
Magnet design studies for RACCAM

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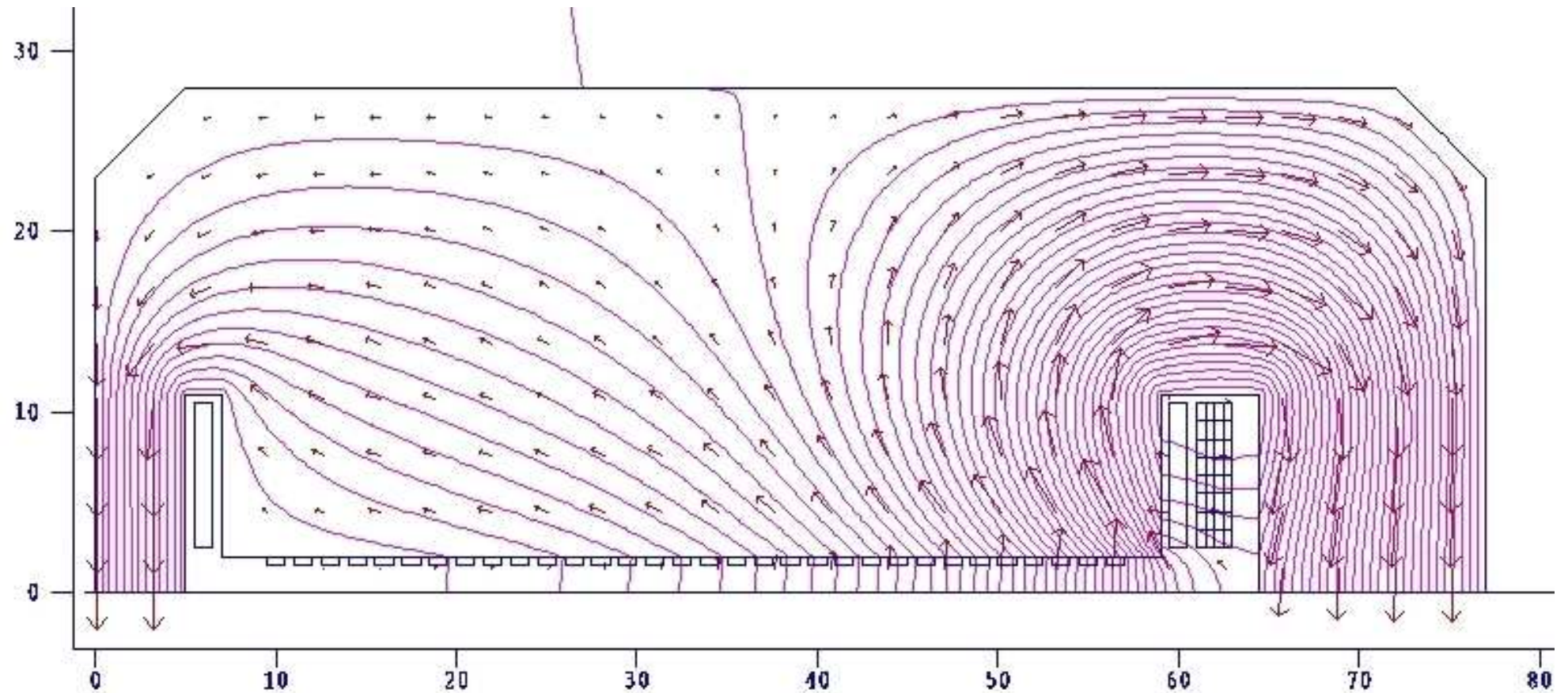
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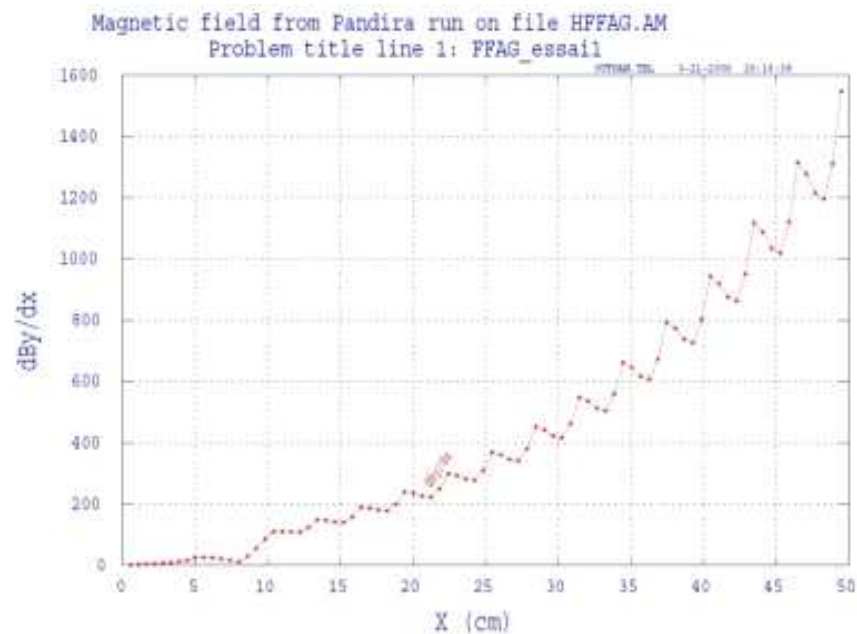
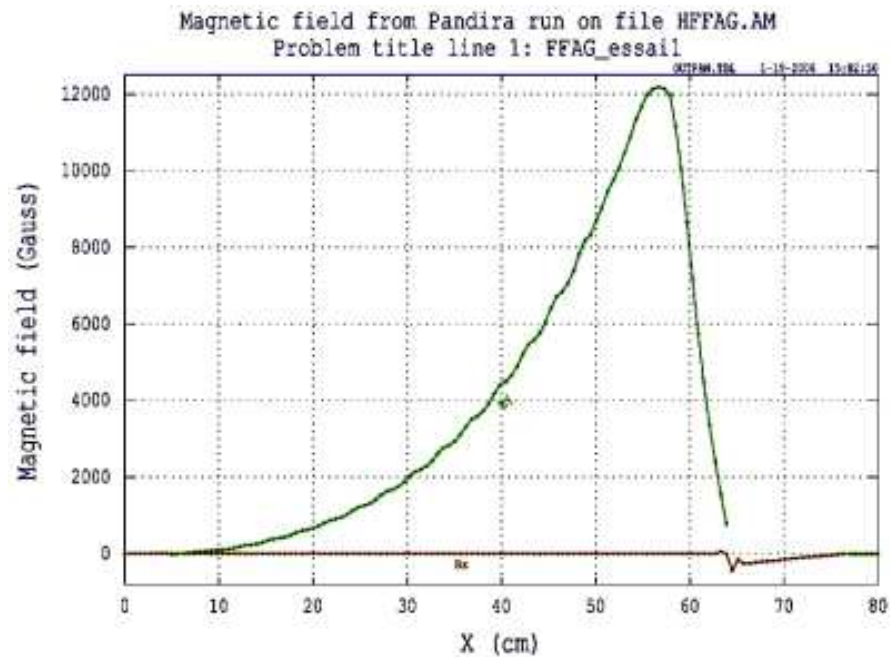
Introduction

