Sharp Edge Effects of the Magnets of a FFAG Accelerator *

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Abstract. The paper discuss the issues, the consequences and the methods for controlling the edge effects caused by particles entering and leaving magnets with trajectories at non-vanishing angles with the edges in FFAG accelerators made of *Non-Scaling Lattices*.

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PARTICLE TRAJECTORIES IN A NON-SCALING FFAG LATTICE

Particles circulating in a Fixed-Field Alternating-Gradient (FFAG) accelerator, made of a *Non-Scaling Lattice* composed of a periodic sequence of FDF triplets of sector magnets, have trajectories that are not parallel to each other [1]. With the possible exception of one, the reference trajectory, all particles travel through regions in the magnets with varying field, longitudinally and transversally, and their trajectories are not made of arcs of circles. The Lorentz condition $eBr = pc$ is of course always satisfied, but locally as the particle momentum p is constant, moving in a region of varying field B will cause the path to adjust also by varying the bending radius r locally and accordingly.

In the case the magnets of the FDF triplets are sector magnets with adjacent entrance and exit planes parallel to each other, trajectories are then bound to leave and to enter the edges of the magnets at a substantially non-vanishing angles. Figure 1 gives an illustration of the situation with two extreme trajectories shown. The injection trajectory, taken here also as the reference trajectory, is the only one that is made precisely of arcs of circle, and moreover enters/leaves magnet at zero angle with the edges. As one moves away from the reference orbit, trajectories of particles with larger momentum values distort significantly from pure arcs of circles. Moreover the entrance/exit angles also increase with the particle momentum.

On the other end it is well known that in FFAG accelerators with *Scaling Lattices* all trajectories are essentially parallel to each other and the trajectories enter and leave magnets edges always at zero angle.

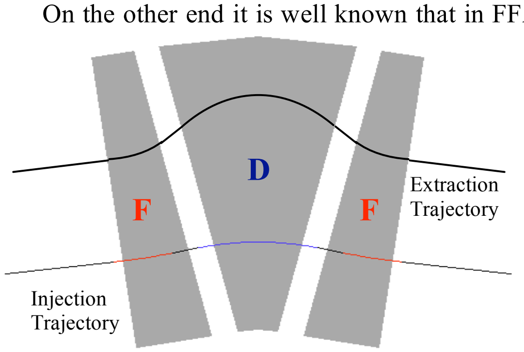

FIGURE 1. The FFAG FDF Period with Inject. (Reference) and Extract. Orbits

EXAMPLE OF THE 1.5-GEV FFAG INJECTOR TO BNL-AGS

A *Non-Scaling Lattice* was recently considered for the design of the 1.5-GeV FFAG accelerator proposed [2, 3] as a new injector to the Brookhaven National Laboratory (BNL) Alternating-Gradient Synchrotron (AGS). The lattice was designed with four rules [1] that were devised to control the chromatic behavior and

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to reduce the magnet physical aperture. In particular, one rule requires the application of the *Adjusted Field Profile* [4] that is highly non-linear along the length and the across the width of the magnets. Figure 2 shows the bundle of trajectories with kinetic energies ranging from 400 MeV (the reference/injection trajectory) and 1.5 GeV across the length of half of the triplet period. It is seen that whereas the injection trajectory is indeed made of arcs of circle (rectified!), the other trajectories have not a circular shape. Moreover, whereas the reference trajectory enters and leaves magnet at zero angle, the other trajectories form angles with the magnet edges that increase with the particle momentum. Leaving and/or entering a magnet at an angle with the edges creates locally a focusing/defocusing effect on the particle motion. If the edge effect is not taken into account the (fractional) betatron tunes are as shown in Fig. 4 across the momentum aperture. Because of the application of the *Adjusted Field Profile* rule, the vertical tune is essentially flat, whereas there is a variation in the horizontal tune due to the residual focusing effect of the trajectory curvature. Nevertheless the picture will change when the edge effects are properly taken into account. The edge angles are given by the space derivative x_{co}' of the closed orbit x_{co} and are displayed in Fig. 3.

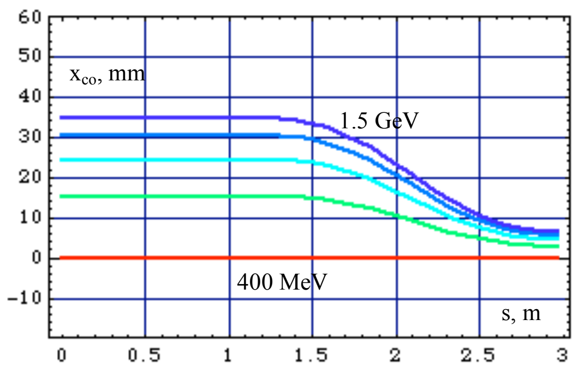

FIGURE 2. Trajectories for different momenta along half the length of a period.

