Innovative RF acceleration in scaling FFAG accelerator

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Features of scaling FFAG

- A fixed magnetic field

  → Rapid acceleration with high repetition.

- Based on strong focusing

- Zero chromaticity

  → A beam is stable during acceleration.

Beam acceleration

There are two way of acceleration.

- RF frequency is variable → For proton acceleration
- RF frequency is fixed → Harmonic number jump
  Stationary bucket acceleration
  For electron or muon acceleration.
  (proton in the future)

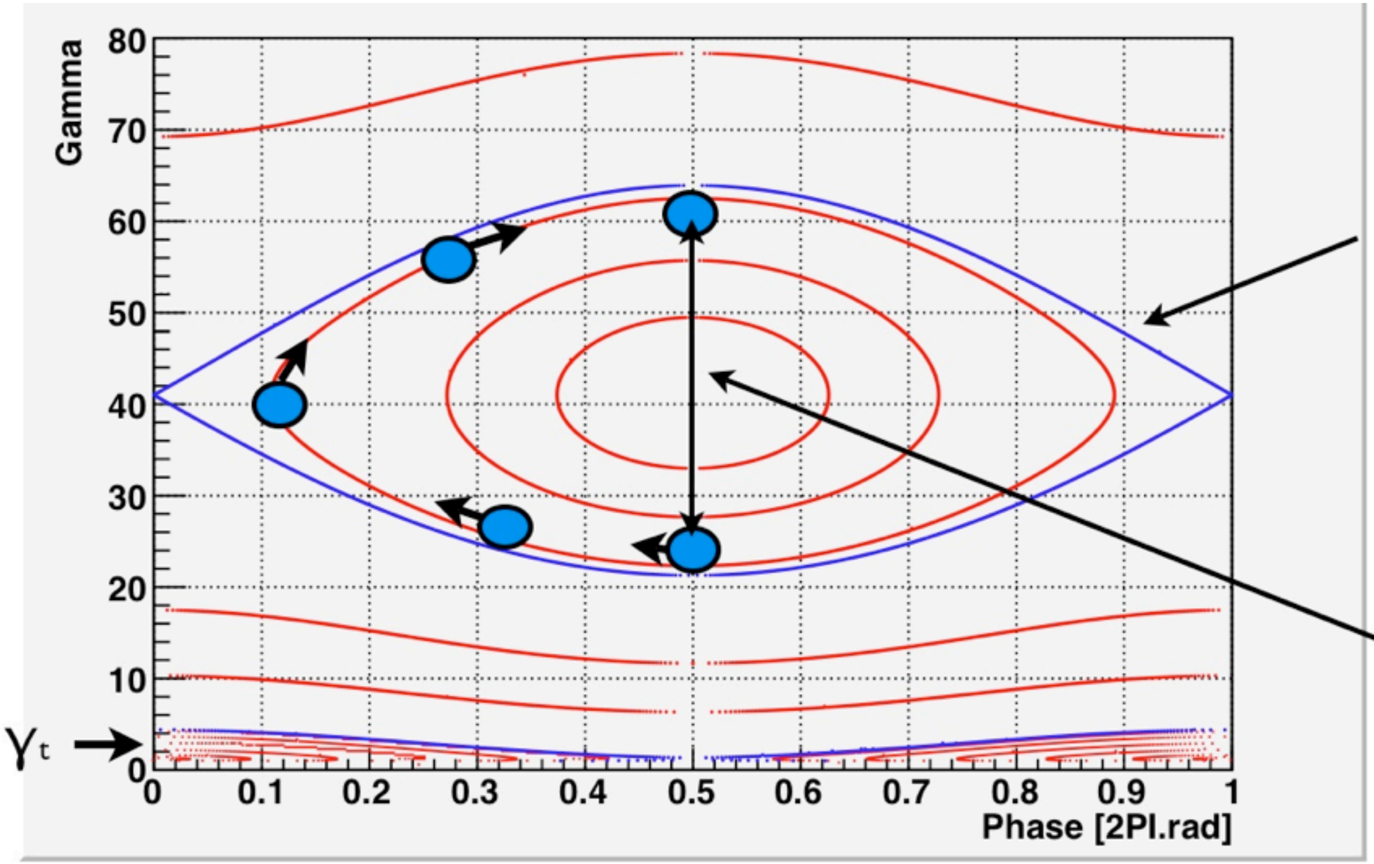
< The merit of using fixed RF frequency >

- We can get high RF voltage rather than variable frequency

→ Rapid acceleration can be achieved.

The problem of stationary bucket acceleration

- The bucket hight is limited by RF voltage.