- Pointed out the possibility of using a distribution of conductors which allow to vary the k
- Started from the KURRI spiral dipole design



■ First POISSON simulation shows :

- @ current law doesn't follow field law $B_0\left(\frac{x}{x_0}\right)^k$,
- @ primary uniform field necessary,
- @ field oscillations along the median plane.



Field components of a series of conductors

- $\{\xi, \eta\}$ are the conductor coordinates and $\{x, 0\}$ the observation coordinates in the median plane

- Field components deduced from basic Biot and Savart law :

$$\text{@ vertical : } B_y(x, \xi, \eta) = \frac{\mu_0 I}{4\pi} \frac{x - \xi}{(x - \xi)^2 + \eta^2}$$

$$\text{@ horizontal : } B_x(x, \xi, \eta) = \frac{\mu_0 I}{4\pi} \frac{\eta}{(x - \xi)^2 + \eta^2}$$

- For a series of N conductors :

$$\text{@ vertical : } B_y(x, \{\xi_i, \eta_i\}) = \frac{\mu_0}{4\pi} \sum_{i=1}^N I_i \frac{x - \xi_i}{(x - \xi_i)^2 + \eta_i^2}$$

$$\text{@ horizontal : } B_x(x, \{\xi_i, \eta_i\}) = \frac{\mu_0}{4\pi} \sum_{i=1}^N I_i \frac{\eta_i}{(x - \xi_i)^2 + \eta_i^2}$$

Field produced in the median plane

- The system is simply reduced to a linear matrix equation : $A_{x,y} \times I + B_y = 0$

- with a series of P observation points : $B_{x,y} = \begin{pmatrix} B_{x,y1} \\ B_{x,y2} \\ \vdots \\ B_{x,yP} \end{pmatrix}$

- with a series of N conductors : $I = \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix}$

