Acceptance and Injection/Extraction Studies of PRISM

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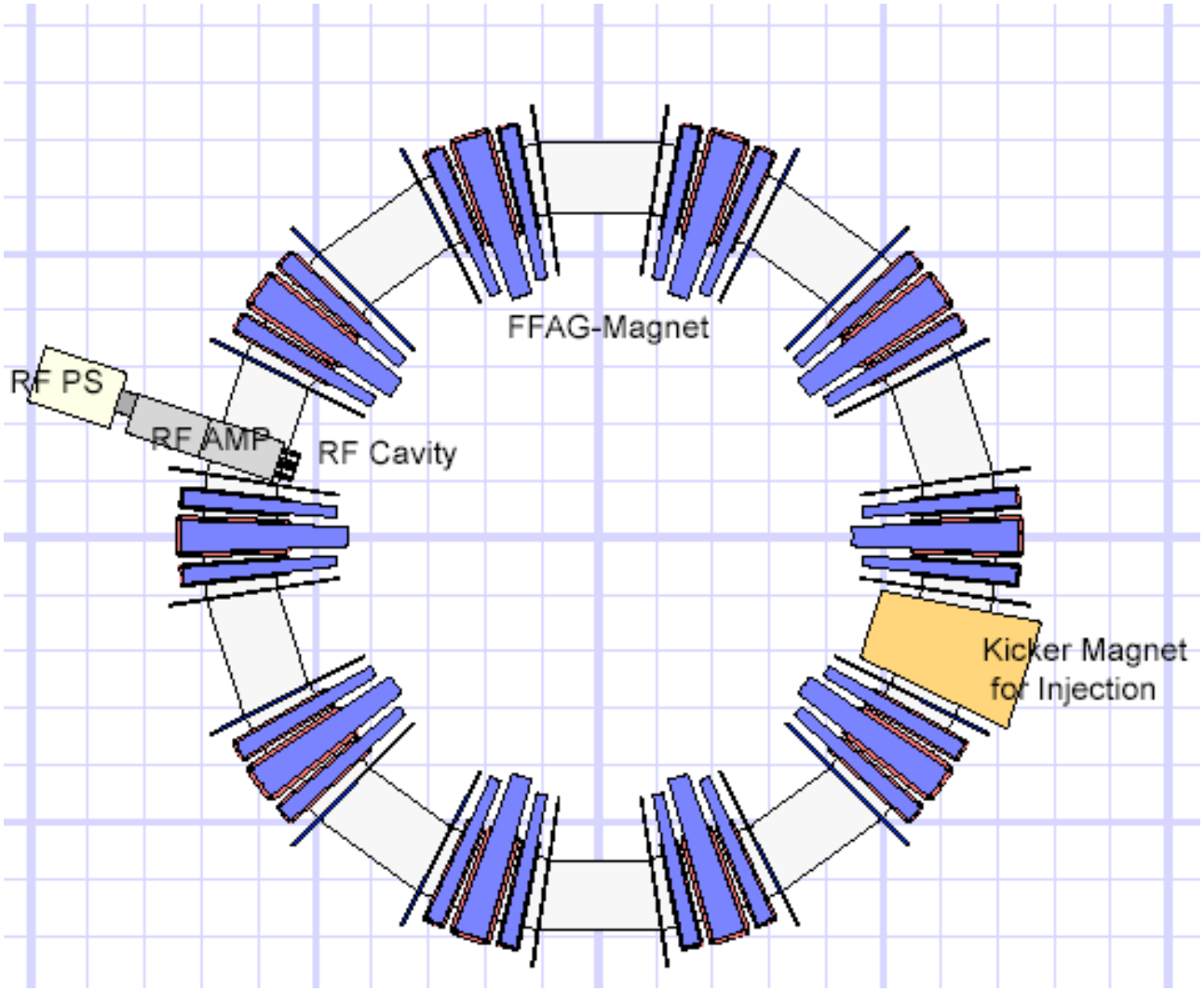
KEK FFAG Workshop

10/15/04

See also: MUC-NOTE-ACCELERATION-297

1. Acceptances for two B vs. azymuth assumptions
2. Acceptance for linearized PRISM lattices
3. Horizontal Injection/extraction
4. Vertical Injection/extraction

PRISM



PRISM Layout

Express Field index k as Taylor series of multipoles

$$B(x) \;=\; B_o\left(1+\frac{x}{R}\right)^k \;\approx\; B_o\left\{1 \;+\; k\left(\frac{x}{R}\right) \;+\; \frac{k(k-1)}{2!}\left(\frac{x}{R}\right)^2 \;+\; \text{etc}\right\}$$

$$= \; B_o \; + \; \sum_{n=1}^{n=m} M_n \; \left(\frac{r}{R}\right)^n$$

$$M_n = B_o \; \frac{\prod_{i=0}^{i=n-1}(k-i)}{n!}$$

ICOOL allows up to $m = 5$ (Dodecapole)

Approximation for fields vs azymuth for each magnet

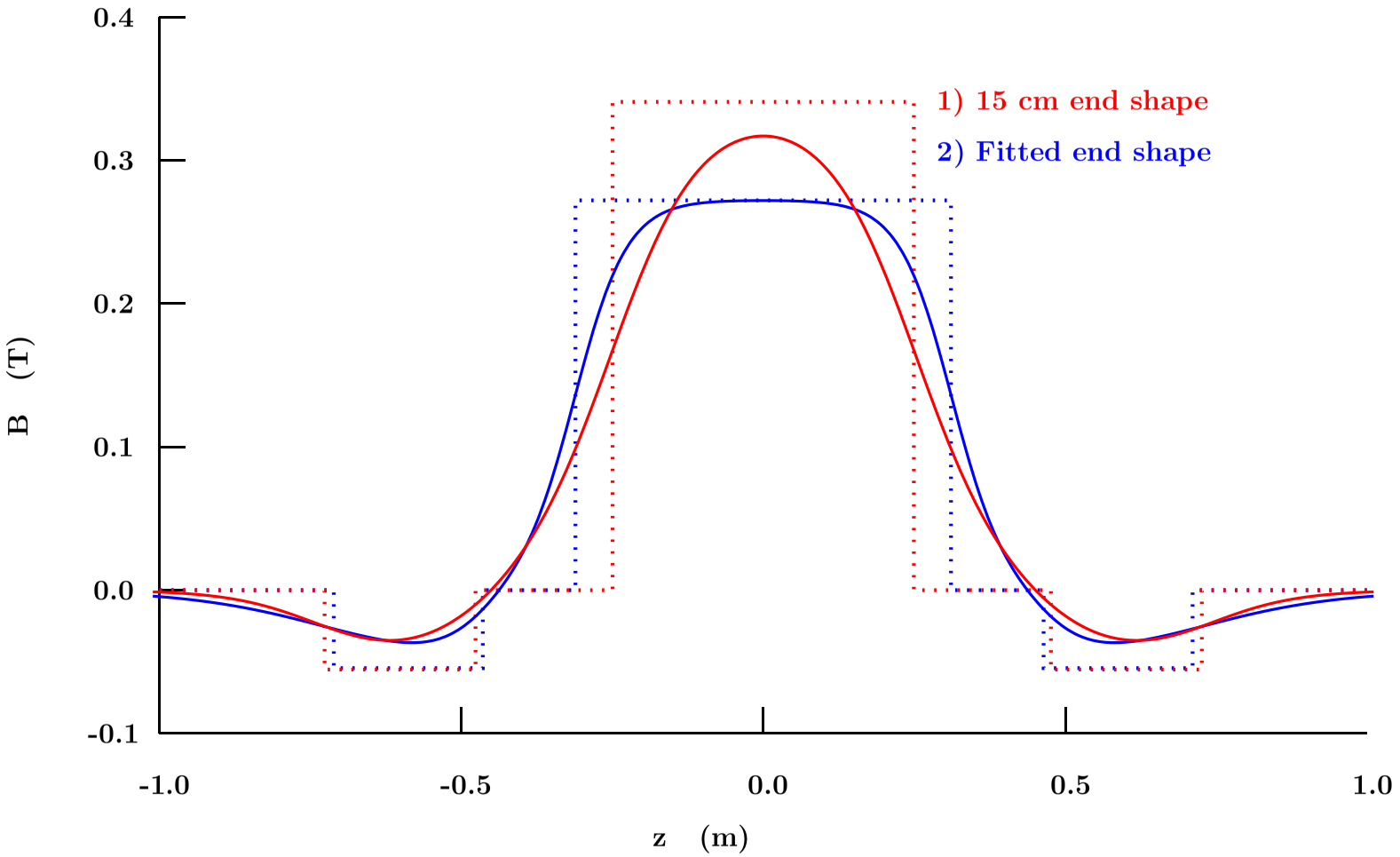
$dz_i = z_i/\Gamma$ **and nominal field** B_o**:**

$$B \;=\; \frac{B_o}{2}\left\{\frac{(e^{dz_1}-e^{-dz_1})}{(e^{dz_1}+e^{-dz_1})} \;-\; \frac{(e^{dz_2}-e^{-dz_2})}{(e^{dz_2}+e^{-dz_2})}\right\}$$

