Design Criteria of a Proton FFAG Accelerator*

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Abstract. There are two major issues that are to be confronted in the design of a Fixed-Field Alternating-Gradient (FFAG) accelerator, namely: (i) the stability of motion over the large momentum range needed for the beam acceleration, and (ii) the compactness of the trajectories over the same momentum range to limit the dimensions of the magnets. There are a number of rules that need to be followed to resolve these issues. In particular, the magnet arrangement in the accelerator lattice and the distribution of the bending and focusing fields are to be set properly in accordance with these rules. In this report we describe four of these rules that ought to be applied for the optimum design of a FFAG accelerator, especially in the case of proton beams.

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FEATURES OF THE FFAG ACCELERATORS

The main feature of the FFAG accelerators is that they are essentially based on conventional room-temperature magnet technology with constant field. As the beam is accelerated by RF cavities, its trajectory spirals from an inner orbit where injection occurs toward an outer orbit from which the beam is extracted. The radial extension of trajectories is entirely confined within the magnet aperture, and the field does not need then to be ramped neither for bending nor for focusing.

In principle, this mode of operation requires a large momentum excursion that, for instance from 200 MeV to 1.0 GeV or from 400 MeV to 1.5 GeV is about ±40% around the central momentum value. To avoid that the momentum range gets exceedingly too large, FFAG accelerators require a relative large injection energy, of few hundred MeV. We shall not here been concerned with the nature of the injector, and simply assume that there is one with the proper energy.

Another feature of the FFAG accelerator is the use of magnets with combined function for simultaneous bending and focusing. The field *profile* is not constant but varies across the magnet width and may vary from magnet to magnet. Moreover, the bending and the focusing alternate providing strong focusing and a more compact momentum aperture when compared to cyclotrons. Nevertheless the reverse bending subtracts from the total bending increasing the circumference of the ring.

Different types of magnet lattice configuration have been proposed and investigated, namely: FODO, Doublets, and Triplets [1]. But the Triplets have been found to be the most advantageous, especially in the FDF configuration [2].

Finally, though there is some freedom in choosing the magnet edge configuration and the relative orientation of the magnets with respect to each other, we shall only consider here the case that the magnets are combined-function sector magnets with the exit plane of one parallel to the entrance plane of the next. For a certain reference momentum p_0, to be chosen conveniently, the entrance and exit angles the corresponding reference trajectory makes

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with each magnet vanish. Essentially, the FFAG accelerator is made of a sequence of identical periods of triplets as the one shown in Fig. 1.

EQUATIONS OF MOTION IN THE FFAG ACCELERATOR

After a complete linearization, the equations of motion in a periodic alternating sequence of bending and focusing are

$$x'' + h^2 (1 + n) x / (1 + \delta) = h d / (1 + \delta) \tag{1}$$
$$y'' - h^2 n y / (1 + \delta) = 0 \tag{2}$$

where x and y are respectively the radial and the vertical displacement of any particle with momentum $p = p_0 (1 + \delta)$ from the reference trajectory with momentum p_0. A prime denotes derivative with respect to the path length s measured along the reference trajectory. The curvature of the trajectory is indicated by h (the inverse of the radius of curvature) that is zero in the drift regions joining magnets, and otherwise constant within a magnet, though it can change sign and magnitude from magnet to magnet. Thus in general h = h(s). The focusing is expressed by the field index n = n(s) that also may vary both in sign and magnitude from magnet to magnet. The field index is related to the local field B and gradient dB/dr by hn = dB / Bdr. Since we are assuming a planar motion with no vertical bending, there is dispersion in the horizontal plane represented by the term at the r. h. side of Eq. (1), but not in the vertical plane.

