Medical FFAG Rings

E. Keil, A.M. Sessler, D. Trbojevic

FFAG Workshop

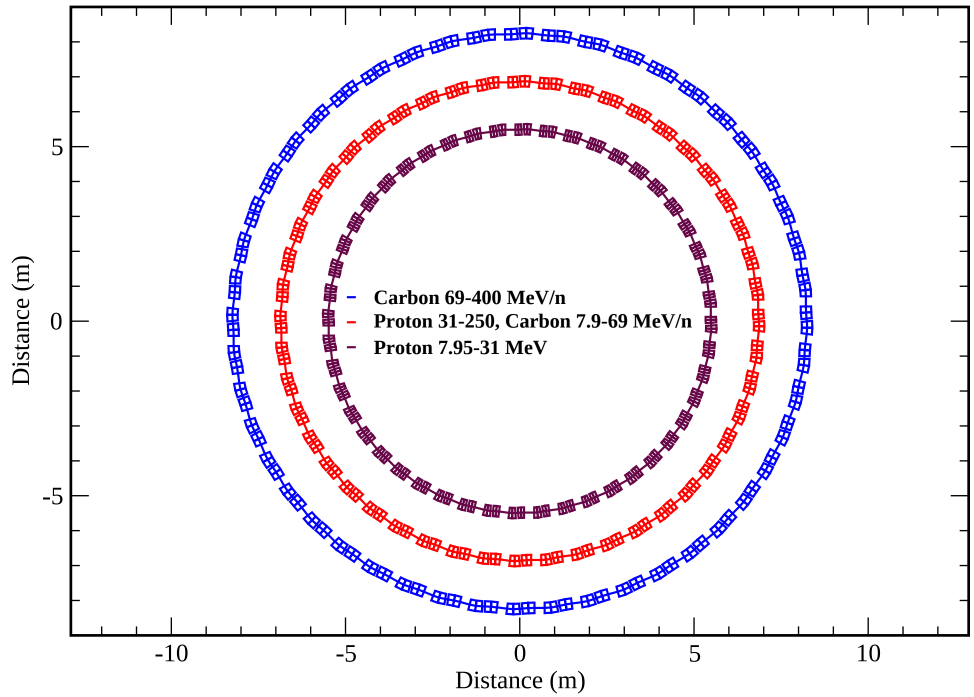
Kyoto Nov 2006

My WWW home directory:

`http://keil.home.cern.ch/keil/`
`MuMu/Doc/FFAG06/FFAG_Nov06/Slides05Nov06.pdf`

Layout of Three Rings

- Compact layout with three concentric rings
- Smallest ring for H^+
- Middle ring for H^+ and C^{6+} operates in the same range of $B\rho$ for H^+ and C^{6+} and accelerates by a factor 3 in $B\rho$
- Injection of H^+ and C^{6+} at the same speed $c\beta$
- Largest ring for C^{6+}



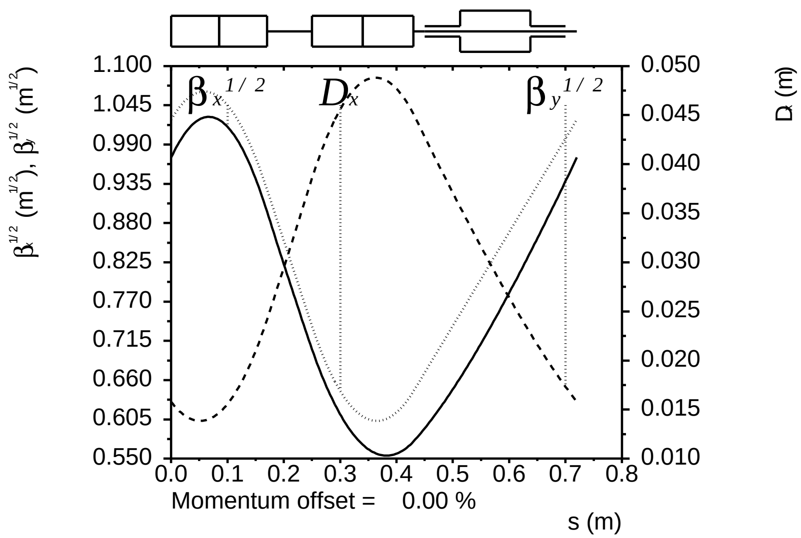
- Ratios of radii 4:5:6 much smaller than ratios of reference momenta, a historical relic from low-frequency RF system
- B increases from smallest to largest ring
- Alternative layout has rings on either side of a straight line that start with RFQ and linac, includes all transfer lines, and ends at the gantry

Table of Parameters

Particle	H^+			
Ring	1-Inj	1-Extr	2-Inj	2-Extr
Kin. En./u(MeV)	7.951	30.97	30.97	250
β	0.1294	0.2508	0.2508	0.6136
$B\rho$(Tm)	0.4083	0.8107	0.8107	2.432
$\delta p/p$	-0.3301	$+0.3301$	-0.5	$+0.5$
Harmonic no.	1155	596	745	305
Particle	C^{6+}			
Ring	2-Inj	2-Extr	3-Inj	3-Extr
Kin. En./u(MeV)	7.8934	68.801	68.801	400
β	0.1294	0.3645	0.3645	0.7145
$B\rho$(Tm)	0.8107	2.432	2.432	6.3472
$\delta p/p$	-0.5	$+0.5$	-0.4459	$+0.4459$
Harmonic no.	1451	515	618	315

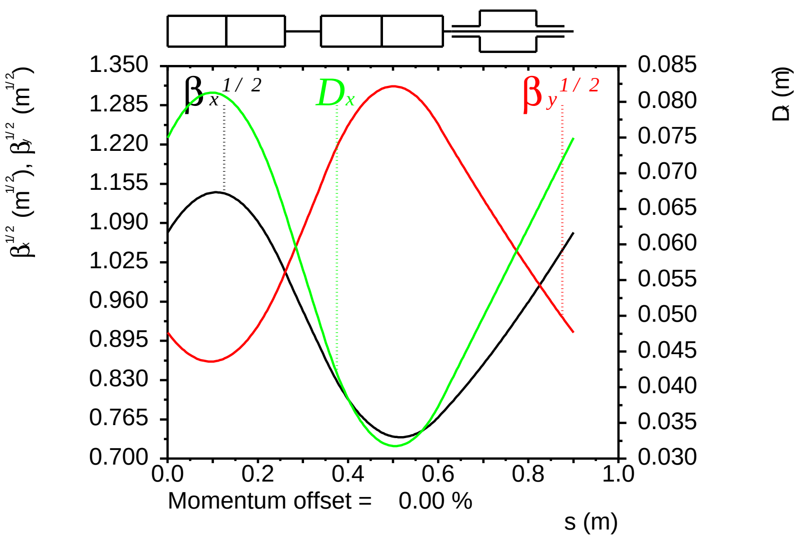
Orbit Functions in Ring 1

- Orbit functions at $\delta p/p = 0$
- Doublets of combined-function magnets
- Same space for RF cavities at 1.3 GHz in all rings



