

Status of 6-D transmission simulations in FFAGs

Contents

1	Code developments - FFAG needs	2
2	Scaling FFAG, 6-D tracking simulations	5
3	Tracking in 3-D field maps	8
4	6-D transmission in the ISS-spec, linear FFAGs	12
5	6-D transmission in I-FFAG[-I]	18
6	On computing precision	21
7	Various temporary conclusions	22

1 Code developements - FFAG needs

**HYPOTHESIS [THEOREM] : 6-D TRANSMISSION SIMULATIONS
ARE [TO BE] BASED ON RAY-TRACING METHODS.**

● **Optics :**

- **Scaling FFAG simulations need special elements, like**

- [sector, spiral,...] **dipoles with “arbitrary” axial (including fringe field effects) and radial dependence of magnetic field**

$$B_{zi}(r, \theta) = B_{z0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r)$$

- **Scaling FFAG, NC magnets (“FFAG” procedure) :**

$$\mathcal{R}_i(r) = (r/R_{0,i})^{K_i}$$

- **Scaling FFAG, SC magnets (“DIPOLES” procedure) :**

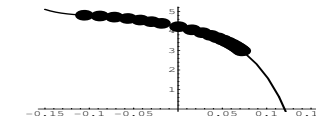
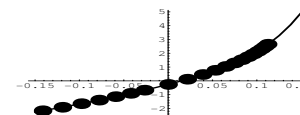
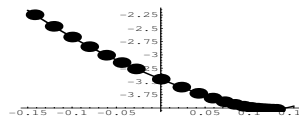
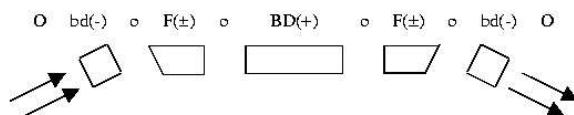
$$\mathcal{R}_i(r) = b_{0,i} + b_{1,i}(r - R_{0,i})/R_{0,i} + b_{2,i}(r - R_{0,i})^2/R_{0,i}^2 + \dots$$

- **accounting for possible variable gap $g = g_0(R_0/r)^K$, and overlapping of fringe fields**



- **Linear FFAGs are built from quadrupole fields, the “MULTIPOLE” procedure in Zgoubi is employed to simulate that. These FFAGs did not necessitate any particular developement.**

- **The isochronous FFAG lattice combines the use of “MULTIPOLE” and “DIPOLES” procedures.**



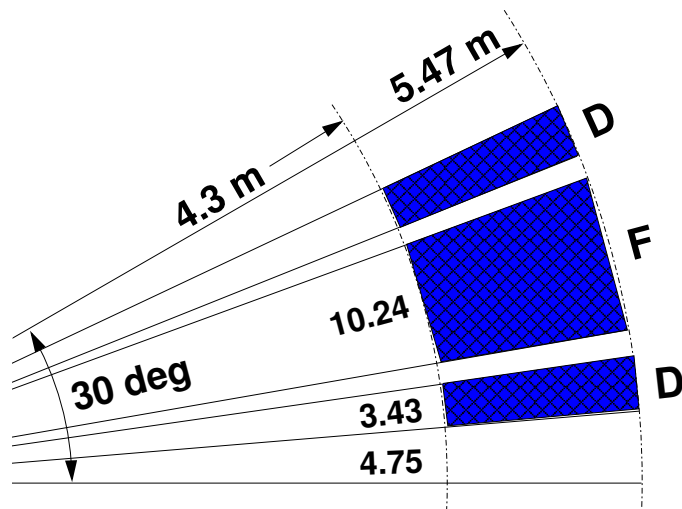
- **RF : not a real issue for the moment. Regular point transforms are ok. The question of TOF in terms of a reference particle, or absolute TOF computation sometimes arise - well managed by the ray-tracing methods up to now.**

Code developments (cont'd) - the “FFAG” and “DIPOLLES” procedures

- Main goals : simulate $B_{zi}(r, \theta) = B_{z0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r)$ AND allow for possible overlapping of fringe fields.

Main apps : scaling and isochronous FFAGs.

- An example : simulation of a scaling FFAG triplet :

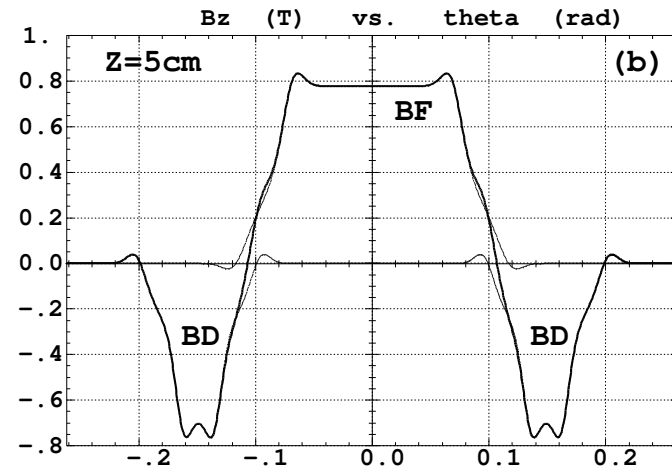
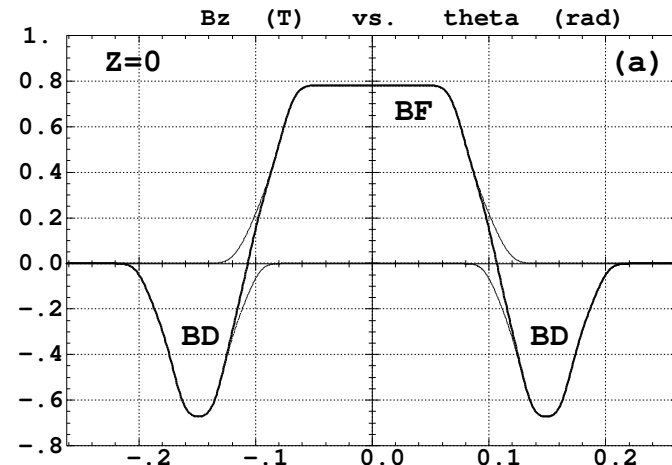


DFD triplet.

The geometrical model is based on the superposition of the independent contributions of the N dipoles :

$$B_z(r, \theta) = \sum_{i=1, N} B_{z0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r)$$

at all (r, θ) in the mid-plane. Field off mid-plane is obtained by Taylor expansion.



Field experienced for $r_0 = 4.87$ m in the DFD dipole triplet.

A superposition of $N = 3$ independent contributions, at all (r, θ, z) .

Code developements (cont'd) - "FIT" procedure

FIT VARIABLES : all data !

FFAG05 Wkshp, KURRI-599 Dec. 2005

An indispensable tool for

- preliminary adjustments (tunes, etc.) prior to 6-D simulations
- considered very useful for further assessment and optimisation of higher order behavior, DA, transmission, ...

FIT CONSTRAINTS :

Trajectory coordinates (e.g., final coordinates)

Several other types of quantities that are deduced from trajectories, e.g. :

- first and higher order transport coefficients
- beam matrix coefficients (waist, divergence)
- particle fluxes through ellipses (→ transmission efficiency)

In the case of periodic structures :