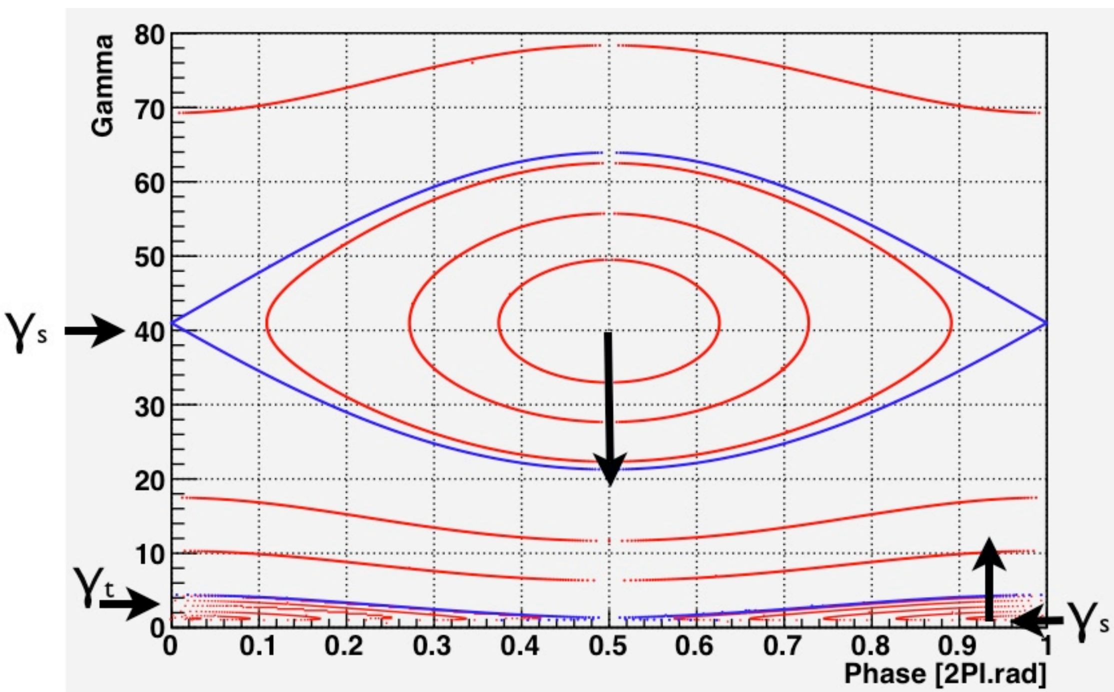
Put particles in stationary bucket below/above transition energy(Et).

The energy change is small.

[Fig.1 Stationary bucket acceleration using one bucket]

Improve the problem

- To get large energy change even if use stationary bucket acceleration



[Fig.2 Hamiltonian contour]

To make the two stationary energies approach each other which is below/above the transition energy.

The two stationary buckets can be close.

If we can accelerate particles using two stationary buckets, the energy change will be large.

The way of making two stationary energies close each other

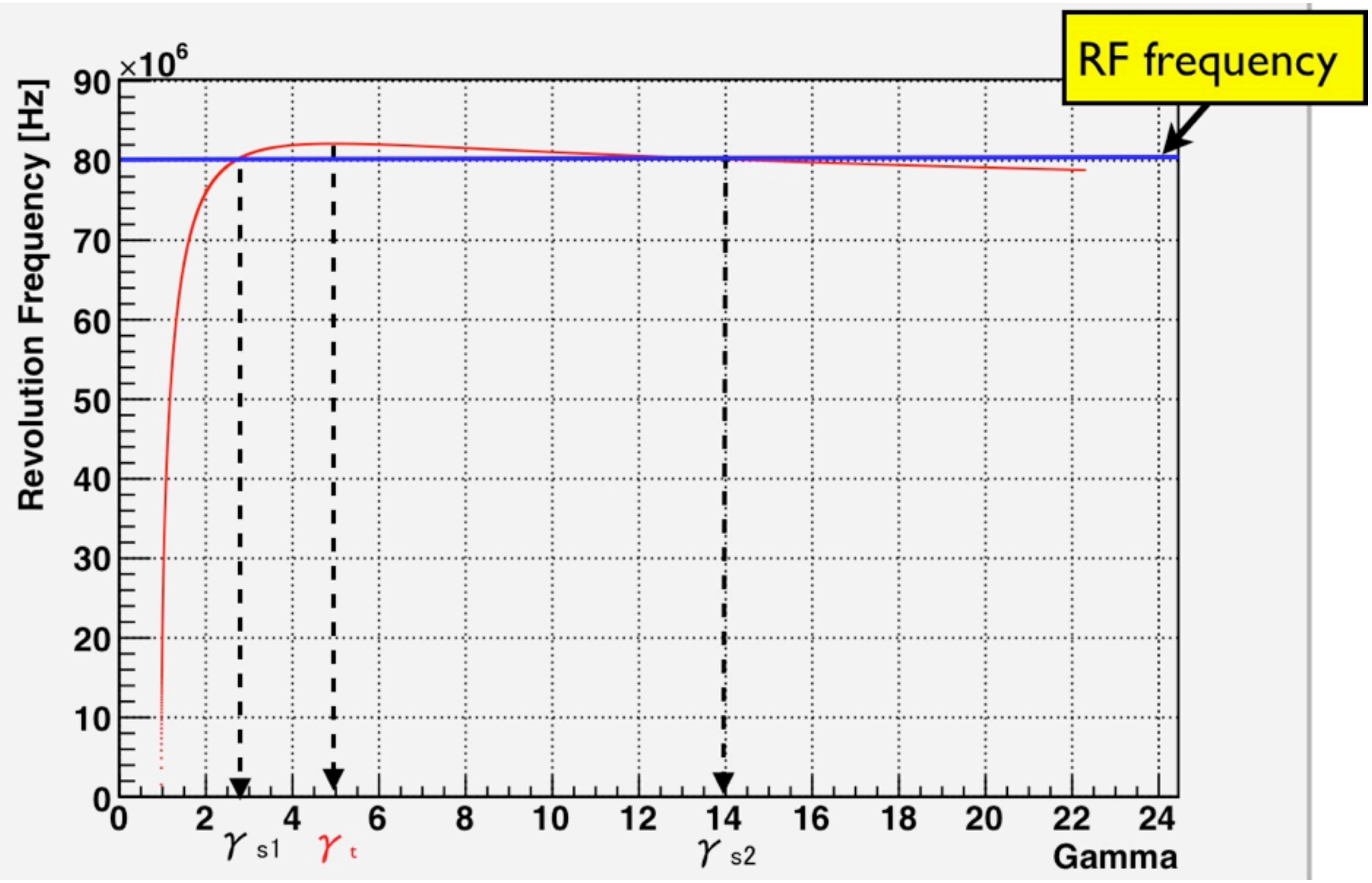
$$R = R_0 \left(\frac{P}{P_0} \right)^{\frac{1}{k+1}}$$

$$f = \frac{\beta c}{2\pi R}$$

$$k = 24$$

$$R_0 = 0.58[m]$$

$$E_0 = 6500[MeV]$$



γ_{s1} : Stationary gamma@under transition energy

γ_{s2} : Stationary gamma@upper transition energy

If we choose the RF frequency near the top of the curve, two stationary gamma(energy) close to each other.

