Recent Results from Optimization Studies of Linear Non-Scaling FFAGs for Muon Acceleration

J. Scott Berg

FFAG04 Workshop, KEK

14 October 2004

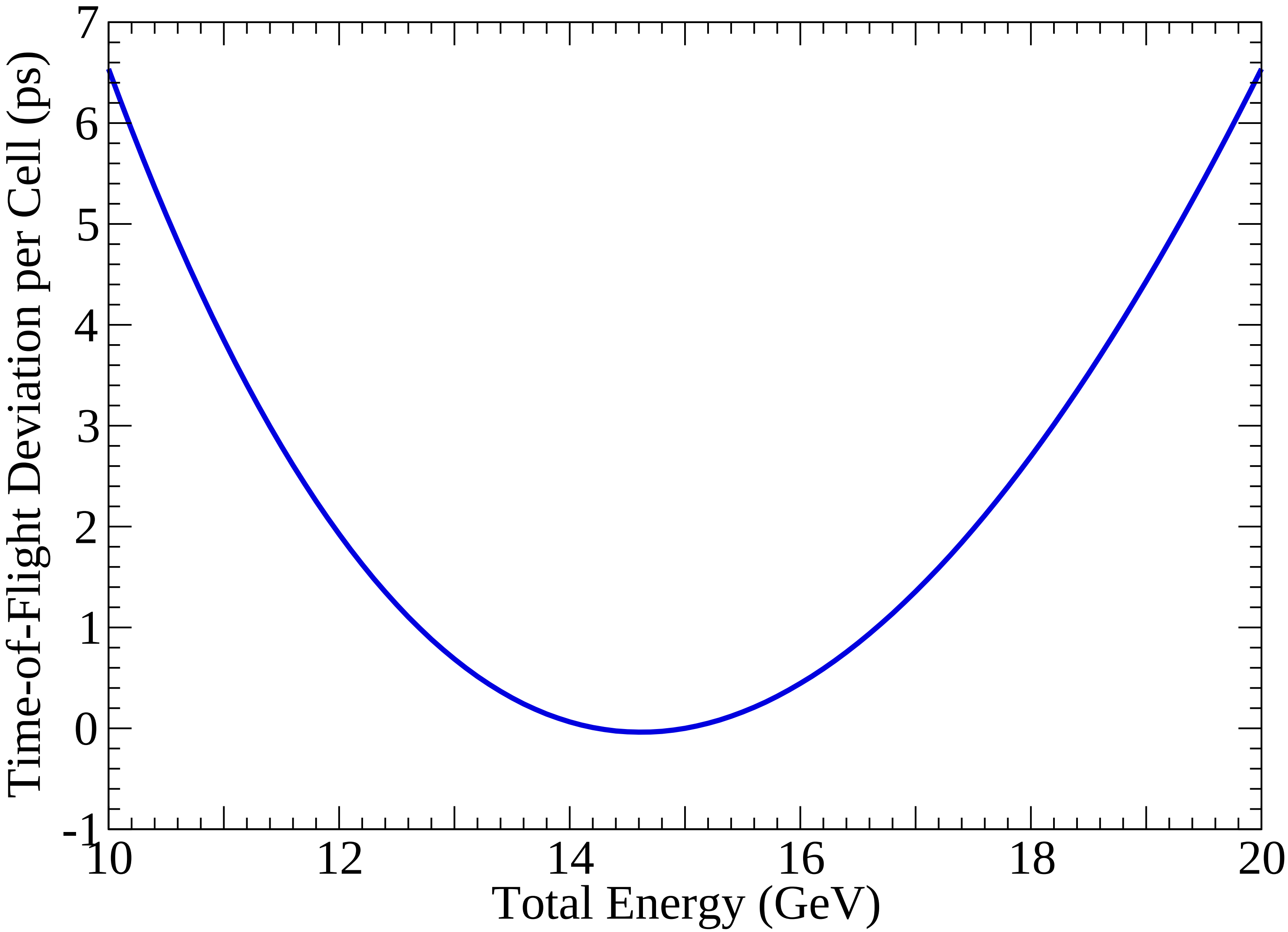
Outline

- Review of optimization process
- Review of previous results
- Updated Cost Model
- Characteristics of optimal lattices
- Minimum cost rings
- Decay cost
- Parametric dependencies of lattices
- New lattices
- Remaining work
- Conclusions

Review of Optimization Process

- Muon FFAG lattices consist of several identical cells of a particular type (doublet, FDF triplet, FODO)
- Assume 201.25 MHz RF
- A drift of at least 2 m is specified for the RF cavity
  - Purpose: keep field on superconducting cavities below 0.1 T
- Leave 0.5 m of space between magnets in doublet/triplet
- Time-of-flight vs. energy is parabolic-like; set height of parabola at min and max energy to be same
- For longitudinal acceptance, constrain $w = V/(\omega \Delta T \Delta E)$
  - ΔT is height of parabola (one turn), V is total voltage installed
  - Value of w depends on energy range, empirically chosen, increases with decreasing energy
- Factor of 2 in energy: 2.5–5 GeV, 5–10 GeV, 10–20 GeV