- closed orbits
- tunes, chromaticities, anharmonicities

```

'OBJET' * c.o., constant Gap *
226.8235847      68MeV/c muon
2
2 1
499.377      0.      0.      0.      0.      1.2      'b'
1 1 1 1 1 1 1 1 1
'FFAG'
0
3 45.      500.      NMAG, Sector angle, R
18.17      0.      -0.717      5.      mag 1 : ACNT, dum, B0, K
6.3 0.      EFB 1 : lambda, gap const/varbl
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
1.23 0.      1.E6      -1.E6      1.E6      1.E6
6.3 0.      EFB 2
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
-1.23 0.      1.E6      -1.E6      1.E6      1.E6
0. -1      EFB 3 : inhibited by iop=0
0 0.      0.      0.      0.      0.      0.      0.
0. 0.      0.      0.      0.      0.
22.5 0.      3.2      5.      mag 2 : ACNT, B0, K, dums
6.3 0.      EFB 1
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
3. 0.      1.E6      -1.E6      1.E6      1.E6
6.3 0.      EFB 2
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
-3 0.      1.E6      -1.E6      1.E6      1.E6
0. -1      EFB 3
0 0.      0.      0.      0.      0.      0.      0.
0. 0.      0.      0.      0.      0.
26.83 0.      -0.717      5.      mag 3 : ACNT, dum, B0, K
6.3 0.      EFB 1
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
1.23 0.      1.E6      -1.E6      1.E6      1.E6
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-1.23 0.      1.E6      -1.E6      1.E6      1.E6
0. -1      EFB 3
0 0.      0.      0.      0.      0.      0.      0.
0. 0.      0.      0.      0.      0.
0      2 125.      KIRD anal/num, resol(mesh=step/resol)
.5      integration step size
2 0.      0.      0.      0.

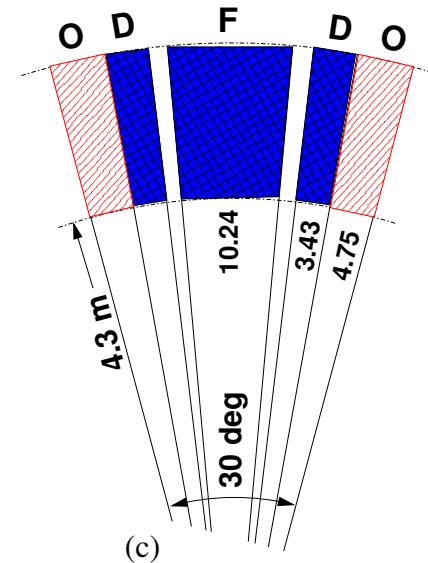
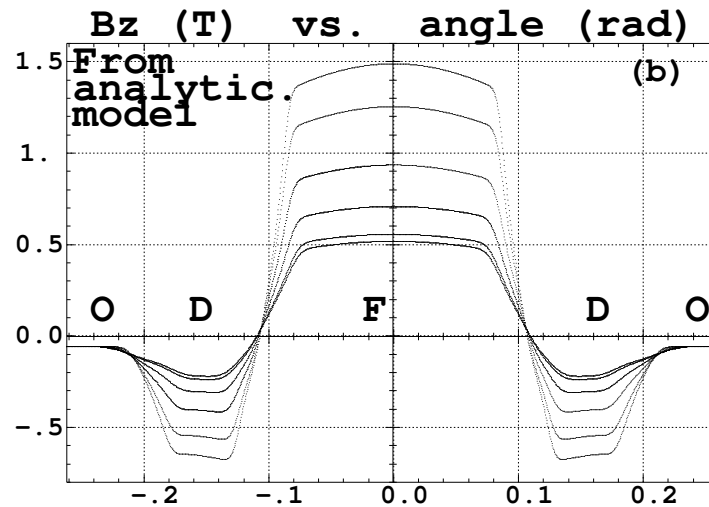
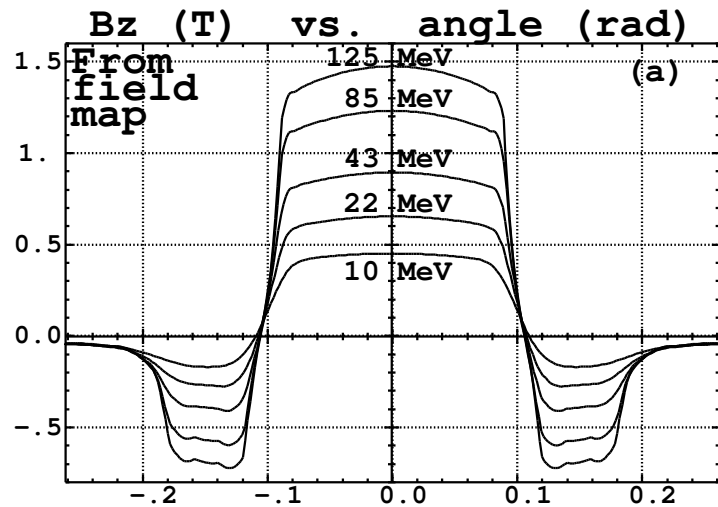
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Now comes the description of the tracking conditions and results

2 Scaling FFAG, 6-D tracking simulations

POP of 6-D tracking, using KEK 150 MeV FFAG parameters

Preliminaries : Real field versus “FFAG” procedure



Comparison of magnetic field along closed orbits in the case of,