Transformed into 50 Fourier components

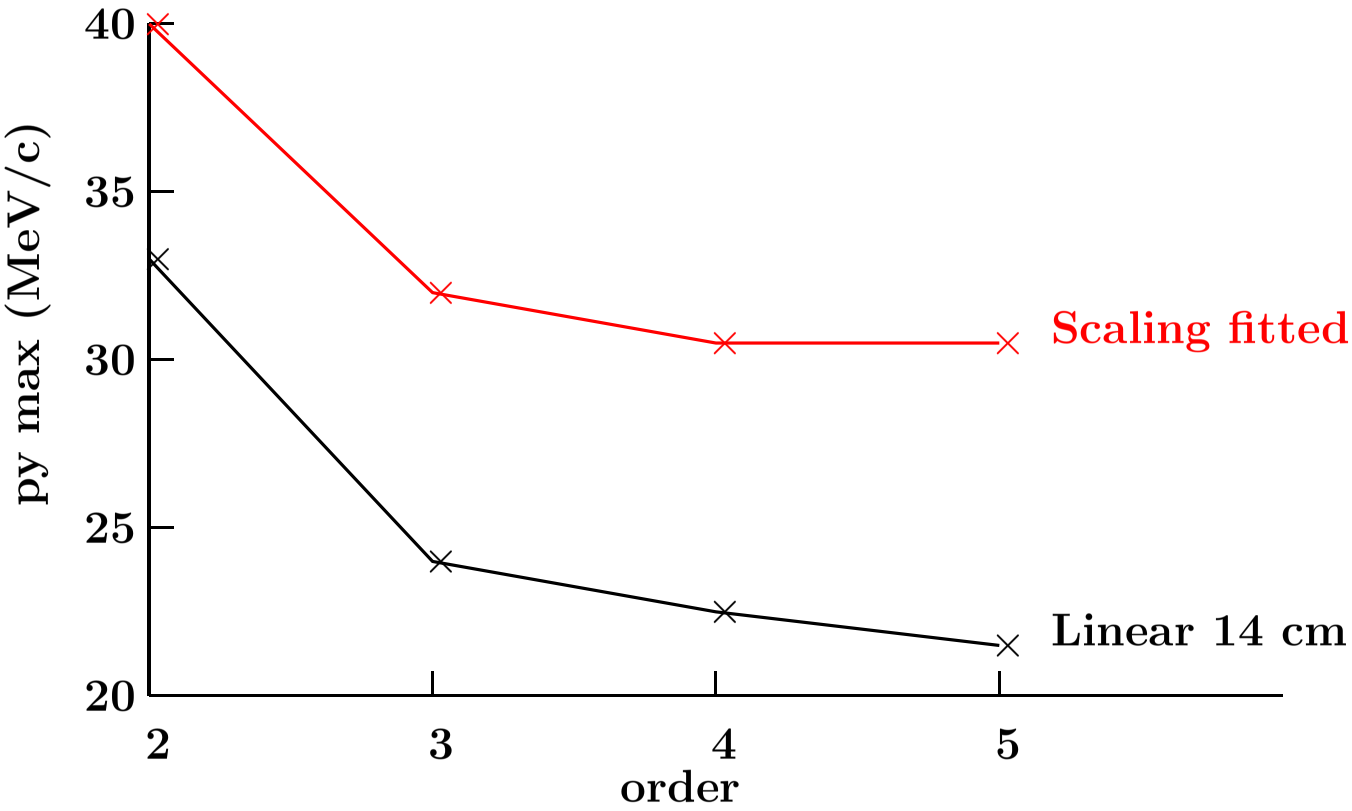
Input Field vs. azymuth for cases 1 and 2

- Case 2: With Ends Fitted to Arimoto's Plot
- Case 1: With more gentle transition F to D



By vs. z plots: Case 1 (red) & Case 2 (blue)

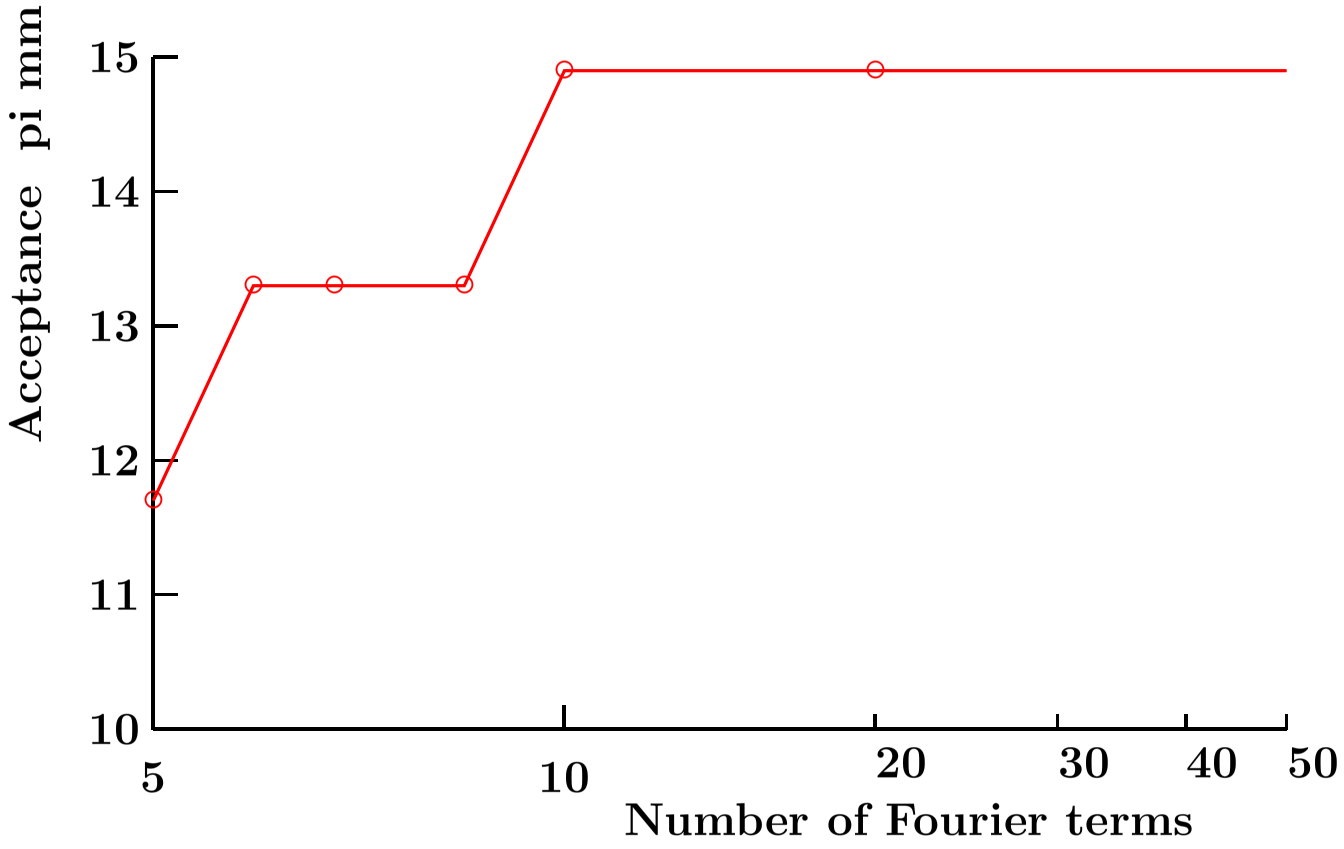
Sensitivity to ICOOL Calculation Order



Final calculations done to 5th order

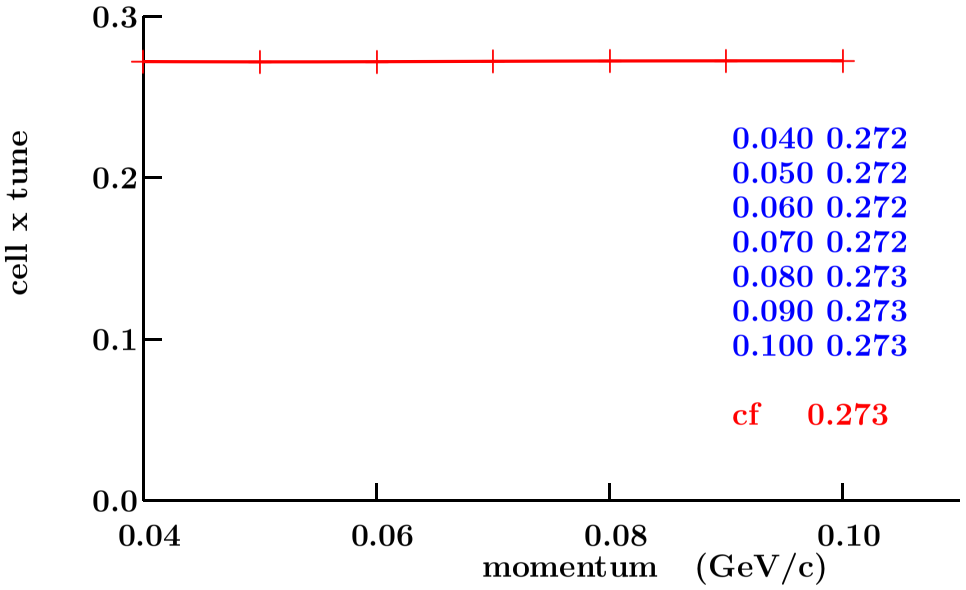
good to of order 1% in amplitude 2% in acceptance