Longitudinal Hamiltonian for scaling FFAG accelerator

We can analytically write Hamiltonian for scaling FFAG.

$$\frac{T_{rev}}{T_{rev(s)}} = 1 + \frac{\Delta\phi}{2\pi h} = \left(\frac{R}{R_{s2}}\right) \Big/ \frac{P/E}{P_{s2}/E_{s2}}$$

$$= \left(\frac{P}{P_{s2}}\right)^{\frac{1}{k+1}} \frac{P_{s2}}{E_{s2}} \left(\frac{E}{P}\right)$$

$$= \left(\frac{E^2 - m^2}{P_{s2}^2}\right)^{\frac{\alpha}{2}} \frac{P_{s2}}{E_{s2}} \left(\frac{E^2}{E^2 - m^2}\right)^{\frac{1}{2}}$$

$$= P_{s2}^{1-\alpha} \frac{E}{E_{s2}} (E^2 - m^2)^{\frac{\alpha - 1}{2}}$$

$T_{rev(s)}$: Revolution time of reference particle
T_{rev} : Revolution time of each particle
E_{s1} : Stationary Energy@under transition energy
E_{s2} : Stationary Energy@upper transition energy
$\alpha = \frac{1}{k+1}$

Exact for scaling FFAG

$$R = R_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

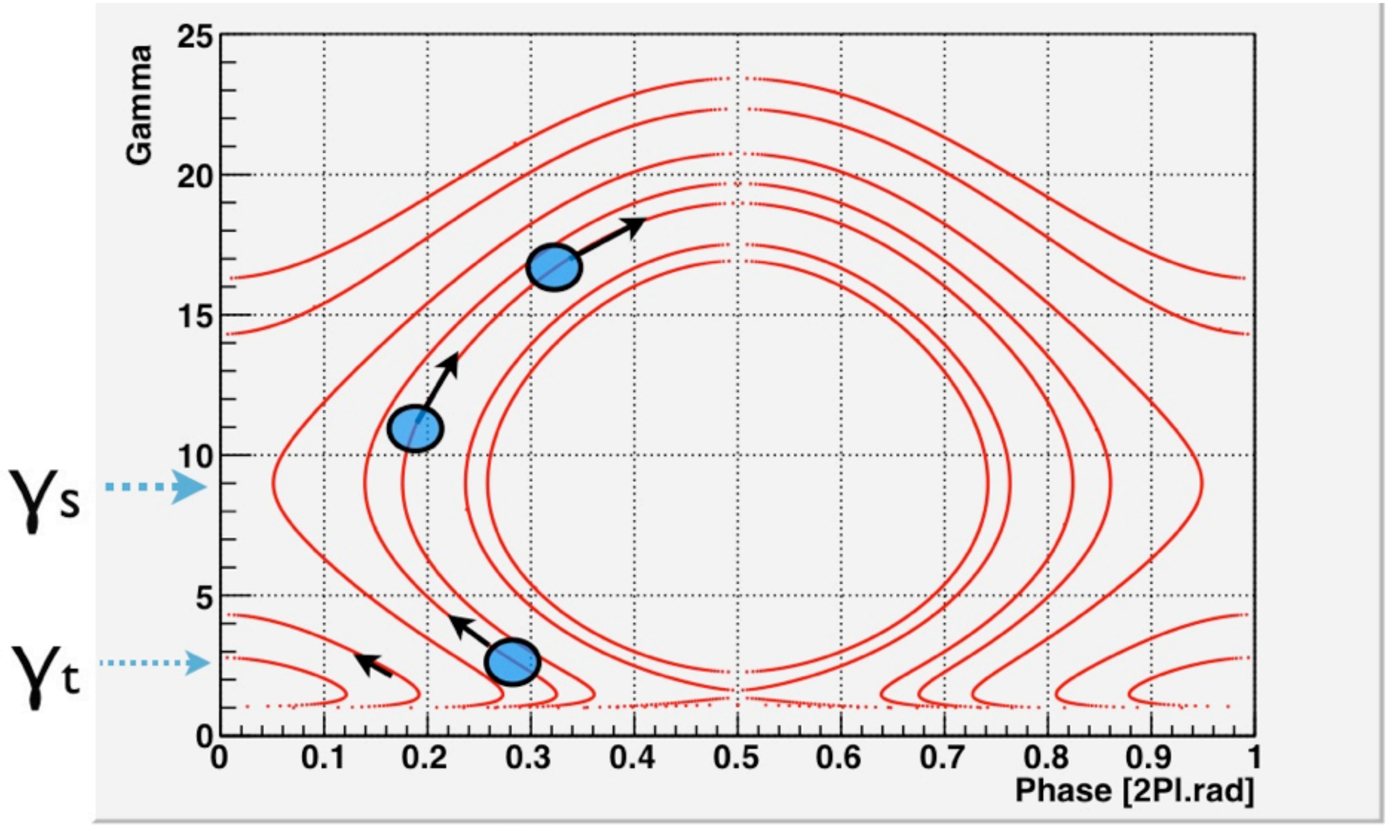
$$\Delta \leftrightarrow \frac{1}{f_{rf}} \frac{d}{dt}$$

$$\begin{cases} \dfrac{d\phi}{dt} = 2\pi f_{rf} h \left[\dfrac{P_{s2}^{1-\alpha}}{E_{s2}} E (E^2 - m^2)^{\frac{\alpha-1}{2}} - 1 \right] \\ \dfrac{dE}{dt} = f_{rf} V_0 \sin\phi \end{cases}$$

$$H = 2\pi h \left[\frac{1}{\alpha + 1} \frac{(E^2 - m^2)^{\frac{\alpha+1}{2}}}{E_{s2} P_{s2}^{\alpha - 1}} - E \right] + V_0 \cos\phi$$

Normalized by f_{rf}

Longitudinal phase space in scaling FFAG



[Fig.3 Hamiltonian contour]

γ is changed from 1 to 20 !! in this scheme

- Some particle can move up beyond transition energy(equivalent to transition gamma:γt)
- From injection to extraction, the energy change is big.
- Rapid acceleration from injection to extraction.