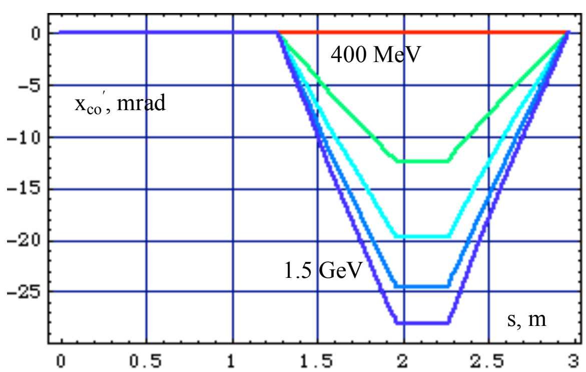

FIGURE 3. Trajectory angles for different momenta along half the length of a period.

ESTIMATE OF THE EDGE EFFECTS

In the sharp-edge approximation the edge effect is represented by the following set of kicks in correspondence of each edge crossing ($\Delta x = \Delta y = 0$)

$$\Delta x' = B_y(x_{co}) (\tan x_{co}') \, x / r \, B_y(0) (1 + \delta) \quad (1)$$

$$\Delta y' = - B_y(x_{co}) (\tan x_{co}') \, y / r \, B_y(0) (1 + \delta) \quad (2)$$

where x_{co}' is the edge angle as derived by inspection of Fig. 3, r is the curvature radius and B the bending field at the edge location of the particle. Both r and B clearly depend on the actual location x_{co}.

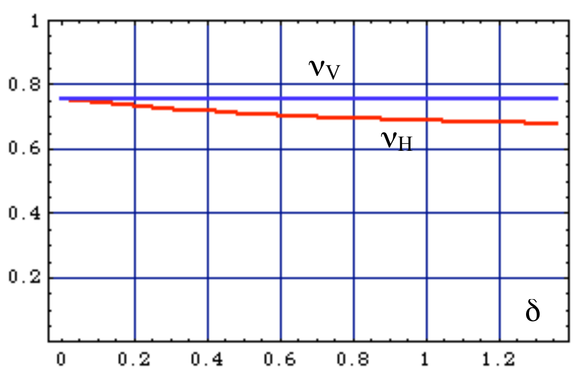

FIGURE 4. Fractional Betaron Tunes versus momentum without Edge Effect.

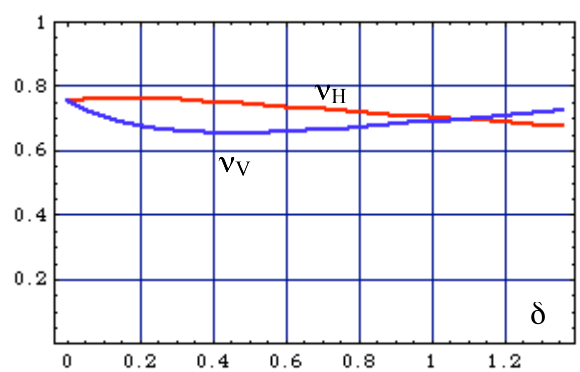

FIGURE 5. Fractional Betaron Tunes versus momentum with Edge Effect included.

In our notation $p = p_0(1 + \delta)$, and p_0 is the reference momentum taken here to correspond to injection ($\delta = 0$). Also, according to the usual convention, a zero edge angle corresponds to the particle trajectory entering or leaving *perpendicularly* with magnet edge in question.

The edge effect is introduced with an iteration procedure that is repeated until a satisfactory converging solution is found. The result is shown in Fig. 5 that shows the new betatron tunes across the momentum range of acceleration. Usually only few iterations are needed to achieve convergence. It is seen that the betatron tune plots are significantly distorted in both planes. Fortunately, in this case, the variation across the momentum aperture is no more than 0.1, and thus acceptable.

CONCLUSIONS

To control the edge effect in a FFAG accelerator with Non-Scaling Lattice is important to minimize the entrance and exit angles x_{co}'. In turn this is achieved with reducing the dispersion, that is the off-momentum closed orbit x_{co}. Since the dispersion increases about linearly with the square of the bending per period (a triplet), this can be achieved with a ring circumference as large as possible and a large periodicity. That in particular constitutes the 4th rule [1] devised for the design of a FFAG lattice.

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