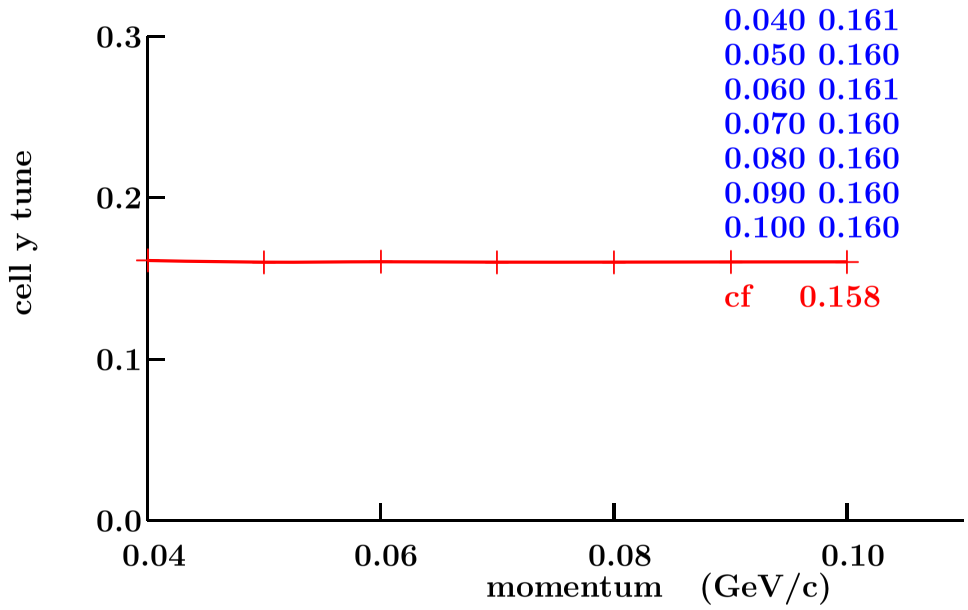
Sensitivity to Number of Fourier Terms



50 terms seems more than adequate

Tunes



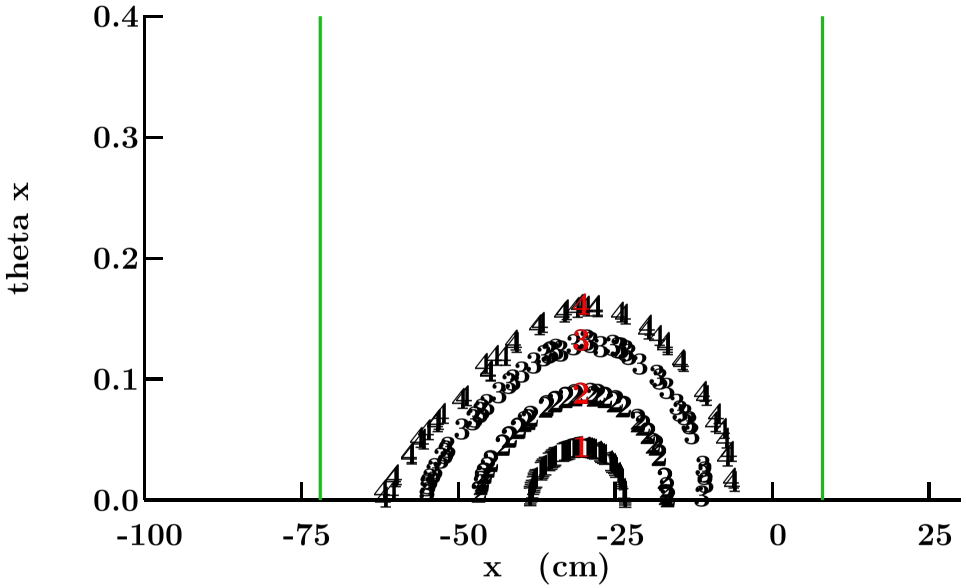


Tunes vs. momentum in the a) x and b) y directions

- **Flat tunes indicate good approximation to scaling**

Horizontal Acceptance

- Particles with larger p_x are unstable in y



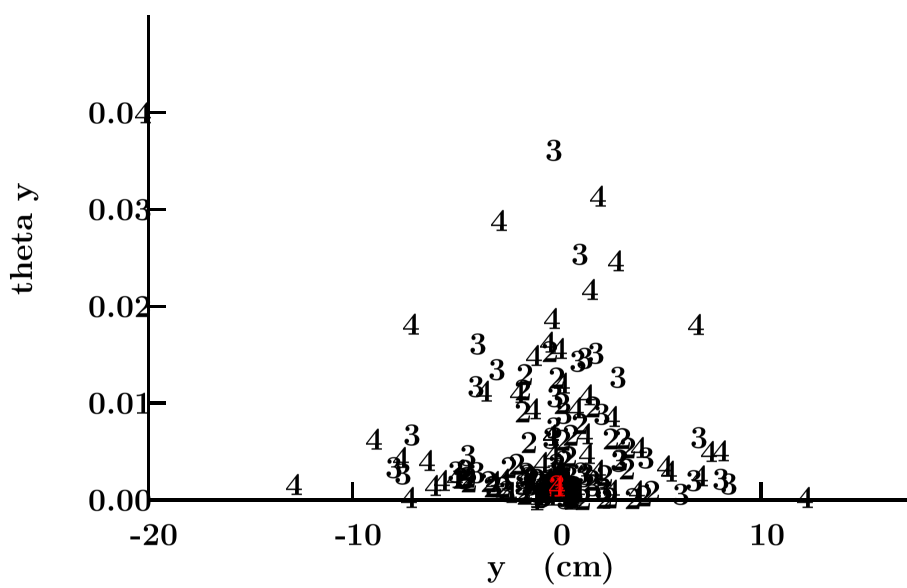
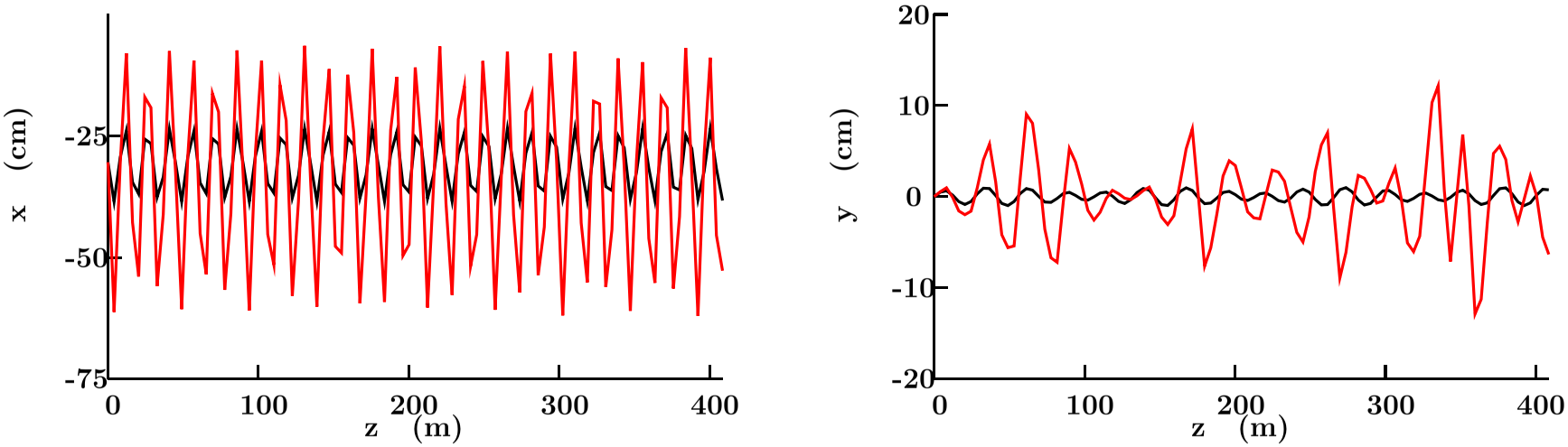


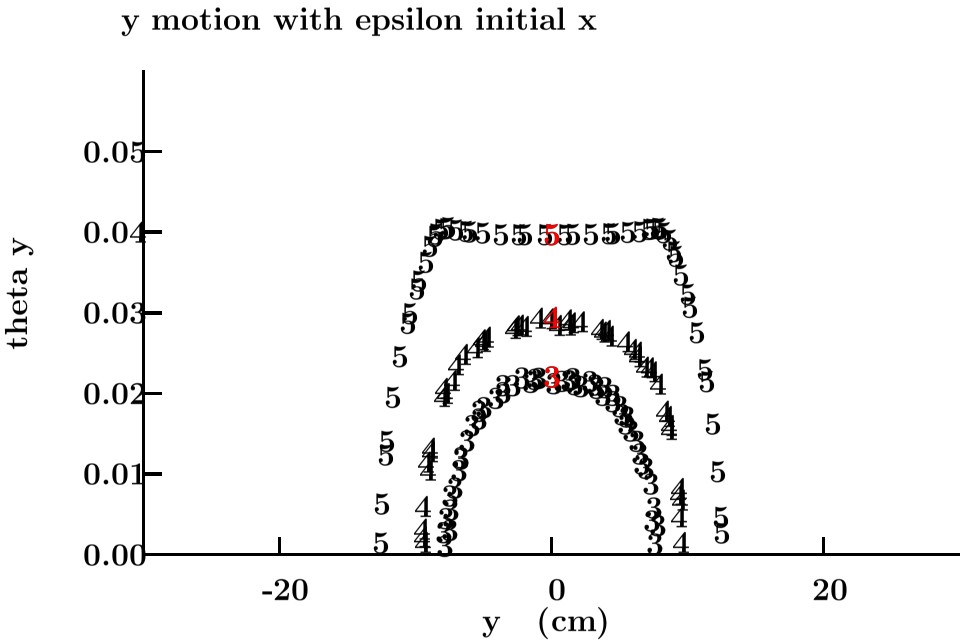
Figure 7: a) x' x phase plot and b) y' y phase plot for very small initial vertical amplitude

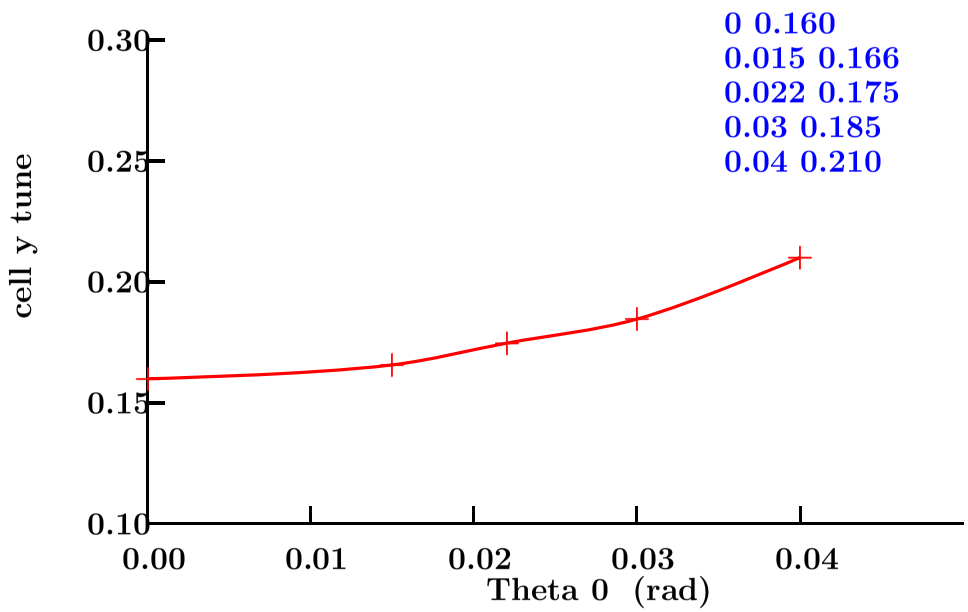


a) x vs z and b) y vs. z for very small initial vertical amplitude, and two initial x amplitudes

- x-y coupling generates beat in y
- Unstable for large initial x amplitude
- Acceptance still large: of order 20 pi mm