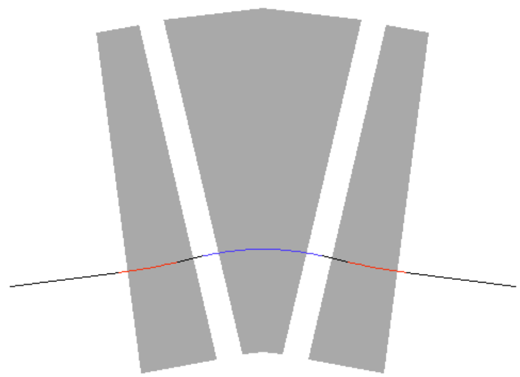

FIGURE 1. The FFAG FDF Period and Injection Orbit

The stability of motion is determined by the phase advance per period that is given by the focusing parameter averaged over one period*

$$K = h^2 n / (1 + \delta) \tag{3}$$

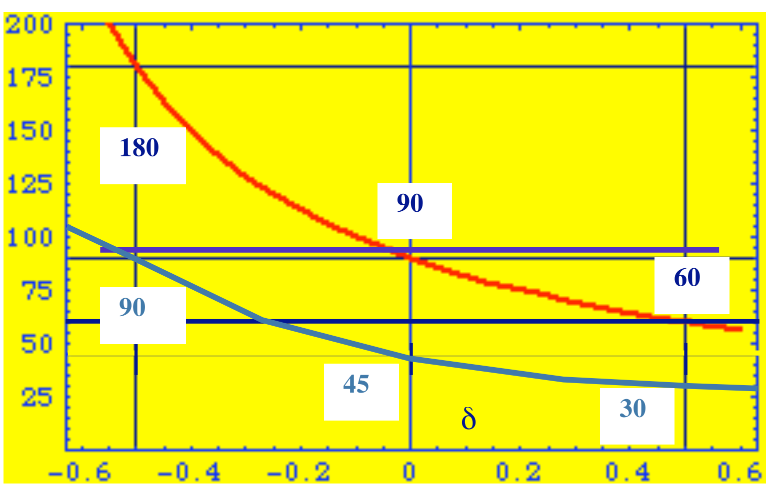

FIGURE 2. Phase Advance across a Period vs. momentum deviation δ

Figure 2 is the plot of the expected phase advance per period versus the momentum deviation δ, assuming that h and n themselves are not functions of δ. We display a large momentum range of ±50% around the central momentum value taken also as the reference momentum. Usually one chooses parameters so that the phase advance per period is close to 90^o at $\delta = 0$. But inspection of Eq. (3) shows that approaching $\delta \sim -0.5$ the phase advance doubles to 180^o and the motion reaches a limit of stability. At the other end $\delta \sim +0.5$ the phase advance is lowered to around 60^o. The motion is stable but in the meantime the lattice functions and betatron tunes have varied significantly. Thus when tuning the accelerator in the middle of the momentum range the largest stable spread that one can expect is ±50%.

* In the horizontal plane there is an extra focusing term due to the curvature of the reference trajectory. It is usually small, and may be relevant only in rings with small circumference and low periodicity.

Rule # 1. A first major, though trivial, suggestion is to tune the FFAG triplet period at the lower momentum value so that the phase advance is about 90-100°. With constant h and n then the phase advance drops to about 30° at the large momentum end, as shown by the lower curve of Fig. 2. The motion is always stable but the lattice functions still vary considerably from one end to the other. The desire would be to find a way to maintain the lattice functions and the betatron tunes constant over the whole momentum range for instance as shown by the continuous line of Fig. 2.

Rule # 2. The second recommendation is to use the FDF arrangement of the triplets [2] since this, as it was well known from the lattice studies of electron storage rings for the production of synchrotron radiation, yields a considerable lower dispersion when compared to the DFD triplet, and thus a more compact spread of trajectories and smaller magnet width. At the same time most of the bending ought to be done by the central D magnet, while the two F magnets at the ends of the triplet provide reverse bending, that is have the polarity inverted with respect to that of the D magnet.

The major concern of any type of FFAG accelerator is the variation of the betatron tunes and functions that can be exceedingly too large for the required momentum range. There are two methods to reduce and to control such large tune variation.

