

# Transverse beam blowup in non-scaling FFAG “particle loss and its mechanism”

Shinji Machida

CCLRC/RAL/ASTeC

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[http://hadron.kek.jp/~machida/doc/nufact/ffag/machida\\_20061106.ppt](http://hadron.kek.jp/~machida/doc/nufact/ffag/machida_20061106.ppt) & [pdf](#)

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# Error study (1)

## Definition of errors

- Alignment error
  - Horizontal and vertical directions. No longitudinal or tilt.
  - Assume each QD and QF are on a single support table. QD and QF are aligned (or misaligned) together.
  - Alignment errors produce additional dipole kicks.
- Gradient error
  - Random errors of quadrupole strength.
  - Gradient errors produce additional quadrupole kicks.
- Distribution of both errors is Gaussian and there is a cut at 2 sigma.
- Use 20 seeds to see statistics.

## Error study (2)

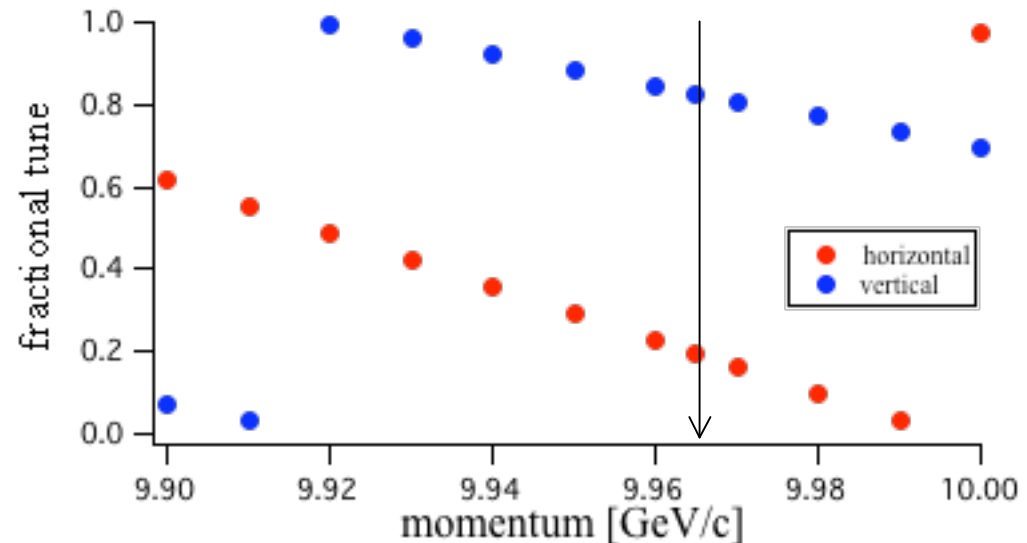
### Tracking conditions

- Constant energy gain (independent of RF phase) per turn is assumed to avoid time of flight variation due to transverse amplitude.
- 10 to 20 GeV acceleration is finished in 17 turns.
- $30 \pi$  mm rad emittance in horizontal and vertical (ellipsoidal in 4-D).
- Distribution of 500 macro particles is Waterbag.
- Zero  $dp/p$  or phase spread (initially) are assumed.

## Error study (3)

### Initial matching

- Assume the distortion of beta functions are small. Particle distribution is matched unperturbed lattice function.
- To make sure the above assumption is reasonable, choose the initial tune away from half-integer resonance.
- Initial momentum is 9.965 [GeV/c] in the present study.



## Error study (4)

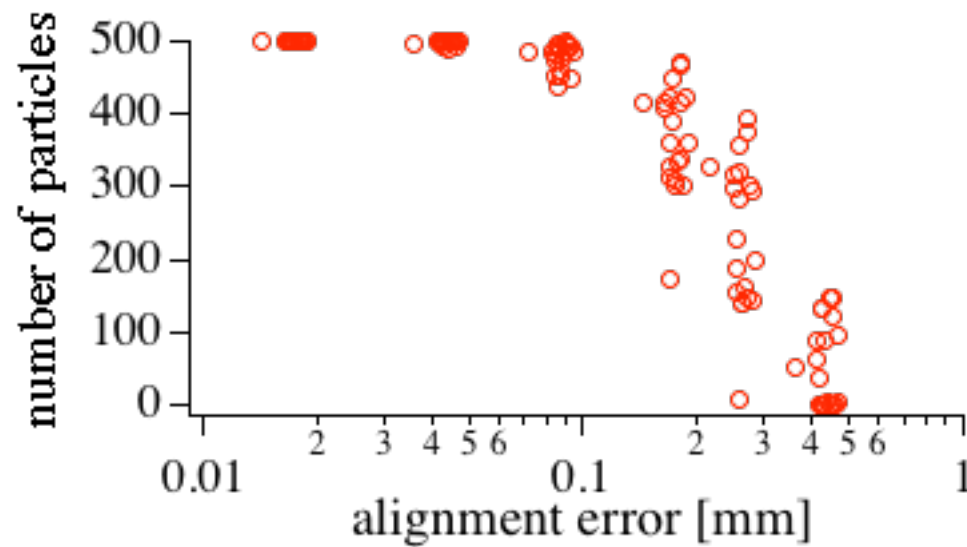
### Loss criteria

- At every cell, calculate amplitude (single particle emittance) of each macro particle.
- If the amplitude is more than  $45 \pi$  mm rad (1.5 times of the initial emittance), the particle is regarded as lost.
- This is a “phase space collimation” concept introduced by Meot.

# Tracking results (1)

## alignment error

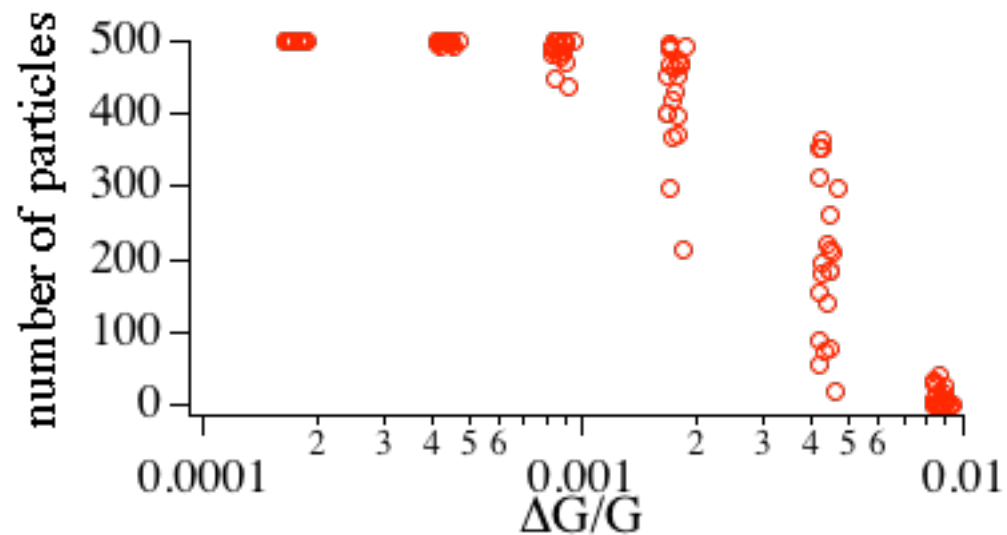
- No beam loss at 20  $\mu\text{m}$  (rms) or below.
- 100  $\mu\text{m}$  (rms) seems to be tolerable.