Dynamic aperture in vertical (y) plane

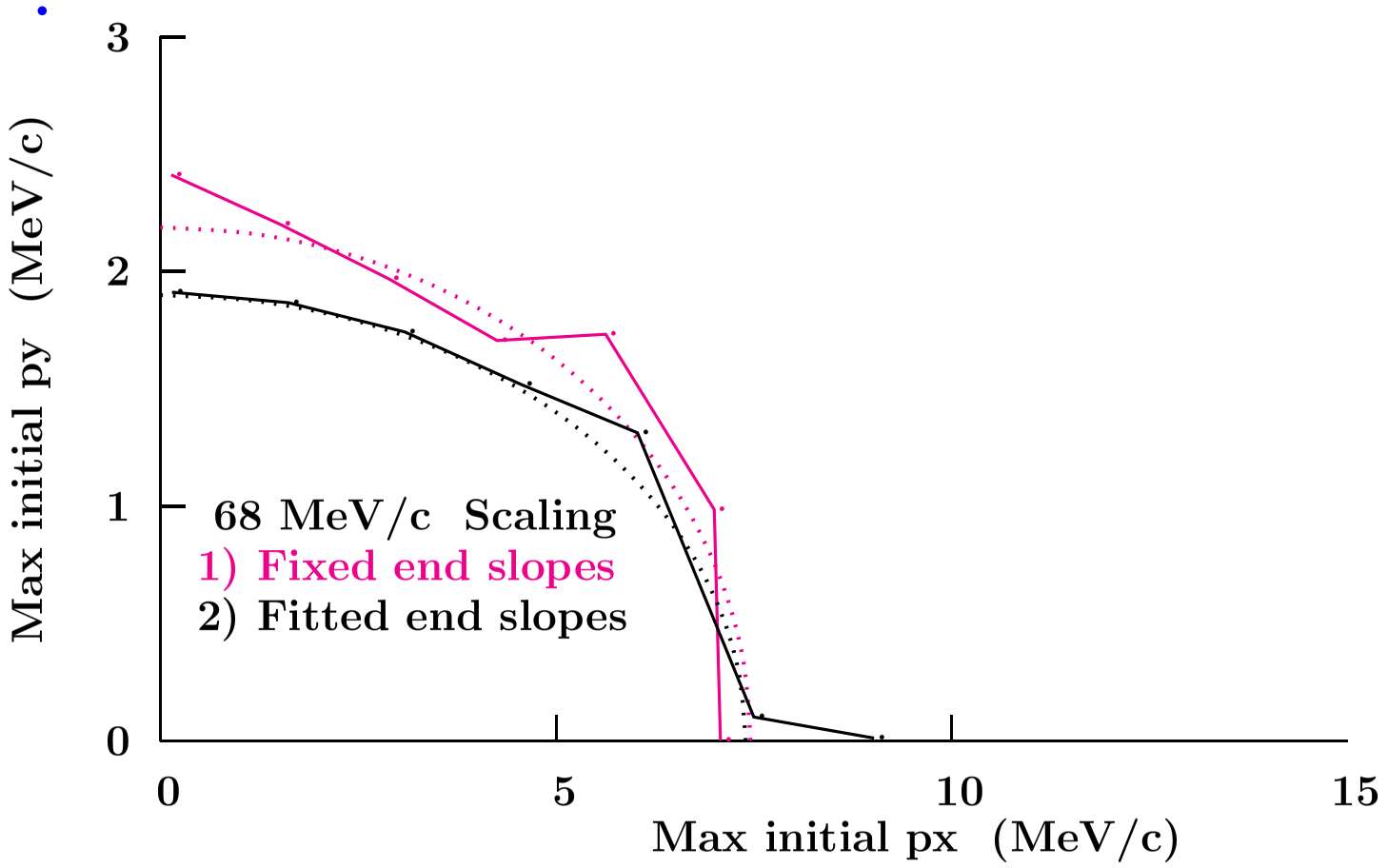




a) y' y phase plot and b) y tune vs. initial y amplitude for small horizontal amplitude

- Large tune vasriation with amplitude
- Little mixing with x
- Acceptance limited by hitting a resonance
- Acceptance less than in x: of order 3 pi mm

Acceptances and ellipses for cases: 1) and 2)

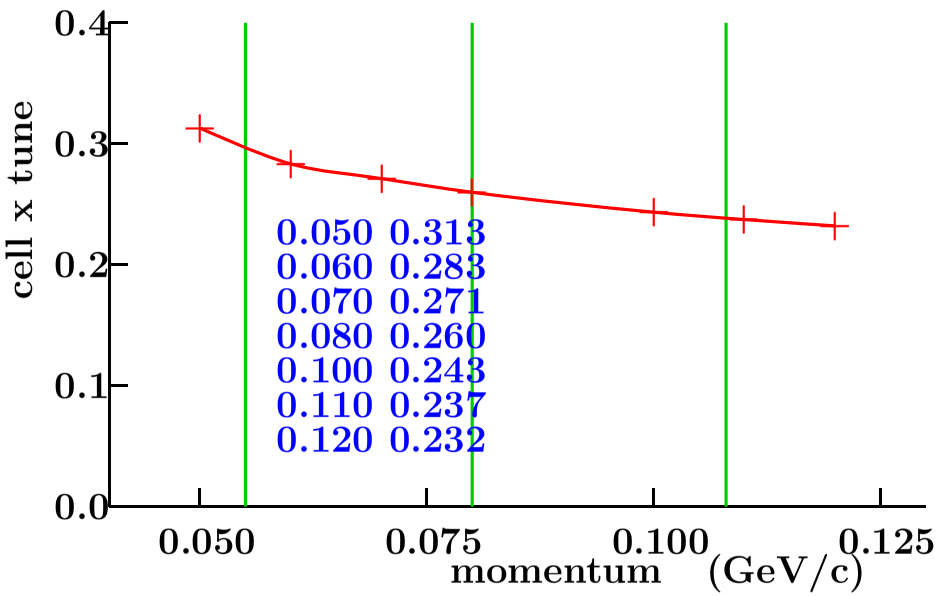


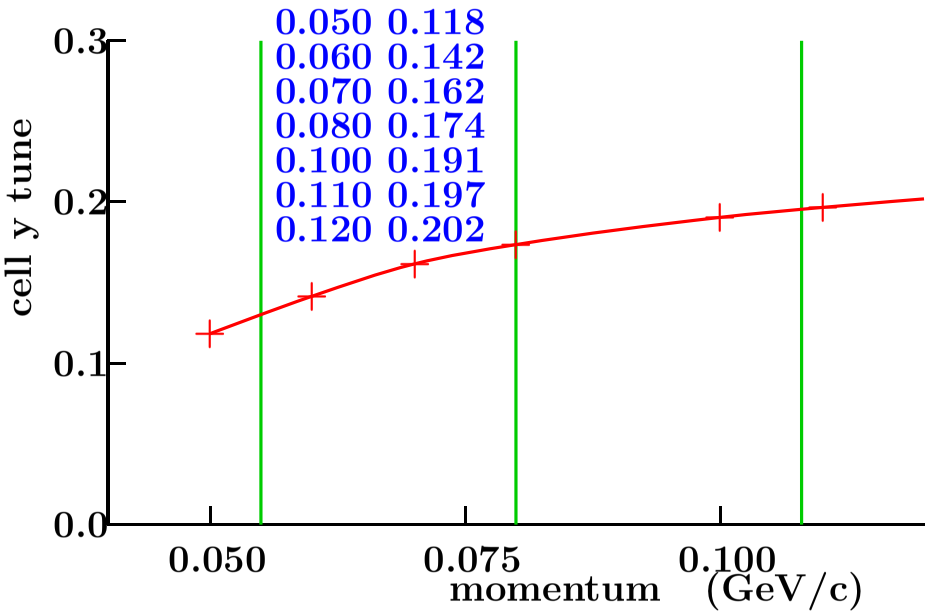
- Case 1 greater acceptance than case 2.
- Less rapid field change between F and D magnets

Cases 3 & 4: Linear Non-Scaling lattices

Set all multipoles above quadrupole to zero

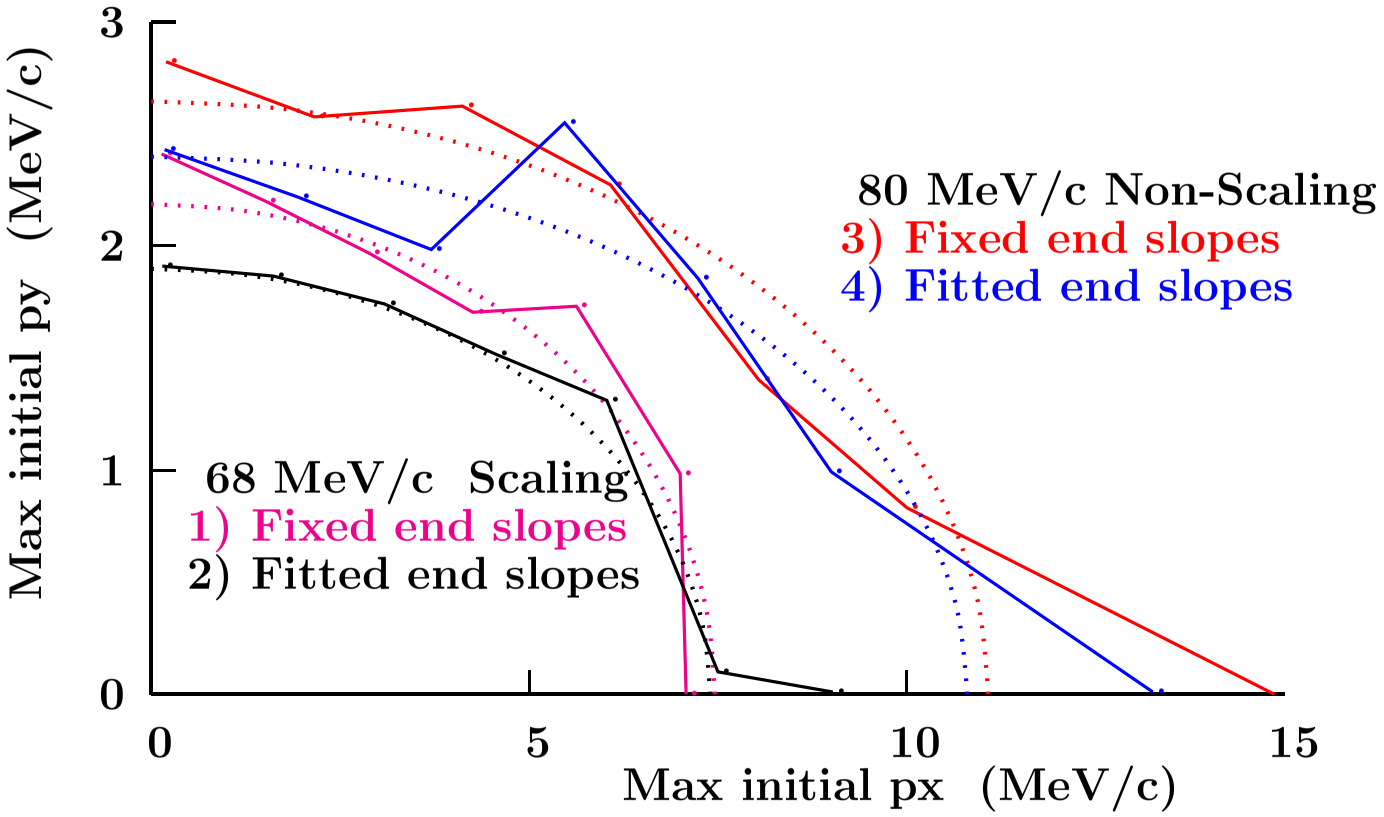
Tunes





- Tunes are no longer constant
- Non-Scaling
- Acceptances not independent of mom
- But we have only looked at central mom