SCALING FFAG LATTICE

One method is the use of the so-called *Scaling FFAG Lattice*. Inspection of Eq.s (1 and 2) shows that the momentum dependence is at the denominator of the focusing and dispersion parameters. It is possible to conceive a geometrical arrangement of trajectories so that the momentum dependence is absorbed by the curvature h. Indeed Eq.s (1 and 2) are a particular form of the equations of motion for a particle with electric charge q and momentum p

$$x'' = q\, B_y\, (1 + hx) / pc \qquad (3)$$

$$y'' = -\, q\, B_x\, (1 + hx) / pc \qquad (4)$$

where again the derivative is with respect to the path length s of the actual trajectory with curvature h. B_x and B_y are respectively the radial and vertical components of the guiding magnetic field. The Lorentz condition imposes that at any location

$$q\, B / pc = 1 / r \qquad (5)$$

where r is the local radius of curvature of the particle with momentum p at the location where the bending field is B_y. With an arbitrary origin corresponding to the curvature $h = 1 / r_0$, we can assume that B_y is a function of r, and that by expansion

$$B_y = B\, [1 + (r - r_0)\, dB / B\, dr] \qquad (6)$$

Then Eq. (3) becomes

$$x'' = q\, B\, (1 + hx) / pc + q\, (dB / dr)\, x / pc \qquad (7)$$

with $x = r - r_0$, and we have neglected a higher order term in x^2. Neglecting also the contribution of the curvature term hx, the focusing parameter is then

$$K = (r\, dB / B\, dr) / r^2 \qquad (8)$$

If one requires K to be independent of location and of particle momentum, then the field index

$$n = -\, r\, dB / B\, dr \qquad (9)$$

is constant across the momentum (radial) aperture. The corresponding field profile is then

$$B \quad = \quad B_0\,(r / r_0)^{-n} \qquad (10)$$

A lattice made of sector magnets with this profile is called a *Scaling Lattice*. The trajectory of any particle with momentum p is an arc of circle with radius r on which the bending field is given according to the Lorentz condition Eq. (5) all along the length of the magnet. An example of trajectories in a triplet arrangement is displayed in Fig. 3, with only half period shown. As one can see the trajectories are truly parallel to each other, with entrance and exit angle always vanishing independently of their momentum value. Of course the same profile is to be adopted in both F and D magnets, though the bending field B, the radius of curvature r_0, and the field index n are different in magnitude and sign. One can easily demonstrate that this configuration works only with a DFD triplet, and usually requires large field strength and larger magnet aperture. But, on the other end, one has the benefit to have eliminated chromatic effects.

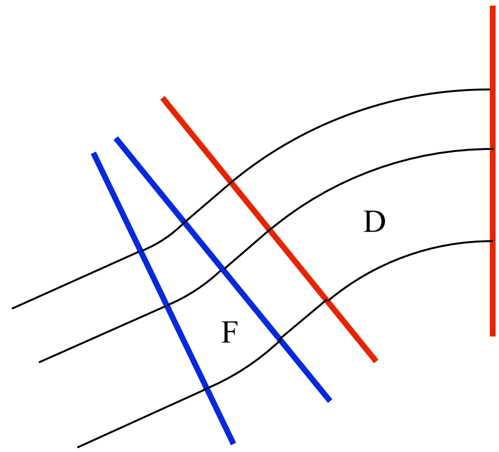

FIGURE 3. Scaling FFAG Lattice

NON-SCALING FFAG LATTICE

A *Non-Scaling FFAG Lattice*, on the other end, is the one by which only for the reference trajectory corresponding to the curvature h and momentum p_0 the Lorentz condition Eq. (5) is exactly satisfied uniformly along the length of a sector magnet. On the reference trajectory there is a uniform constant curvature h and bending field B_y. Moreover, this reference trajectory makes zero angle at the entrance and exit of the magnets. But this is not true for any other particle with a different momentum value $p = p_0\,(1 + \delta)$. For this particle the trajectory is not an arc of circle with constant curvature, but crosses regions with varying bending and focusing field as shown in Fig. 4. In the special case of a constant field gradient G = dB / dr it is then not possible to preserve constant tunes and lattice functions across the momentum aperture.