## Tracking results (2)

### gradient error

- No beam loss at  $dG/G=5 \times 10^{-4}$  (rms) or below.
- $dG/G= 1 \times 10^{-3}$  (rms) seems to be tolerable.

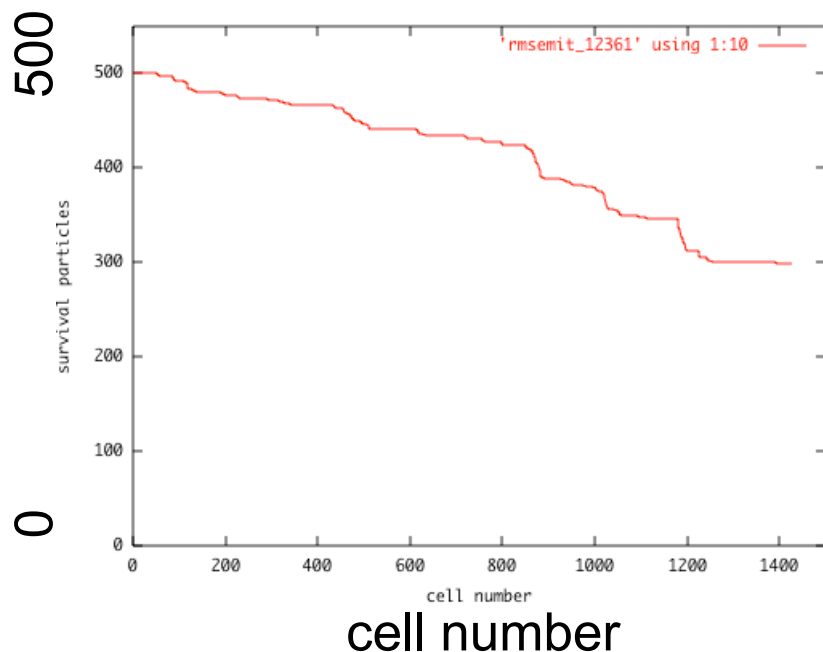


# Tracking results (3)

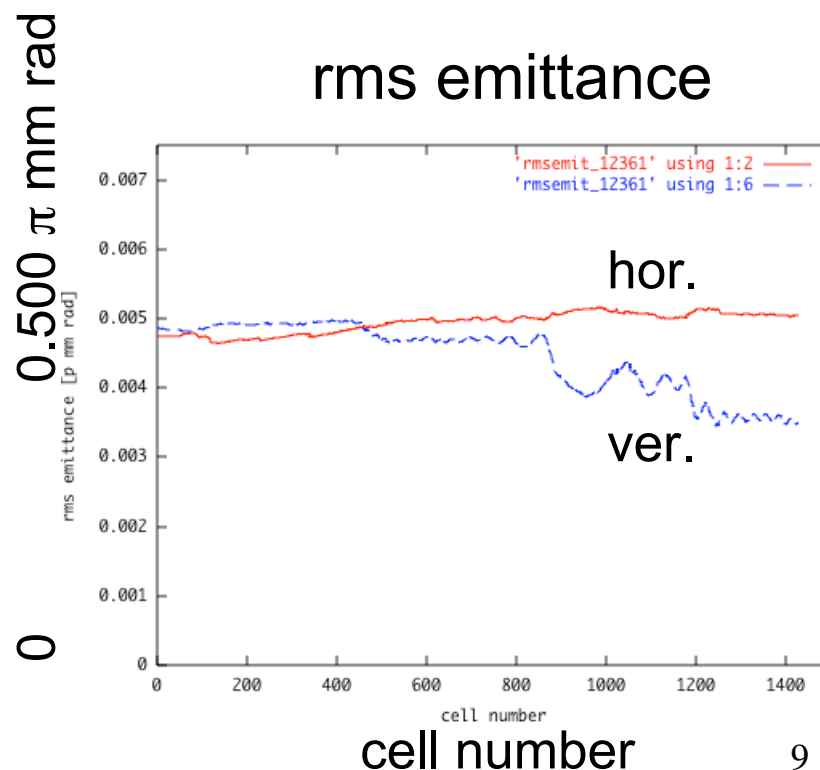
## Example of particle loss and emittance evolution

- There are both slow loss and sharp drop at some points.
- Rms emittance does not have obvious structure.