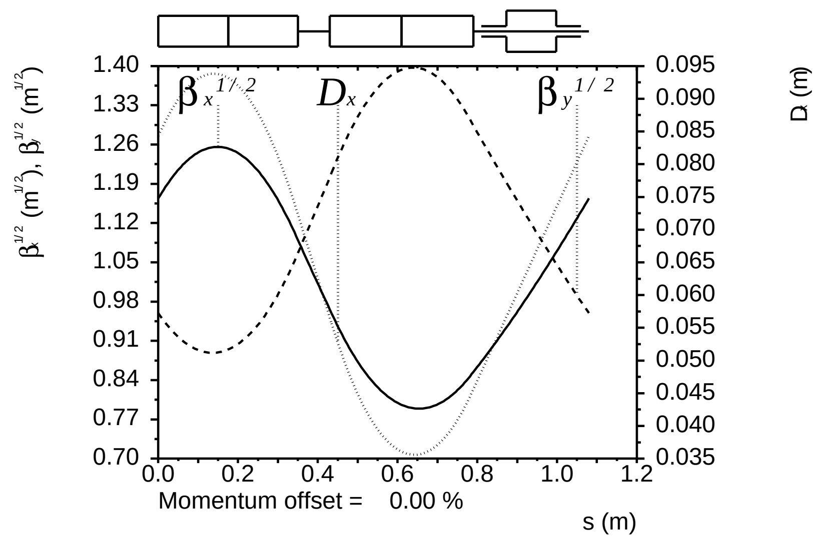
Orbit Functions in Ring 2

- Orbit functions at $\delta p/p = 0$
- Doublets of combined-function magnets
- Same space for RF cavities at 1.3 GHz in all rings



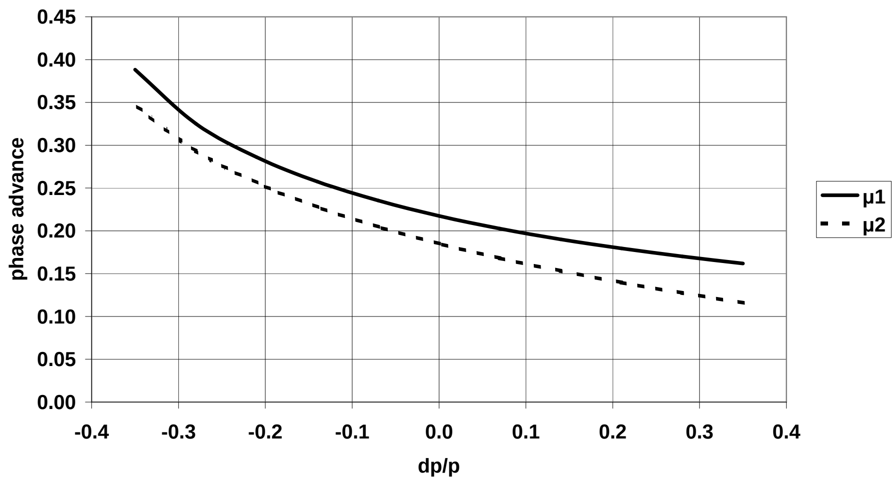
Orbit Functions in Ring 3

- Orbit functions at $\delta p/p = 0$
- Doublets of combined-function magnets
- Same space for RF cavities at 1.3 GHz in all rings



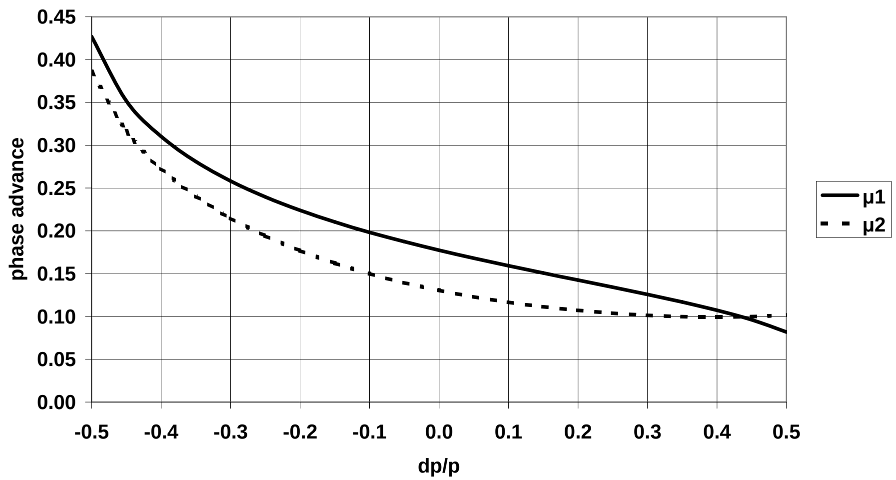
Phase Advances in Ring 1

- Phase advances per cell μ_1 and μ_2 in cycles
- Index 1 for horizontal, 2 for vertical
- Range of $\delta p/p$ adapted to range needed
- Adjust shape of combined-funtion magnets, μ_1 and μ_2 at reference energy such as to achieve $\delta p/p$ range needed



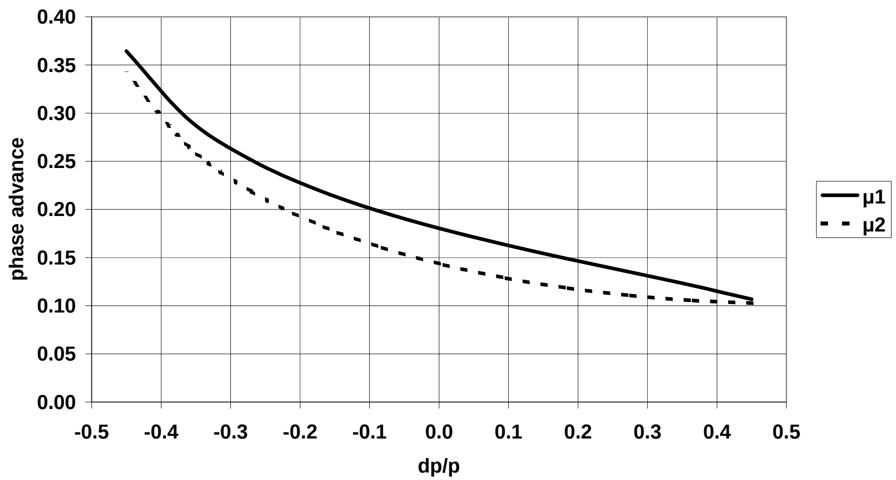
Phase Advances in Ring 2

- Phase advances per cell μ_1 and μ_2 in cycles
- Index 1 for horizontal, 2 for vertical
- Range of $\delta p/p$ adapted to range needed