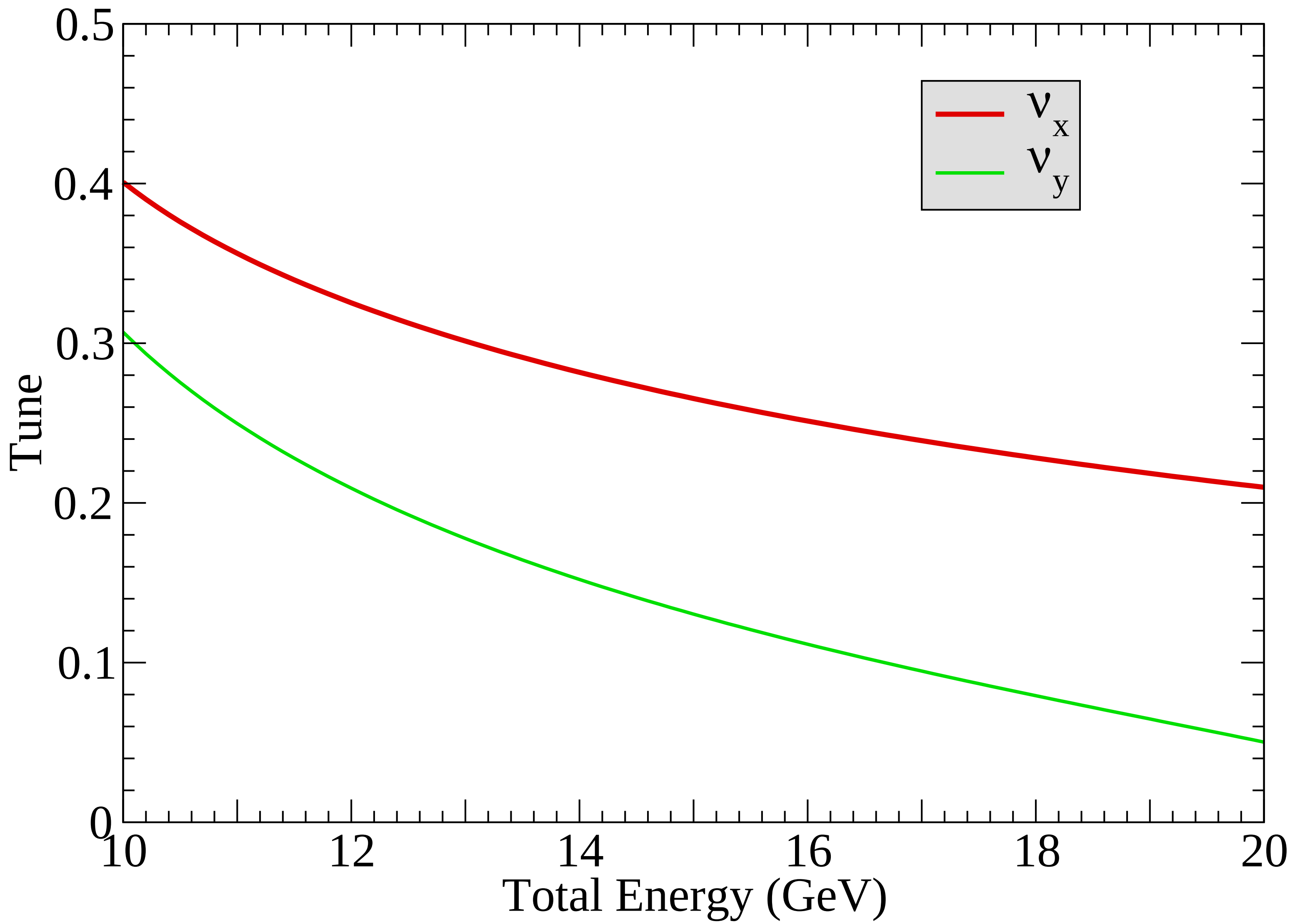
Time-of-Flight vs. Energy



Review of Previous Results of Optimization

- Doublet lattice is most cost effective
  - Triplet lattice has lowest voltage requirement, but
  - Three magnets per cell drives up magnet cost
  - Difference FD $\longrightarrow$ FDF $\longrightarrow$ FODO is around 5% each
- Tunes for optimal lattice are well split over the entire energy range
  - Horizontal tune is higher
- Cost per GeV of acceleration increases rapidly as energy decreases
  - 2.5–5 GeV of questionable cost value for muon acceleration

Tune vs. Energy



Updated Cost Model (Palmer)

- Compared to previous model
  - Cost at zero field for fixed magnet size does not go to zero
  - A new symmetry factor (quad/dipole/combined function) is used
    - Proportional to amount of coil needed
    - Factor is identical for dipoles and quadrupoles
    - Factor is less than 1 for combined function
- Basic formula: product of 4 factors

$$f_B(\hat{B})f_G(\hat{R}, L)f_S(B_-/B_+)f_N(n)$$

  - f_B: dependence on field
  - f_G: geometric dependence: magnet length L
  - f_S: symmetry dependence
  - f_n: dependence on number of magnets being made n

Updated Cost Model (cont.)

- For linear midplane field profile $B_y = B_0 + B_1 x$,

$$B_\pm = |B_0| \pm |B_1|\, k_R R$$

- Peak field and larger radius it requires

$$\hat{B} = B_+ + |B_1|\, k_C B_+ \qquad \hat{R} = k_R R + k_M \hat{B}$$

- The factors

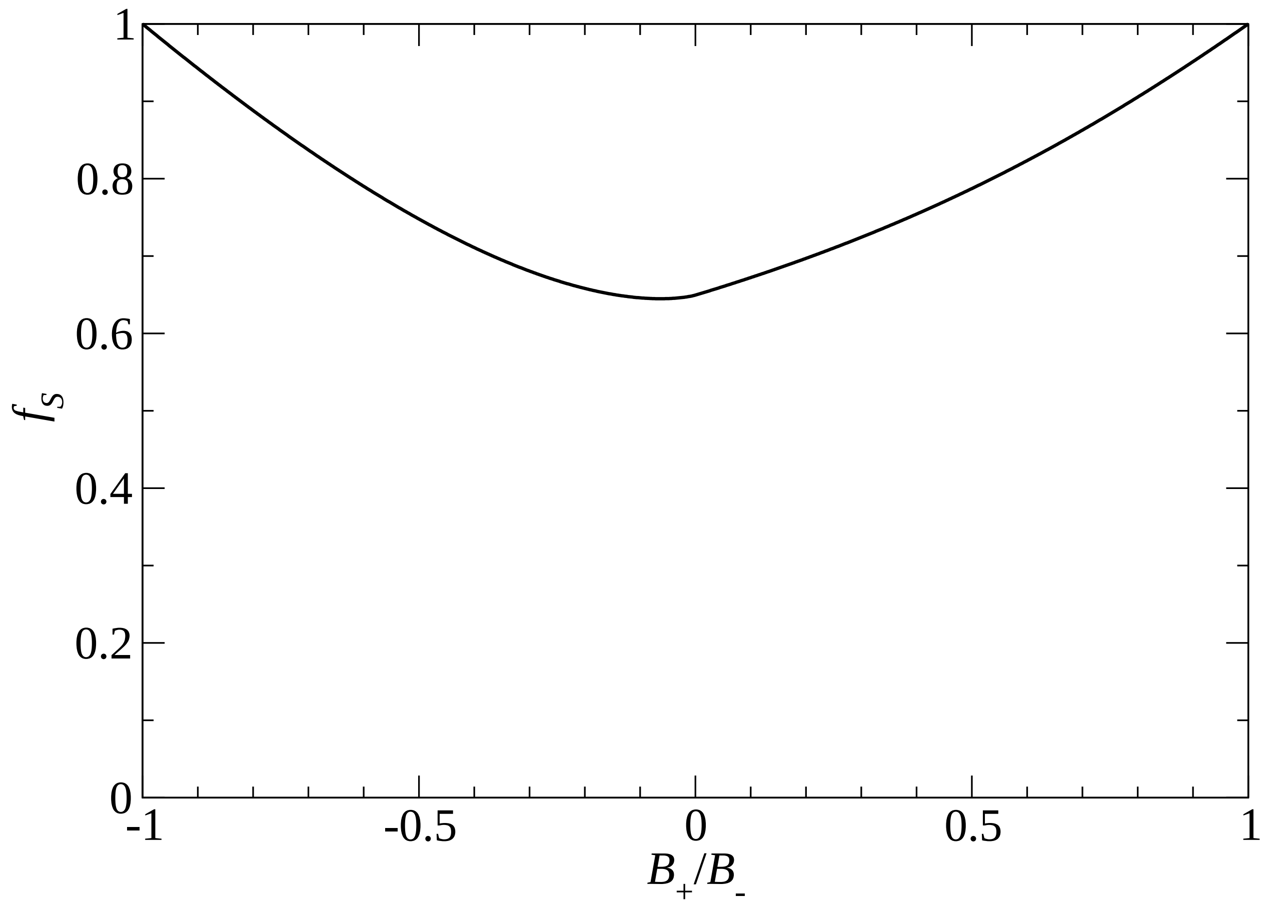
$$f_B(\hat{B}) = C_0 + C_1 \hat{B}^{k_B} \qquad f_G(\hat{R}, L) = \hat{R}(L + k_G \hat{R})$$