particle loss



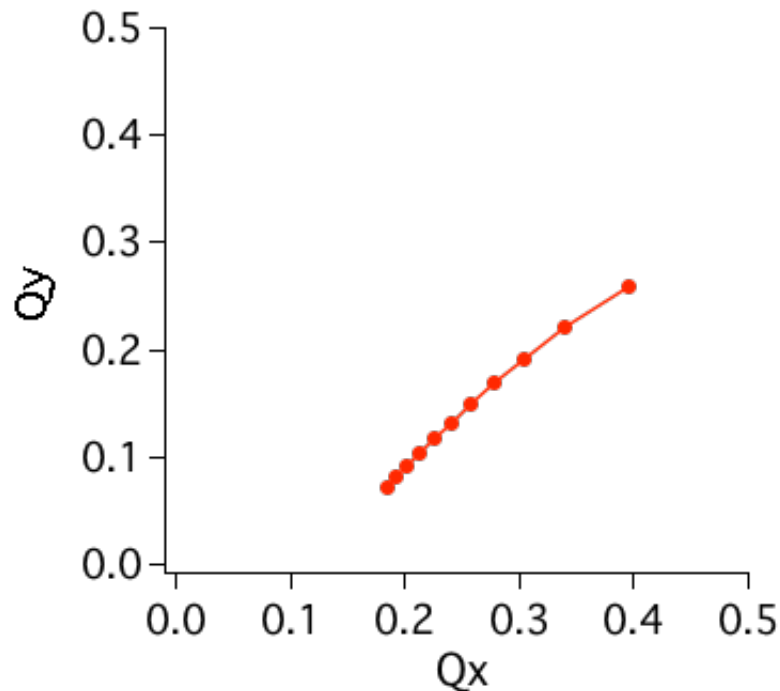
rms emittance



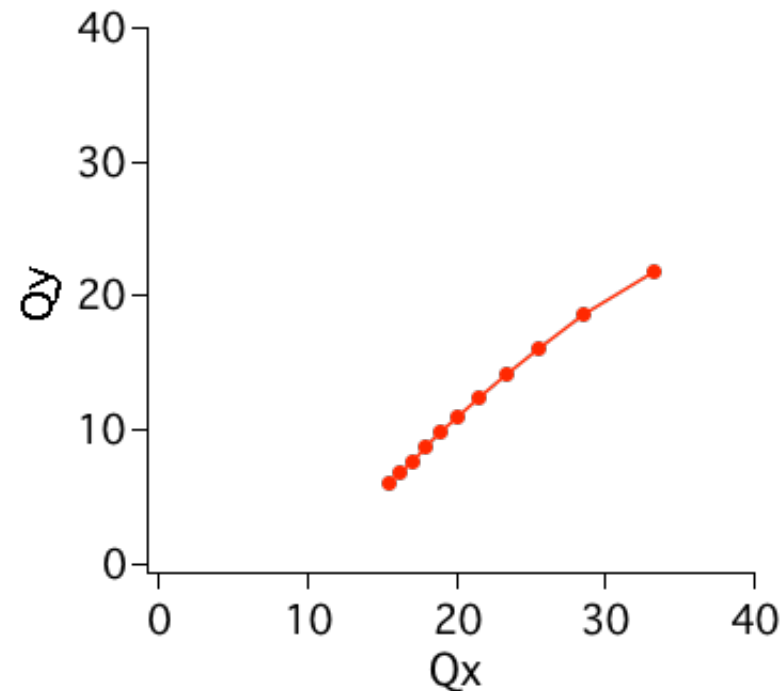
# What causes particle loss? (1)

- Do alignment errors excite integer “resonance” and gradient errors excite half-integer “resonance”?
- If a particle can see periodic structure of a turn?

cell tune



total tune (84 cell)

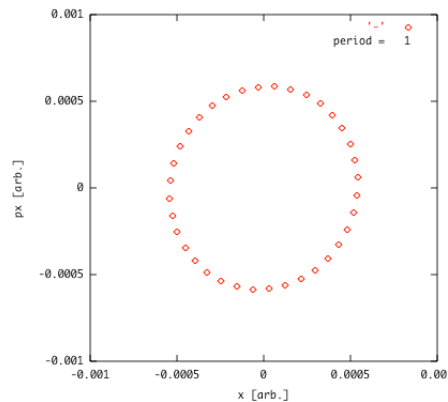


# What causes particle loss? (2)

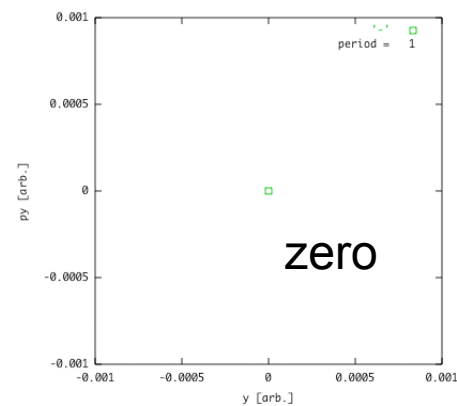
## 1D model

- Track only horizontal envelop and see how it evolves.
- Initially horizontal emittance is 0.003, 0.3, and  $30 \pi$  mm rad.
- Vertical emittance is zero.
- Initially  $dp/p$  and phase spread is zero.  $dp/p$  remains zero through the process.

horizontal



vertical



# What causes particle loss? (3)

## lattice beta and beam beta

- Lattice beta
  - Beta function as a “periodic” boundary condition.
  - It is a function of momentum.
- Beam beta
  - Beam ellipse parameter calculated with multi particles.

$$\beta_x = \frac{\langle x_i^2 \rangle}{\epsilon_{x,rms}}$$

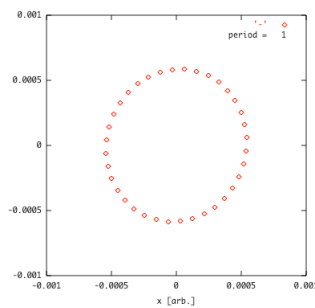
$$\epsilon_{x,rms} = \sqrt{\langle x_i^2 \rangle \langle p_{x,i}^2 \rangle - \langle x_i p_{x,i} \rangle^2}$$

# What causes particle loss? (4)

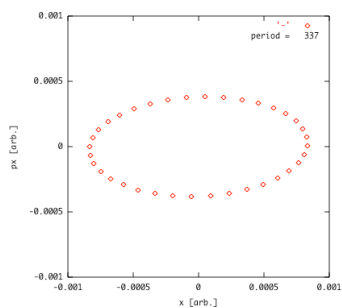
## Envelop evolution normalized with “lattice beta”