(a) : TOSCA 3-D map representative of the 150 MeV FFAG, and,
(b) : field from the “3+2”-dipole geometrical model.

(c) geometry of the “3+2”-dipole design, including two additional dipole regions (hatched) that simulate 700 G field extent over the two end drifts.

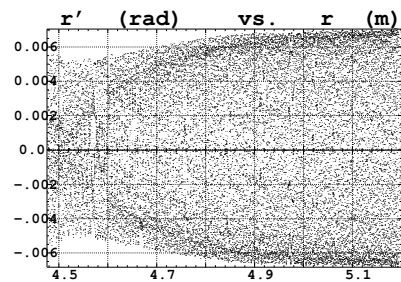
Scaling FFAG, 6-D tracking (cont'd) [*Details in : NIM A 547 (2005)*] : a particle with $z_0 = 3$ cm (a large value !) accelerated from 12 to 125 MeV.

A/ Case of analytical calculation of field derivatives (the Zgoubi method needs the $d^n B/ds^n$)

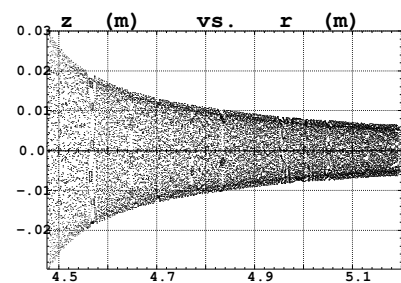
Method : $d^n B/ds^n$ computed from the geometrical model of the (r, θ) field dependence, and from distances to EFBs.

Interest : accurate (good for symplecticity), fast (larger step size).

x-z coupling, yet $\epsilon_x \approx 0$



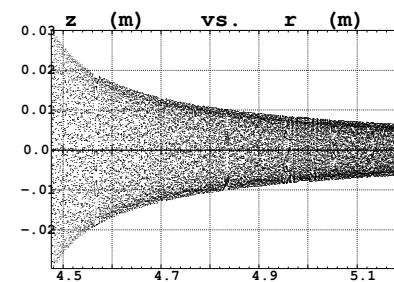
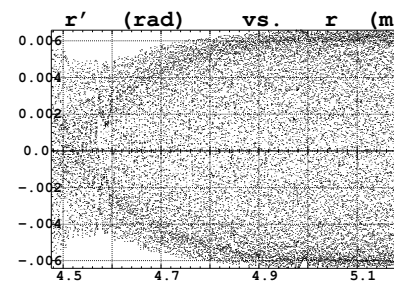
ϵ_z -damping $\propto \sqrt{\beta\gamma}$



B/ Case of numerical calculation of field derivatives

Method : $d^n B/ds^n$ computed from numerical interpolation between small set of field values surrounding the particle.

Interest : allows easier implementation of new (exploratory !) types of (r, θ) field dependences, in Zgoubi.

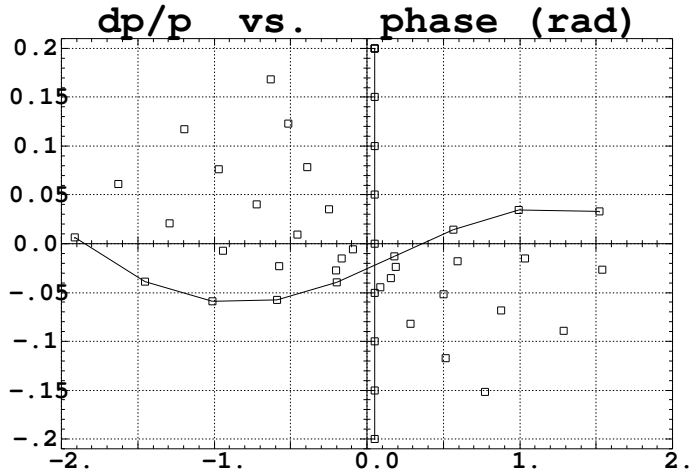


A large step size is used, in both cases : 0.5 cm, with success.

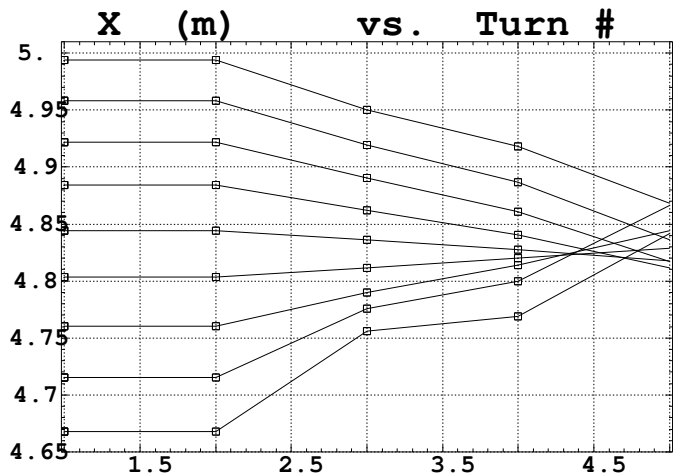
This is allowed thanks to the accuracy of the integration method.

NuFact needs : a rough test of bucket acceleration, using “FFAG” procedure

In fact, a similar problem : PRISM
 Bunch rotation in 5 turns (simplistic
 RF wave, sine-like) :



Radial compression :

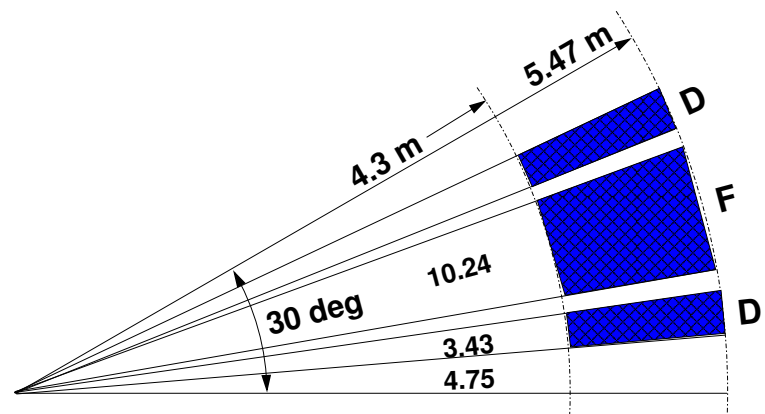


Data file - 45deg. period (drift-DFD-drift) :

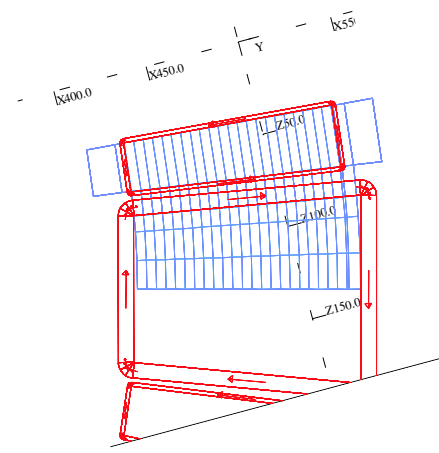
```
'OBJET' * c.o., constant Gap *
226.8235847      68MeV/c muon
2
2 1
499.377      0.      0.      0.      0.      1.2      'b'
1 1 1 1 1 1 1 1 1
'FFAG'
0
3 45.      500.      NMAG, AT=tetaF+2tetaD+2Atan(XFH/R0), R0
18.17      0.      -0.717      5.      mag 1 : ACNT, dum, B0, K
6.3      0.      EFB 1 : lambda, gap const/var=0/>0