$$D = (1 + B_-/B_+)/2 \qquad Q = (1 - B_-/B_+)/2 = 1 - D$$

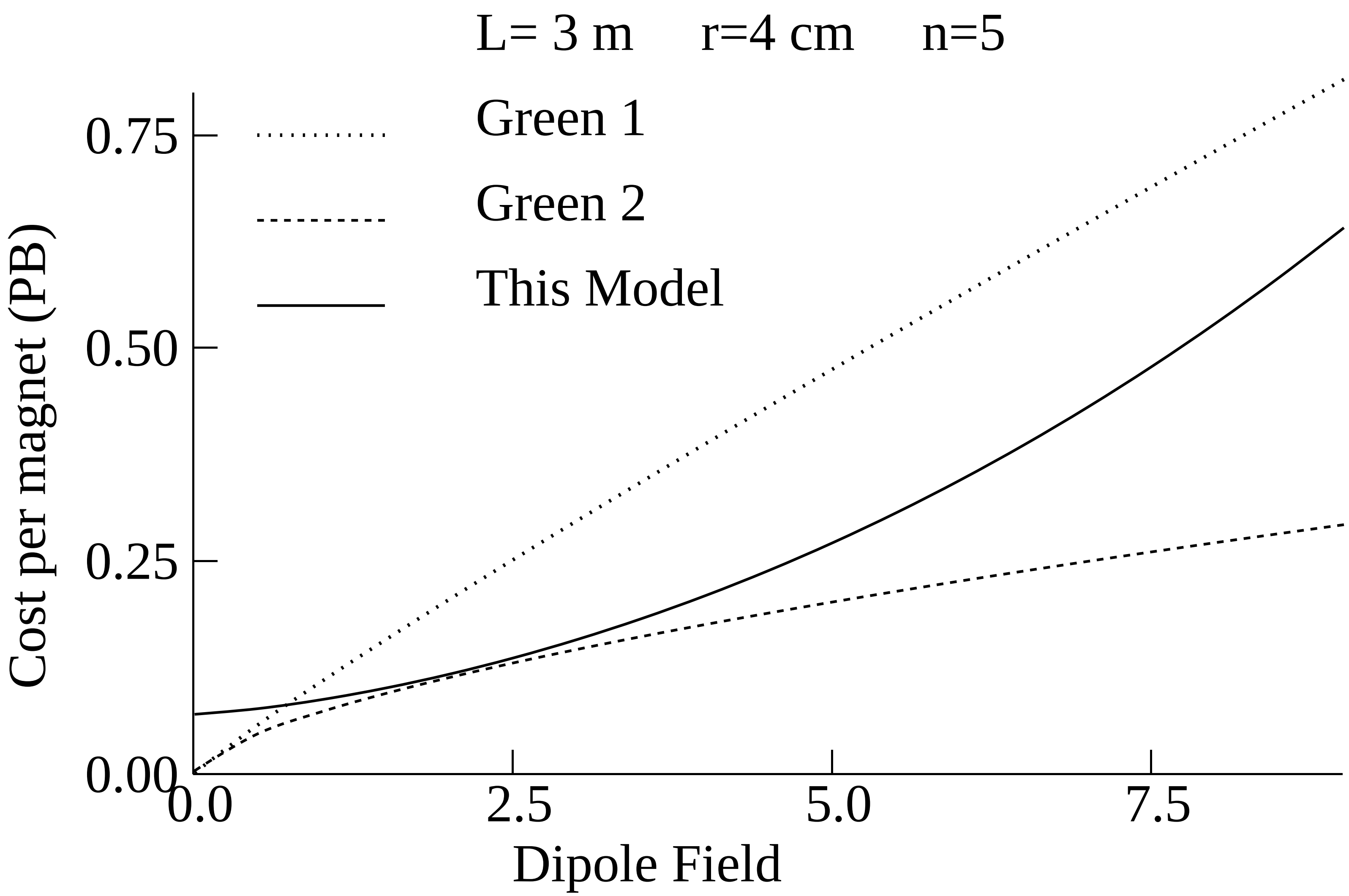
$$f_S(B_-/B_+) = \frac{\int_0^\pi |D\cos\theta + Q\cos 2\theta|\, d\theta}{\int_0^\pi |\cos\theta|\, d\theta} \qquad f_N(n) = (n_0/n)^{k_N}$$

Updated Cost Model (cont.)

k_R	1.3
k_C	2.47 mm/T
k_M	2 mm/T
C_0	0.101 PB/m^2
C_1	16.78 mPB/T$^{1.5}$/m^2
k_B	1.5
k_G	36
k_N	1/3
n_0	300