0.003 p mm rad

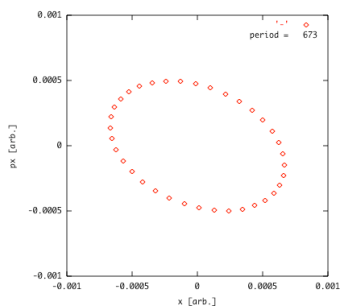
0 turn



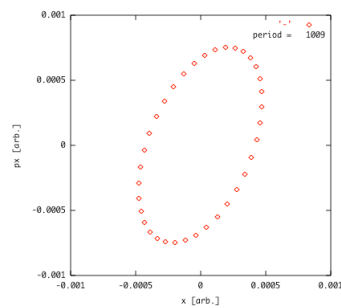
4 turn



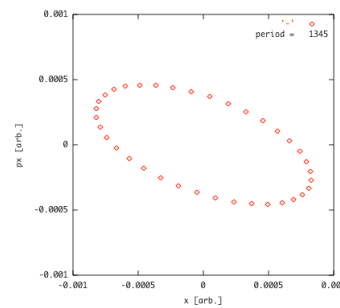
8 turn



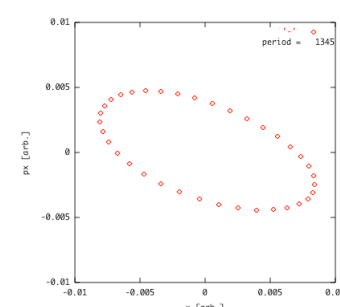
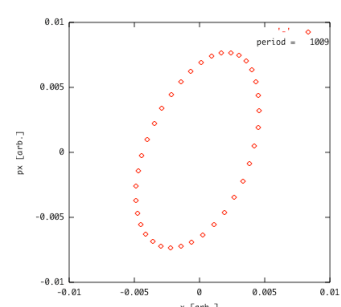
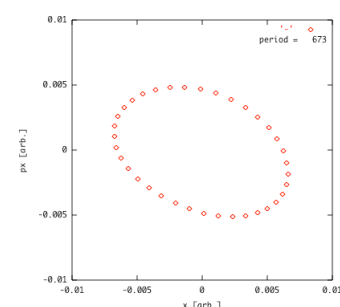
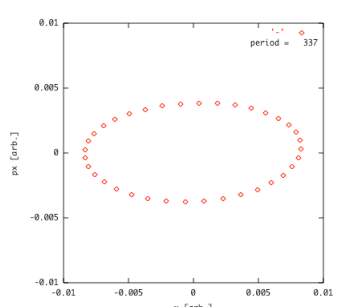
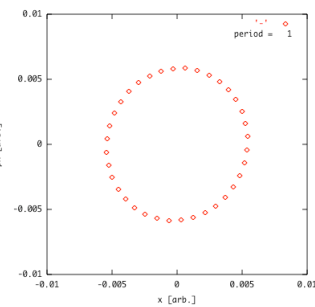
12 turn



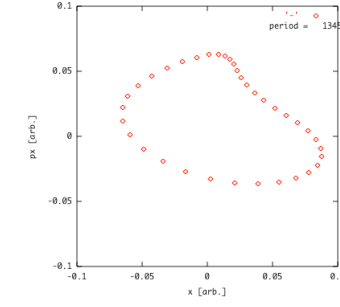
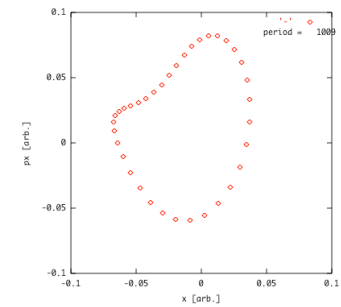
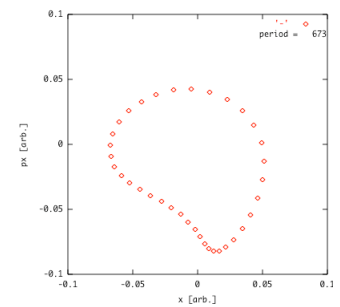
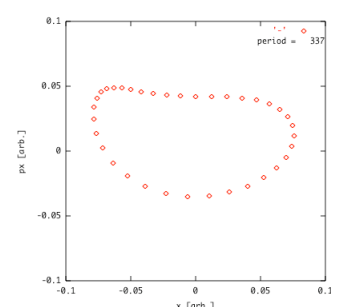
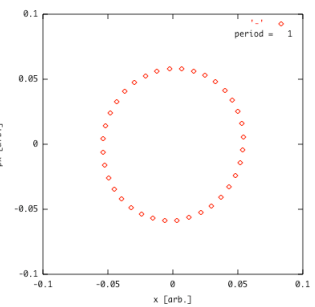
16 turn



0.3 p mm rad



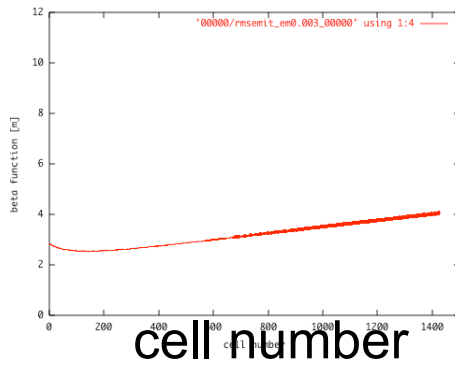
30 p mm rad



# What causes particle loss? (5)

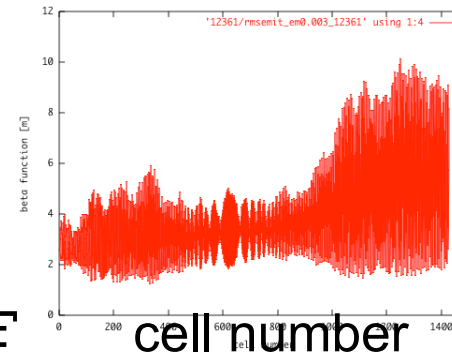
## Beam beta without and with gradient errors

without errors



0.003 p mm rad

with errors

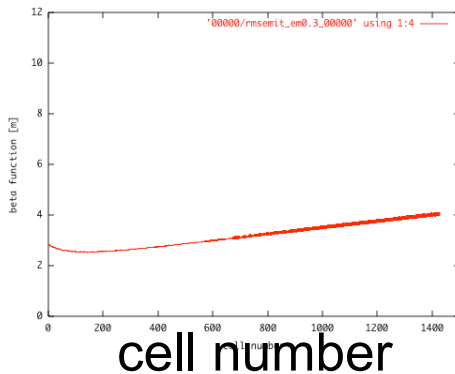


beta function [m]

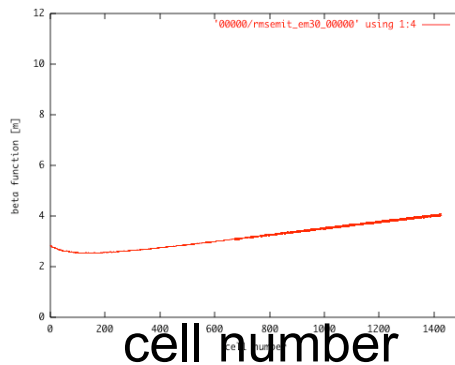
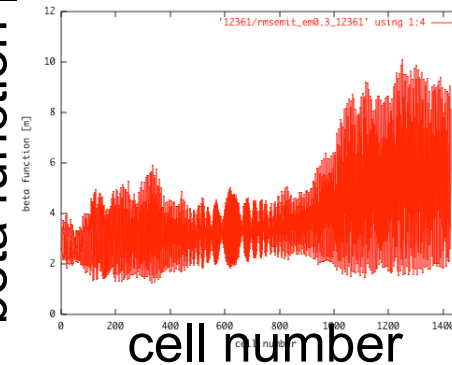
cell number

beta function [m]