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
1.23      0.      1.E6      -1.E6      1.E6      1.E6
6.3      0.      EFB 2
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
-1.23      0.      1.E6      -1.E6      1.E6      1.E6
0. -1      EFB 3 : inhibited by iop=0
0 0.      0.      0.      0.      0.      0.      0.
0.      0.      0.      0.      0.      0.
22.5      0.      3.2      5.      mag 2 : ACNT0.3927rad, m, B0, K,dummies
6.3      0.      EFB 1
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
3.      0.      1.E6      -1.E6      1.E6      1.E6
6.3      0.      EFB 2
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
-3      0.      1.E6      -1.E6      1.E6      1.E6
0. -1      EFB 3
0 0.      0.      0.      0.      0.      0.      0.
0.      0.      0.      0.      0.      0.
26.83      0.      -0.717      5.      mag 3 : ACNT, dum, B0, K
6.3      0.      EFB 1
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
1.23      0.      1.E6      -1.E6      1.E6      1.E6
6.3      0.      EFB 2
4 .1455      2.2670      -.6395      1.1558      0.      0.      0.
-1.23      0.      1.E6      -1.E6      1.E6      1.E6
0. -1      EFB 3
0 0.      0.      0.      0.      0.      0.      0.
0.      0.      0.      0.      0.      0.
0      2      125.      KIRD anal/num (=0/2,25,4), resol(mesh=step/re
5      integration step size (cm)
```

3 Tracking in 3-D field maps

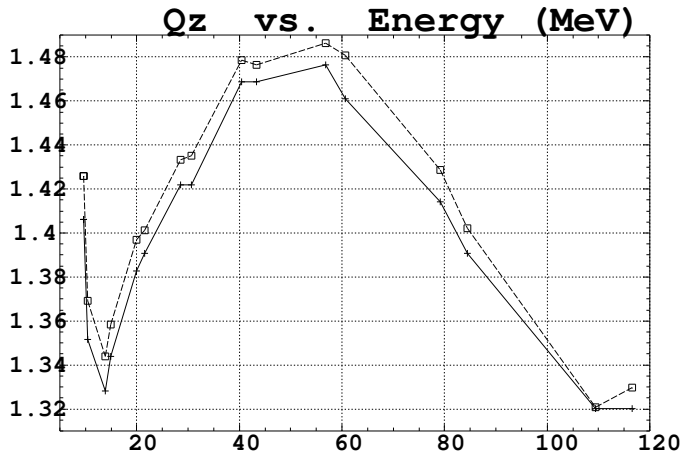
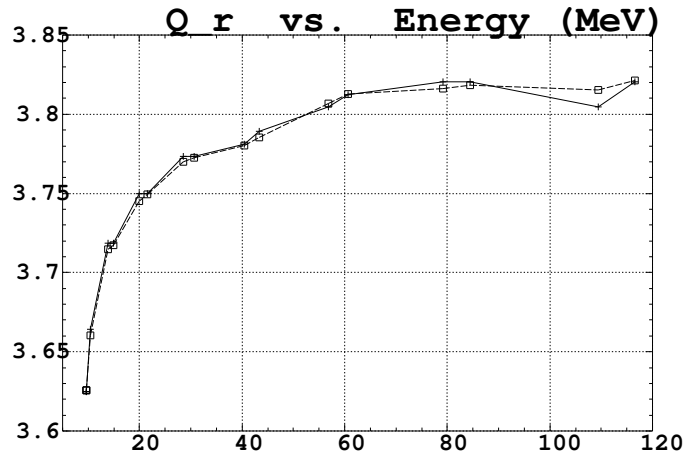
[Details in : CERN NuFact Note 140 (2004)]



Geometry of the DFD sector triplet and 30 degrees sector cell.



Geometry of TOSCA field map, covering half the angular extent.

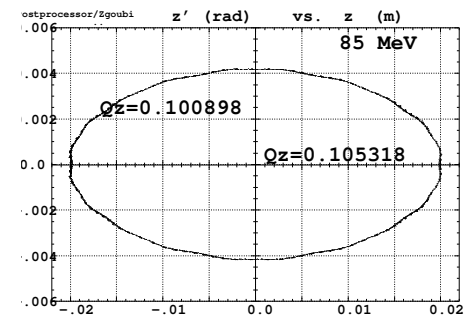
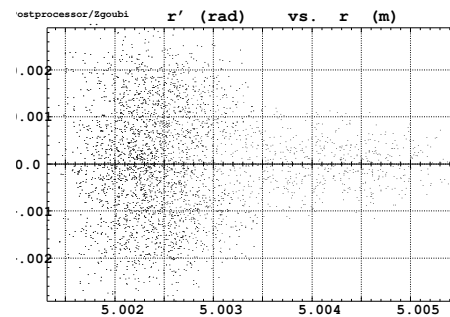
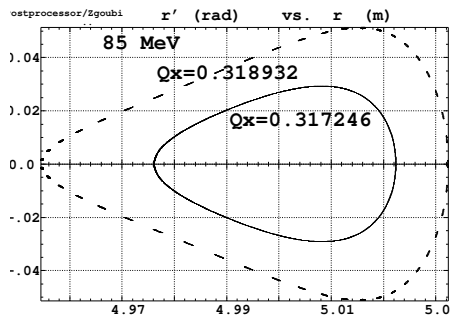
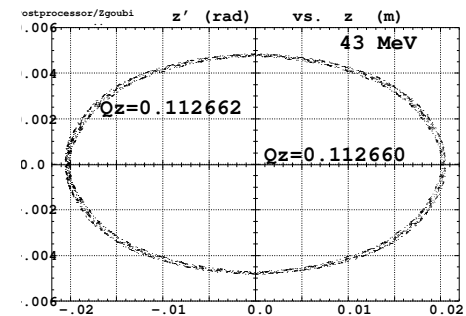
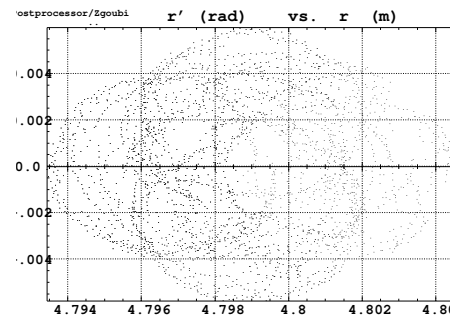
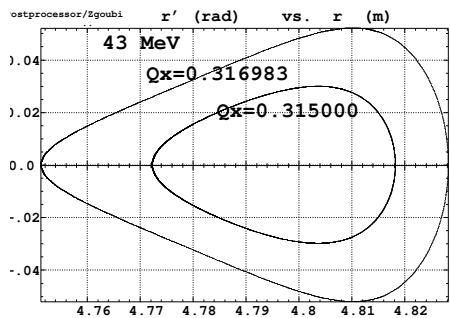
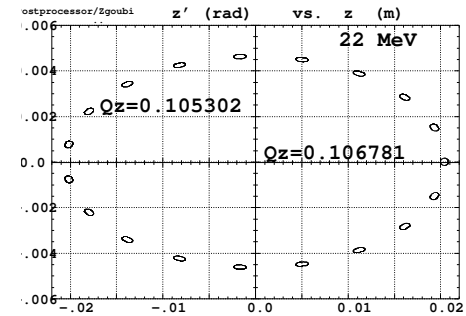
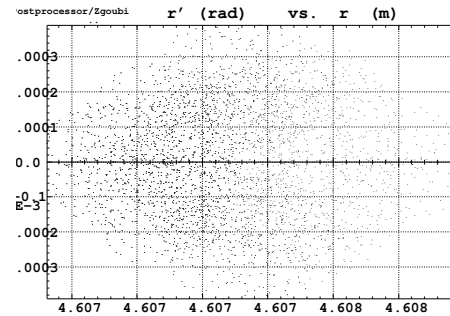
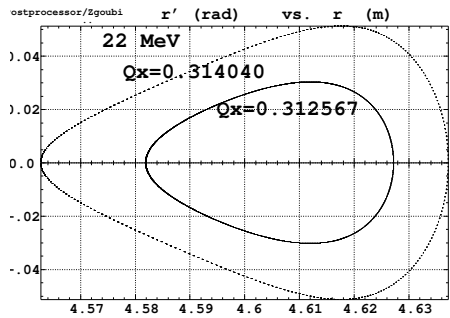