Acceptances of all cases



Acceptances for scaling lattices 1) and 2); and linear lattices 3) and 4)

- Cases 3 & 4 greater acceptance than cases 1 & 2
- Linear focus helps

Summary

Acceptances in order:

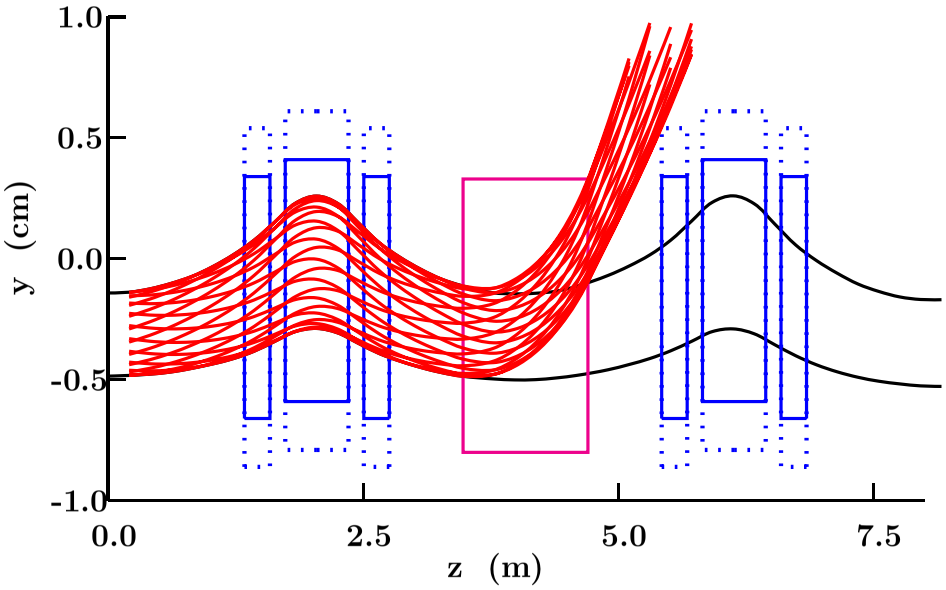
Cse	End shapes	B vs x	x π mm	y π mm	xy $(\pi \text{ mm})^2$
2	**fitted**	**scaling**	**12.57**	**2.04**	**25.6**
1	**14 cm**	**scaling**	**12.77**	**2.70**	**34.5**
4	**fitted**	**linear**	**21.65**	**1.95**	**42.3**
3	**14 cm**	**linear**	**22.78**	**2.37**	**54.1**

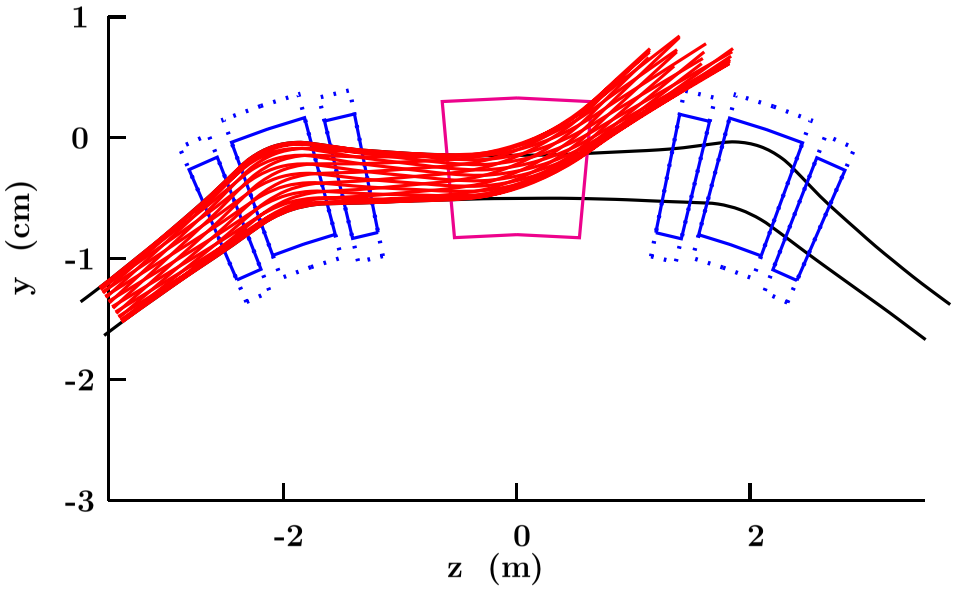
Conclusions of PRISM Acceptance Study

- 5 multiploles can well represent the field index k
- Horizontal acceptance with zero vertical assymetry is unphysical
- Fit to the Arimoto's field shape gave somewhat less acceptances than more gentle field end shapes.
- Linear combined function lattice gave
  - Almost the same momentum acceptance
  - Lower peak fields
  - $\approx$ 2 times acceptance at central momentum
  - Acceptance vs. momentum not yet studied

Horizontal Injection/Extraction

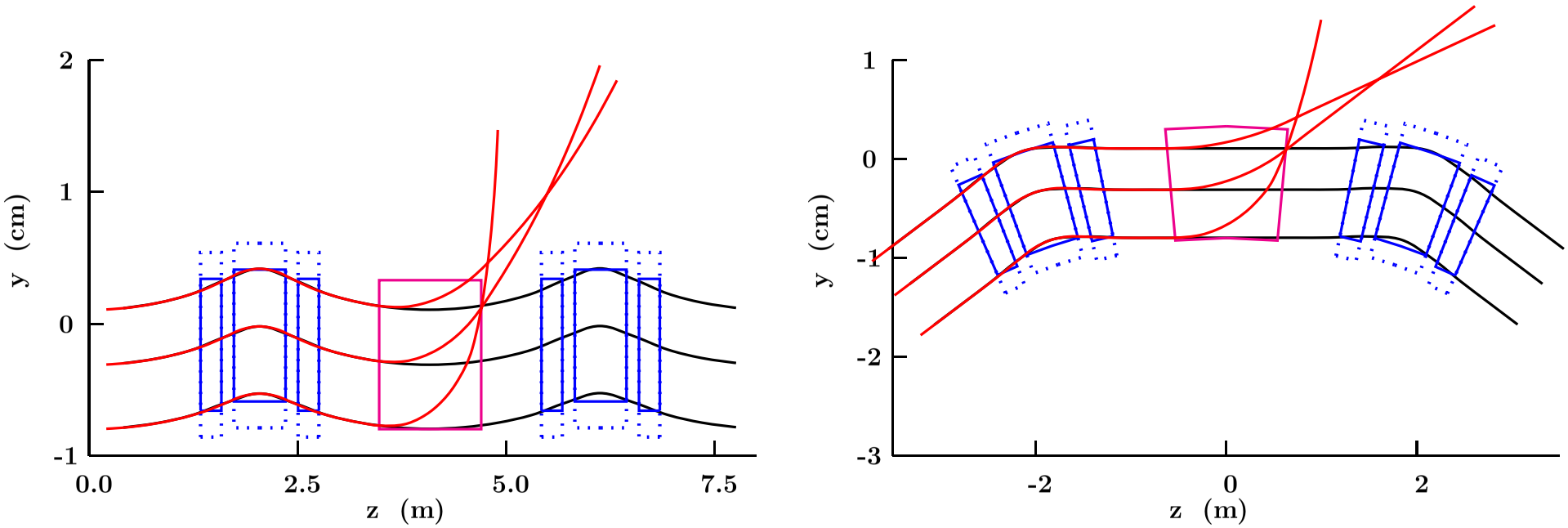
- Look first at central momentum
- Only solution is to inject within the same straight





Check max and min momenta at zero amplitude

Apertures for 43 MeV (-37%) to 98 MeV (+44%)