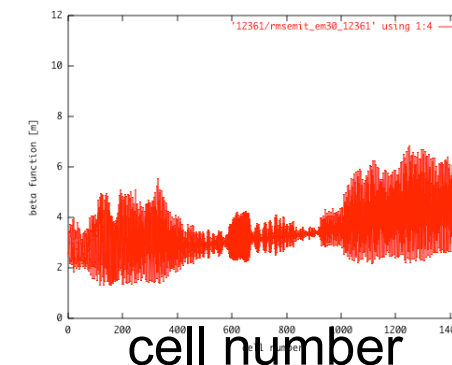
cell number



0.3 p mm rad



30 p mm rad



cell number

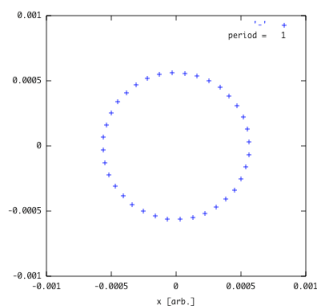
cell number

# What causes particle loss? (6)

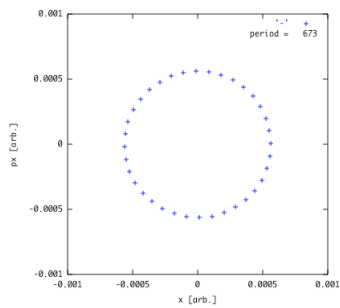
## Envelop evolution normalized with “beam beta”

0.003 p mm rad

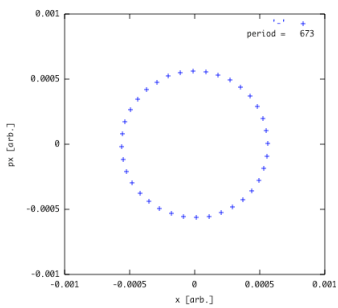
0 turn



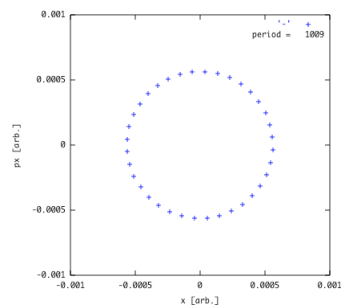
4 turn



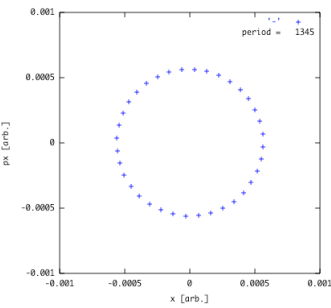
8 turn



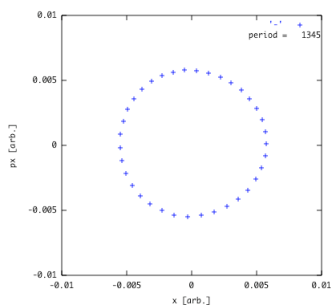
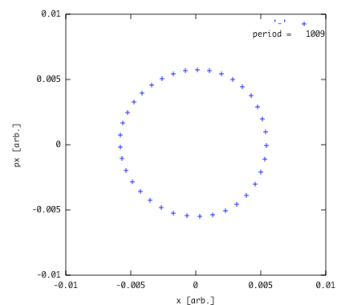
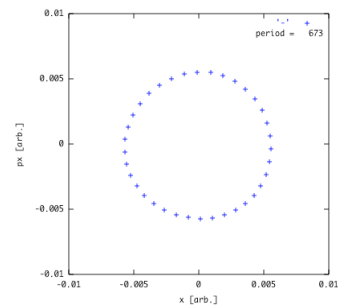
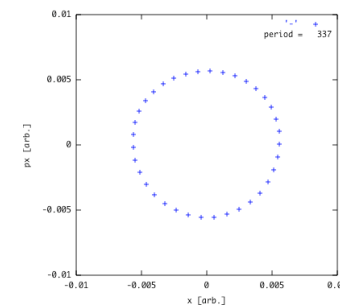
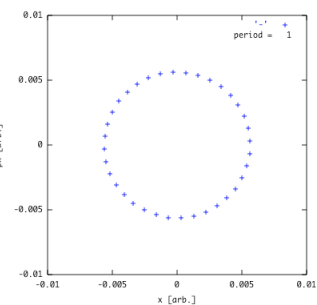
12 turn



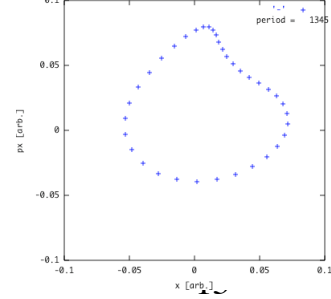
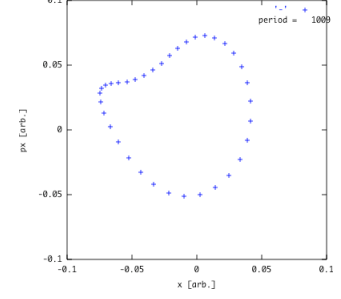
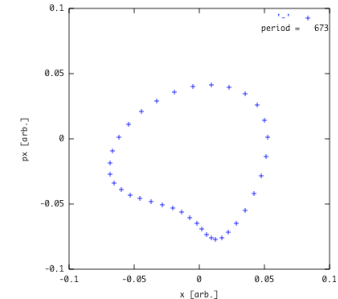
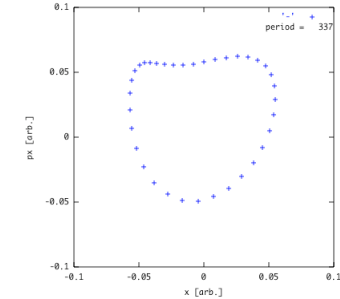
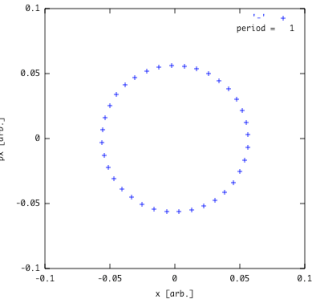
16 turn



0.3 p mm rad



30 p mm rad



# What causes particle loss? (7)

## 1D summary

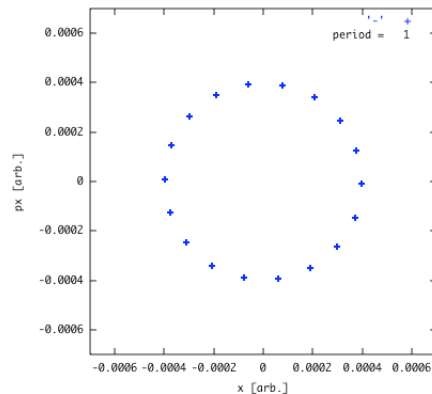
- Beam envelop tumbles due to modulation of beta function or mismatch (they are the same thing?) .
- Although the emittance does not change, amplitude of some particle becomes large and hit walls.
- Large amplitude beam ( $30 \pi$  mm rad) is distorted by nonlinearity of lattice. Non structure harmonic is excited.