Radial tune (left plot) and axial tune (right) as a function of energy, as obtained using RK4 integration (solid lines/crosses), or using Zgoubi (dashed line/squares).

6-D tracking, using 3-D magnetic field maps (cont'd)

The results below make clear that the symplecticity is very good (“precision to order Δs^6 ”) even though using field maps.

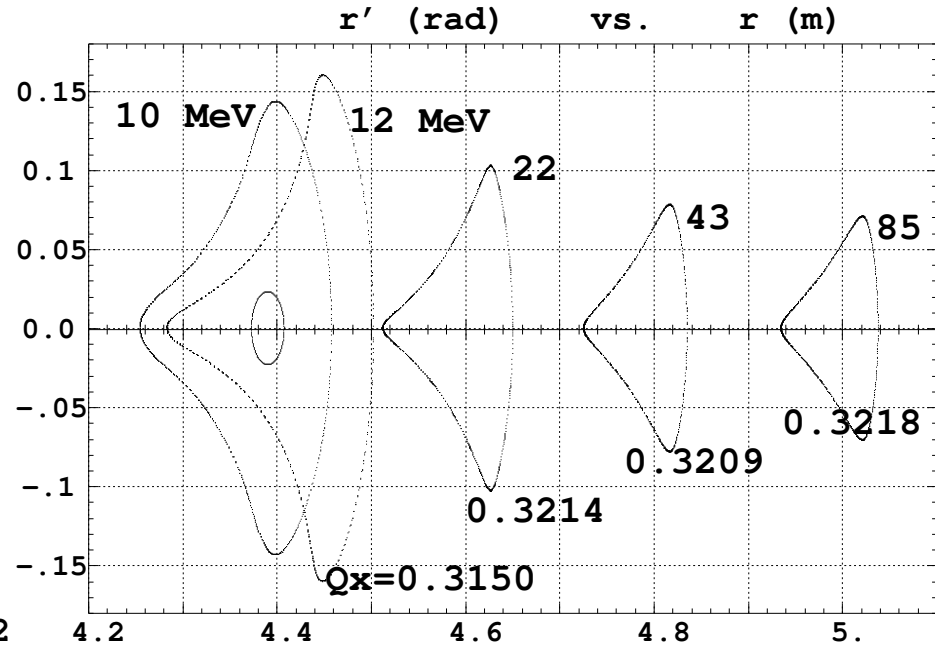
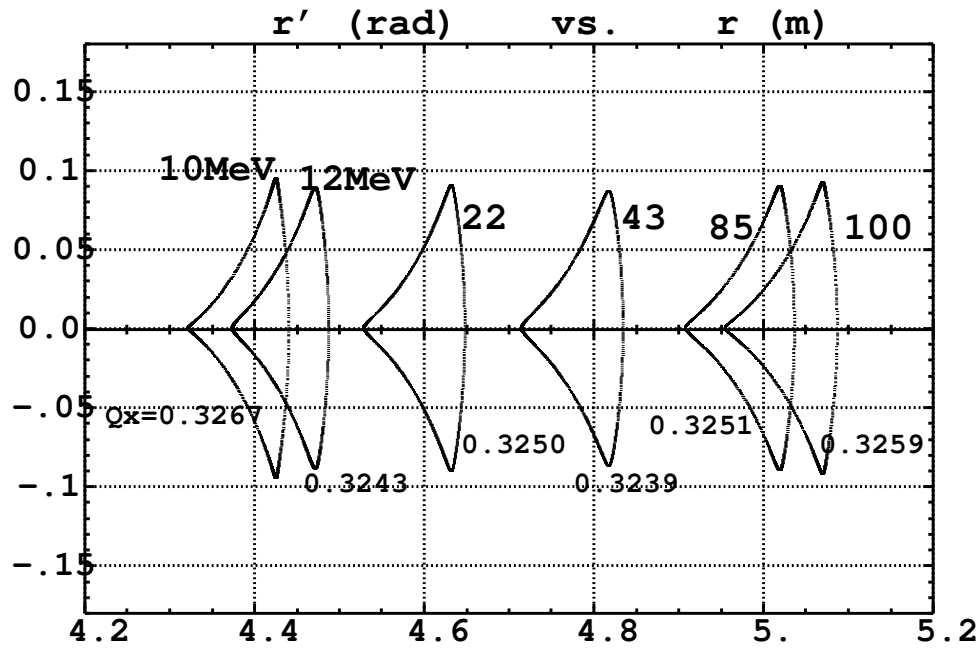
Note that, the mesh size needs be very small, \approx mm, and the integration step size must stick.



Horizontal motion, near stability limit.
The inner motion is 3500 pass in a cell
the outer one is 4700.

Right column : vertical phase-space for $z_0 = 2$ cm with $r_0 = r_{closed\ orbit}$.
Left column : corresponding horizontal motion. 3200 periods.

Tracking in scaling FFAG(cont'd) - Comparison between “FFAG” procedure and 3-D field maps



**150MeV FFAG : horizontal phase space, the limits of stable motion, for 5 energies.
For comparison : tracking with geometrical model (left), or using TOSCA map (right).**

Scaling FFAG, 6-D tracking simulations - CONCLUSIONS

Tools ready for more :

muon acceleration, non-relativistic β (capture, cooling, etc.)
as well as -relativistic (e.g., using fixed RF freq.)

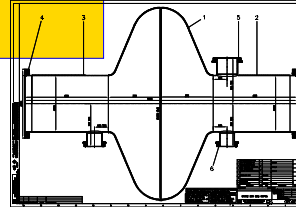
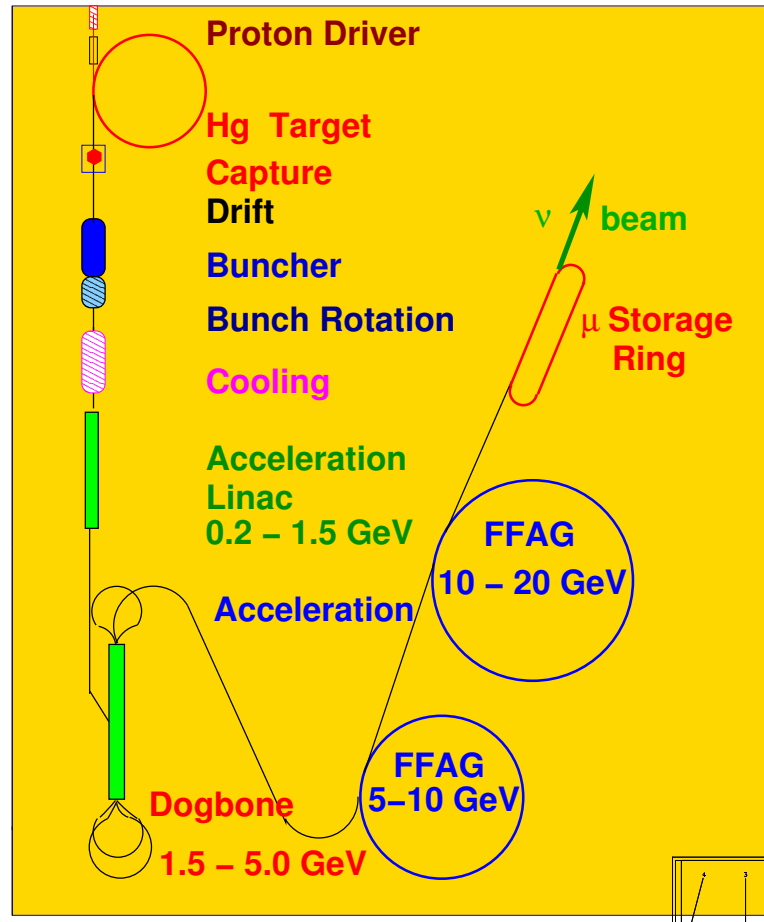
hadron machines, non-relativistic β (therapy, C, p)
as well as relativistic (p-driver)

4 6-D transmission in the ISS-spec, linear FFAGs

Lattice parameters

ISS data :

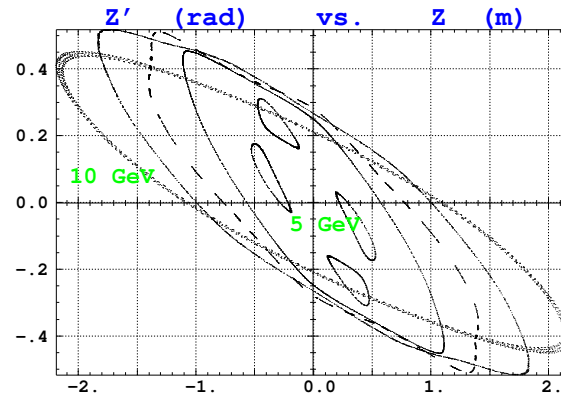
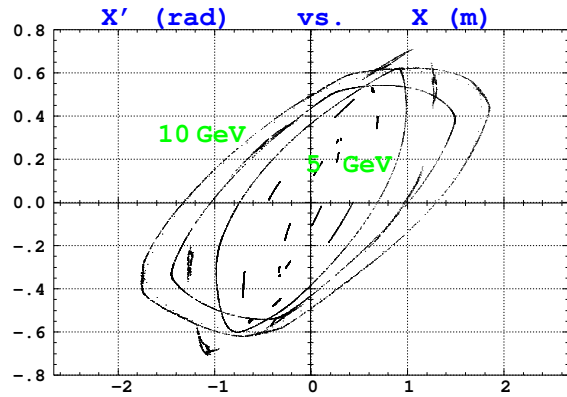
Energy (GeV)	2.5→5	5→10	10→20
No. of cells	52	64	84
No. of turns	4.8	9	15.5
Circumference (m)	210	285	410
D magnet :			
length (m)	0.6	0.75	0.93
radius (cm)	13.2	9.8	7.5
pole tip field (T)	4.7	6.1	7.5
F magnet :			
length (m)	0.95	1.20	1.45
radius (cm)	20.7	16.6	13.3
pole tip field (T)	2.7	3.4	4.2
No. of cavities	42	(46)	54
Zgoubi :			
E gain/cav. (MeV)	12.5	12.4	12.7



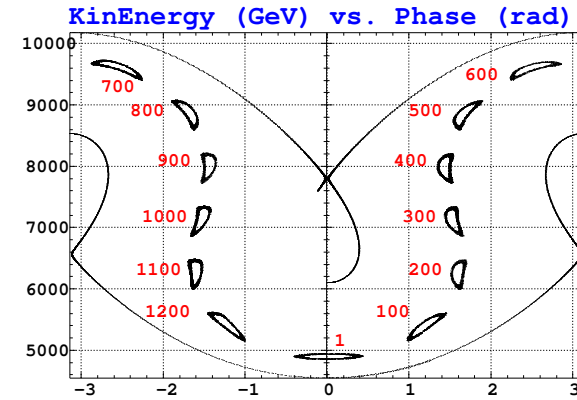
- F/D quadrupole doublet ⇒ ‘ ‘MULTIPOLE’ ’
- Acceleration : one cavity every cell. $\hat{V} = 9.928$, Rf freq. 202.332.