Details of innovative RF acceleration

- Requirement for RF voltage
- Stationary energy above transition energy vs acceptable path
- K value VS bucket height (1)
- K value VS bucket height (2)

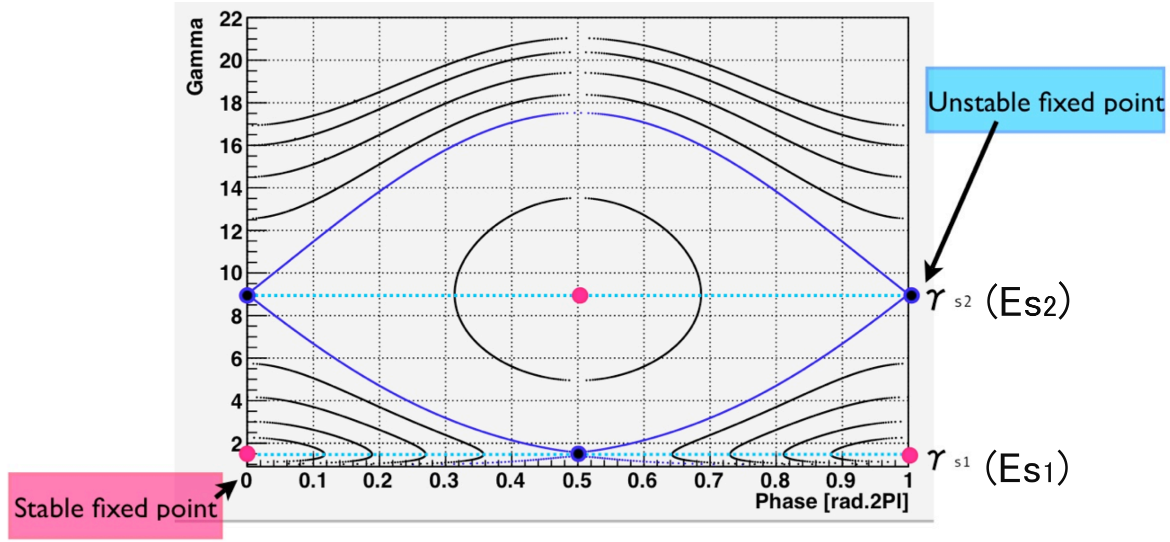
Requirement for RF voltage

$$H = 2\pi h\left[\frac{1}{\alpha+1}\frac{(E^2-m^2)^{\frac{\alpha+1}{2}}}{E_{s2}P_{s2}^{\alpha-1}} - E\right] + V_0\cos\phi$$

Normalized by f_{rf}

T_0 : Revolution time of reference particle
T : Revolution time of each particle
E_{s1} : Stationary Energy@under transition energy
E_{s2} : Stationary Energy@upper transition energy
$\alpha = \frac{1}{k+1}$

The Hamiltonian contour which through the point (E_{s2},π) and (E_{s1},0) is the limit contour which is going up across the transition energy.



$$H(E_{s2},0) = H(E_{s1},\pi)$$

$$V = \pi h\left[\frac{1}{\alpha+1}\left(\frac{P_{s1}^2}{E_{s1}} - \frac{P_{s2}^2}{E_{s2}}\right) + (E_{s2}-E_{s1})\right]$$

Once we set the parameters of stationary energy(E_{s1}/E_{s2}), α and harmonic number, we can get minimum RF voltage.

Requirement for RF voltage

$$H = 2\pi h\left[\frac{1}{\alpha+1}\frac{(E^2-m^2)^{\frac{\alpha+1}{2}}}{E_{s2}P_{s2}^{\alpha-1}} - E\right] + V_0\cos\phi$$

Normalized by f_{rf}

T_0 : Revolution time of reference particle
T : Revolution time of each particle
E_{s1} : Stationary Energy@under transition energy
E_{s2} : Stationary Energy@upper transition energy
$\alpha = \frac{1}{k+1}$

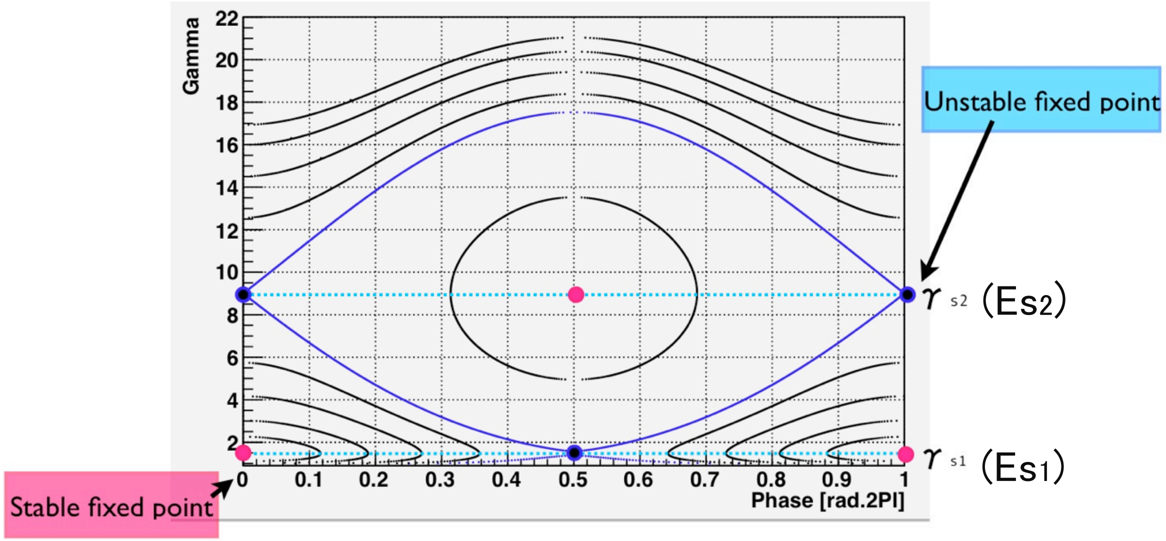
The Hamiltonian contour which through the point (E_{s2},π) and $(E_{s1},0)$ is the limit contour which is going up across the transition energy.