# What causes particle loss? (8)

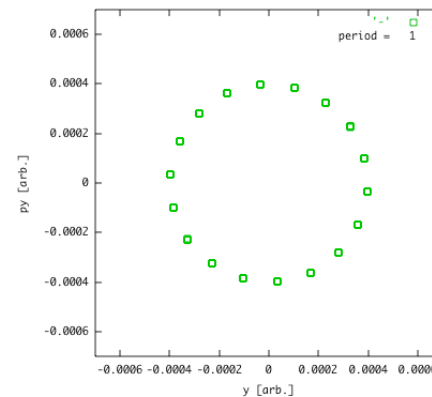
## 2D model

- Track horizontal and vertical envelop and see how it evolves.
- Initially emittance is 0.0015, 0.15, and  $15 \pi$  mm rad. The sum of horizontal and vertical emittance is 0.003, 0.3 and  $30 \pi$  mm rad.
- Initially  $dp/p$  and phase spread is zero.  $dp/p$  remains zero through the process.

horizontal



vertical



# What causes particle loss? (7) envelop evolution with “beam beta”

0.003 p mm rad

0 turn

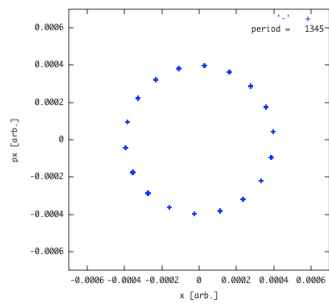
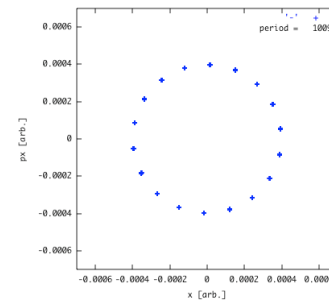
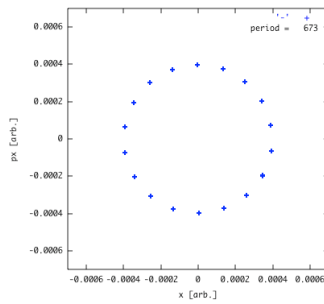
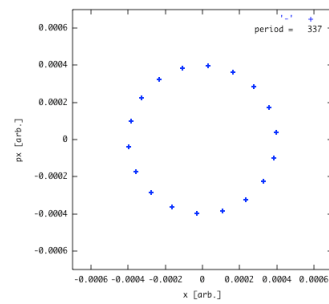
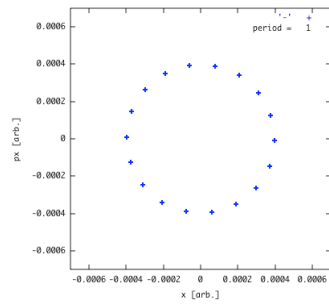
4 turn

8 turn

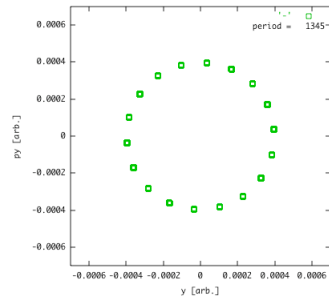
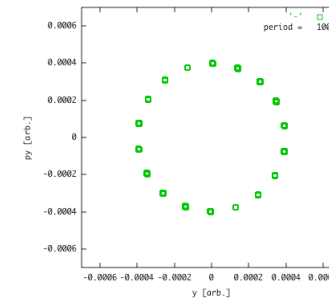
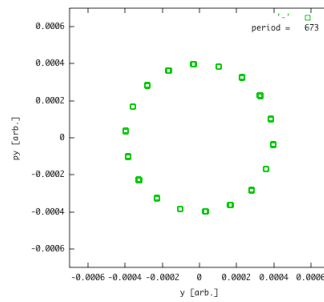
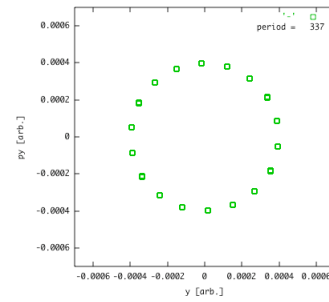
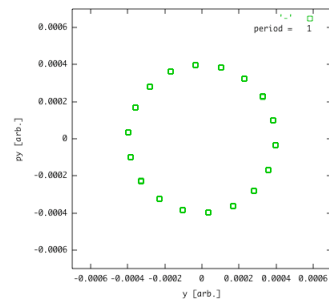
12 turn

16 turn

horizontal



vertical



# What causes particle loss? (9) envelop evolution with “beam beta”

0.3 p mm rad

0 turn

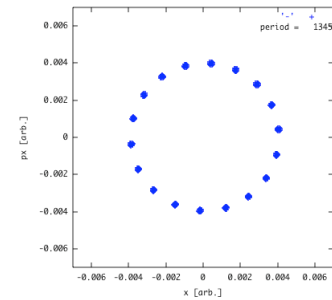
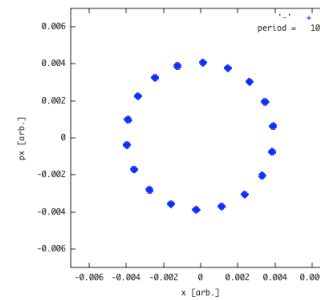
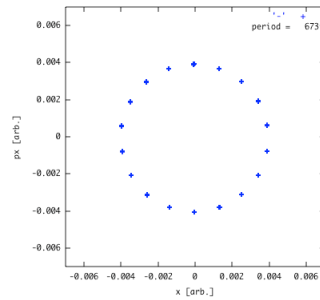
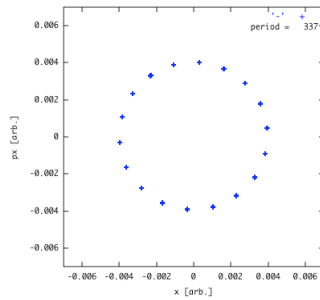
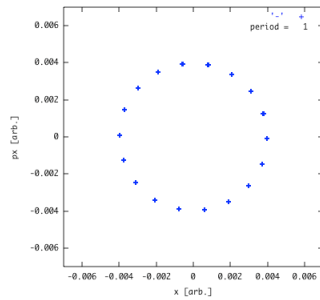
4 turn