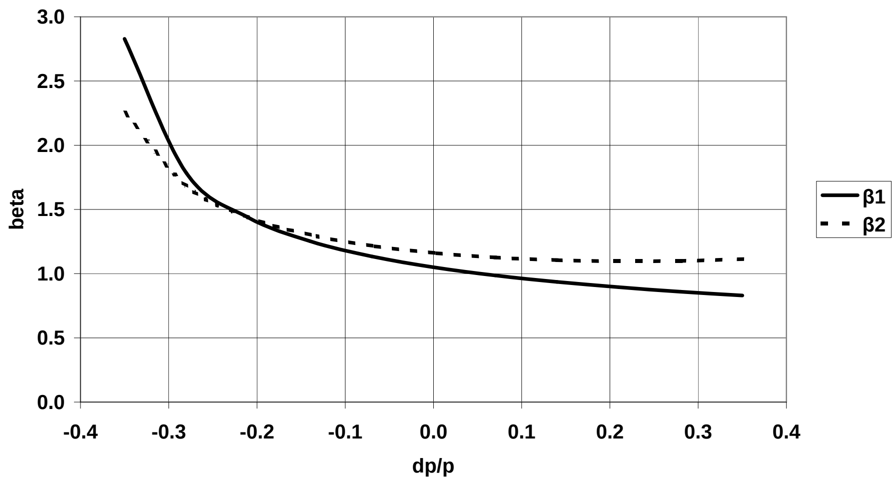
Phase Advances in Ring 3

- Phase advances per cell μ_1 and μ_2 in cycles
- Index 1 for horizontal, 2 for vertical
- Range of $\delta p/p$ adapted to range needed



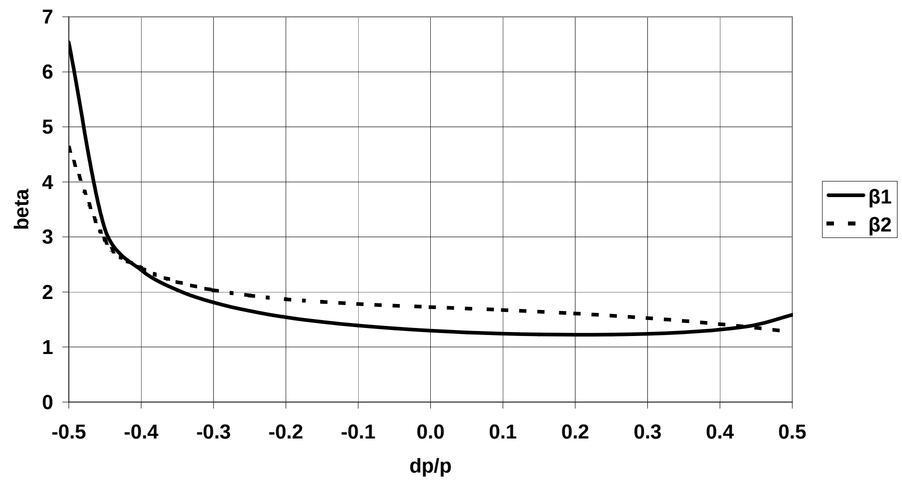
Maximum Beta Functions in Ring 1

- Index 1 for horizontal, 2 for vertical
- Range of $\delta p/p$ adapted to range needed



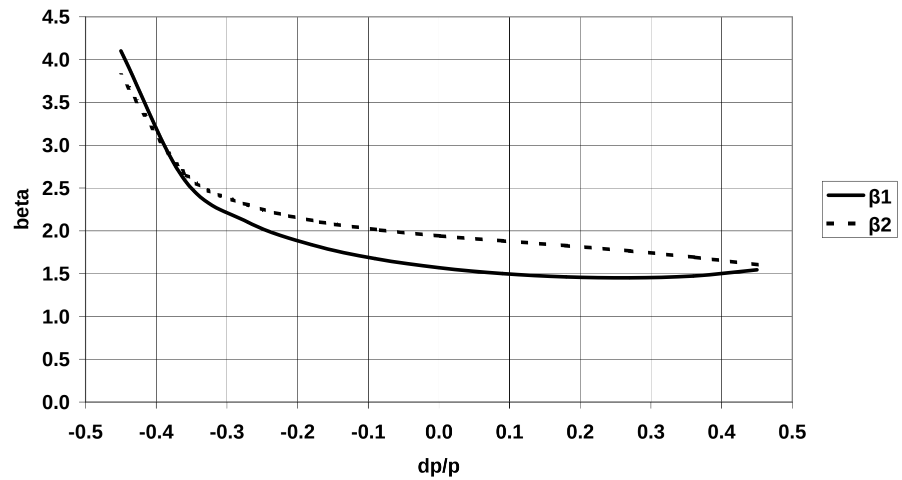
Maximum Beta Functions in Ring 2

- Index 1 for horizontal, 2 for vertical
- Range of $\delta p/p$ adapted to range needed



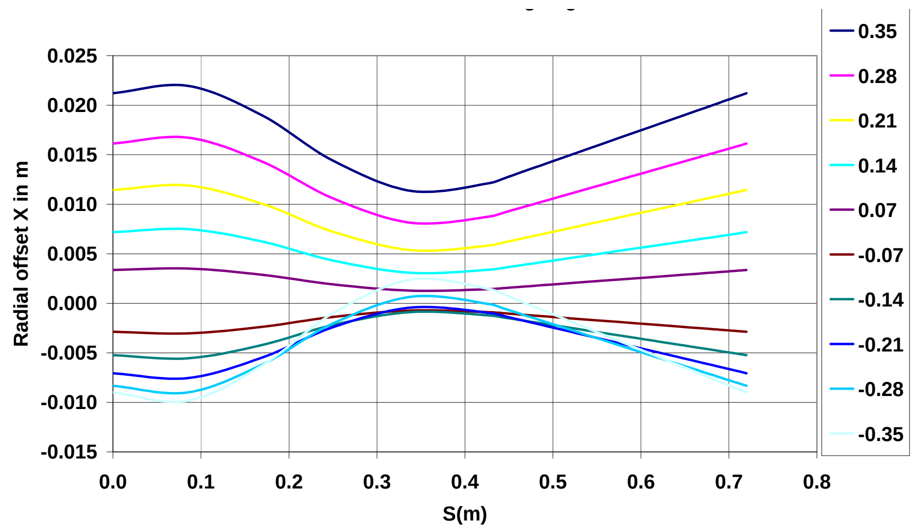
Maximum Beta Functions in Ring 3

- Index 1 for horizontal, 2 for vertical
- Range of $\delta p/p$ adapted to range needed



