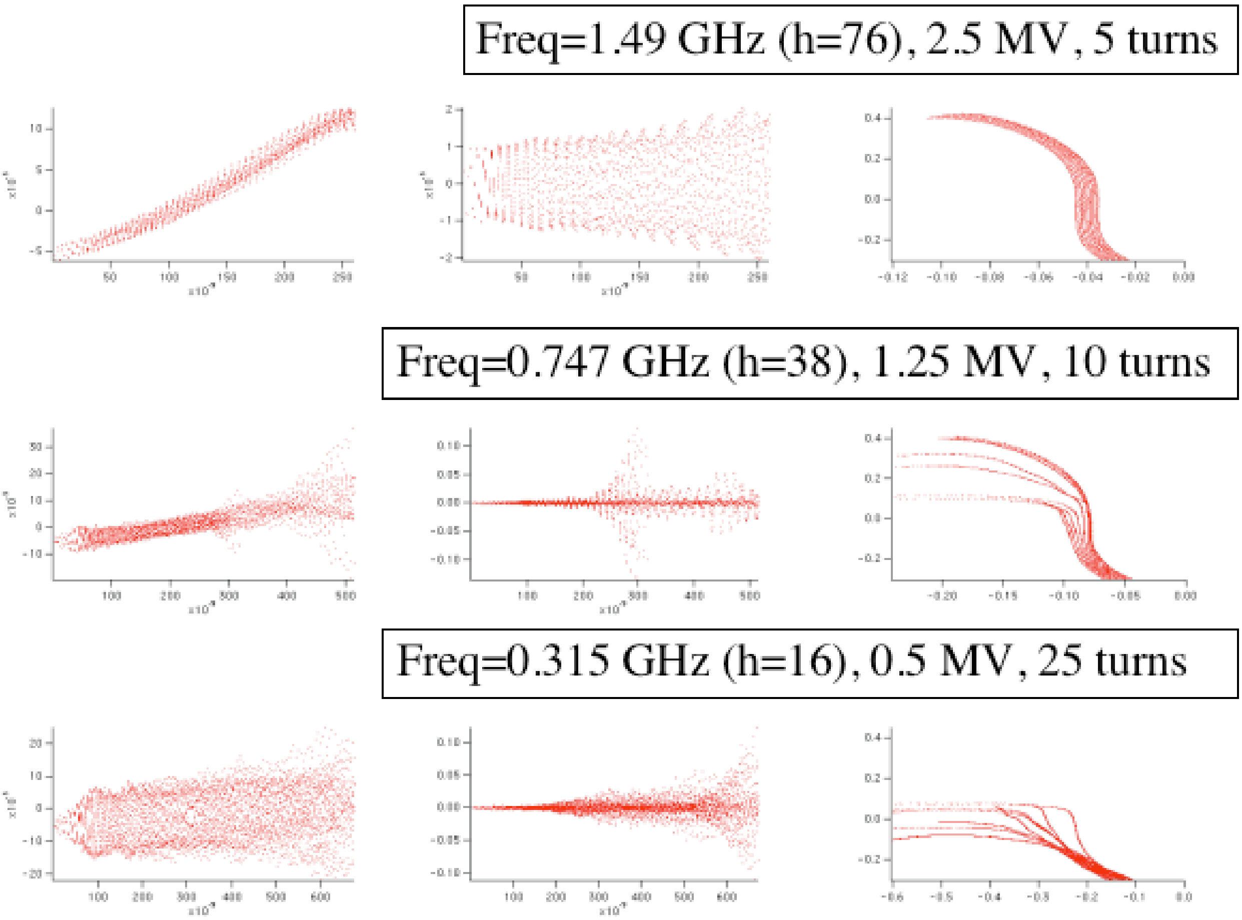


FIGURE 4. Horizontal and vertical amplitude as a function of time at left and center, respectively, and longitudinal phase space at right for each alignment error.

To change crossing speed, we reduce the RF frequency and RF voltage at the same time. For example, if both are halved, time or turn number to complete acceleration is doubled without changing trajectory in longitudinal phase space. In addition to the nominal crossing speed, that is realized with 1.5 GHz or harmonic number of 76 and 2.5 MV, we tested RF parameters of 0.747 GHz or harmonic number of 38 and 1.25 MV, 0.315 GHz or harmonic number of 16 and 0.5MV.

Figure 5 shows that if the crossing speed is 5 times, there is significant amplitude growth even with alignment errors of +-0.05 mm.



Freq=1.49 GHz (h=76), 2.5 MV, 5 turns

Freq=0.747 GHz (h=38), 1.25 MV, 10 turns

Freq=0.315 GHz (h=16), 0.5 MV, 25 turns

FIGURE 6. Amplitude growth in different crossing speed. Alignment errors are fixed at +-0.05 mm.

Half Integer Resonance Crossing

We observed transverse amplitude growth as a function of gradient errors and crossing speed. As gradient errors, we tested 9 cases. In each case, gradient errors are randomly introduced and its maximum amplitude (in both positive and negative) is 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5%, respectively. Figure 6 shows horizontal and vertical amplitude as a function of time at left and center, respectively, and longitudinal phase space at right in each case.