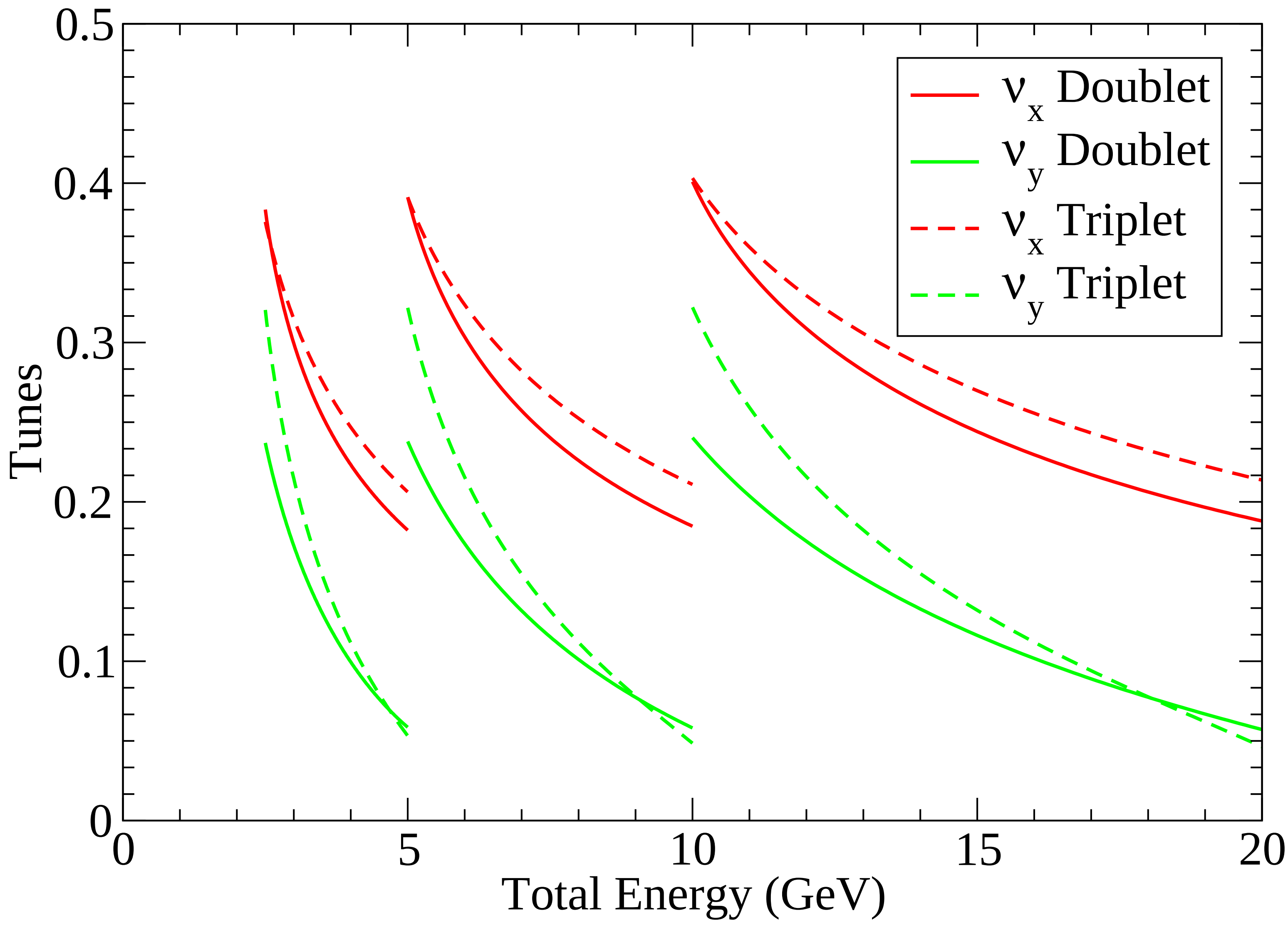
Updated Cost Model (cont.)



Characteristics of Optimal Lattices

- Tune profile depends only on lattice type, factor of energy gain
  - In particular, independent of magnitude of energy
  - This is caused by trying to optimally fit the beam inside the pipe
    - Vertically: low and high energy should have same height
    - Horizontally: same idea, but more complex tradeoff (low and high energy beam sizes, closed orbit swing, time-of-flight)
    - Time-of-flight reduction likely favors higher horizontal tune
- For modest lengths, lattice (magnet+linear) cost decreases with increasing circumference
  - Reduced dispersion reduces aperture requirement
  - Remarkably, this cost reduction is goes down more quickly than inversely in the number of cells
  - At some point, this stops as the nonzero transverse beam size stops the decrease in the aperture
  - The minimum-cost solution does not have every cell filled with RF!