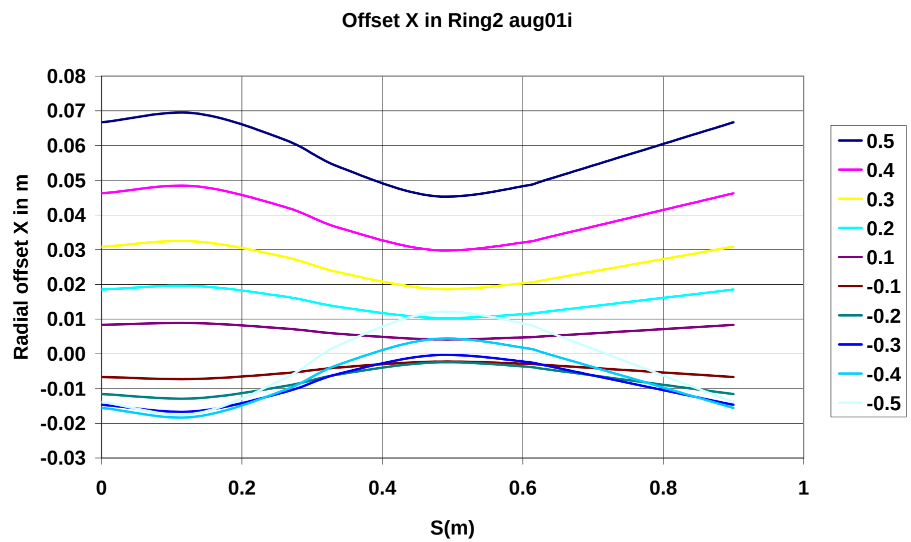
Horizontal Orbit Offsets in Ring 1

- Adjust bending angles in combined-function magnets such that **path length** varies quadratically with $\delta p/p$ near reference energy, and that spread in path length is minimal
- Achieves smallest possible horizontal offsets with $\delta p/p$
- **Time of flight** dominated by variation of speed $c\beta$
- Range of $\delta p/p$ in graph adapted to range needed



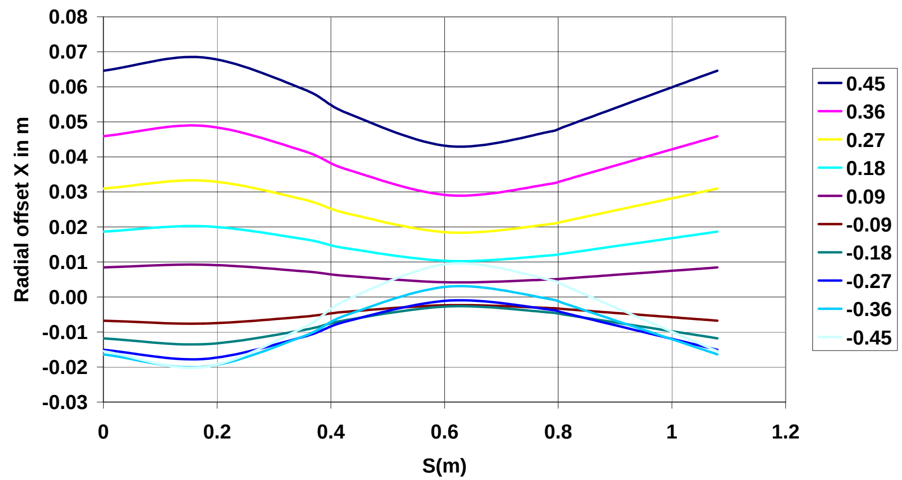
Horizontal Orbit Offsets in Ring 2

- In the F magnets the offsets x are largest, and the trajectories with increasing $\delta p/p$ are progressively more to the outside of the reference orbit
- In the D magnets the offsets x are smallest, and the trajectories with increasing $\delta p/p$ are not in the same order
- The trajectory with the smallest $\delta p/p$ is more outside than trajectories with intermediate values of $\delta p/p$.