↓

$$H(E_{s2},0) = H(E_{s1},\pi)$$

↓

$$\alpha = \frac{\left(\frac{P_{s1}^2}{E_{s1}} - \frac{P_{s2}^2}{E_{s2}}\right)}{\frac{V_0}{\pi h} - (E_{s2}-E_{s1})} - 1$$



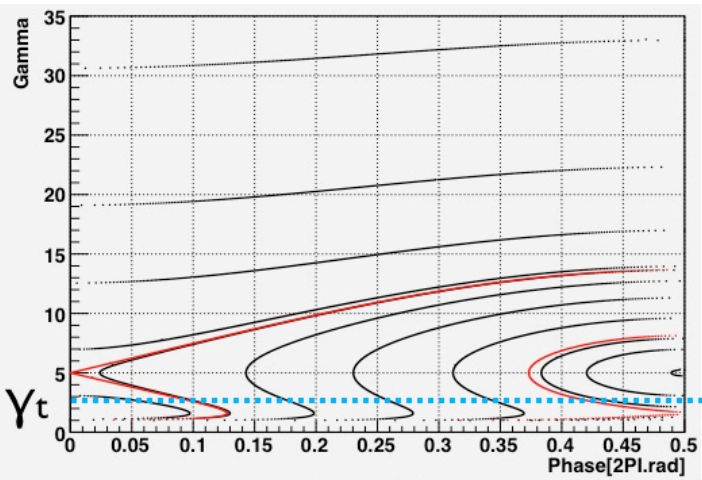
If we get stationary energy(E_{s2}/E_{s1}), harmonic number and RF voltage, we can know α (K value).

Details of innovative RF acceleration

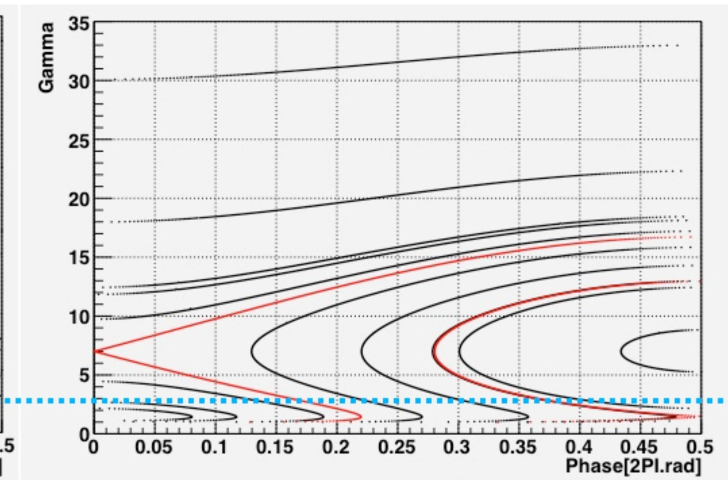
- Requirement for RF voltage
- Stationary energy above transition energy vs acceptable path
- K value VS bucket height (1)
- K value VS bucket height (2)

Stationary energy above transition energy vs acceptable path

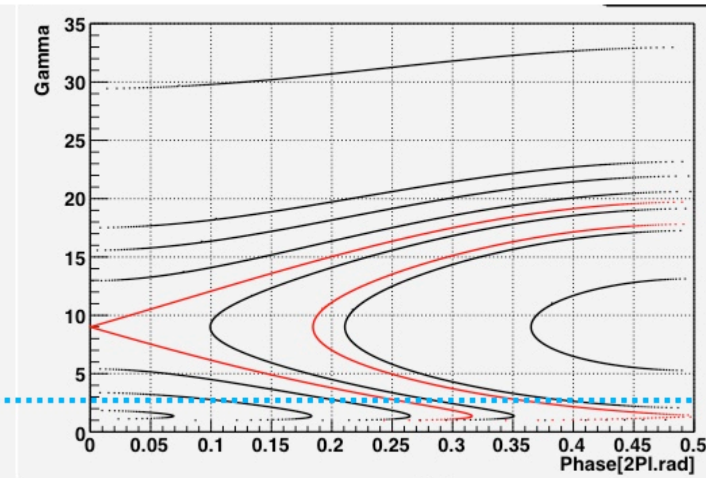
RF voltage	Stationary gamma (E_{s2})	K value	Harmonic number
Fixed	Changing	Fixed	Fixed



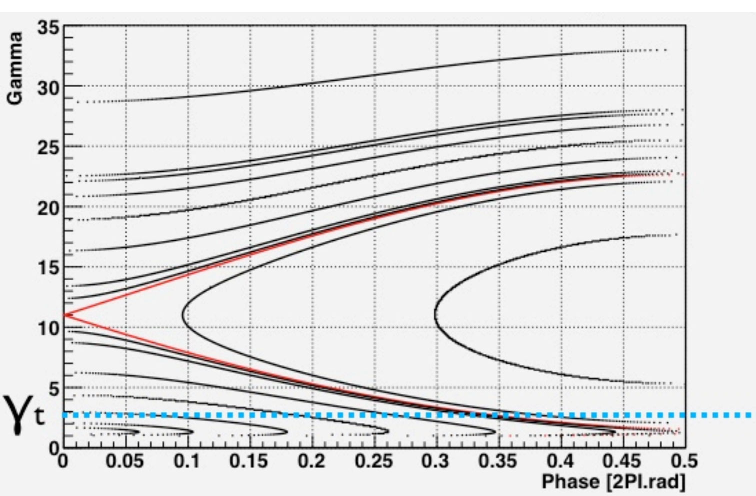
[γ_s=5]