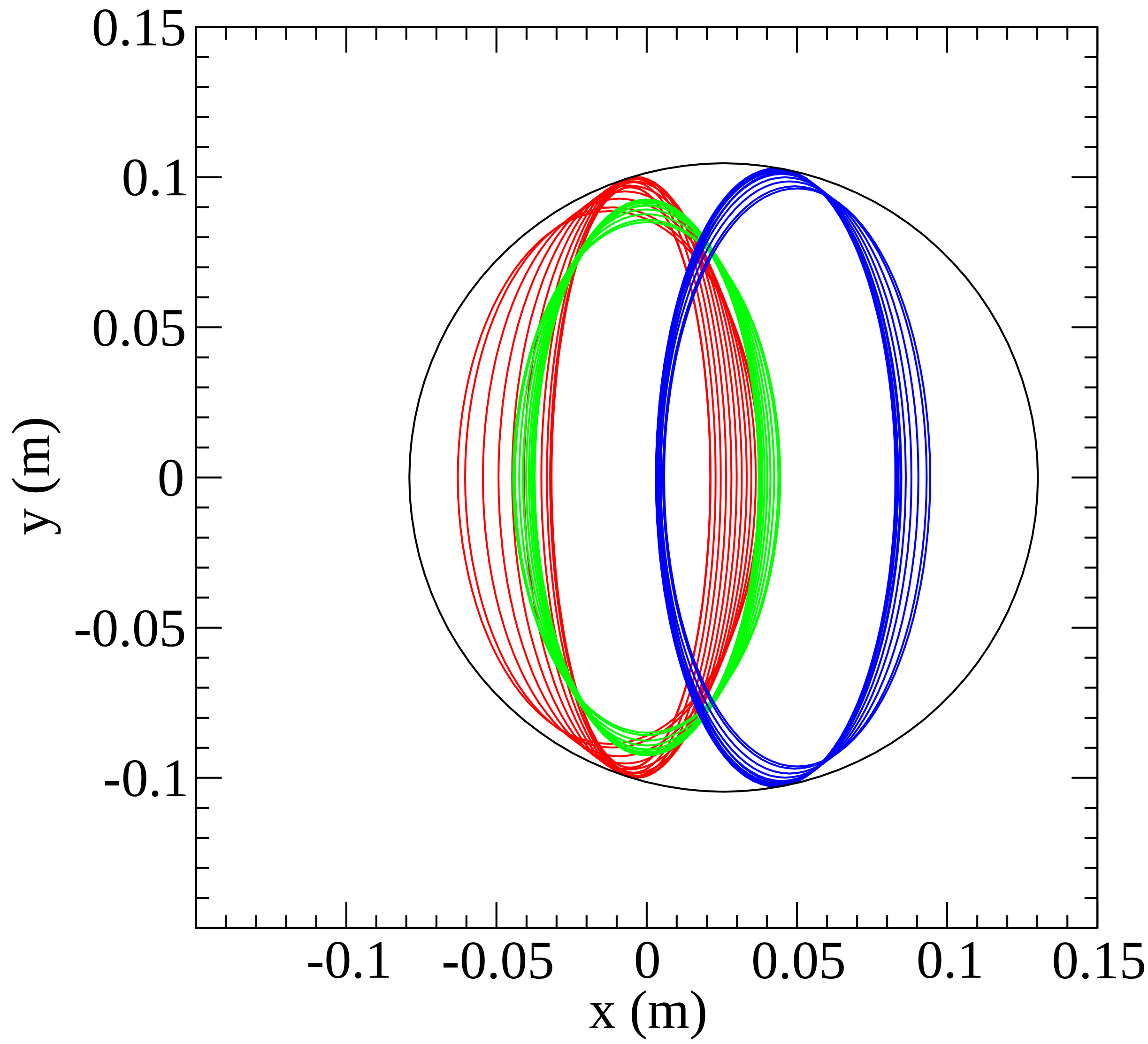
Tune Profiles for Different Energies



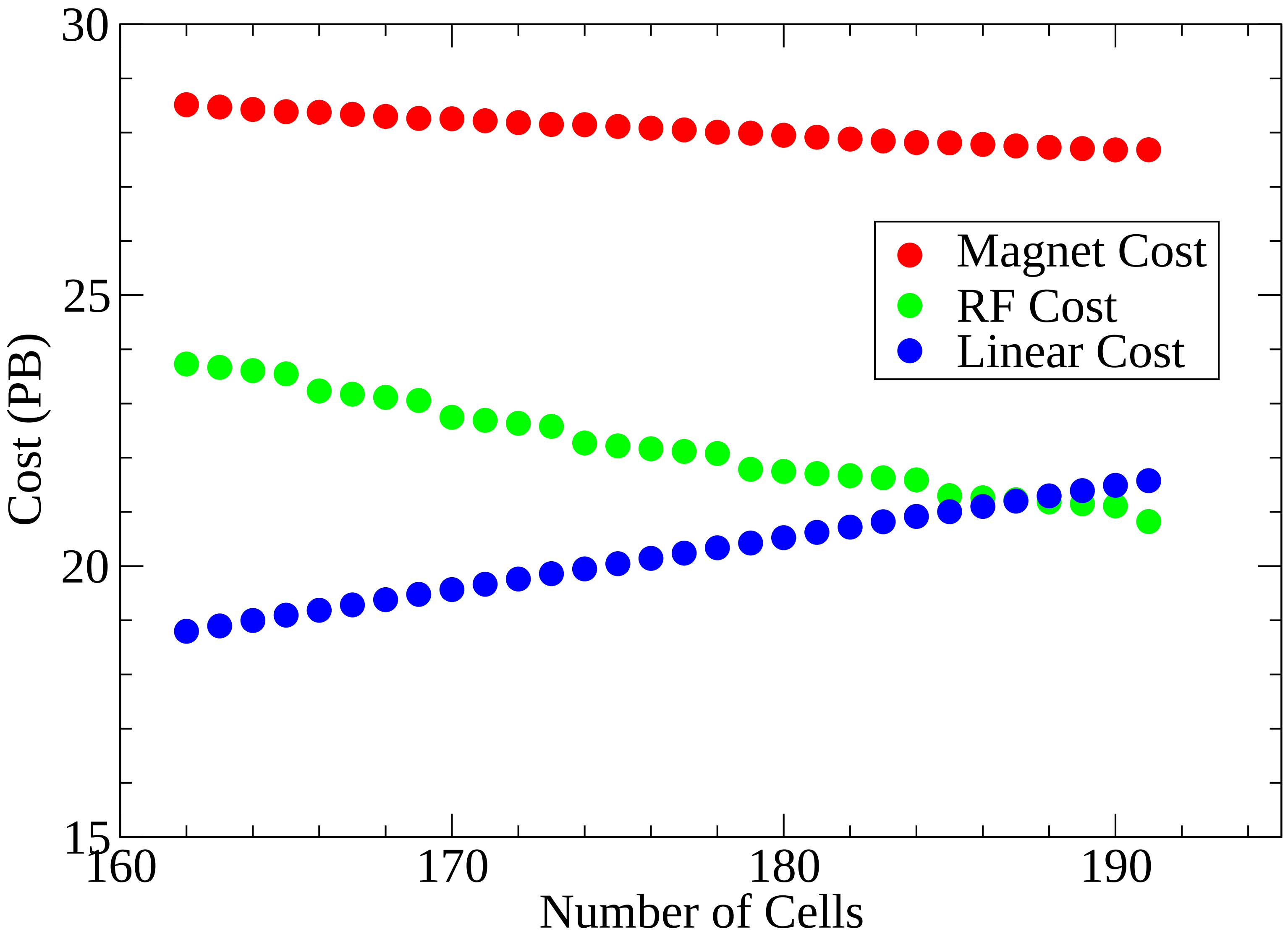
BROOKHAVEN NATIONAL LABORATORY

Beam Ellipses in D Quad

Muon Collaboration



Costs vs. Number of Cells



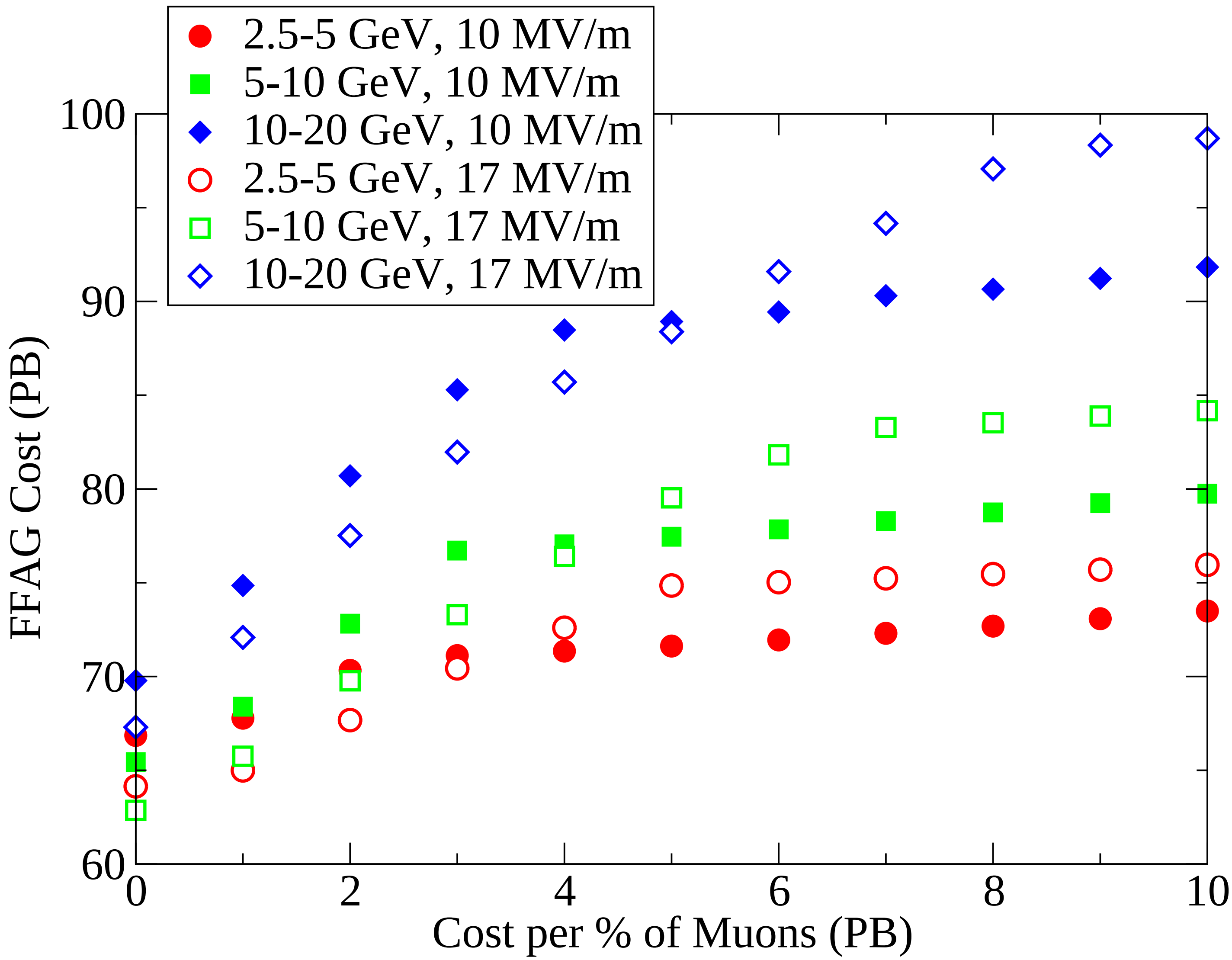
Decay Cost

- The minimum cost rings are extremely long
  - Decays are unacceptably high
- Need to incorporate tradeoff between decays and cost of acceleration into optimization
  - Simplest thinking: can always make detector larger to make up for lost particles
  - Multiply detector cost by fractional loss
  - Over-simplifies things (e.g., as detector gets larger, fractional increase costs more)