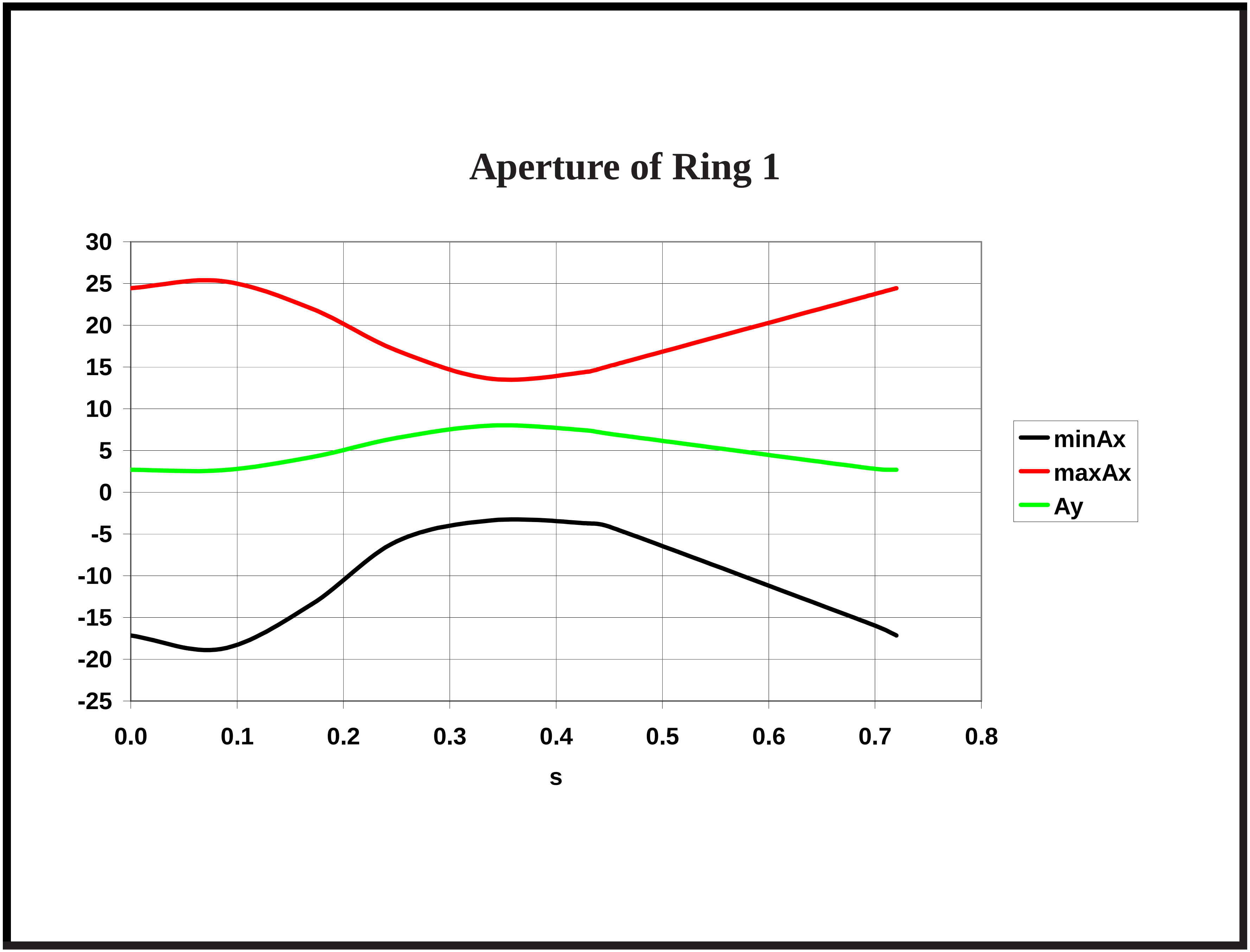
Horizontal Orbit Offsets in Ring 3

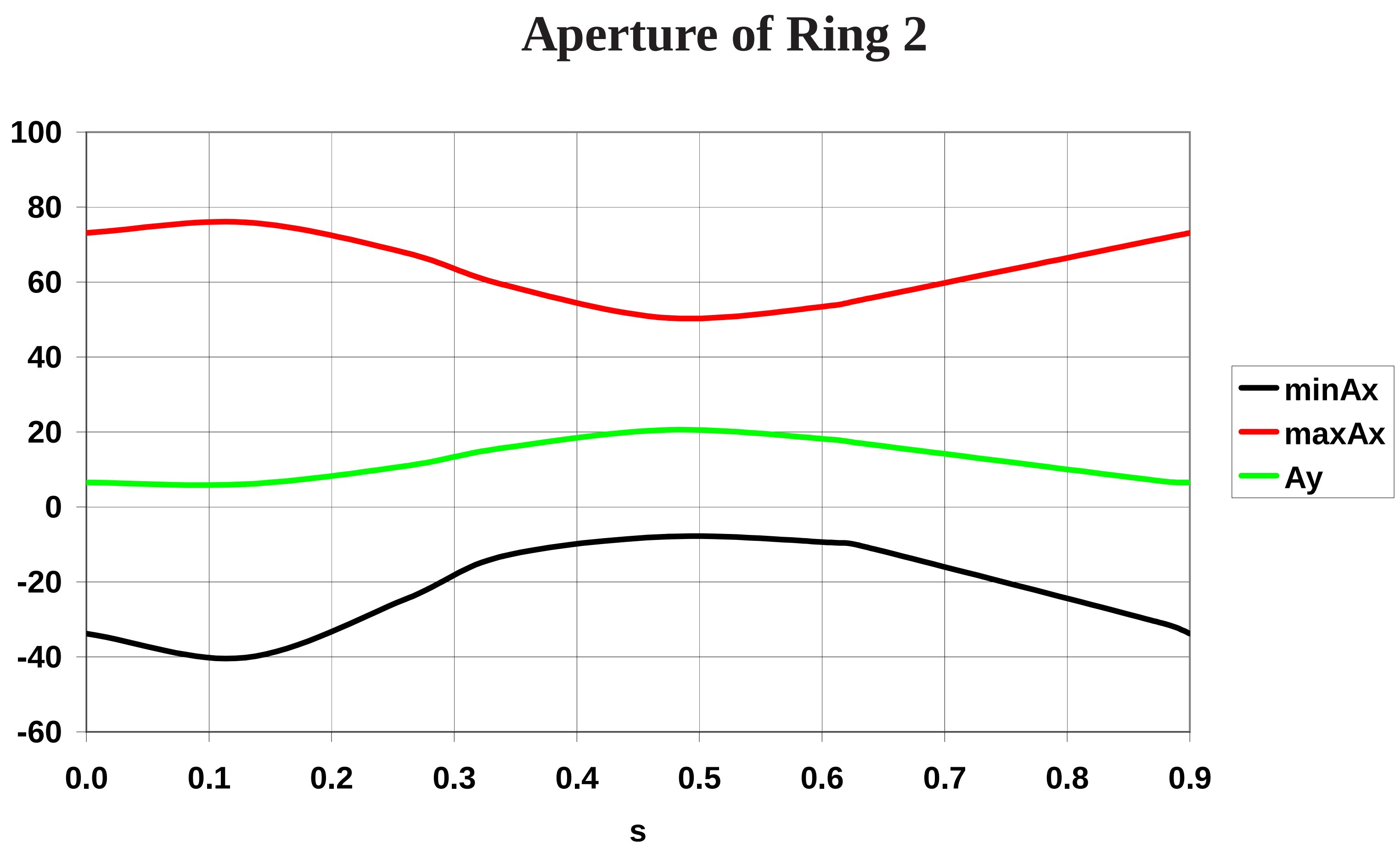
- In the F magnets the offsets x are largest, and the trajectories with increasing $\delta p/p$ are progressively more to the outside of the reference orbit
- In the D magnets the offsets x are smallest, and the trajectories with increasing $\delta p/p$ are not in the same order
- The trajectory with the smallest $\delta p/p$ is more outside than trajectories with intermediate values of $\delta p/p$.