- Beams of different momenta cross
- Difficult matching problem

Kicker Energy and Voltage

$$V = \frac{B_y \; X \; L}{t_{\rm rise}}$$

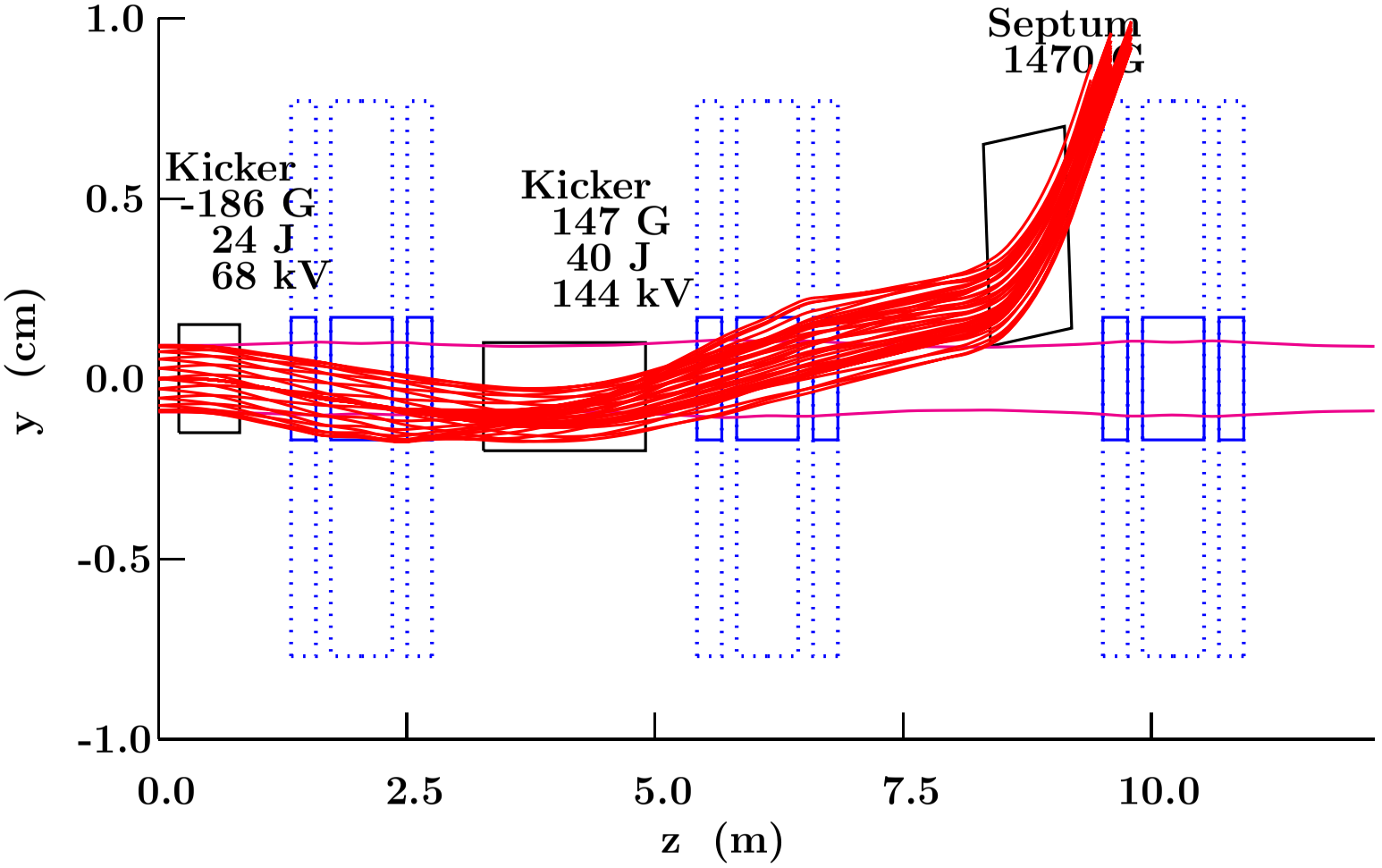
$$U = \frac{B_y^2 \; L \; X \; Y}{2 \; \mu_o}$$

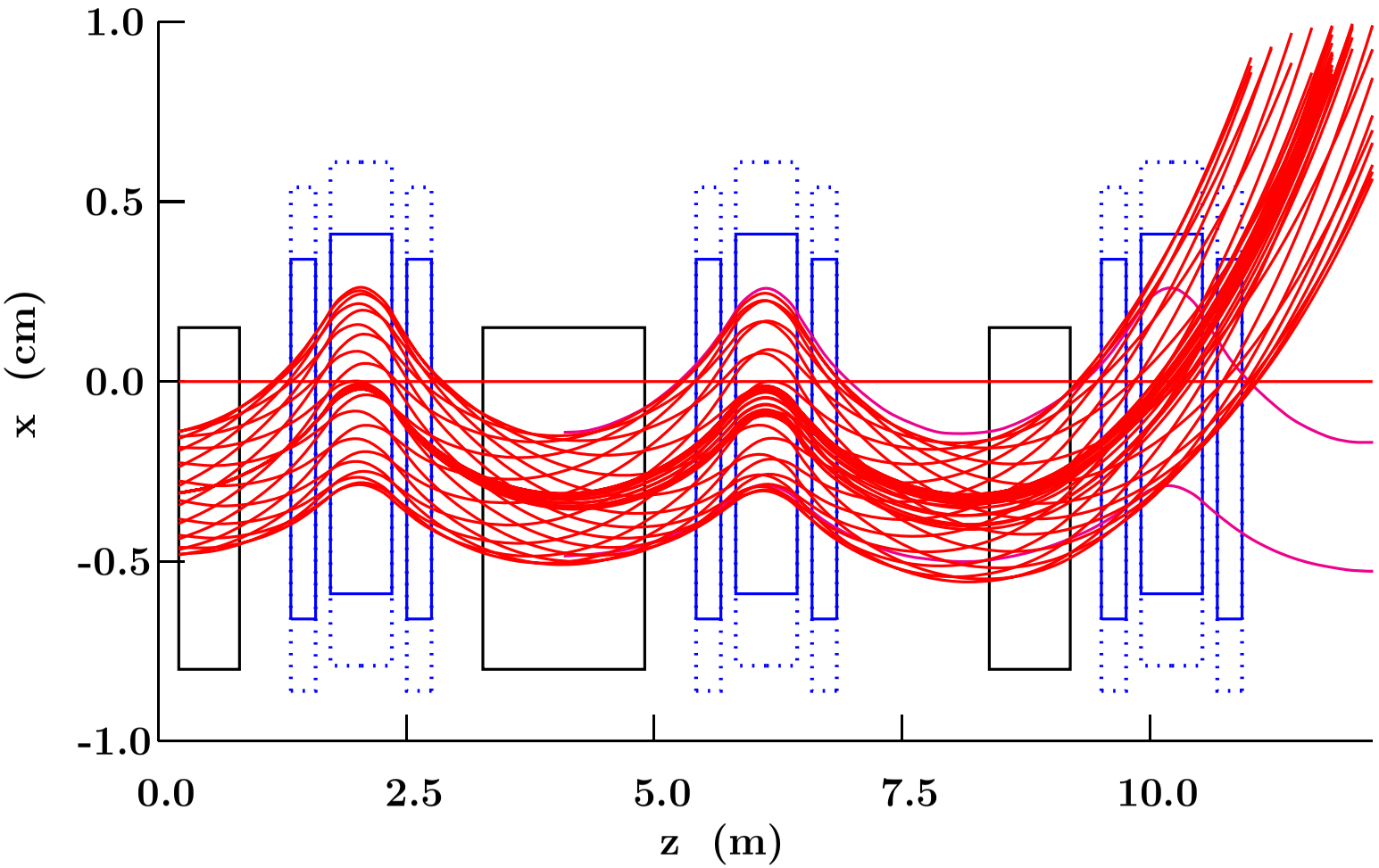
Length	**cm**	**122.5**
Width	**cm**	**120**
Height	**cm**	**34**
Kicker Field	**T**	**0.108**
Rise time	**ns**	**50**
Stored Energy	**J**	**2038**
Single turn Voltage	**kV**	**3162**

- **Not quite as bad as RFOFO Kicker U=8200 J V=5700 kV**
- **Similar U, but higher V than Study 2a FFAG U=3000 J V=240 kV**
- **Much worse than anti-proton Kicker U=13 J V=80 kV**

Vertical Injection/Extraction Work done here

Need 2 Vertical kickers

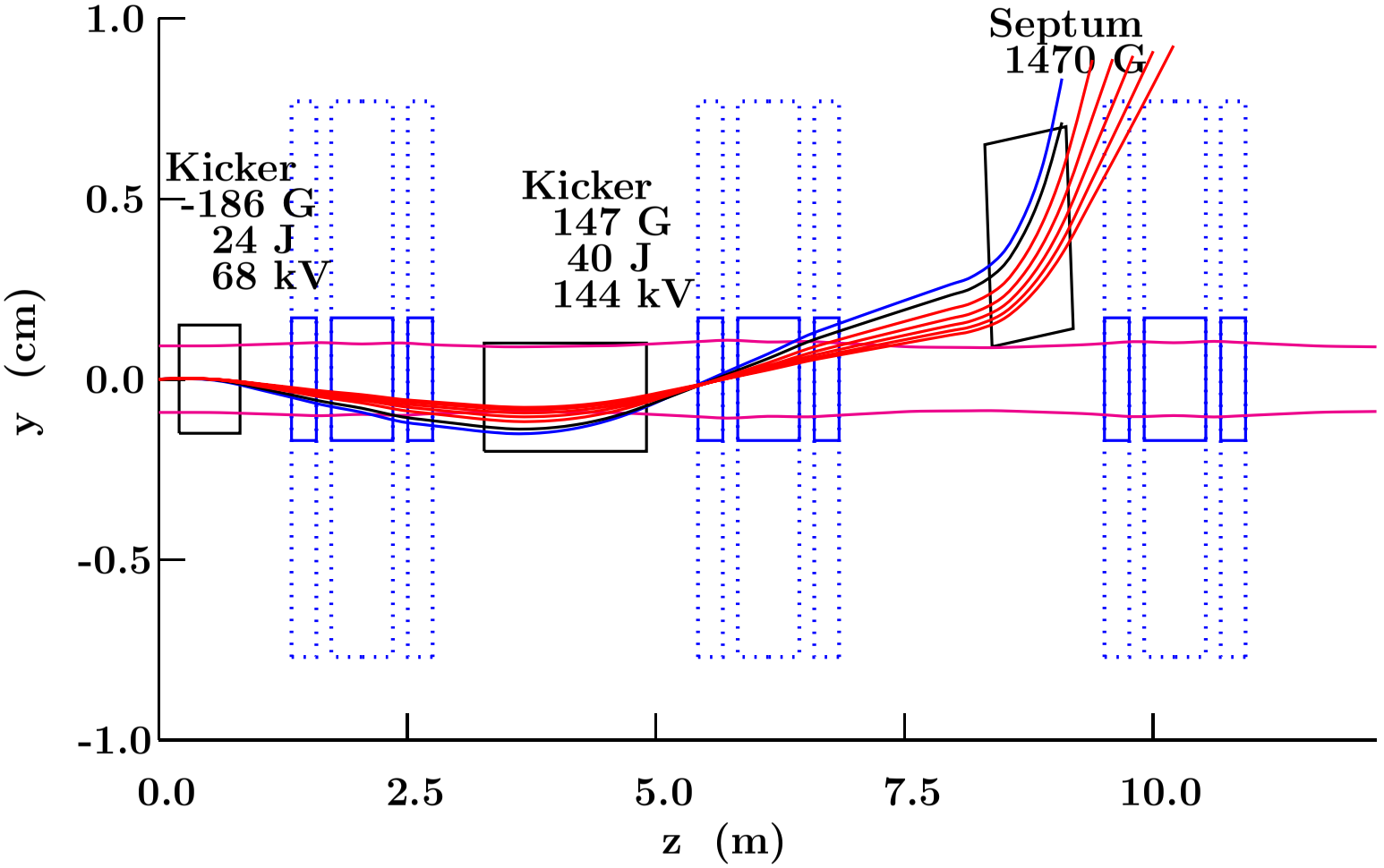



1.0
0.5
0.0
-0.5
-1.0
x (cm)
0.0
2.5
5.0
7.5
10.0
z (m)