6-D transmission in the ISS linear FFAGs (cont'd) - Tracking results

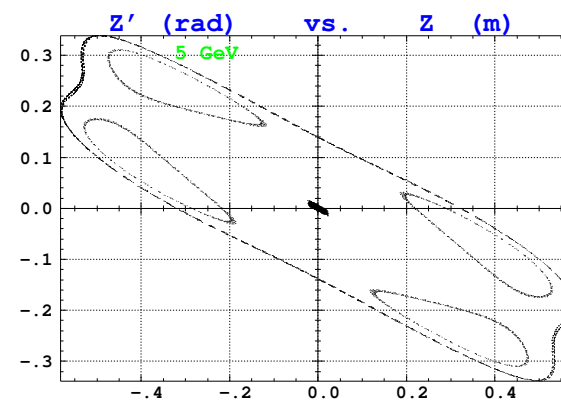
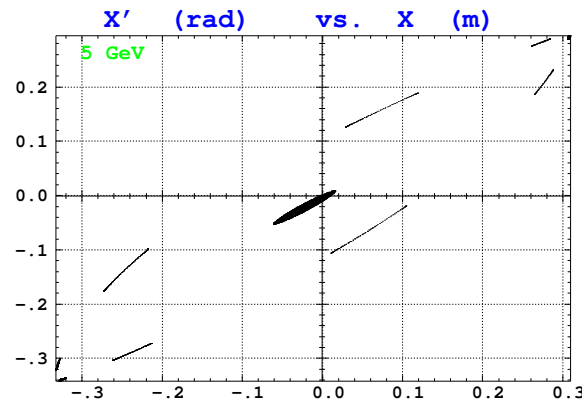
Huge geometrical acceptance



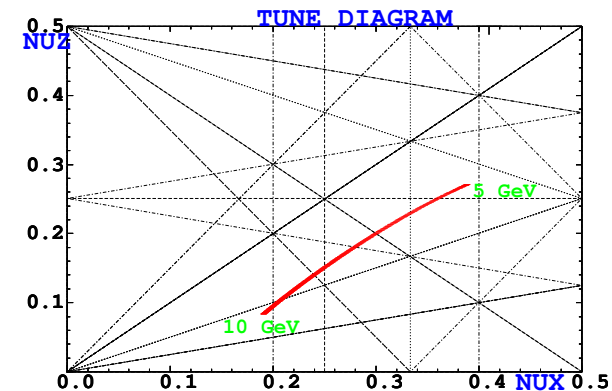
phase-Energy motion



Injected beam ($> 3\pi$ cm/0.05 eV.s) is well within stability limits



Beam path in tune diagram



Transmission in the 5 \rightarrow 10 GeV ring with 3 cm / 0.05 eV.s injected beam yields large momentum spread due to longitudinal/transverse coupling.

In a near future : effects of field/aligment errors \rightarrow DA, tolerances, resonance crossing...

Linear FFAG (cont'd) - 6-D transmission in e-model "EMMA"

Hypothesis, geometry, fields, tof

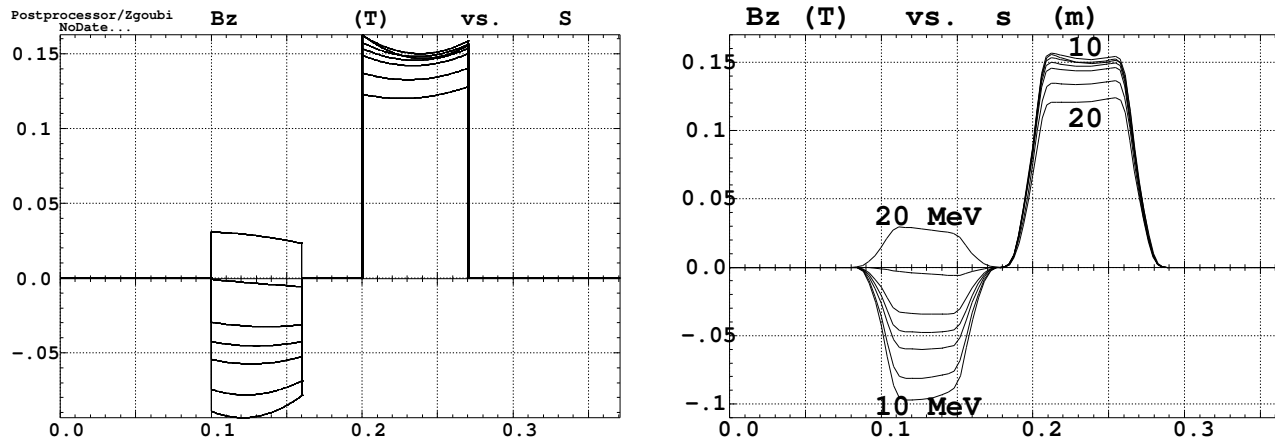


Figure 1: Field on closed orbits along the cell, at various energies, without (left) and with (right) fringe fields.

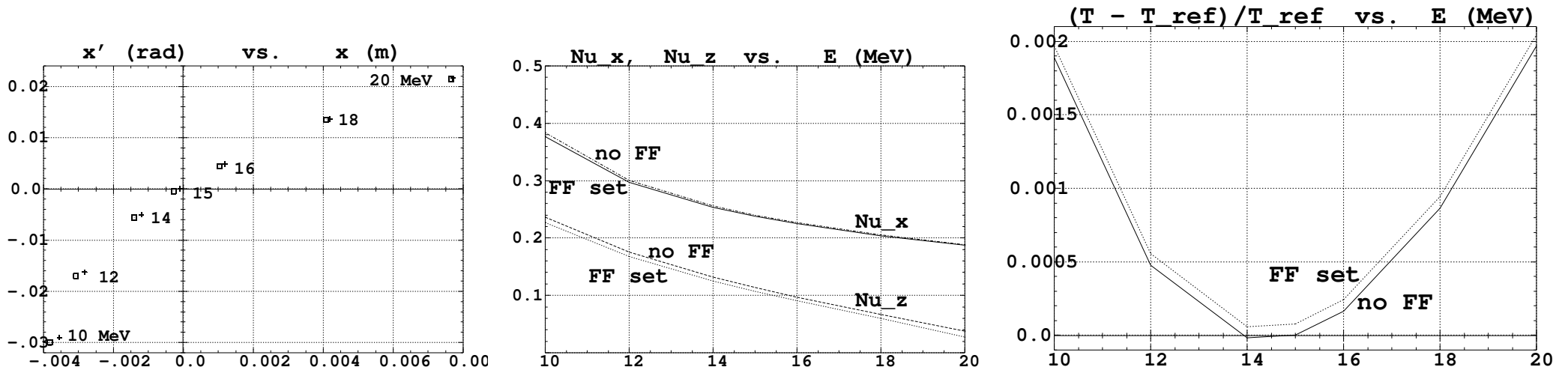


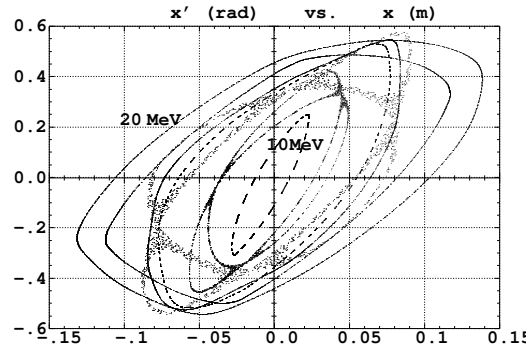
Figure 2: Left : energy dependence of the horizontal closed orbits in (x, x') phase space, with (squares) or without (crosses) fringe fields.

Middle : Tunes as a function of energy. Right : $(T - T_{Ref})/T_{Ref}$ as a function of energy.

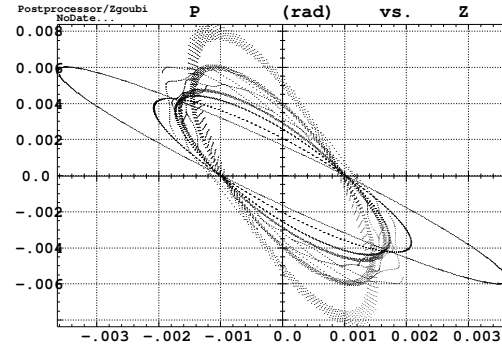
Stability limits

2000-cell H stability limits, about 5% precision in x , at 10, 12, 14, 15, 16, 18 and 20 MeV.

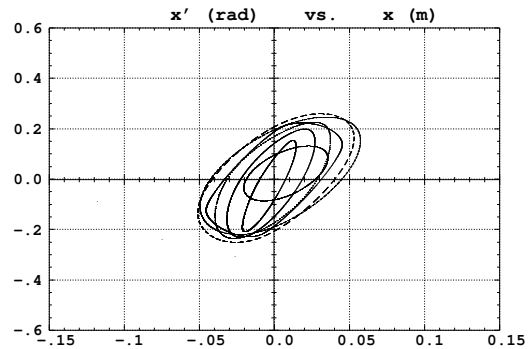
Pure horizontal motion, no fringe fields :



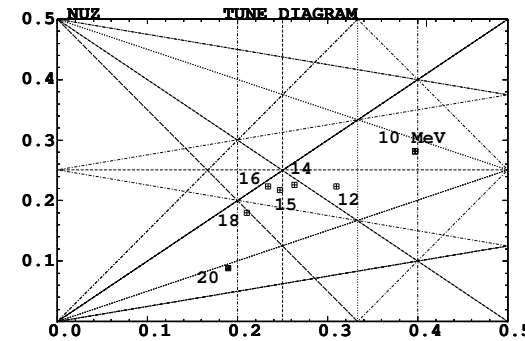
Vertical motion in the $\approx 200 \pi$ (norm.) region :



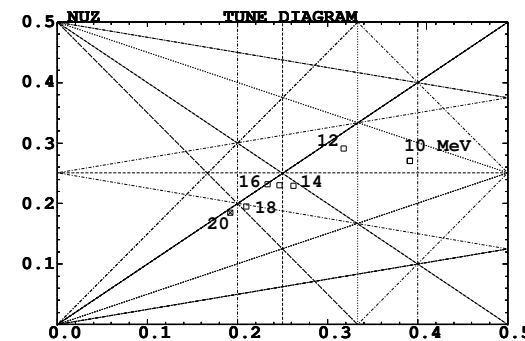
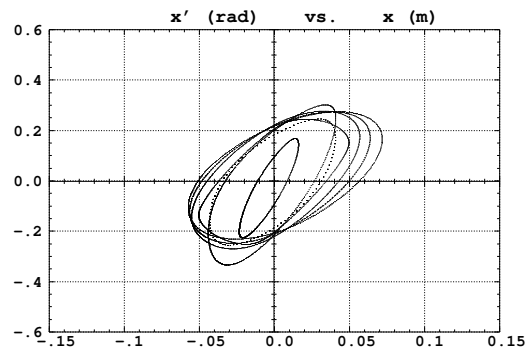
In presence of very small z motion, no fringe fields :



Cell tunes at stability limits (resonance lines up to 5th order are represented) :



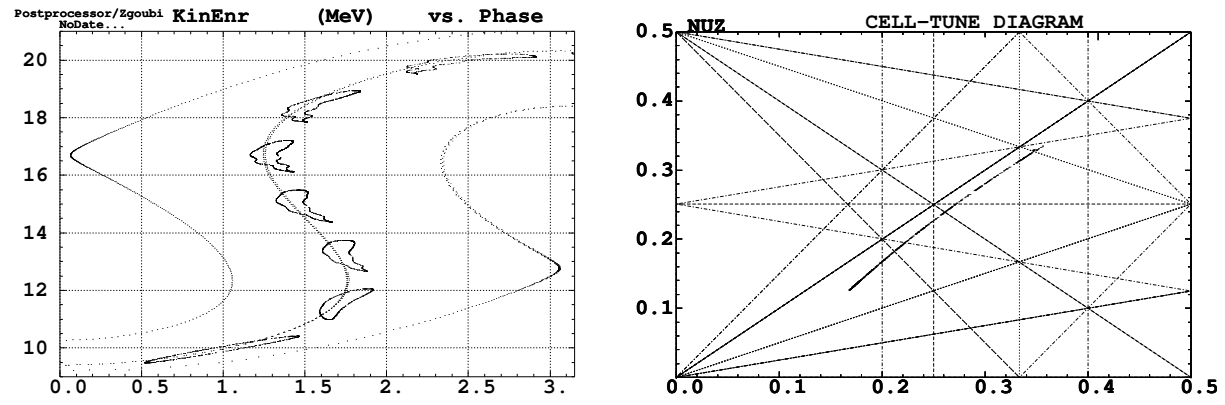
In presence of very small z motion, fringe fields set :



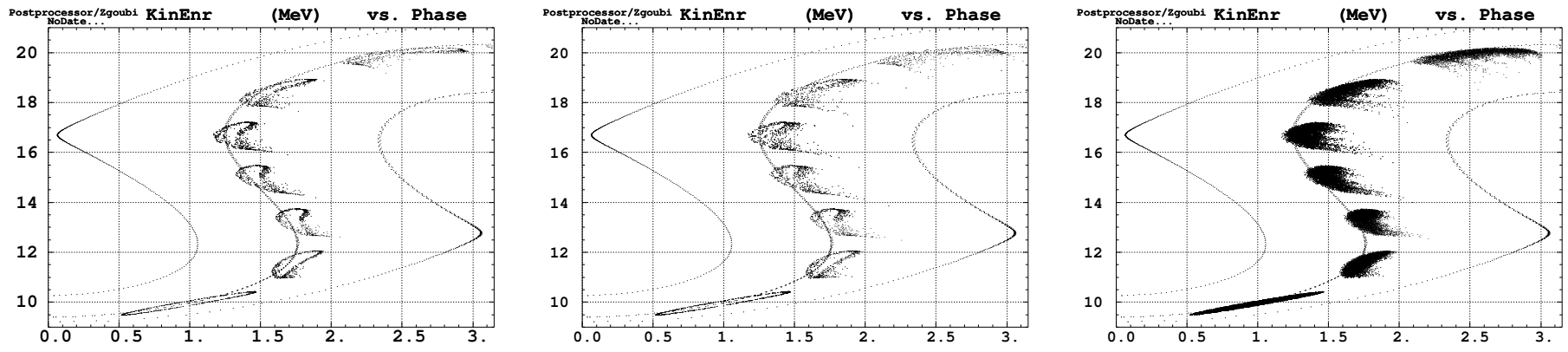
Longitudinal motion

Serpentine

Acceleration of elliptical rings, zero transverse emittance.