  - Baseline: detector costs 500 PB

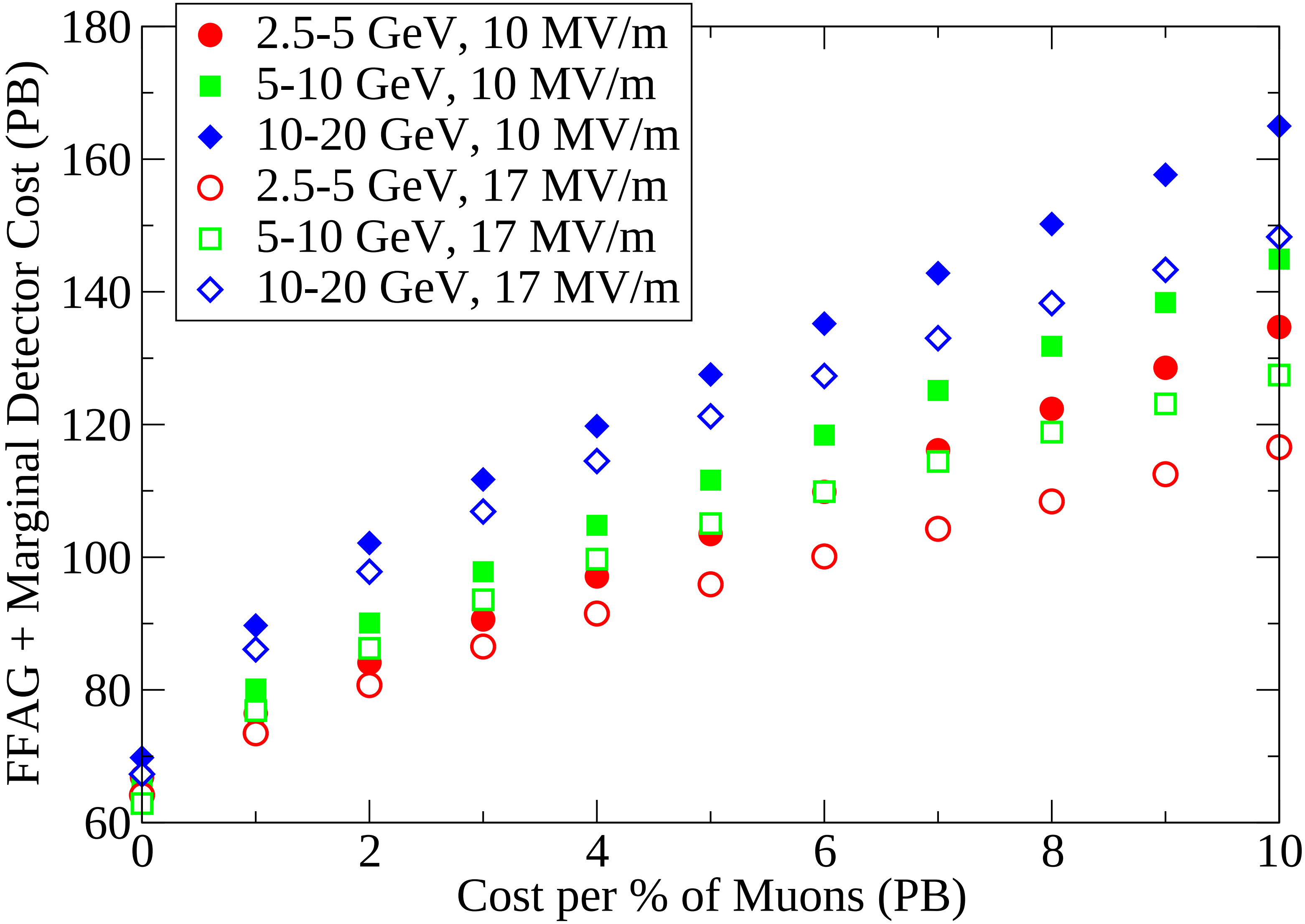
Parametric Dependencies

- Cost vs. decay cost
  - For low decay cost, ring is partially filled
  - As decay cost increases, ring optimized to reduce decay
    - More RF
    - Ring shortens
  - Once ring is filled, can't increase RF or shorten ring easily
    - Ring shortens slightly: magnets shorter, higher field
    - To get little gain, large increase in cost
    - Detector cost increases more rapidly at this point
  - Higher gradient, can go longer before ring is filled
  - Total cost steadily increases with increasing decay cost