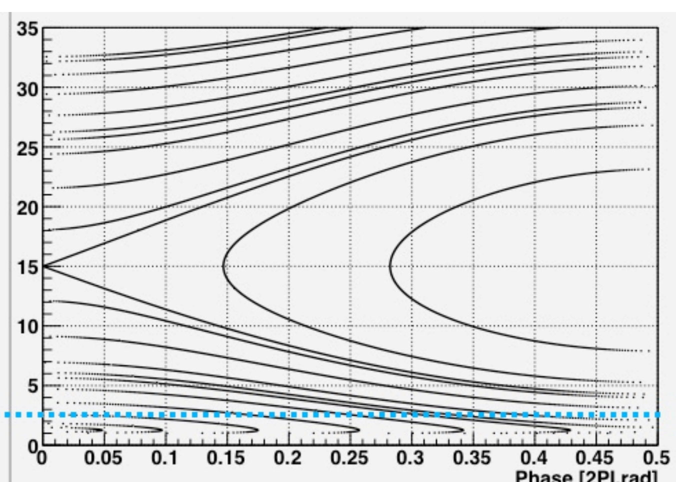
[γ_s=7]



[γ_s=9]



[γ_s=11]



[γ_s=15]

E_{s2} : Stationary Energy@upper transition energy

γ_t : Transition gamma

Acceptable path is becoming small.

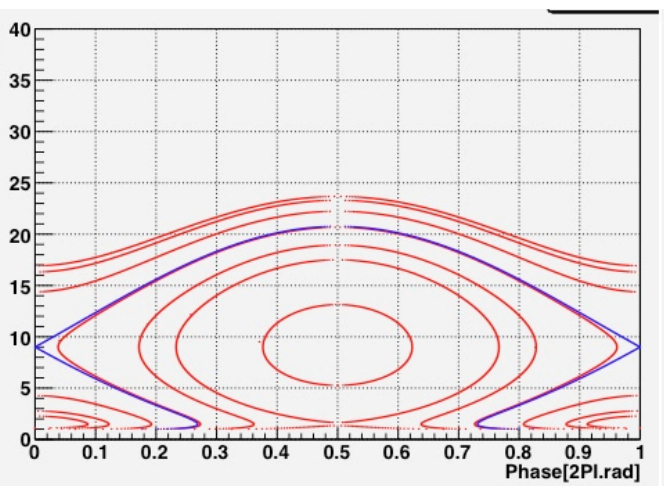
Details of innovative RF acceleration

- Requirement for RF voltage
- Stationary energy above transition energy vs acceptable path
- K value VS bucket height (1)
- K value VS bucket height (2)

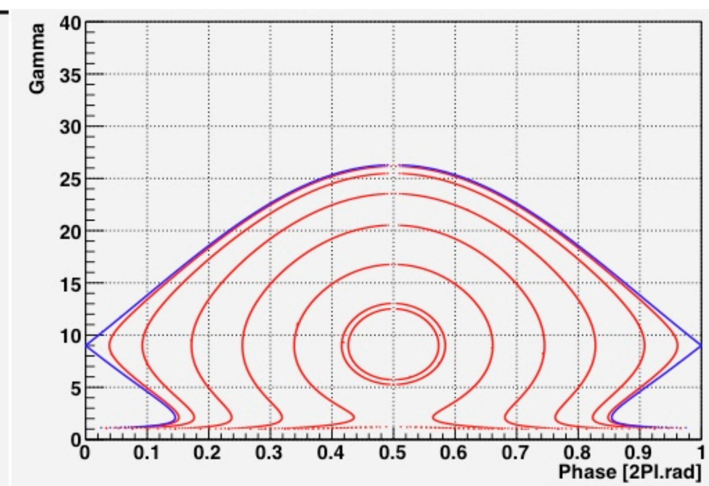
K value VS bucket height (1)

RF voltage	Stationary gamma (E_{s2})	K value	Harmonic number
Fixed	Fixed	Changing	Fixed

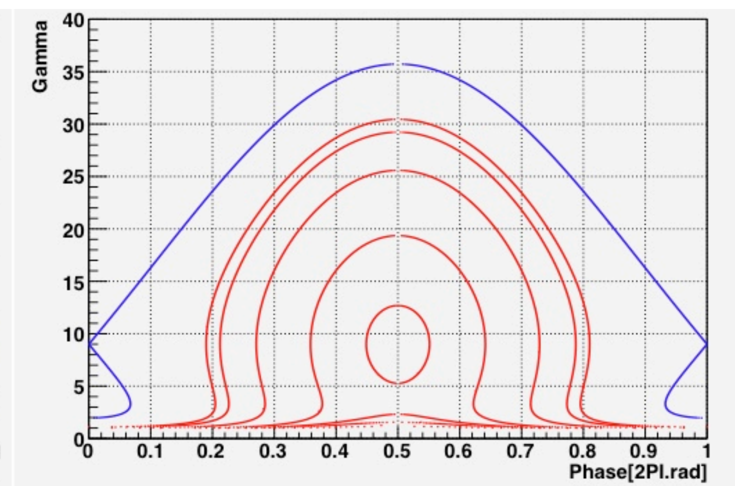
E_{s2} : Stationary Energy@upper transition energy



[k=6]



[k=12]



[k=24]

Changing only k value , the bucket hight becomes large.