Check all momenta with no amplitudes

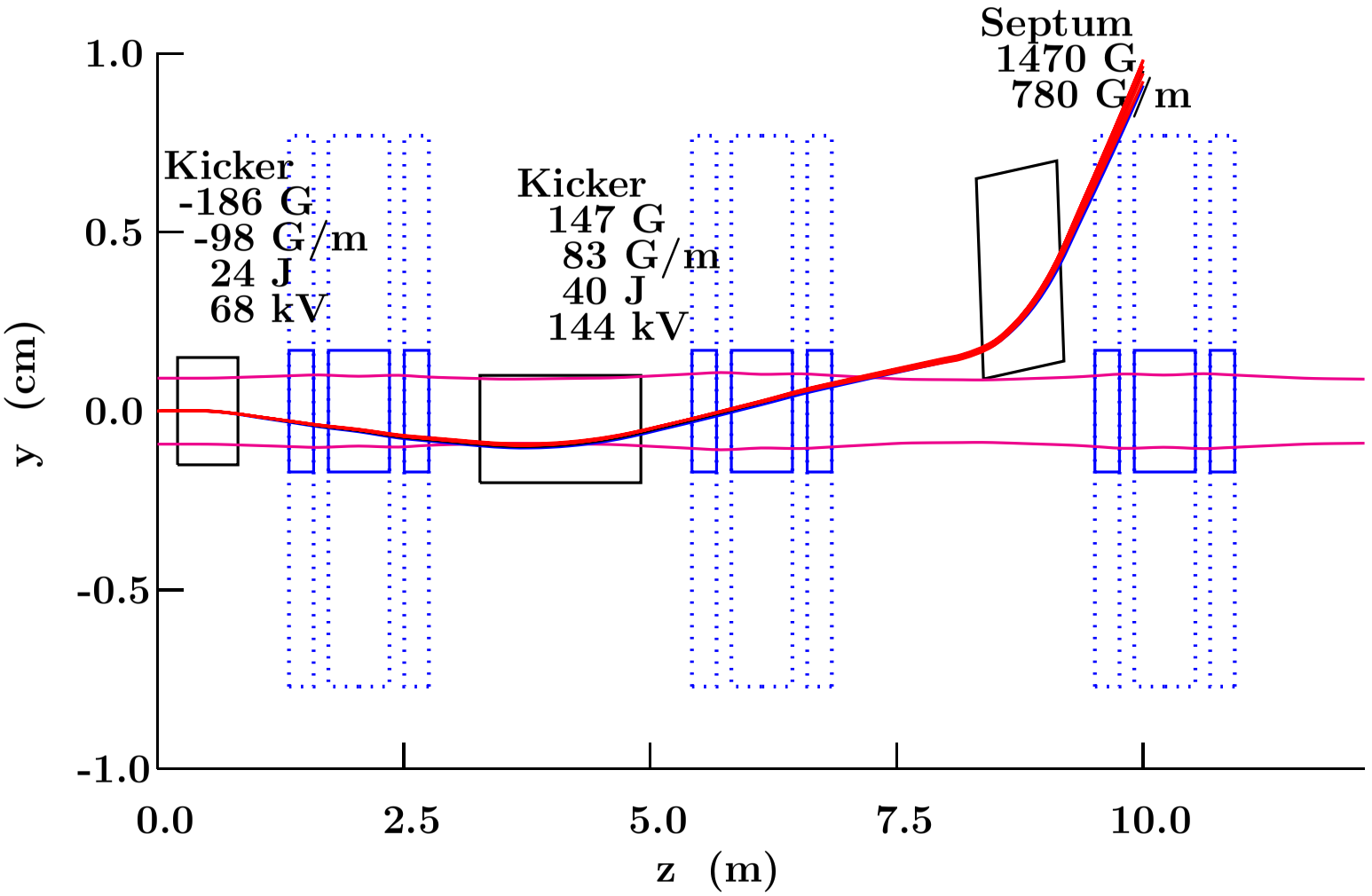
Apertures for 43 MeV (-37%) to 98 MeV (+44%)

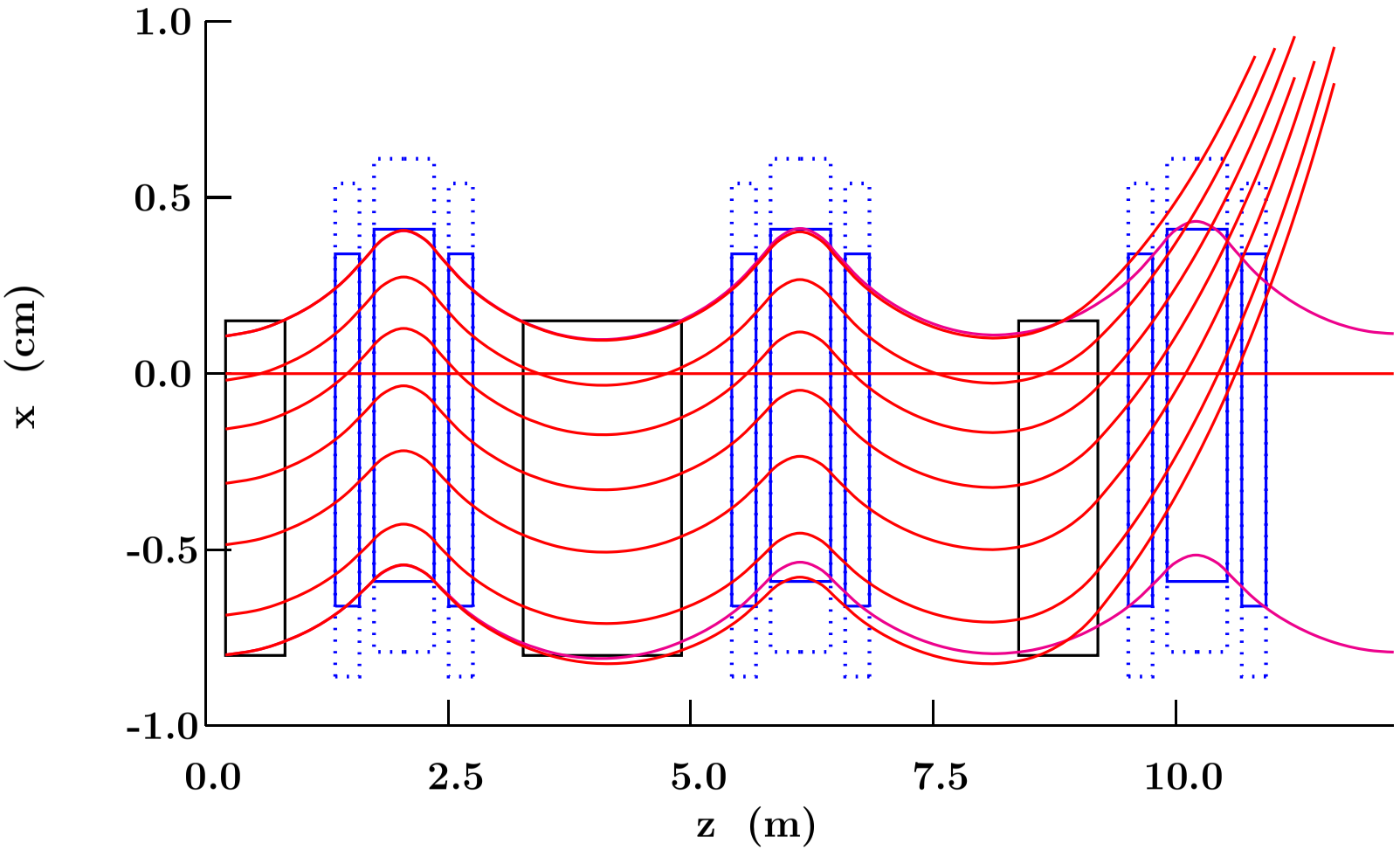
Trouble for injection



Add Skew Quadrupole moment to Kickers

OK now for injection or Extraction

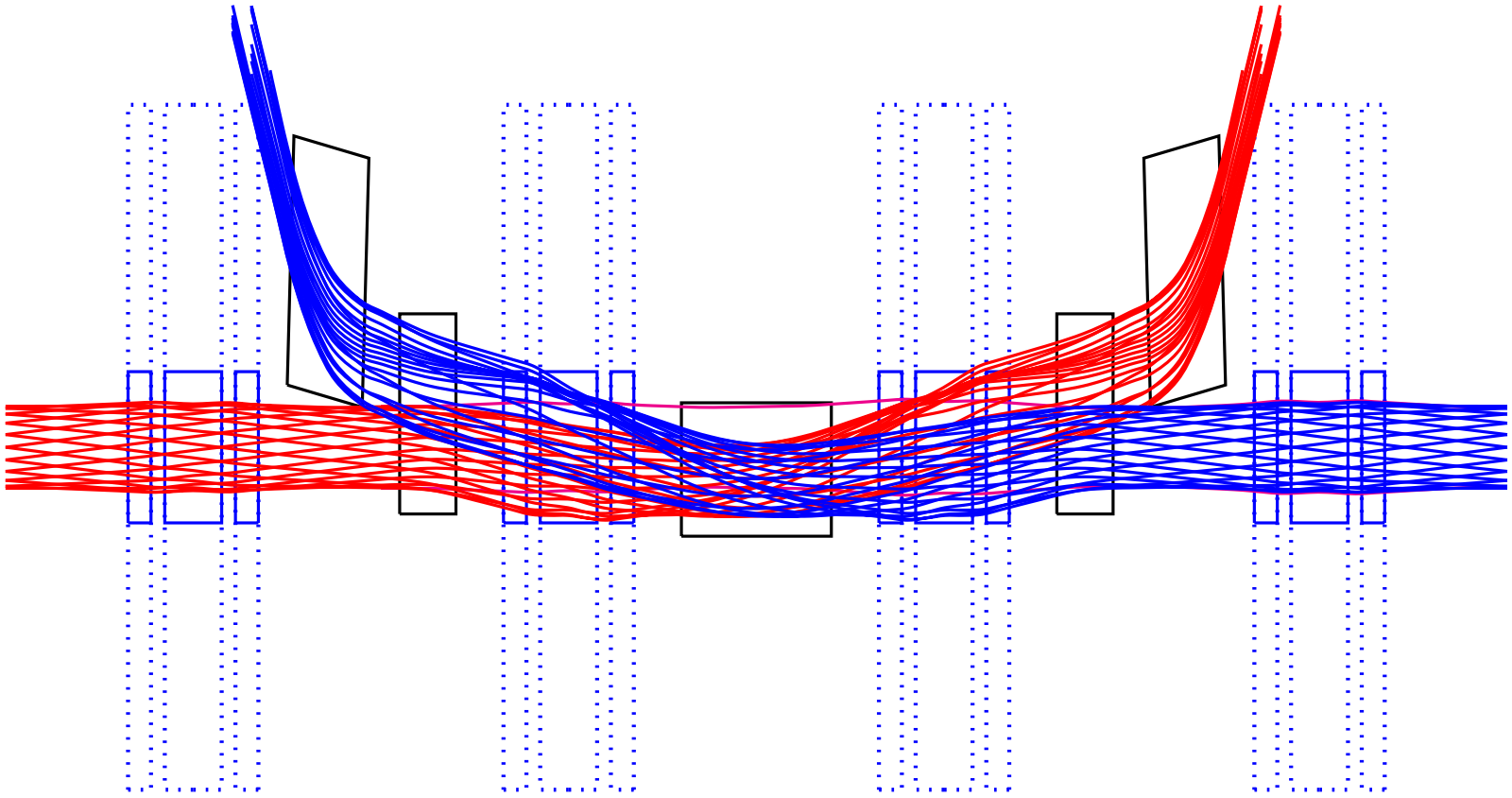



1.0
0.5
0.0
-0.5
-1.0
x (cm)
0.0
2.5
5.0
7.5
10.0
z (m)