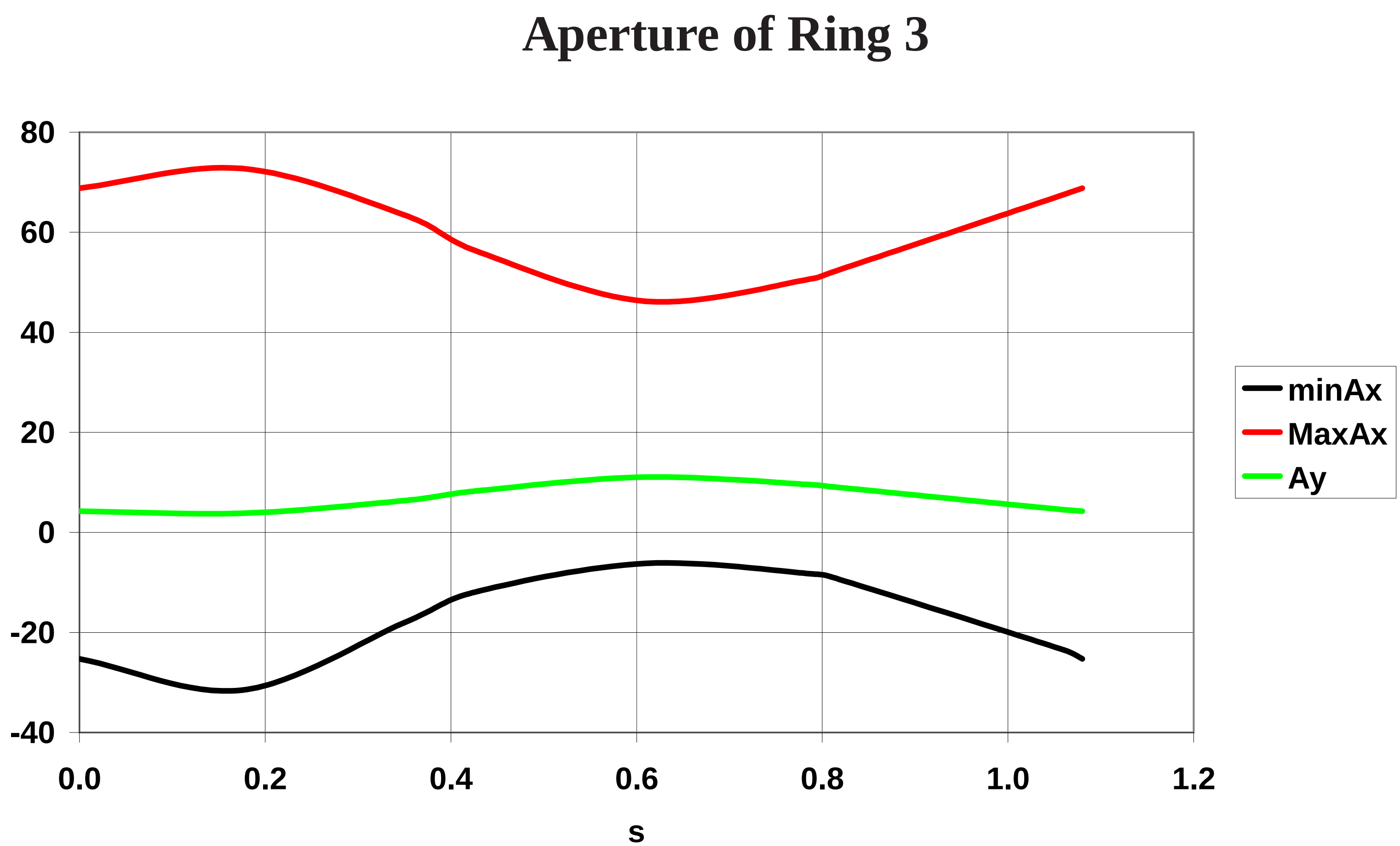


Calculation of Apertures

- Assume normalised emittance $\varepsilon_n = 0.5 \cdot 10^{-6}\pi$ m in both planes, following SPL II Design and an INFN paper on ion sources
- Allow generously for five RMS beam radii in the half apertures
- Calculate the maximum β-functions in $\delta p/p$ range for each element
- Find vertical aperture radius from vertical betatron oscillations
- Calculate the contribution of the horizontal betatron oscillations to the horizontal aperture A_x in a similar manner
- Subtract that contribution from the most negative horizontal offset
- Add that contribution to the most positive horizontal offset
- Pessimistic procedure
- Ring 2 has the largest aperture of the three rings, since range is largest, although cells are not the longest


Aperture of Ring 1
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
s
minAx
maxAx
Ay


Aperture of Ring 2
100
80
60
40
20
0
-20
-40
-60
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
s
minAx
maxAx
Ay


Aperture of Ring 3
80
60
40
20
0
-20
-40
0.0
0.2
0.4
0.6
0.8
1.0
1.2
s
minAx
MaxAx
Ay

Relations between C, N, B, G

- For guidance in choice of the length L and number N of the lattice cells, and hence of the circumference $C = LN$, consider an ensemble of similar lattices, in which all element lengths scale like L, and the reference energies, magnetic rigidities $B\rho$, and phase advances are the same
- The important parameters in the choice are the maximum magnetic field B at the edges of the aperture, which determines the magnet technology, and the aperture A itself
- If B is dominated by the dipole component, we find $C = 2\pi\rho/\bar{B}$, where $\bar{B} < B$ is the average dipole field on the reference orbit
- For given $\bar{B}$ this argument yields C, but neither L nor N
- We find the same scaling law, if B is dominated by the quadrupole component
- At given $B\rho$ gradient G scales like $1/L^2$, field B at the edge of the aperture A scales like GA, and the aperture scales like L/N. Hence, $C = LN$ scales like $1/B$ as before, A decreases with decreasing L and increasing N, but there is no lower limit for L
- Such a limit does arise, when the magnet lengths scale like L^m with $m > 1$, because the lengths of the straight sections, e.g. for the RF system remain constant, as can be seen in the lattice table
- In this case, we find that L^{m-1} scales like $1/BC$, and we get a lower limit on L

Magnet Parameters

- Do not show field B and gradient G at reference energy
- Use horizontal aperture ranges to find B and G at edges of the radial aperture
- In F magnet B changes sign
- F magnet more like a shifted quadrupole than a dipole

Magnet	F			D		
Ring	1	2	3	1	2	3
Inner hor apert radius(mm)	-27	-40	-32	-10	-13	-12
Outer hor apert radius(mm)	28	76	73	19	59	56
B(T) at inner apert radius	-0.68	-1.40	-2.40	0.82	1.69	3.34
B(T) at outer apert radius	0.17	0.57	0.81	0.44	0.50	1.30
Vert half apert(mm)	8	11	7	15	21	11

Acceleration

- Adopt Sandro's scheme involving harmonic number h jumping
- Energy gain in a turn ΔE is adjusted such that the change in $c\beta$ causes a change of the revolution period by an integral number of rf cycles, and hence corresponds to an integral step Δh in h
- Step in energy is

$$\Delta E = -E_0 \beta^2 \gamma^3 \Delta h / h$$

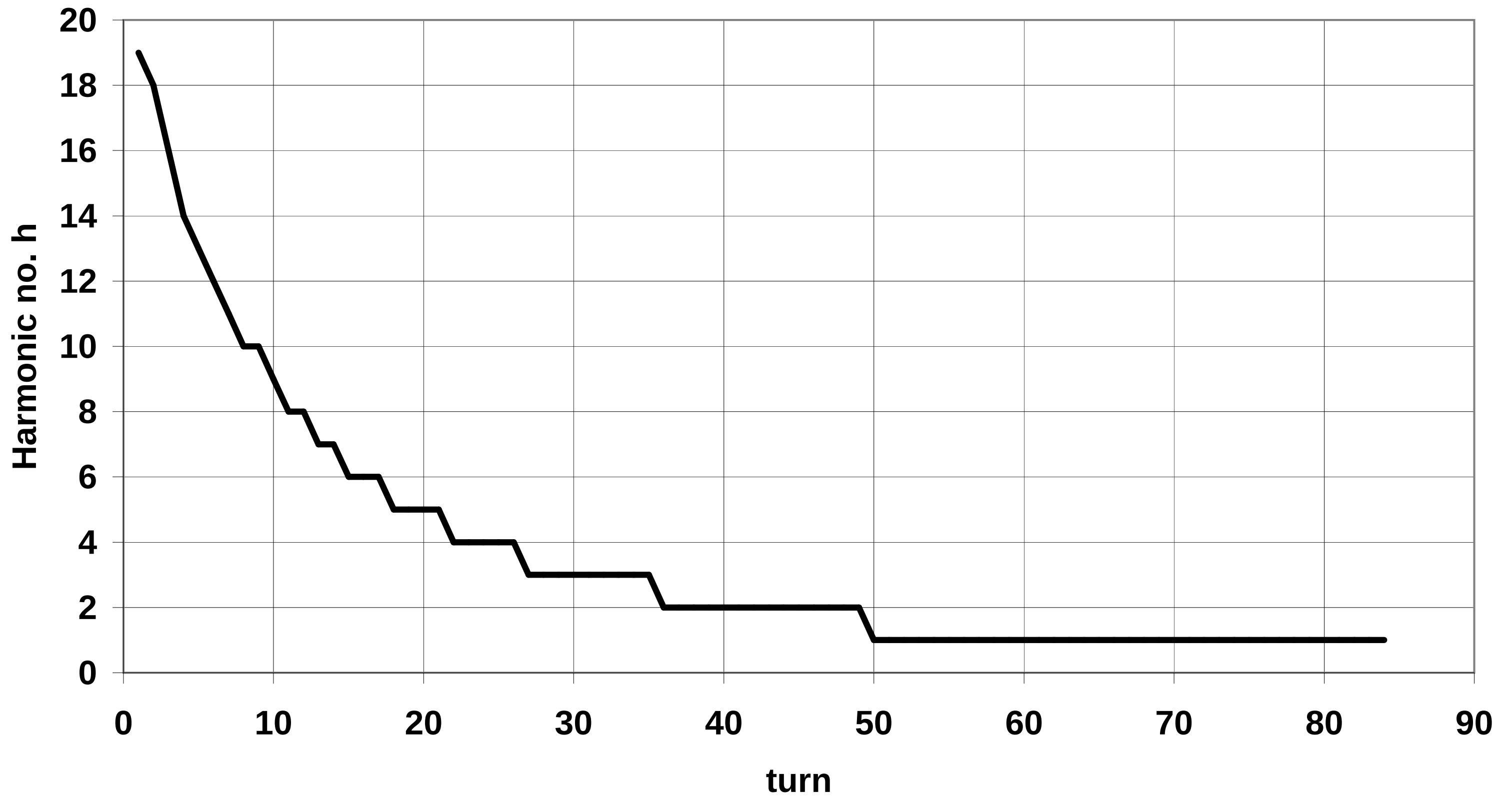
- E_0 is the rest energy of the particle
- High RF frequency and large h advantageous
- Reduce variation of ΔE with γ or number of turns by starting acceleration with $|\Delta h| \gg 1$, gradually decreasing $|\Delta h|$, and switching to $\Delta h = -1$ towards the end