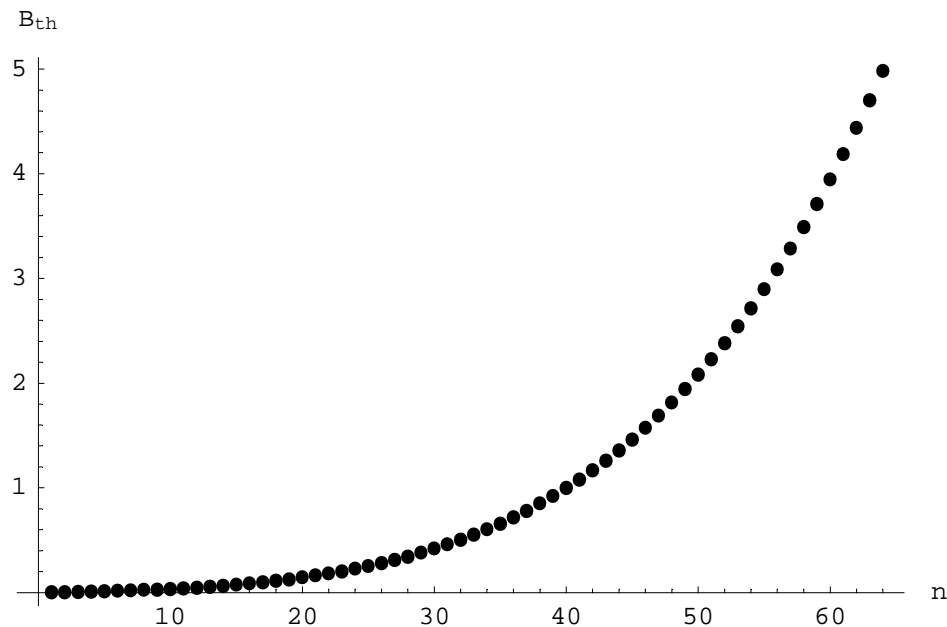
- and $A_y = \begin{pmatrix} \frac{x_1 - \xi_1}{(x_1 - \xi_1)^2 + (y_1 - \eta_1)^2} & \cdots & \frac{x_1 - \xi_N}{(x_1 - \xi_N)^2 + (y_1 - \eta_N)^2} \\ \vdots & \ddots & \vdots \\ \frac{x_P - \xi_1}{(x_P - \xi_1)^2 + (y_P - \eta_1)^2} & \cdots & \frac{x_P - \xi_N}{(x_P - \xi_N)^2 + (y_P - \eta_N)^2} \end{pmatrix}$ or $A_x = \begin{pmatrix} \frac{\eta_1}{(x_1 - \xi_1)^2 + (y_1 - \eta_1)^2} & \cdots & \frac{\eta_N}{(x_1 - \xi_N)^2 + (y_1 - \eta_N)^2} \\ \vdots & \ddots & \vdots \\ \frac{\eta_1}{(x_P - \xi_1)^2 + (y_P - \eta_1)^2} & \cdots & \frac{\eta_N}{(x_P - \xi_N)^2 + (y_P - \eta_N)^2} \end{pmatrix}$

- The matrix has dimensions P×N. If P>N, the system to solve is "overdetermined"

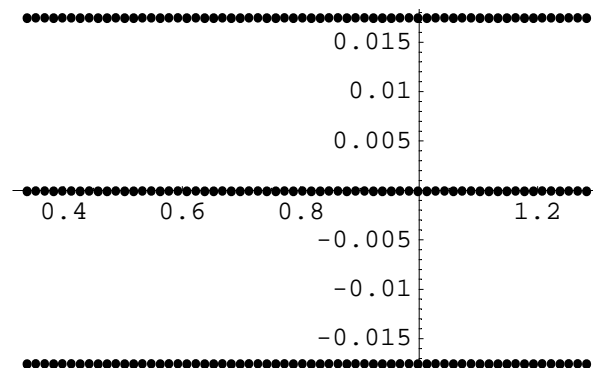
$$\Rightarrow A_{x,y} \cdot I = B \text{ is then transformed into : } A_{x,y}^T \cdot A_{x,y} \cdot I = A_{x,y}^T \cdot B$$

Example of magnetic field to produce

- Example field law from 0.1T to 1.8T (not based on a specific FFAG design)

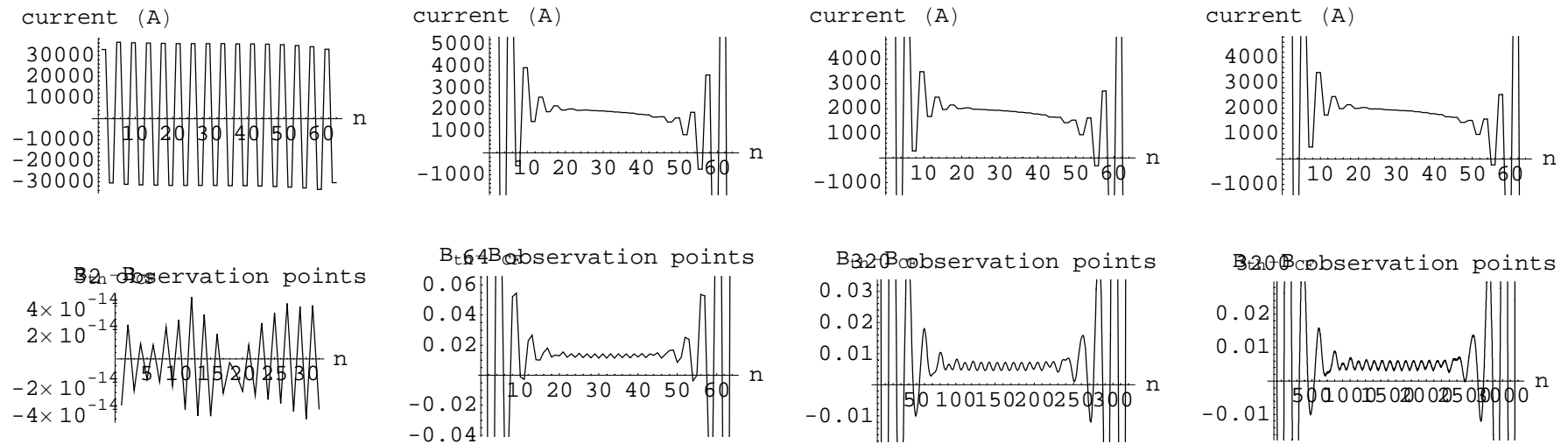


- Geometric structure of the model : conductors at top and bottom and observation points, where the field is calculated in the middle.