When gradient errors are more than 0.5%, amplitude growth becomes significant.

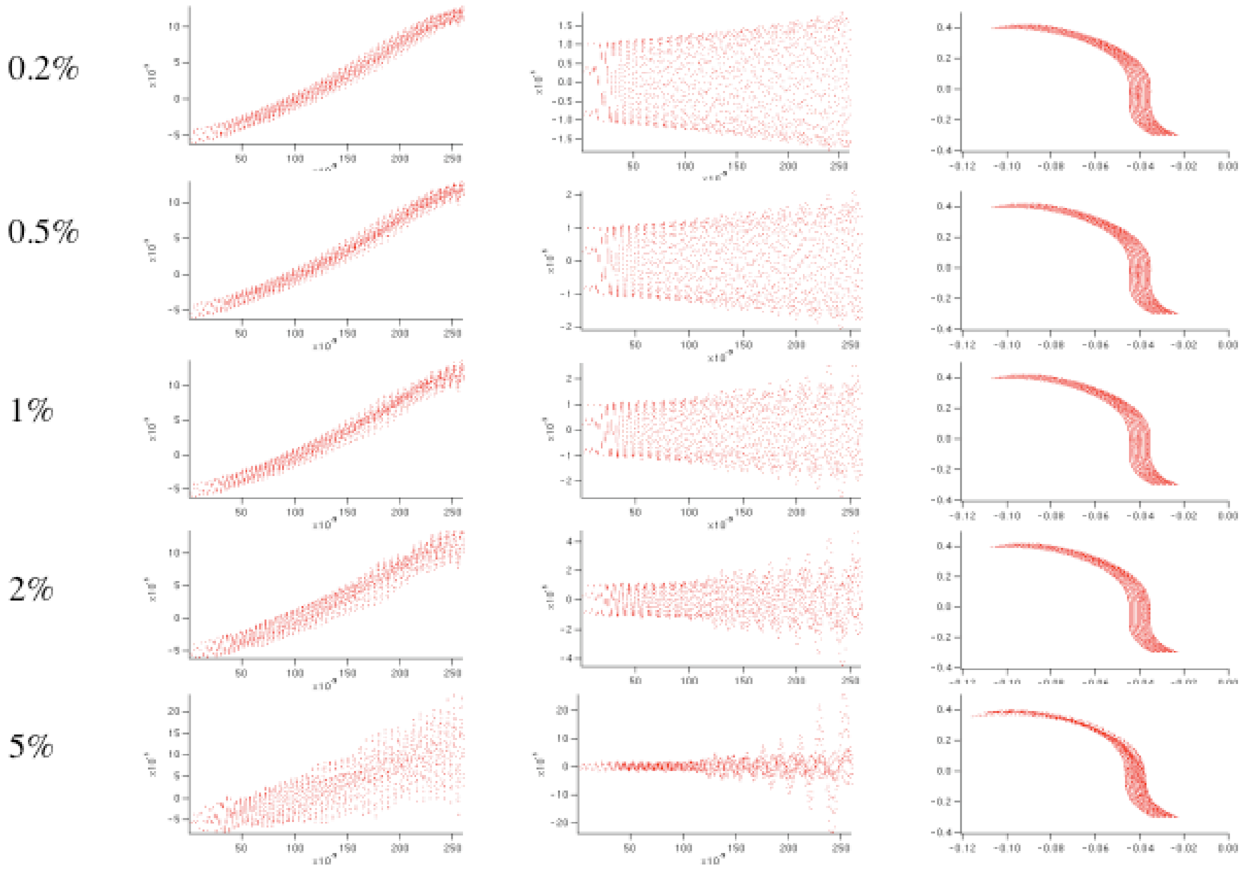


FIGURE 6. Horizontal and vertical amplitude as a function of time at left and center, respectively, and longitudinal phase space at right for each gradient error.

We did the similar exercise on crossing speed for half-integer resonance. The RF frequency and voltage are reduced simultaneously and make the necessary time or turn number more. Figure 7 shows transverse amplitude growth and longitudinal phase space when the crossing speed is 80 times more than nominal value. In that case, alignment errors of 0.05% make noticeable amplitude growth. It is interesting that growth occurs stepwise corresponding to crossing of half-integer resonances.