Injection and Extraction in same 3 cells

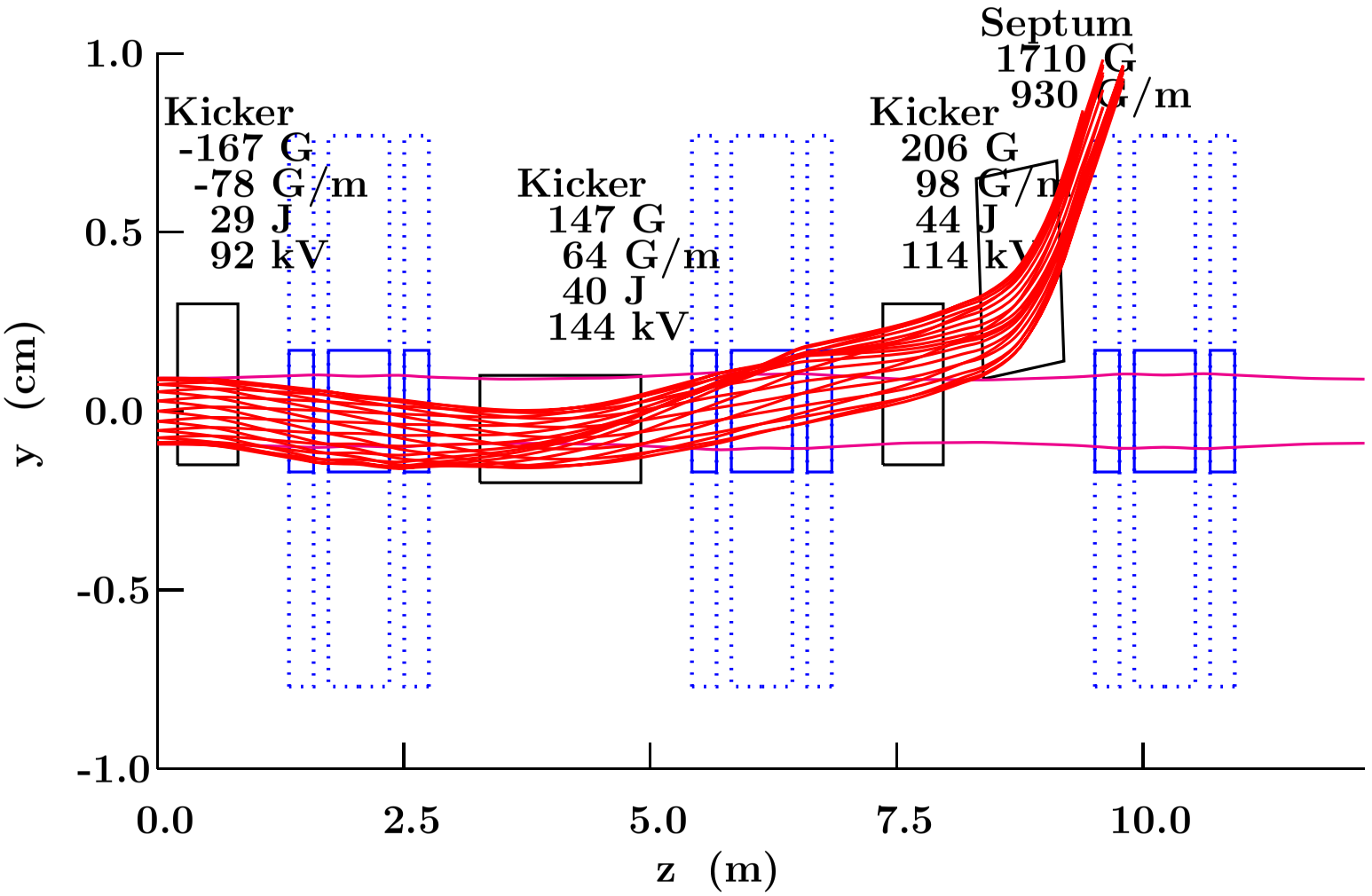
Central kicker must be pulsed twice

End kickers pulsed once



Pulse all kickers twice

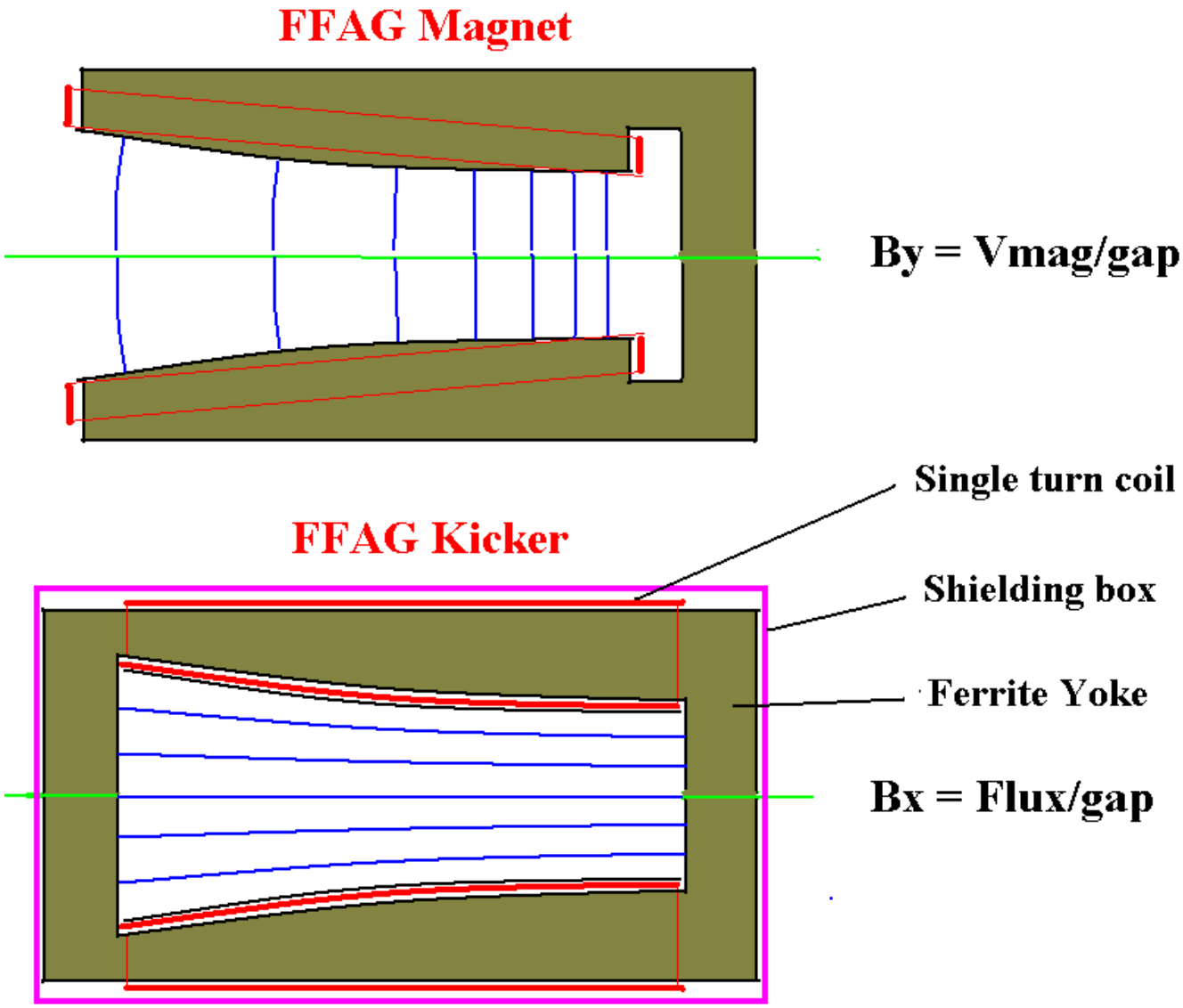
Because they are there



- Slightly improved

What does a Combined Function Kicker Look Like

You already know



Parameters

		dz m	len m	ht m	wid m	tilt deg	B G	Grad G/m	V_o kV	U J
1	Kicker	0.51	0.61	0.45	0.95	0	-167	-78	92	29
2	Kicker	0.00	1.63	0.30	0.95	0	147	64	144	40
3	Kicker	-.51	0.61	0.45	0.95	0	206	98	114	44
4	Septum	0.61	0.82	0.56	0.95	4	1710	930		
Max	(Total)								144	(113)
Horiz		0	1.22	.34	1.2		1080		3160	2038

- dz with respect to center of gap
- V_o is single turn V. If driven from sides V= V_oo/2
- V's and U's are ideal. They will be greater in a real kicker
- Voltage and energy approximately 1/20 of Horizontal kicker

Conclusions on Injection/Extraction

- Vertical injection/extraction much easier than horizontal
  - Needs Much less Magnetic energy
  - Needs much lower Voltage
  - Chromatic correction easy
- But Remaining Design Questions
  - Needs larger vertical apertures in special magnets
  - Kicker Energy still much greater than normal kickers
  - Need two pulses in each kicker
  - Kicker aspect ratio unnatural
  - Needs gradient in kicker field (dipole + skew quadrupole)
- Study needs repeating with real fields and beam
- But this looks plausible