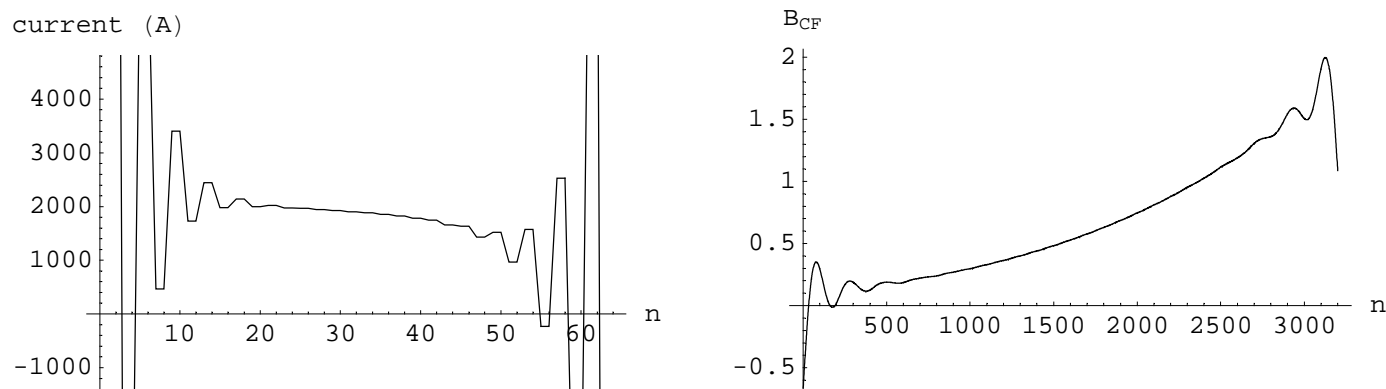


ConductorField example (1)

- The length of the model (with 32 conductors) is exactly equal to the length of the "good field region" l_{gfr} .

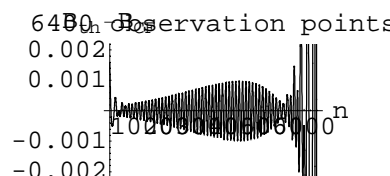
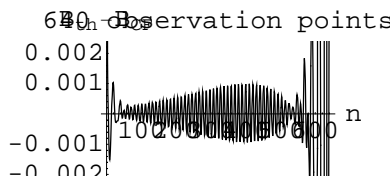
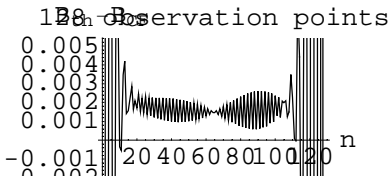
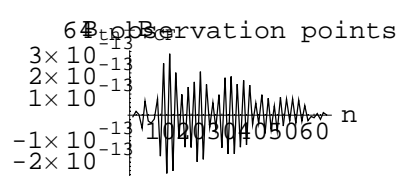
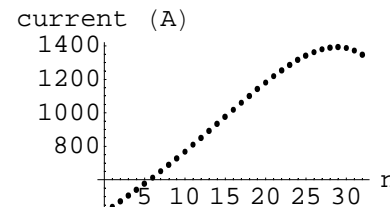
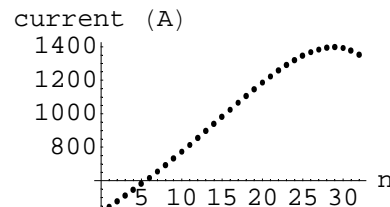
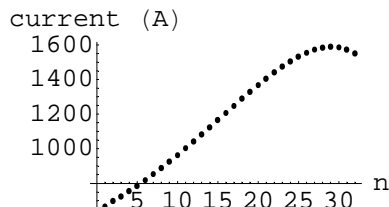
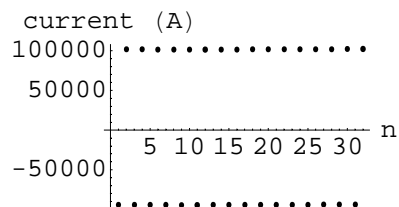


Case with best precision and final current values

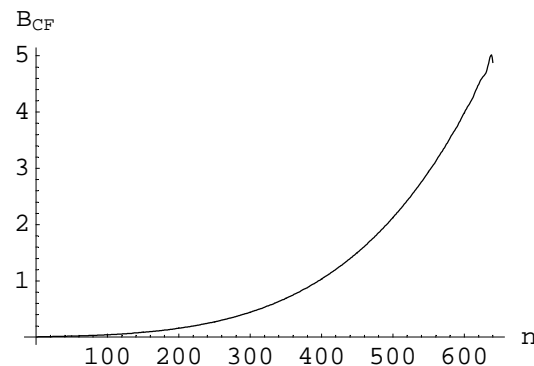
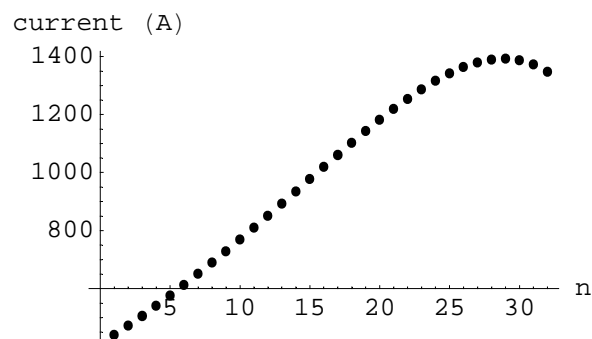


ConductorField example (2)

- The length of the model is enlarged equally in opposite directions, for a total enlargement of l_{gfr}
- The density of conductors don't change.