Acceleration of elliptical rings (zero transverse emittances) from 10 to 20 MeV in 125 cavity passes. Ellipses are represented each 25 cavity passage. Three particular trajectories show the separatrix and the bunch cog. Voltage : 70 kV peak, RF freq. : 1.3552 GHz.



The sensible effect of launching a bunch with non-zero transverse size. Left : $\epsilon_{x,z} = 90 \pi$ mm.mrad norm. Middle : $\epsilon_{x,z} = 200 \pi$ mm.mrad norm. Right : full 6-D acceleration.

Linear FFAG, 6-D tracking simulations - CONCLUSIONS

6-D tracking tools ok, resistant to field linearities

6-D transmission simulations in ISS rings go on

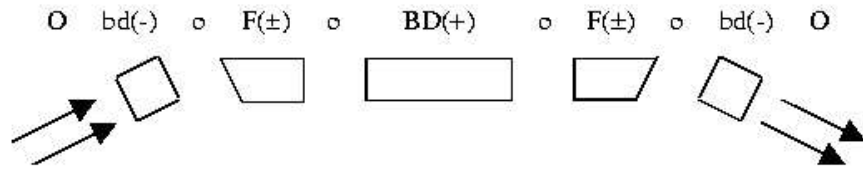
Planning error studies, tolerances, etc.

5 6-D transmission in I-FFAG[-I]

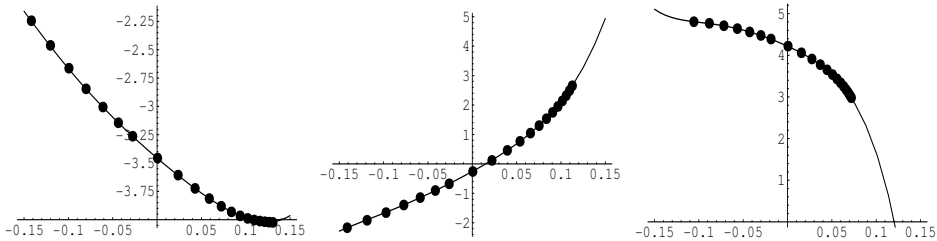
Electron model

Acceleration based on isochronous FFAG lattice
 Best use of the RF : on-crest acceleration
 (cyclotron-like)

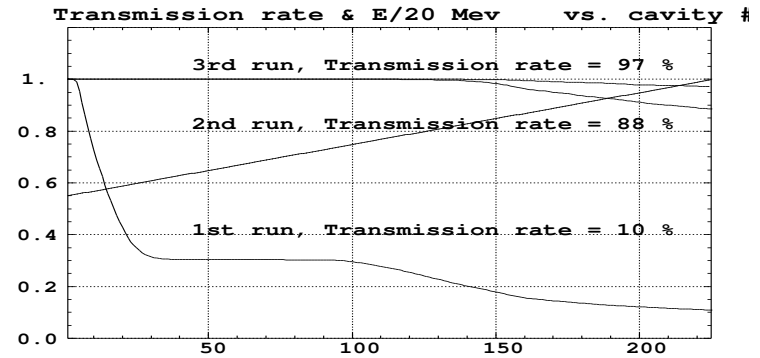
45 cells, 15 turns from 11 to 20 MeV, 40 kV/cell



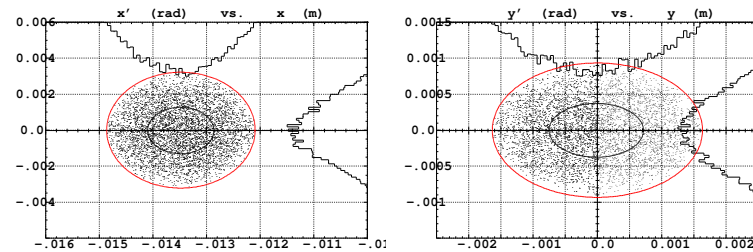
Difficulty for the integrator (the guy and the method) : tracking in highly non-linear fields (yet, comparable to scaling case)



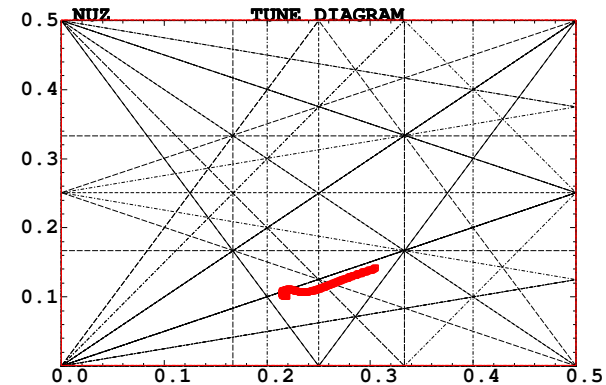
Iterative method



97% transmission for
 $\epsilon_{x/z} = 97/33 \pi$ mm.mrad normalized

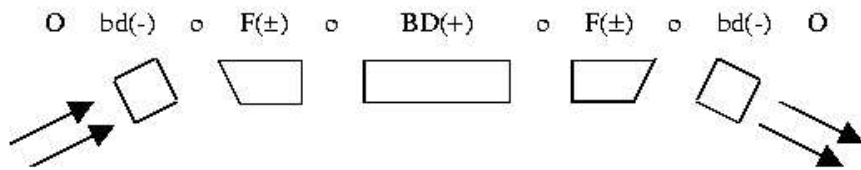


Beam path in tune diagram

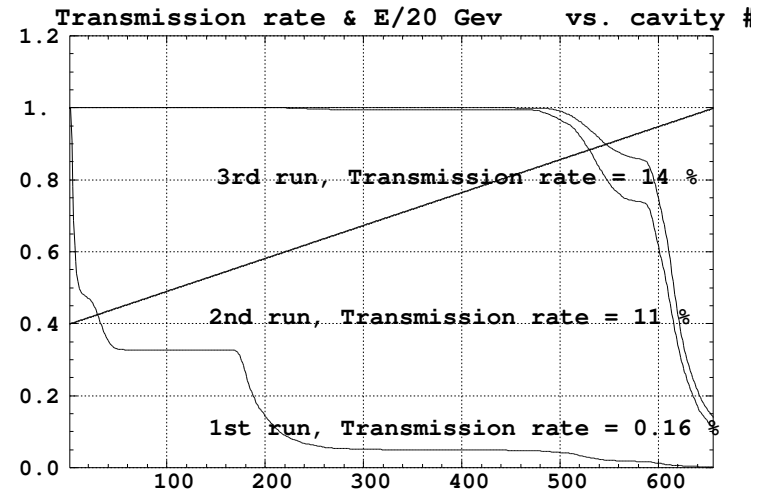


6-D transmission simulations in I-FFAG (cont') - Muon

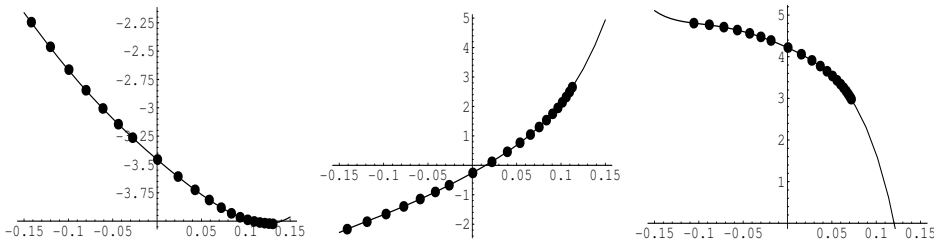
Acceleration based on isochronous FFAG lattice
 Best use of the RF : on-crest acceleration
 (cyclotron-like) using 200 MHz SCRF



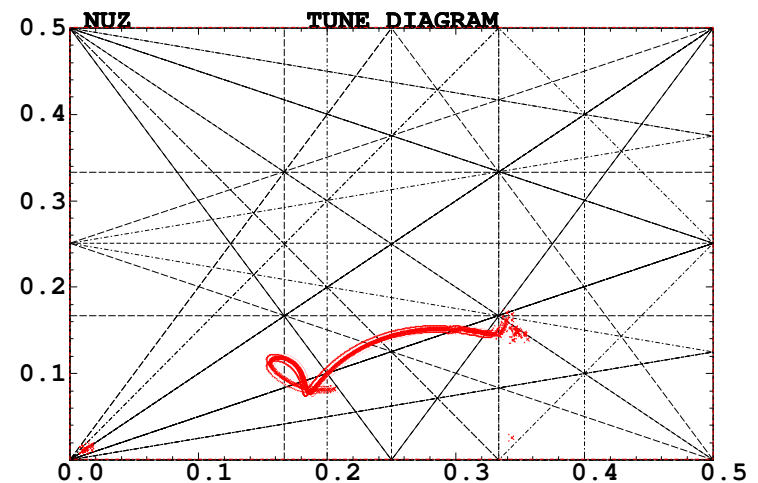
Iterative method



Difficulty for the integrator (the guy and the method) : tracking in highly non-linear fields (yet, comparable to scaling case)



Beam path in tune diagram



Isochronous FFAG, 6-D tracking simulations - CONCLUSIONS

6-D tracking tools ok, resistant to strong non-linearities

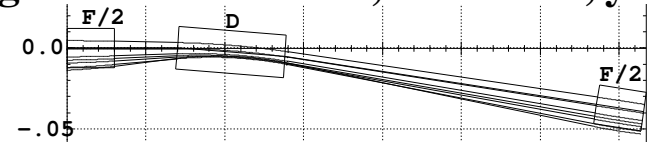
I-FFAG muon acceleration needs lattice optimisations

6-D transmission simulations in isochronous FFAG with insertions (talk by Rees, tomorrow) being worked at.

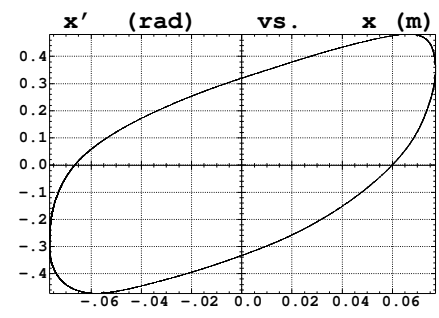