Ring		h_i	h_f	$\|\Delta h\|$	turns	V(MV)
1	H^+	1155	596	8	293	0.11
2	H^+	745	305	25	116	2.4
2	C^{6+}	1451	515	27	253	0.61
3	C^{6+}	618	315	19	84	10.0

Bunch Trains

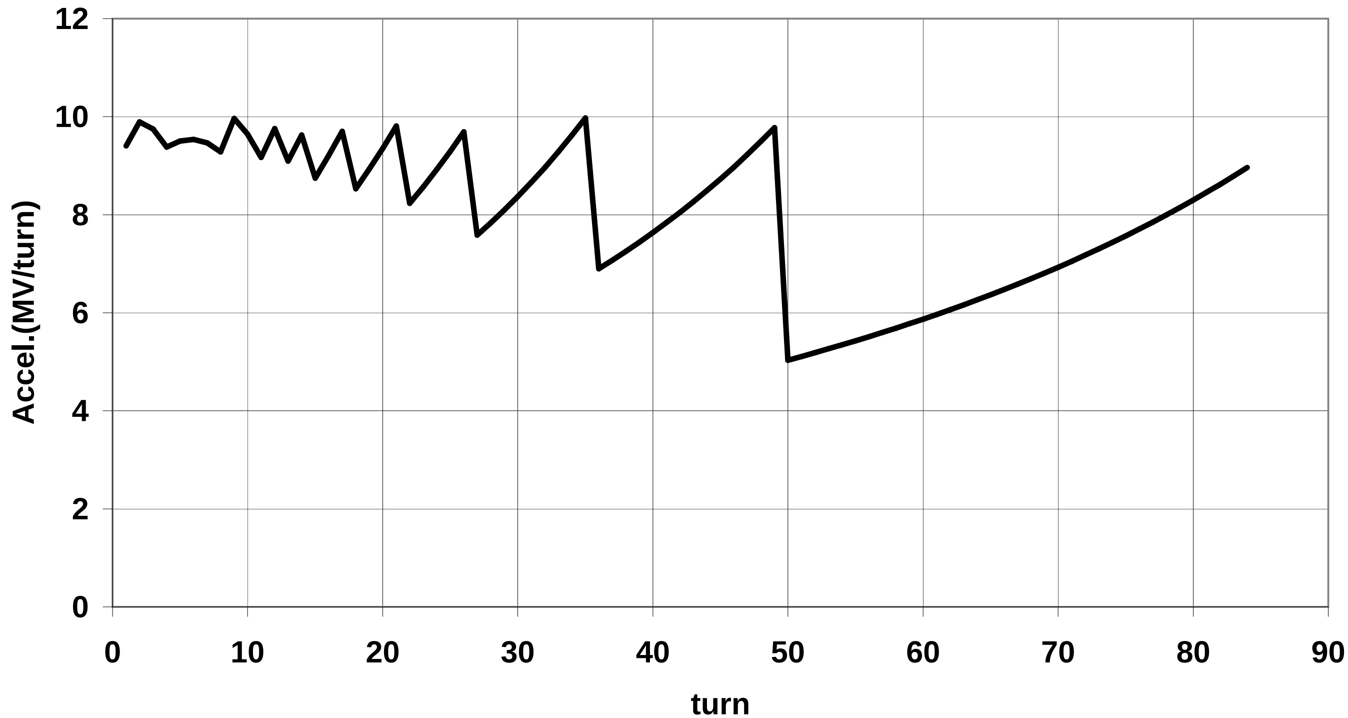
- Bunches arranged in a train
- Train occupies fixed time interval ΔT, which is a small fraction of revolution period at injection into Ring 1 for H^+ and into Ring 2 for C^{6+}
- Fraction of revolution period grows during acceleration
- Remaining gap used for ejection kicker rise time