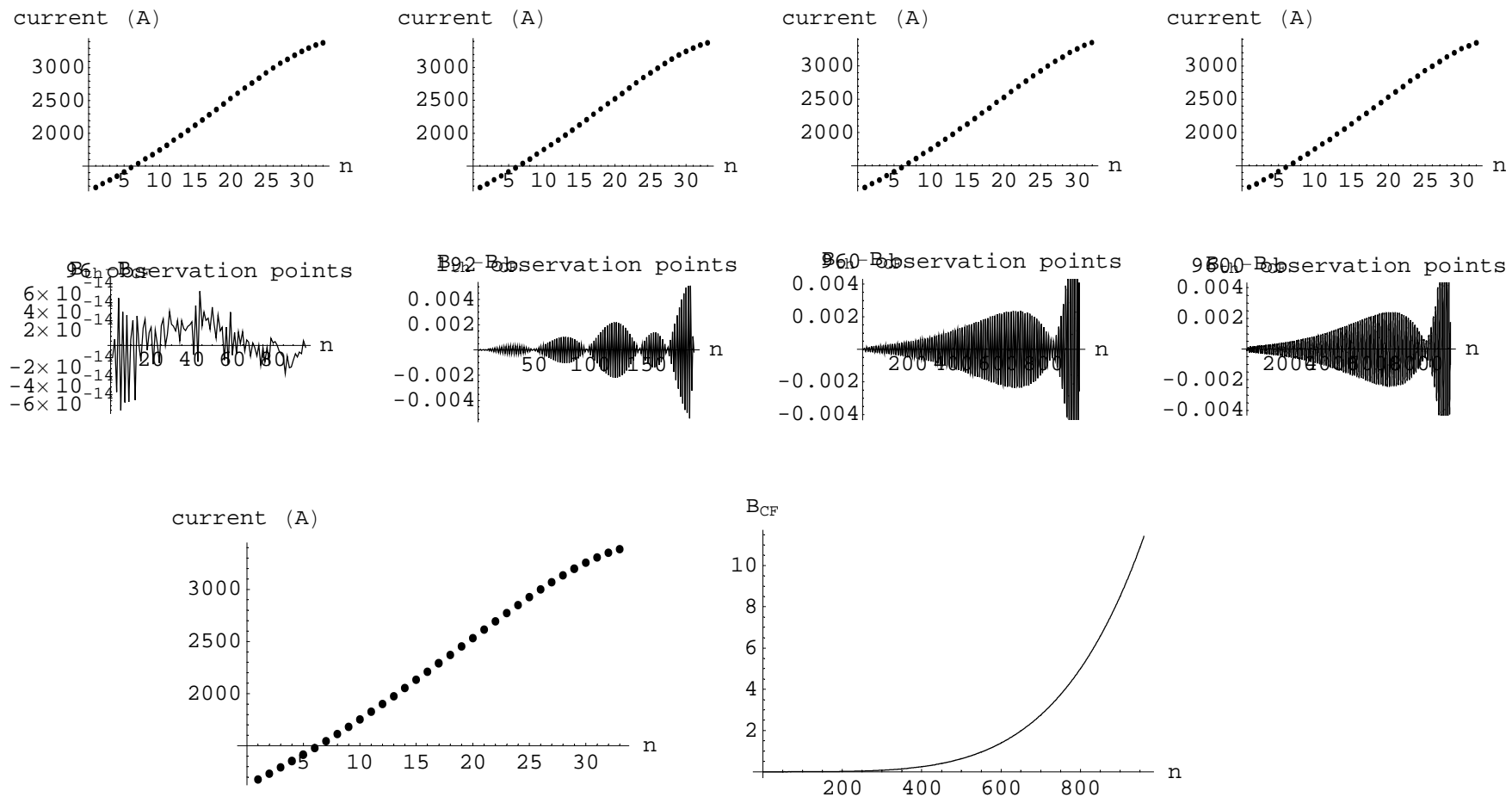


Case with best precision and final current values



ConductorField example (3)

- Enlargement of $2 l_{\text{gfr}}$, density of conductors still the same.