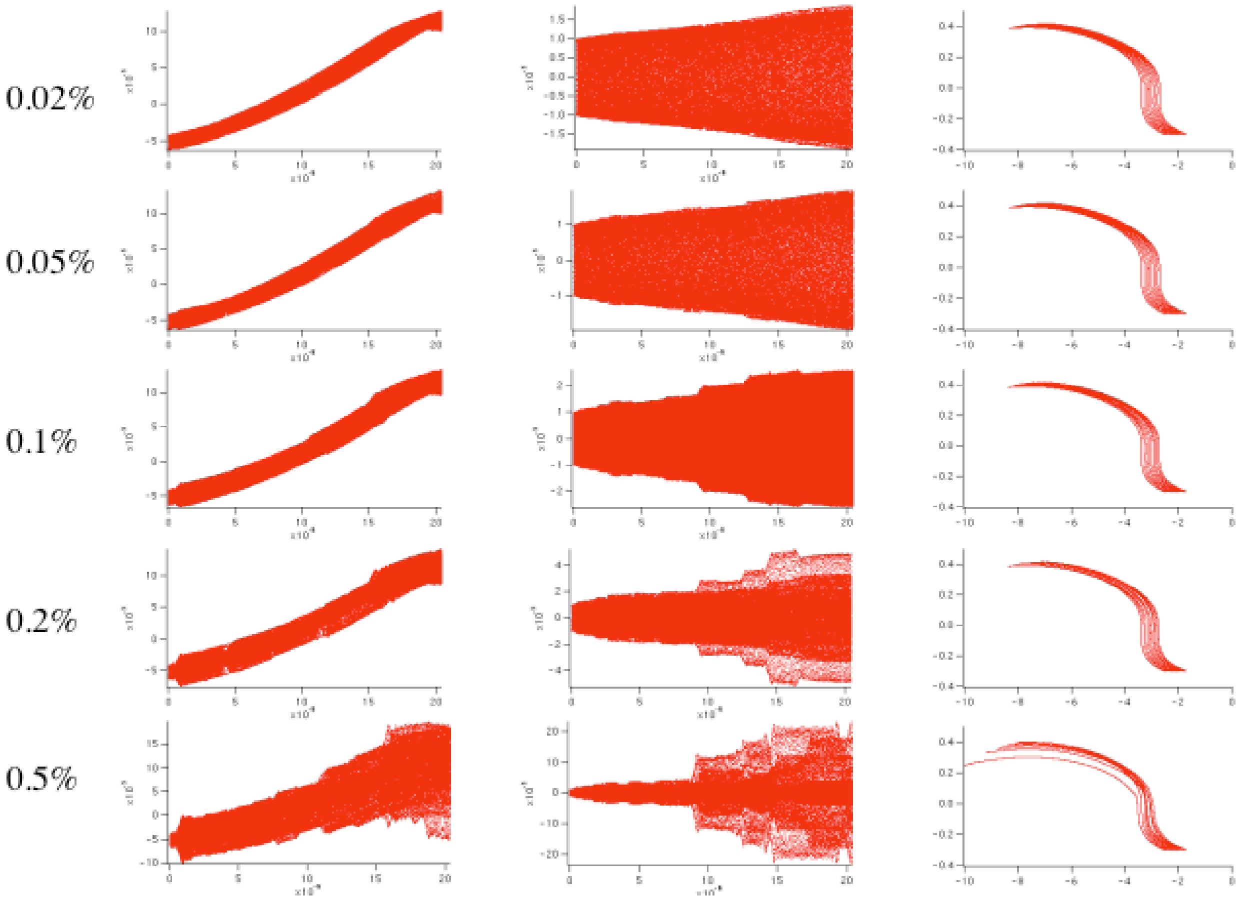


FIGURE 7. Crossing speed is 80 times slower than the nominal parameters. As before, horizontal and vertical amplitude as a function of time at left and center, respectively, and longitudinal phase space at right for each gradient error.

Figure 8 shows threshold of gradient errors vs. crossing speed. The fitted curve is drawn based on scaling law, that is allowable gradient errors is proportional to square root of crossing speed.

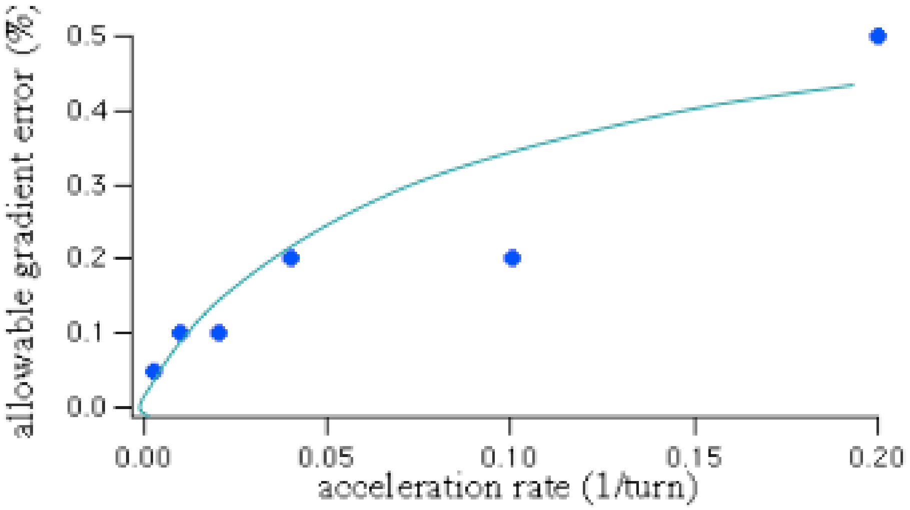


FIGURE 8. Gradient errors vs. crossing speed.

ACKNOWLEDGMENTS

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1. D. Trbojevic, et. al., FFAG04 workshop, http://www.triumf.ca/ffag2004/, 2004.
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