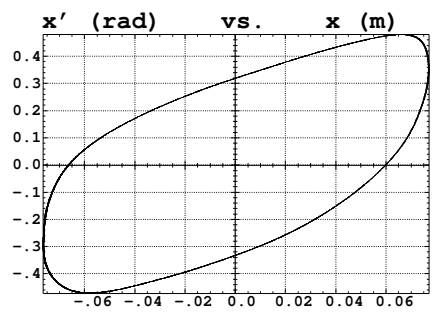
6 On computing precision

In collaboration with E. Forest (KEK), these last weeks
 All serious integration methods, Zgoubi_time, Runge-Kutta_z or_time, Yoshida_z, yield comparable behavior up to motion stability limits.

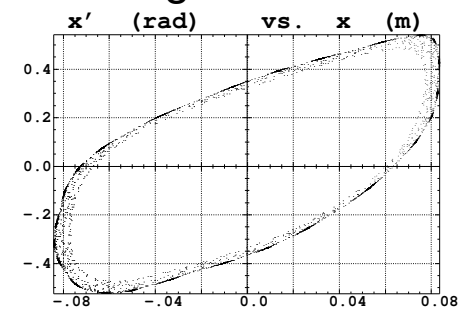
A comparison, which yields excellent agreement : Zgoubi-4 or 6 with RK-4 or -6, close to stability limit in a linear FFAG FODOcell.



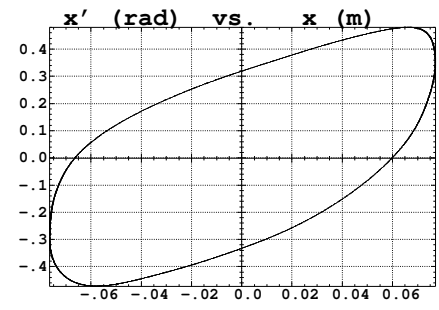
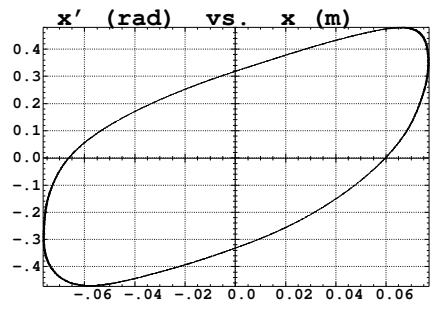
Low precision integration at the last step (usually an iterative process, in order to reach the exact end of the optical element) is liable to yield overall precision which is that of the precision of integration at the last step.



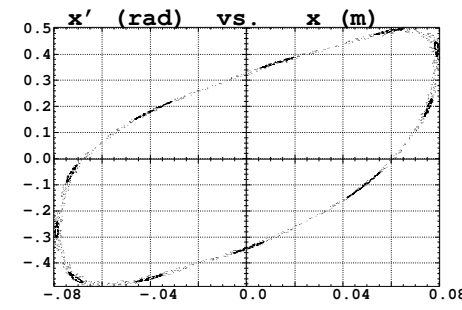
Zgoubi-4 with 40 steps (left), or Zgoubi-6 with 10 steps (right). Both time-integration.



Zgoubi-6 with 20 steps, but for the last step, performed by iterations using with Zgoubi-2.



RK-4 with 10 steps (left), or RK-6 with 10 steps (right). Both z-integration.



RK4 with 10 steps, but for the last step, performed with RK-1.

7 Various temporary conclusions

- The optical elements developed for 6-D simulations are operational, functioning well, and allow high speed tracking
- RF needs / TOF handling needs, are well satisfied : synchronous acceleration, statinary bucket, gutter
- Ray-tracing is routinely used to assess DA in the three types of lattices, linear, isochronous, scaling
- Preliminary 6-D transmission simulations yield results that show good agreement with low order behavior drawn from analytical and matrix methods (tunes, detunings, etc.)

FUTURE

- Give detailed transmission figures and other characterisations concerning presently studied lattices (NuFact, low- β machines)
- Stick to the evolution os lattice studies
- Error studies - field and positioning, tolerances, resonance crossing
- Develop further ray-tracing based optimisation methods

NEEDS :

- Introduce some standardisation in these tracking studies : what are the questions, how are they to be answered.