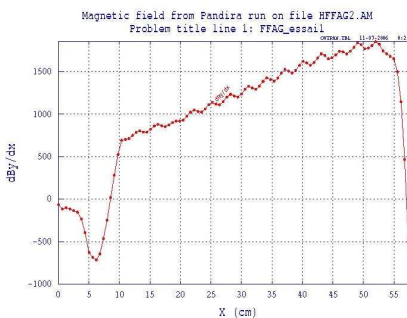
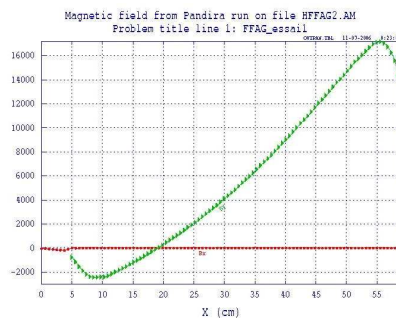


- Conclusion : this function seem to work well with a sufficiently large model compared to the length of the "good field region".

Poisson simulations

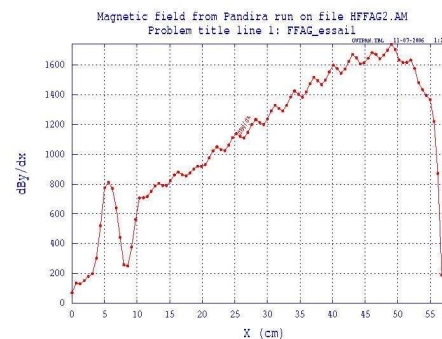
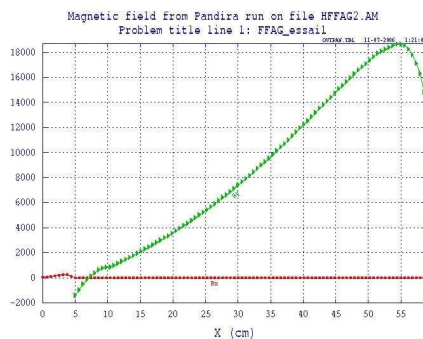
- BUT!... A Poisson simulation shows this can only be considered as a **first approximation**.

*Current distribution from ConductorField function **without correction***



- A primary uniform field has to be applied to give the correct values of the field law.

*Current distribution from ConductorField function **with uniform field added***



- Is improvement necessary?

Field produced by a profiled conducting layer

- Problems with conductors : oscillating field.
- As a first solution to this problem : extend one conductor to the whole surface of the pole.
- Fit calculation on pole shape parameter to match the desired field law.
- Expressions of field components include altitude of conductor surface :

$$\mathbf{B}_{py}[\xi_-] = \mathbf{Factor@Together@Integrate} \left[\frac{-\mathbf{x} + \xi}{\eta^2 + (-\mathbf{x} + \xi)^2}, \xi, \{\eta, h[\xi], g\} \right]$$

$$\mathbf{B}_{px}[\xi_-] = \mathbf{Factor@Together@Integrate} \left[\frac{\eta}{\eta^2 + (-\mathbf{x} + \xi)^2}, \xi, \{\eta, h[\xi], g\} \right]$$

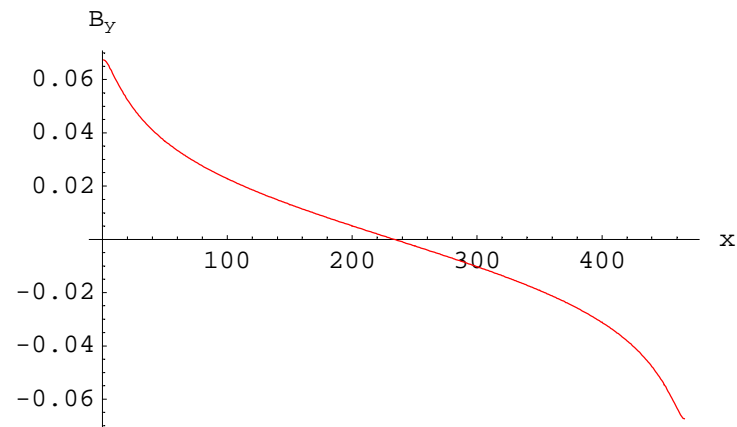
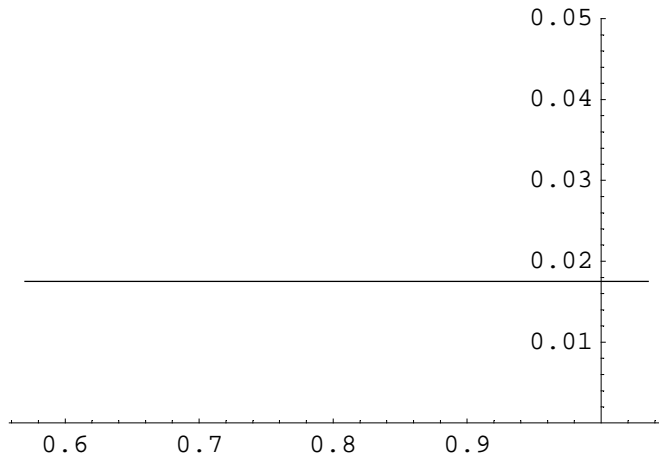
$$\int \left(-\text{ArcTan} \left[\frac{g}{x - \xi} \right] + \text{ArcTan} \left[\frac{h[\xi]}{x - \xi} \right] \right) d\xi$$

$$\frac{1}{2} \int (\text{Log}[g^2 + (x - \xi)^2] - \text{Log}[(x - \xi)^2 + h[\xi]^2]) d\xi$$