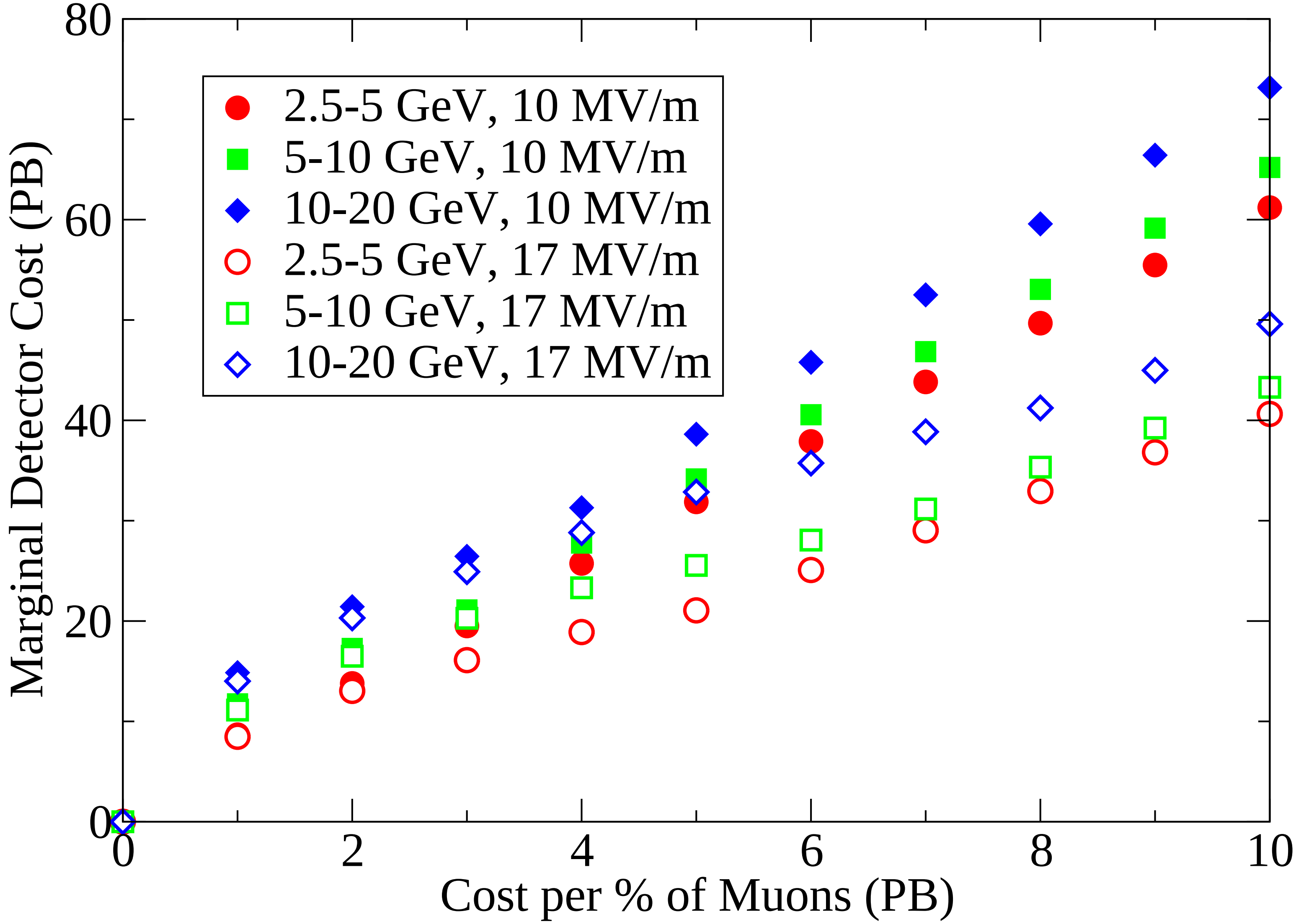
FFAG Cost vs. Decay Cost



Total Cost vs. Decay Cost



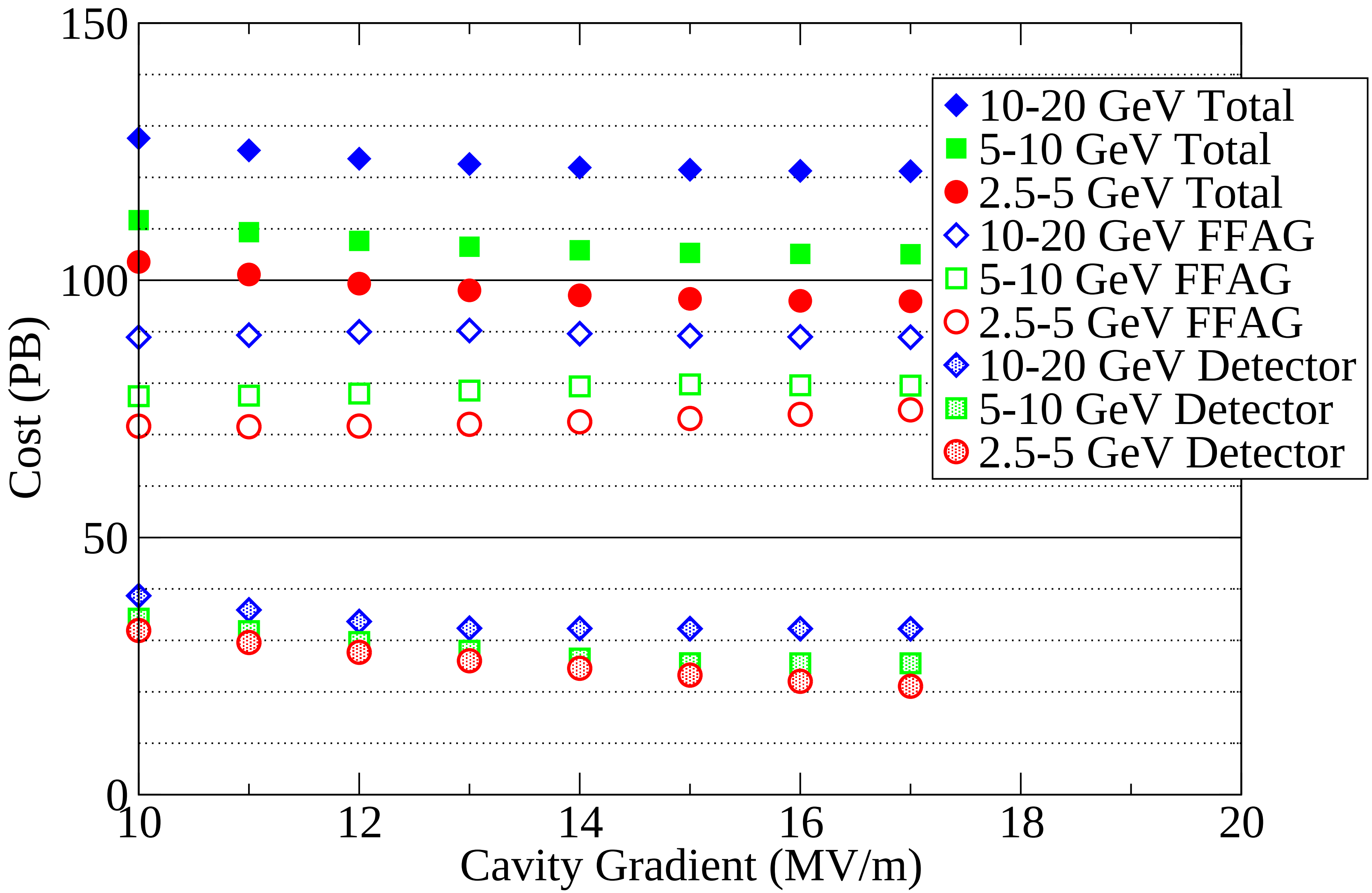
Marginal Detector Cost vs. Decay Cost



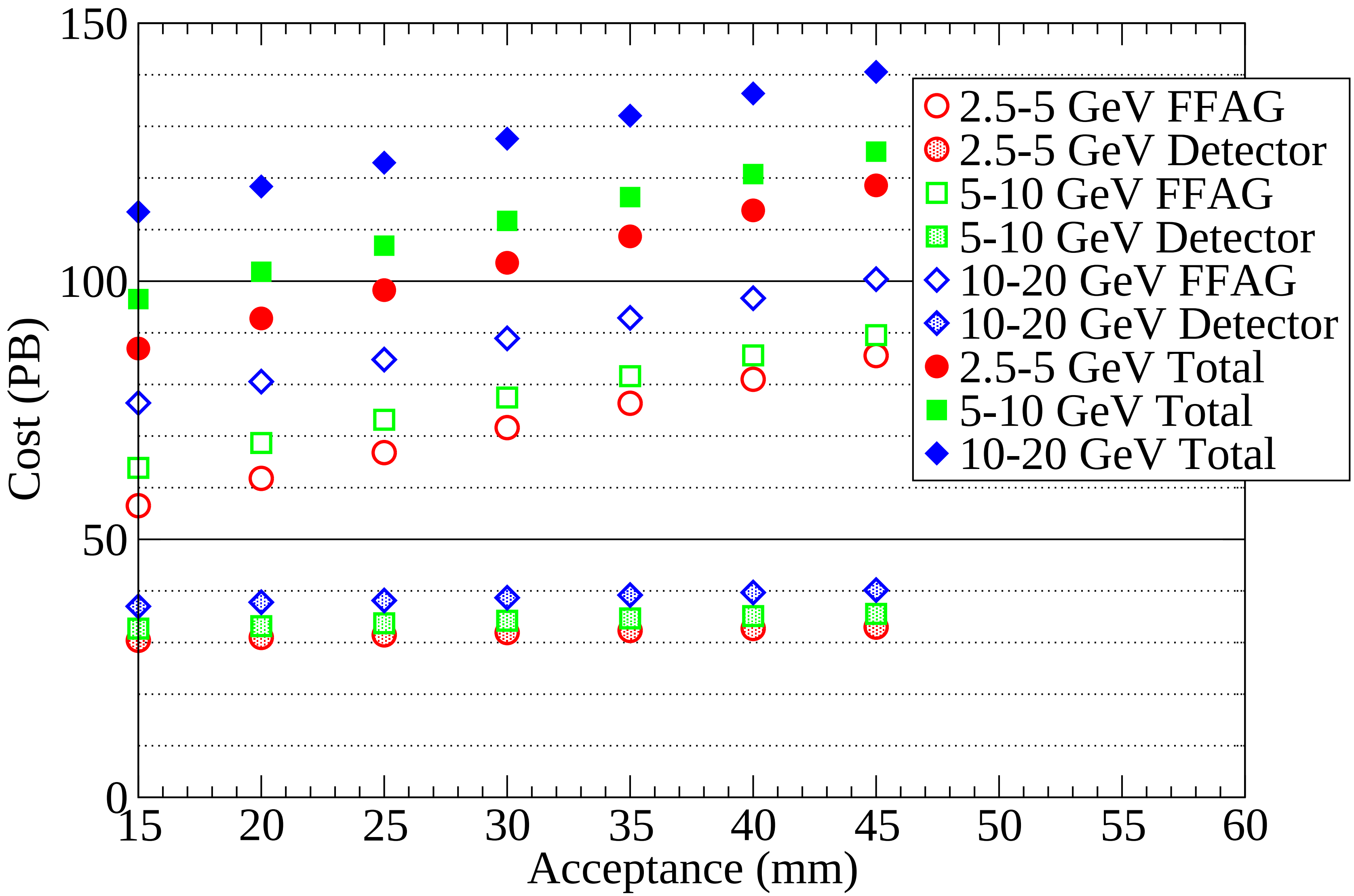
Parametric Dependencies (cont.)

- Cost vs. Gradient
  - Use 5 PB/% for the muon cost
  - Relatively weak dependency
  - FFAG cost increases with increasing gradient for low gradients
    - Total cost decreases since detector cost decreases
    - Ring is filled
      - Total voltage increases faster than cost per voltage
      - Ring circumference decreases, increasing ring cost
  - Higher gradients, can partially fill ring
    - Roughly same voltage and circumference
    - Fewer cavities
- Cost vs. Acceptance
  - Strong dependence of cost on acceptance
  - 10 MV/m: ring filled at these parameters (independent of acceptance)

Cost vs. Gradient



Cost vs. Acceptance



Another Mind-Numbing Lattice Table

Minimum total energy (GeV)	2.5	5	10
Maximum total energy (GeV)	5	10	20
$V/(\omega\Delta T\Delta E)$	1/6	1/8	1/12
No. of cells	64	77	91
D length (cm)	54	69	91
D radius (cm)	13.0	9.7	7.3
D pole tip field (T)	4.4	5.6	6.9
F length (cm)	80	99	127
F radius (cm)	18.3	14.5	12.1
F pole tip field (T)	2.8	3.6	4.4
No. of cavities	56	69	83
RF voltage (MV)	419	516	621
Turns	6.0	9.9	17.0
Circumference (m)	246	322	426
Decay (%)	6.4	6.8	7.7