Variation of Harmonic Number Step Δh in Ring 3



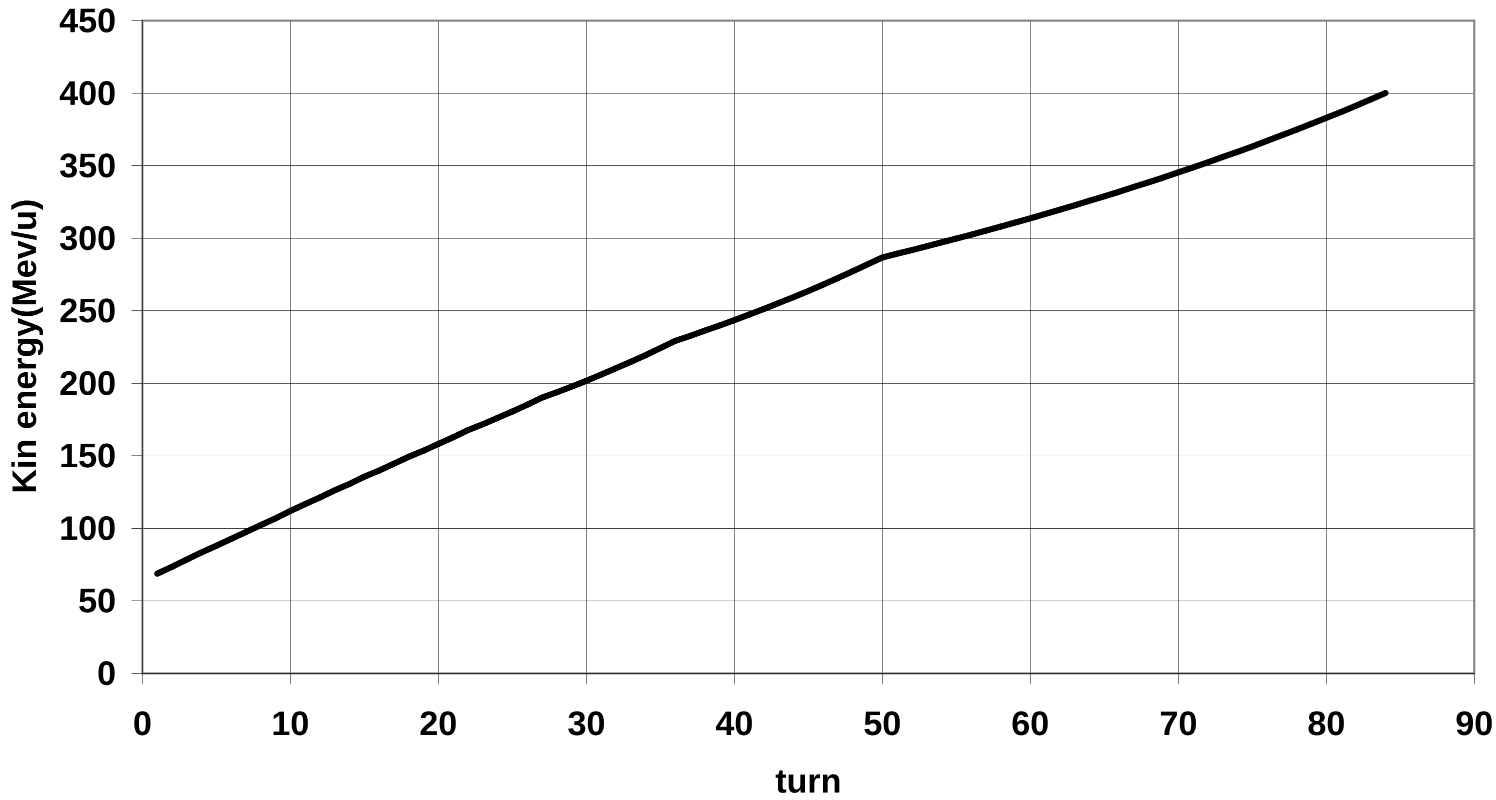
- Reach $\Delta h = -1$ after about half the number of turns
- Number of turns significantly smaller than difference $(h_i - h_f)$, because of the initial step $|\Delta h| > 1$, and much smaller than in a synchrotron

Variation of circumferential acceleration in Ring 3



- Peak accelerating voltage V_{RF} larger than circumferential acceleration by reciprocal of transit time factor
- Many equidistant RF cavities not useful, since step in time of flight then applies to fraction of circumference
- Several RF cavities in neighbouring cells might work

Variation of kinetic energy per nucleon in Ring 3



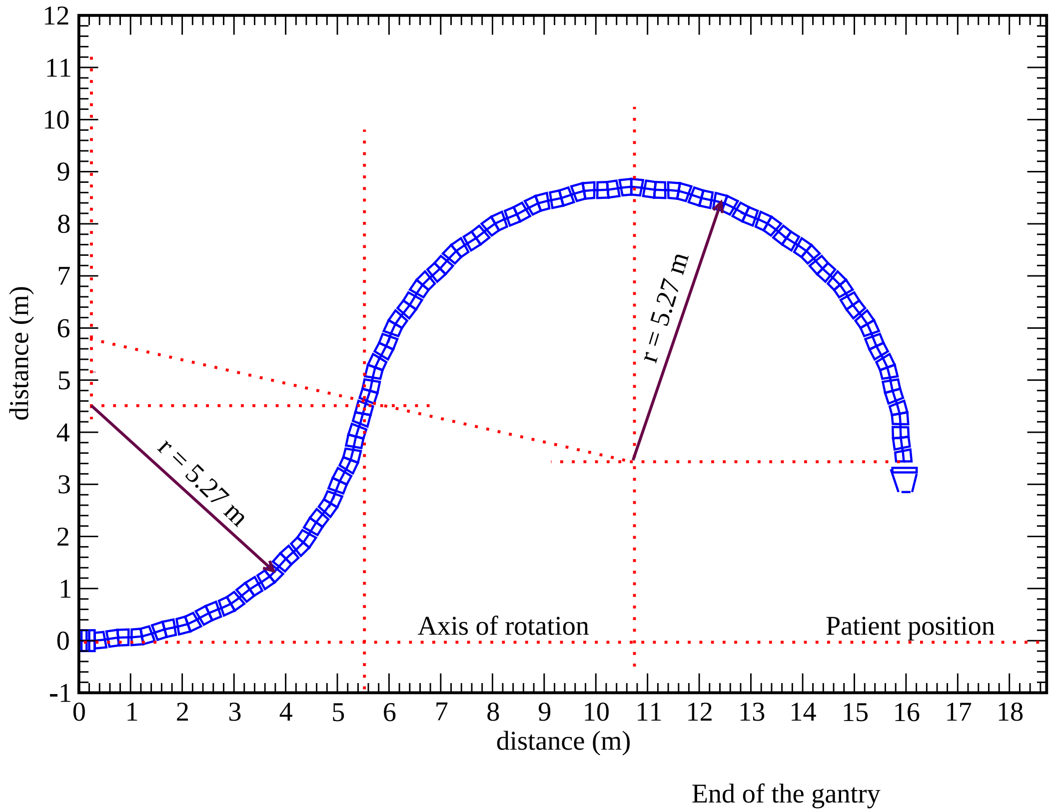
- Energy gain per turn varies by less than a factor of two, but is much larger than in a synchrotron
- High rate of acceleration ensures fast crossing of resonances

Extraction kicker parameters

- Two methods for building and exciting kickers
  - Delay line kickers with square pulse
  - Kickers with half sine wave pulse
- Half sine wave kickers need half the voltage

Ring	2	3
Kick angle(mrad)	10.2	6.8
Rise time(ns)	80	80
Aperture width(mm)	107	94
Aperture height(mm)	38	36
Kicker length(m)	0.2	0.2
Kicker field(T)	0.13	0.22

Schematic layout of FFAG gantry



- 36 cells in full circle
- Distance to patient adjustable, by varying bending angle and number of cells
- Gantry not small, but not heavy either
- Gantry operates d.c.
- Fast energy variation
- Lateral scanning
- Not yet optimised

Optics of FFAG gantry

- Optics must handle beams at any rotation angle
- Round beam at junction between transport line and gantry with equal emittances, equal β-functions, vanishing α-functions, D and D'
- Rotating matching section with two quadrupoles matches equal β-functions at junction to uneqal β-function at gantry cell boundary
- Gantry cells have $D = 0$ and $D' = 0$ at cell boundary
- Change of curvature achieved by changing direction of magnetic field B
- β-functions and D remain matched at $\delta p/p = 0$ for any rotation angle

Conclusions

- Presented a concept of a medical system for H^{+} and C^{6+} therapy consisting of three non-scaling FFAG rings
- Possible staged approach
  - Launch system two rings for H^{+}
  - Expand later with third ring for C^{6+}
- Envisage improvements in various directions
- System can be verfied by an almost unlimited number of simulations
- Enough known about system for cost comparison with other systems
- We are developers of concepts, not engineers for magnets, RF systems, kickers, etc.
- Further studied should be funded by industry