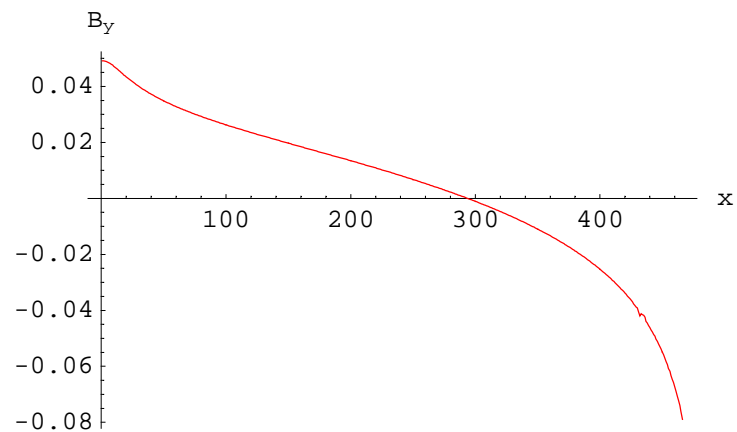
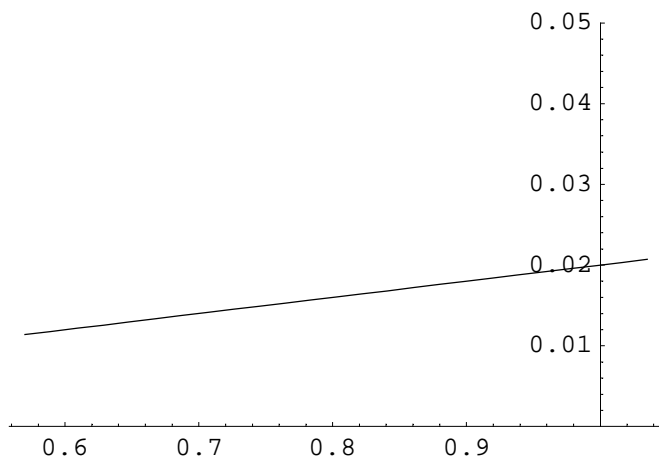
- Variable thickness of the conducting layer placed on a flat pole should lead to increase field gradient by decreasing the gap, as current density don't vary.

SheetField - Example without pole shape fitting (1)

■ Constant altitude

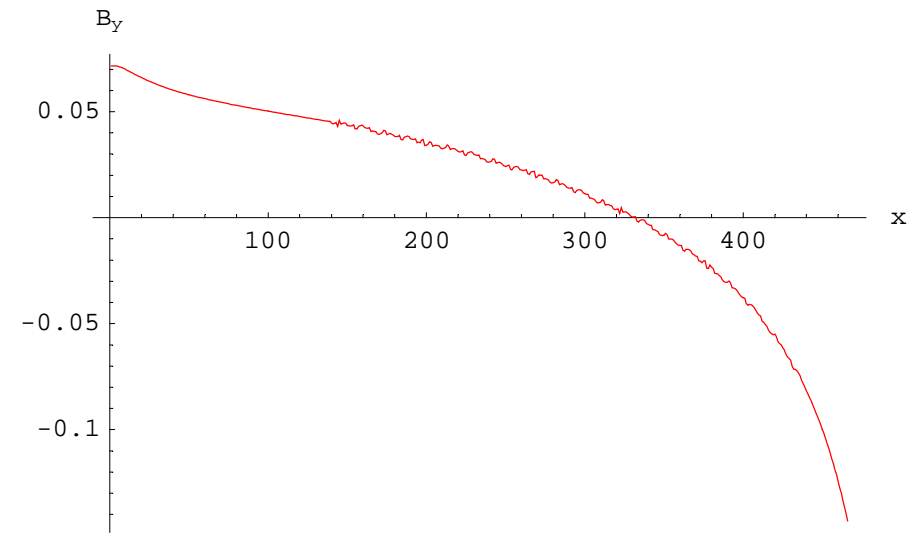
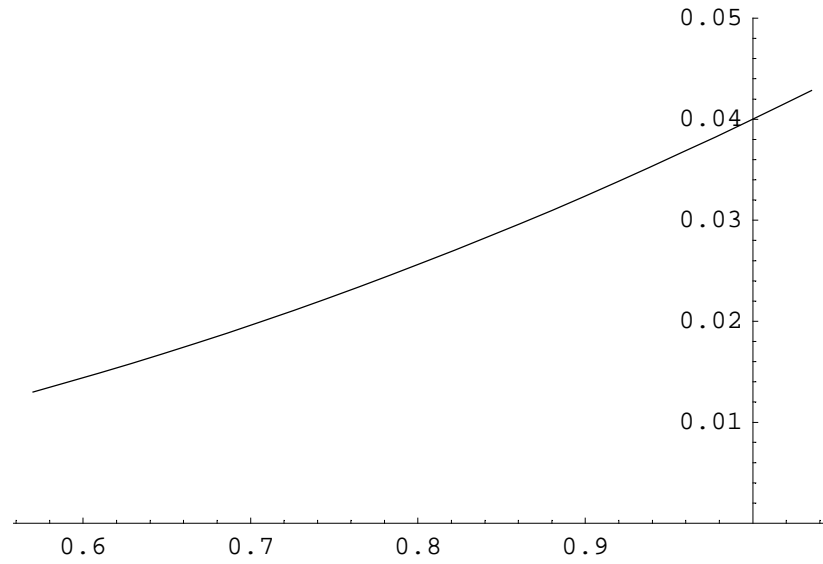