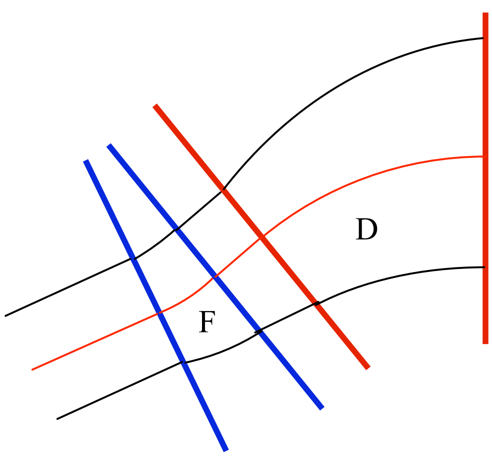

FIGURE 4. Non-Scaling FFAG Lattice

It is nevertheless possible to find an *Adjusted Field Profile* within each of the magnets that cancels the chromatic behavior of the lattice functions over the desired momentum range. The *Adjusted Field Profile* causes the gradient to vary in a specific manner so that the resulting field index at a particular longitudinal location s is now a function of the radial displacement x; that is n = n(x, s) that cancels the momentum dependence at the denominator of the focusing parameter K in Eq. (3). The *Recipe* to derive the *Adjusted Field Profile* requires a careful analysis and an inversion operation [3]. The field so derived satisfies Maxwell's equations correctly. Because now the field configuration varies with the path length s, there is also a solenoid field component B_s that can nevertheless be ignored because it has little consequence on the beam dynamics. On the other end very important are the magnet edge effects [4], because in a *Non-Scaling Lattice* trajectories make non vanishing entrance and exit angles with the magnets, unless $\delta = 0$. The sharp magnet edge effect introduces a tune variation across the momentum aperture that is not compensated by the *Adjusted Field Profile*.

Rule # 3. The search of the *Adjusted Field Profile* is the third recommendation needed for the design of a radial compact FFAF accelerator.

Rule # 4. One more recommendation, the fourth, is to chose a large circumference and a large periodicity as possible compatible with the application in consideration. The physical aperture in the magnets required to accommodate the large momentum range varies about quadratically with the bending per period. A large periodicity not only reduces the beam transverse dimensions for the same emittance, but also reduces considerably the entrance and exit angles trajectories make with the magnets and therefore reduces also the range of the residual betatron tunes not corrected by the *Adjusted Field Profile*.

It was determined empirically (that is numerically) that there is an optimum bending ratio defined as the ratio of the bending angle in the D-magnet to the bending angle of both F-magnets; this ratio is around 2.5. There is also an optimum packing factor, defined as the ratio of the total bending magnet length to the period length; this ratio is around 0.5.

CONCLUSIONS

In summary there are four rules that ought to be applied for an optimum lattice of a FFAG accelerator, namely:

1. Tuning of the lattice at the lowest end of the momentum range; if $p = p_0 (1 + \delta)$, p_0 is the reference momentum taken to correspond to the injection orbit for which $\delta = 0$;
2. Use of a *Non-Scaling* FDF triplet lattice configuration to reduce the amount of global dispersion;
3. Use of the *Adjusted Field Profile* for the cancellation of the chromatic effects;
4. Chose a large circumference and a large periodicity to minimize the betatron tune variation during acceleration caused by the magnet sharp edge effects.

We have applied these rules to the design of (1) a 1.5-GeV FFAG proton accelerator as a new injector to the Alternating-Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) [5], (2) a 1-GeV 10-MWatt Proton Driver in a independent field environment [6], and (3) a 25-MeV proton FFAG accelerator for Medical Applications.

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