8 turn

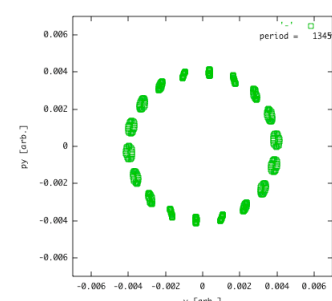
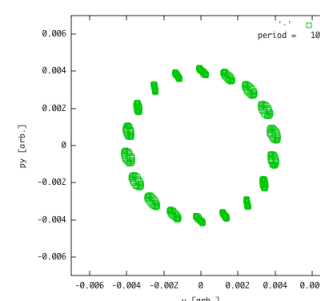
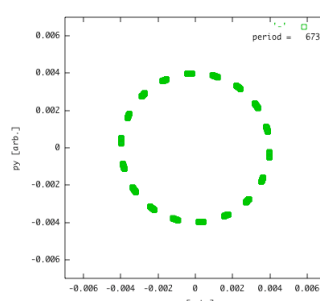
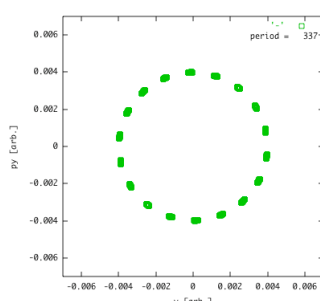
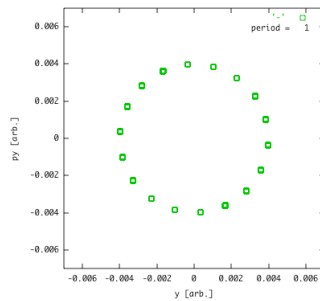
12 turn

16 turn

horizontal



vertical



# What causes particle loss? (10)

## envelop evolution with “beam beta”

30 p mm rad

0 turn

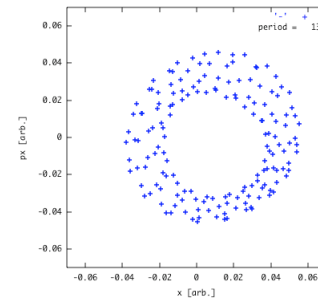
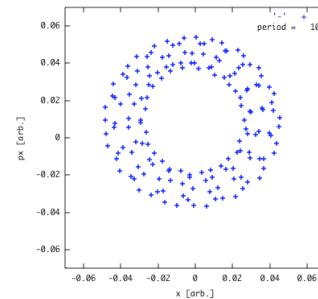
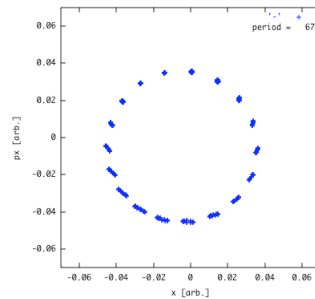
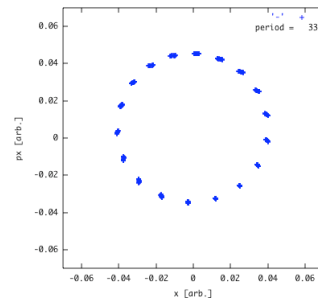
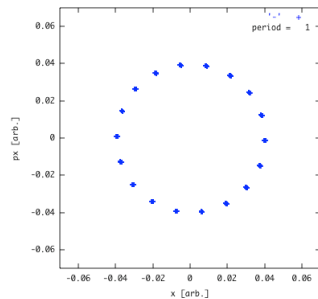
4 turn

8 turn

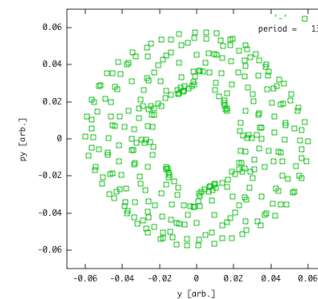
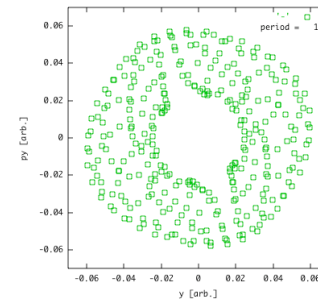
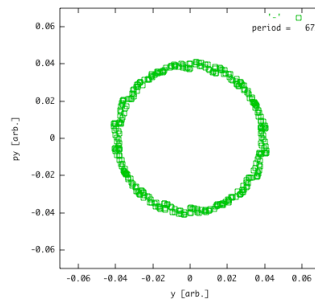
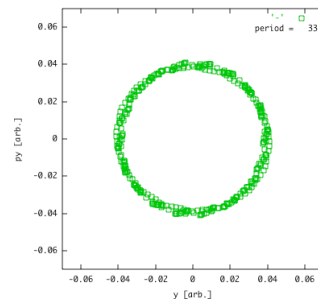
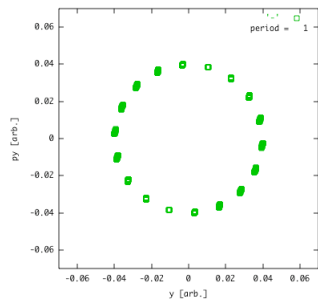
12 turn