■ Linear variation of altitude



SheetField - Example without pole shape fitting (2)

- Parabolic variation of altitude



Stripes - Principle

- The matrix contains integrals for each stripe as $p_j < \xi < q_j$ at each observation point $x_i = \int_{p_j}^{q_j} f_i(\xi) d\xi$. Il y a P points d'observation et N bandes de courant. La matrice a donc les dimensions P x N. Exemple :

```
x = .; p = .; q = .; xi = .; eta = .; h[xi_] = .; g = .; (*Clear["context`*"]*) p = Table[{p1, p2, p3, p4}];
q = Table[{q1, q2, q3, q4}]; f = Table[{f1[xi, eta], f2[xi, eta], f3[xi, eta], f4[xi, eta]}];
Table[Integrate[f[[j]], {xi, p[[i]], q[[i]]}, {eta, h[xi], h[xi] + e}], {j, 1, 4}, {i, 1, 4}] // MatrixForm
```

$$\begin{pmatrix} \int_{p1}^{q1} \int_{h[xi]}^{e+h[xi]} f1[xi, eta] d\eta d\xi & \int_{p2}^{q2} \int_{h[xi]}^{e+h[xi]} f1[xi, eta] d\eta d\xi & \int_{p3}^{q3} \int_{h[xi]}^{e+h[xi]} f1[xi, eta] d\eta d\xi & \int_{p4}^{q4} \int_{h[xi]}^{e+h[xi]} f1[xi, eta] d\eta d\xi \\ \int_{p1}^{q1} \int_{h[xi]}^{e+h[xi]} f2[xi, eta] d\eta d\xi & \int_{p2}^{q2} \int_{h[xi]}^{e+h[xi]} f2[xi, eta] d\eta d\xi & \int_{p3}^{q3} \int_{h[xi]}^{e+h[xi]} f2[xi, eta] d\eta d\xi & \int_{p4}^{q4} \int_{h[xi]}^{e+h[xi]} f2[xi, eta] d\eta d\xi \\ \int_{p1}^{q1} \int_{h[xi]}^{e+h[xi]} f3[xi, eta] d\eta d\xi & \int_{p2}^{q2} \int_{h[xi]}^{e+h[xi]} f3[xi, eta] d\eta d\xi & \int_{p3}^{q3} \int_{h[xi]}^{e+h[xi]} f3[xi, eta] d\eta d\xi & \int_{p4}^{q4} \int_{h[xi]}^{e+h[xi]} f3[xi, eta] d\eta d\xi \\ \int_{p1}^{q1} \int_{h[xi]}^{e+h[xi]} f4[xi, eta] d\eta d\xi & \int_{p2}^{q2} \int_{h[xi]}^{e+h[xi]} f4[xi, eta] d\eta d\xi & \int_{p3}^{q3} \int_{h[xi]}^{e+h[xi]} f4[xi, eta] d\eta d\xi & \int_{p4}^{q4} \int_{h[xi]}^{e+h[xi]} f4[xi, eta] d\eta d\xi \end{pmatrix}$$

$$B_{y,i}(\xi) = \int \left(-\text{ArcTan}\left[\frac{g}{x-\xi}\right] + \text{ArcTan}\left[\frac{h[\xi]}{x-\xi}\right] \right) d\xi$$

$$B_{x,i}(\xi) = \frac{1}{2} \int (\text{Log}[g^2 + (x-\xi)^2] - \text{Log}[(x-\xi)^2 + h[\xi]^2]) d\xi$$

- e : thickness of a stripe (can be different from to another).
- $h[\xi]$: altitude of the surface of the stripes can vary to allow pole shaping.
Example : linear shaping to obtain constant focusing.
- Comparison with conductors (constant thickness and constant altitude) shows same current values are obtained, up to machine precision.
- However stripes should smooth oscillations.

Stripes - Low thickness compared to width

- When stripe thickness is very small compared to the gap :

- vertical component has linear dependence on thickness,

- thickness disappears from horizontal component.

$$B_{y,i}(\xi) = \int -\frac{(x-\xi) h[\xi]}{g^2 + (x-\xi)^2} d\xi$$

- If $h[\xi]$ has any expression \Rightarrow numerical integration.
- If $h[\xi]$ has polynomial expression \Rightarrow analytical integration.
- Thickness disappears from horizontal component and integration remains possible :

$$B_{x,i}(\xi) = \frac{1}{2} \left(-2g \operatorname{ArcTan}\left[\frac{x-\xi}{g}\right] - (x-\xi) \operatorname{Log}\left[\frac{g^2 + (x-\xi)^2}{(x-\xi)^2}\right] \right)$$