Magnet cost (PB)	38.4	36.0	38.1
RF cost (PB)	27.1	33.4	40.2
Linear cost (PB)	6.1	8.0	10.6
Total cost (PB)	71.6	77.5	88.9
Cost per GeV (PB/GeV)	28.7	15.5	8.9

- Decay cost: 5 PB/%
- Acceptance 30 mm
- Choose 10 MV/m: already achieved, cost savings of higher maybe not realized
- Pole tip fields are higher than previously
  - Shortened magnets to improve decay
- 2.5–5 GeV is borderline

Remaining Work for Optimization

- Choice of $V/(\omega \Delta T \Delta E)$ still empirical
- Work on choice of cavity drift length and inter-magnet drift
  - Let it depend on the magnet fields/apertures? How?
- Choice of aperture: should be coupled to cooling design
  - Can compute cooling cost vs. aperture when muon cost is included
  - Cooling cost decreases with increasing aperture
  - Add cooling cost and acceleration cost vs. aperture
  - Presumably there is an optimum aperture

Conclusion

- I am using an improved cost model from Palmer
- We have a better understanding of what optimal lattices will look like
- An earlier notion that magnet costs increase with increasing number of cells was wrong. This has been addressed by including decay costs in the model.
- I have a set of lattices which are optimal to my current understanding
- I can produce "optimal" lattices at will for given constraints
- There are always improvements to be made…