16 turn

horizontal

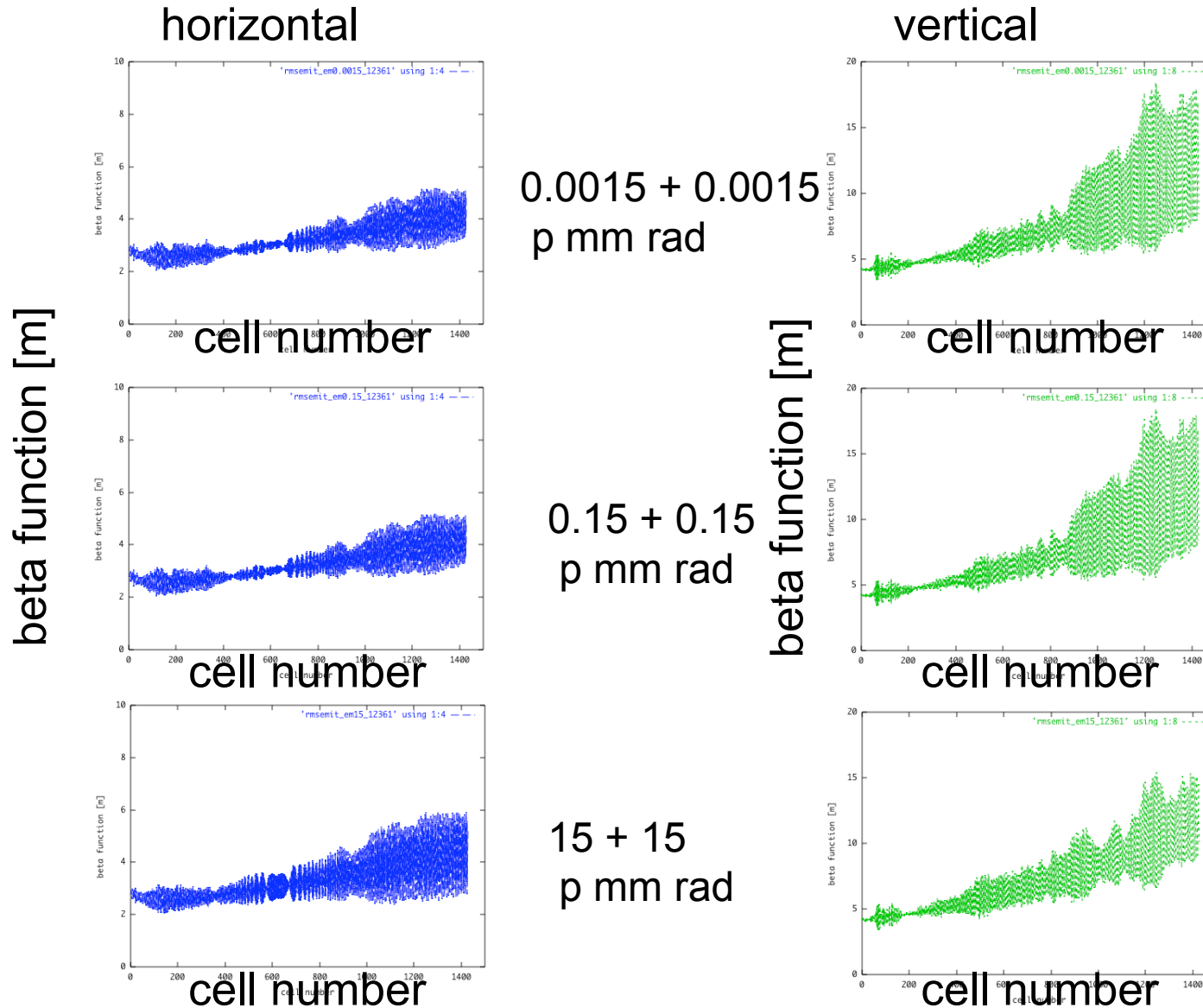


vertical



# What causes particle loss? (11)

## Horizontal and vertical beam beta with gradient errors



# What causes particle loss? (9)

## envelop evolution with “beam beta” without errors

30 p mm rad

0 turn

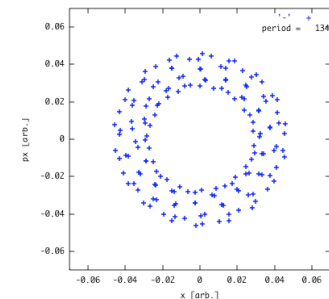
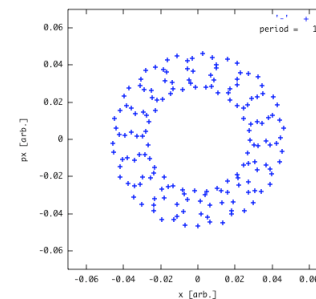
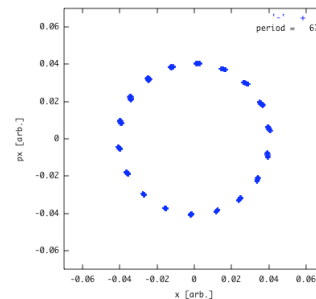
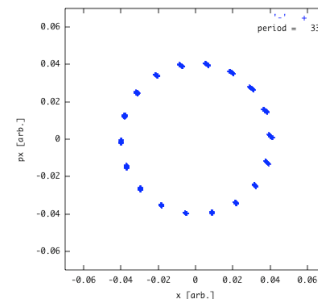
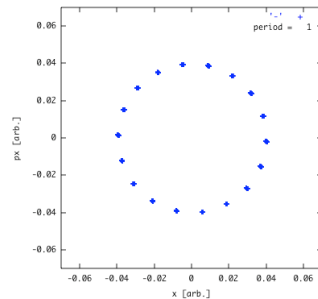
4 turn

8 turn

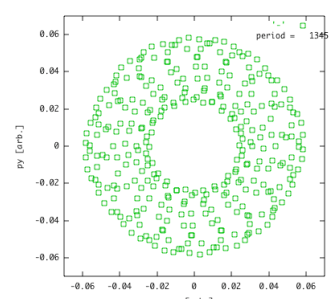
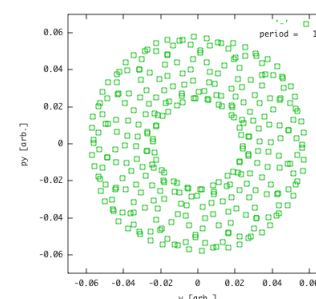
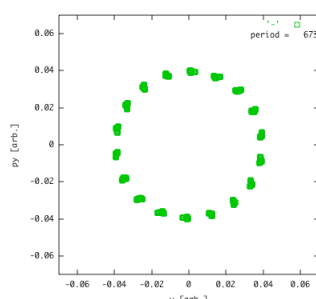
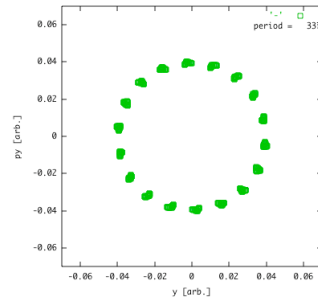
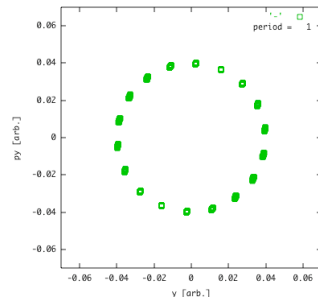
12 turn

16 turn

horizontal



vertical



# What causes particle loss? (10)

## 2D summary

- As seen in 1D case, beam envelop tumbles due to modulation of beta function or mismatch.
- Large amplitude beam ( $30 \pi$  mm rad) is suffered from coupling resonance:  $\nu_x - 2\nu_y = 0$ . That is independent of errors.

# Summary

- When aperture is 1.5 times of emittance (30 p mm rad) in 10 to 20 GeV muon ring.
  - Alignment error of 100  $\mu\text{m}$  (rms) seems to be tolerable.
  - Gradient error of  $dG/G = 1 \times 10^{-3}$  (rms) seems to be tolerable.
- Particle loss mechanism with gradient errors is the following.
  - Beta function or envelop of a beam is modulated. Beam size blows up gradually during acceleration.
  - Nonlinear driving term is excited due to breakdown of symmetry. Large amplitude particle is suffered and beam size increases.
  - Independent of errors, coupling resonance:  $\nu_x - 2\nu_y = 0$  makes vertical emittance growth.