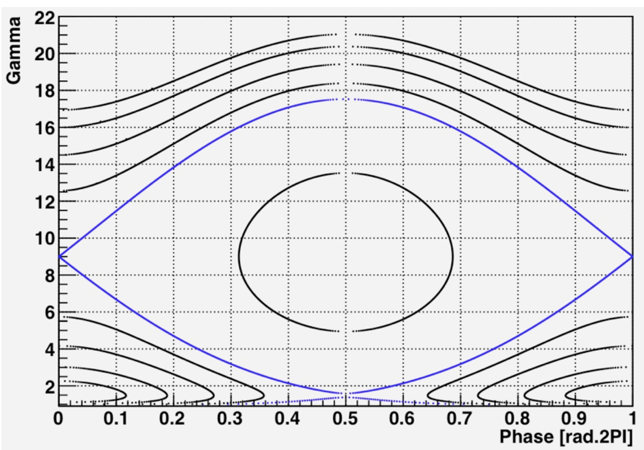
Details of innovative RF acceleration

- Requirement for RF voltage
- Stationary energy above transition energy vs acceptable path
- K value VS bucket height (1)
- K value VS bucket height (2)

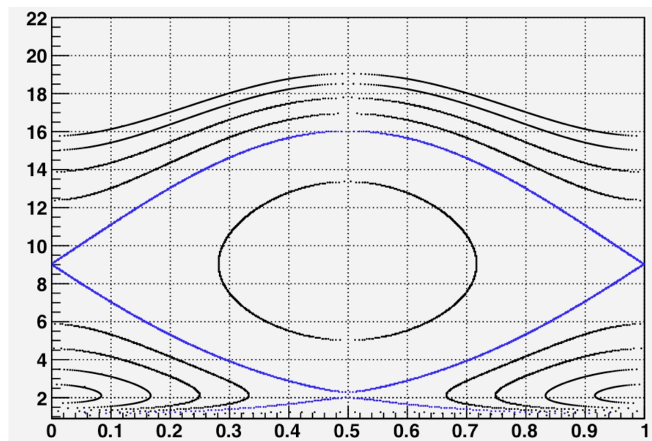
Relation between K value and bucket height (2)

RF voltage	Stationary gamma (E_{s2})	K value	Harmonic number
Minimum RF voltage with each k	Fixed	Changing	Fixed

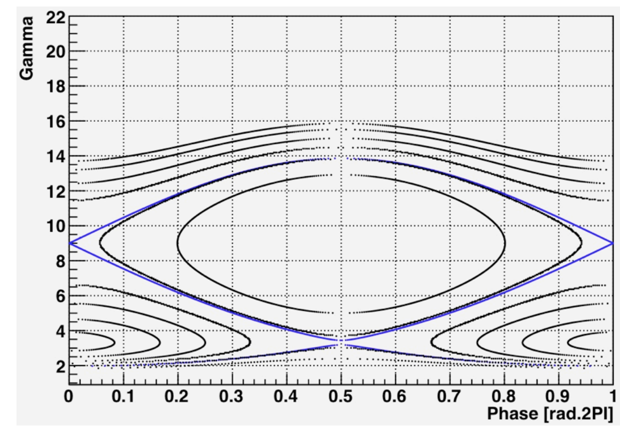
E_{s2} : Stationary Energy@upper transition energy



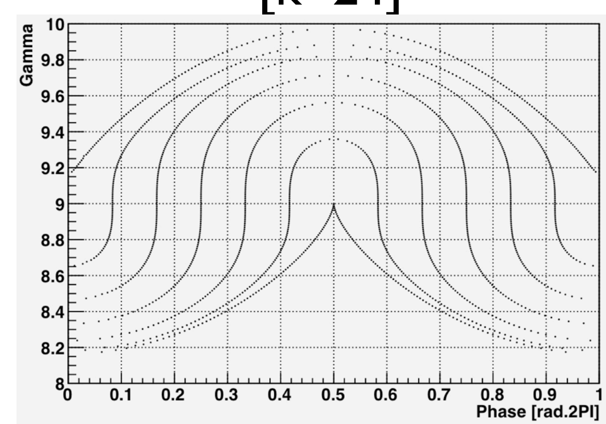
[k=6]



[k=12]



[k=24]



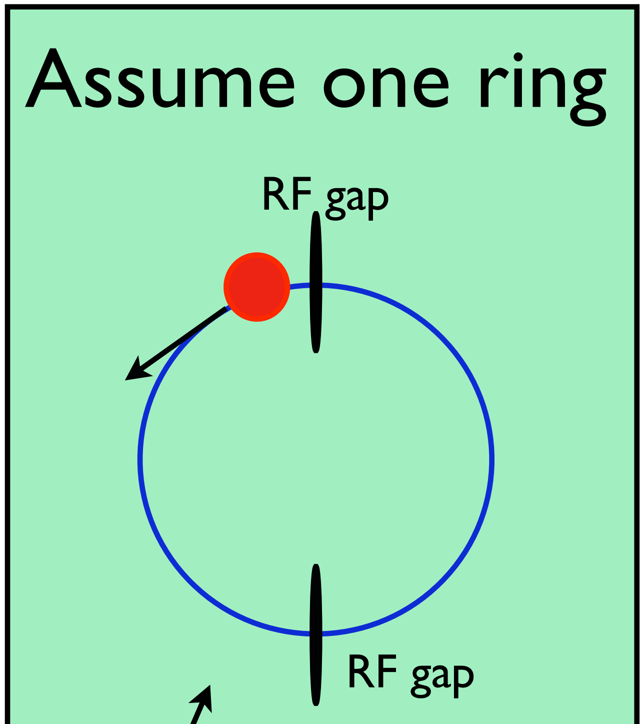
[k=80]

Depending on large k value, the bucket hight becomes small.

※Input RF voltage : 1[MV]

Longitudinal tracking

To Know how many turns does a particle go around the ring from injection energy to the most high energy.

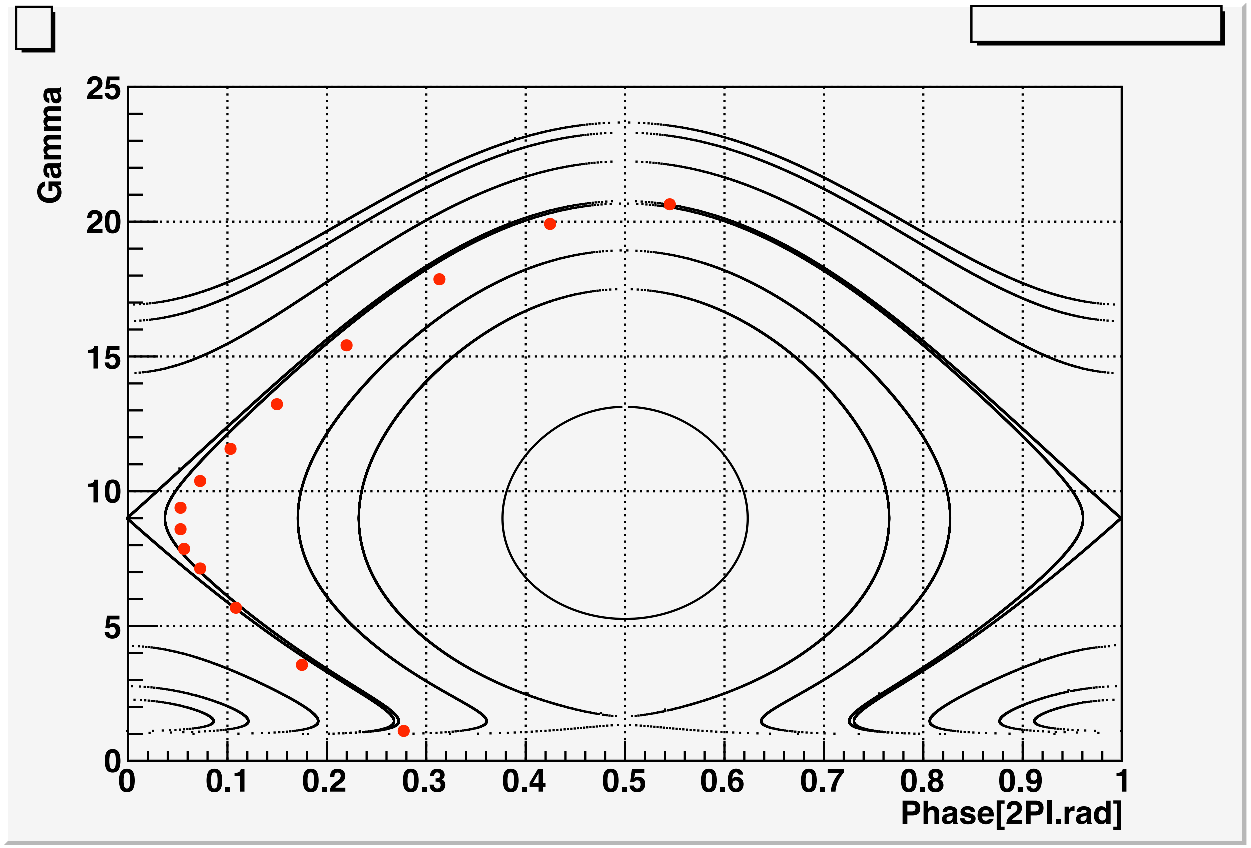


There are 2 RF gap in the ring

Particle	muon
k	6
RF frequency	15 [MHz]
Stationary gamma	9
Each RF voltage	$1.2 \times m_0c^2$
Harmonic number	1
Reference orbit	3.1 [m]
Initial phase	100 [deg]
Initial gamma	~1

Longitudinal tracking result

The red dot is one particle every one turns in the ring.



[Fig.4 Tracking result]

In this scheme

- The number of turn to reach the max energy is 13 turns.

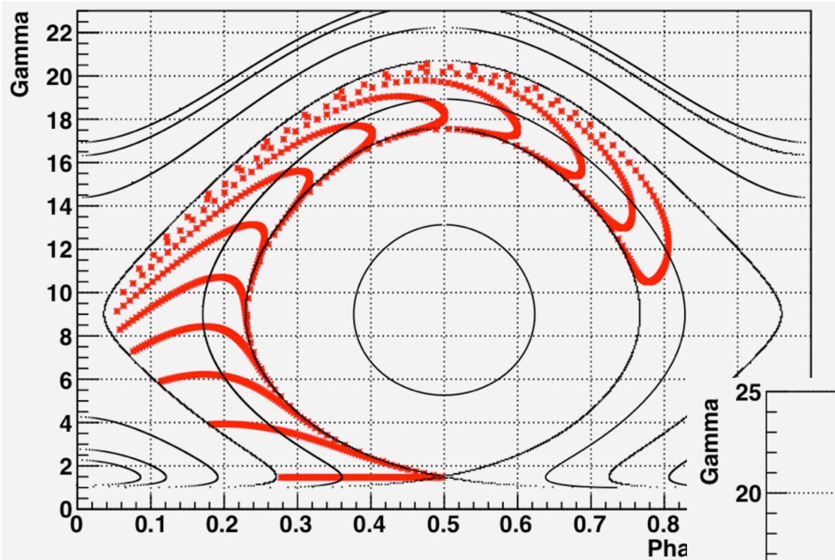
Summary

* I studied of innovative RF acceleration in scaling FFAG accelerator.
* The point is to make the two stationary energies approach each other.
* Using two stationary buckets for acceleration, we can get large energy change.

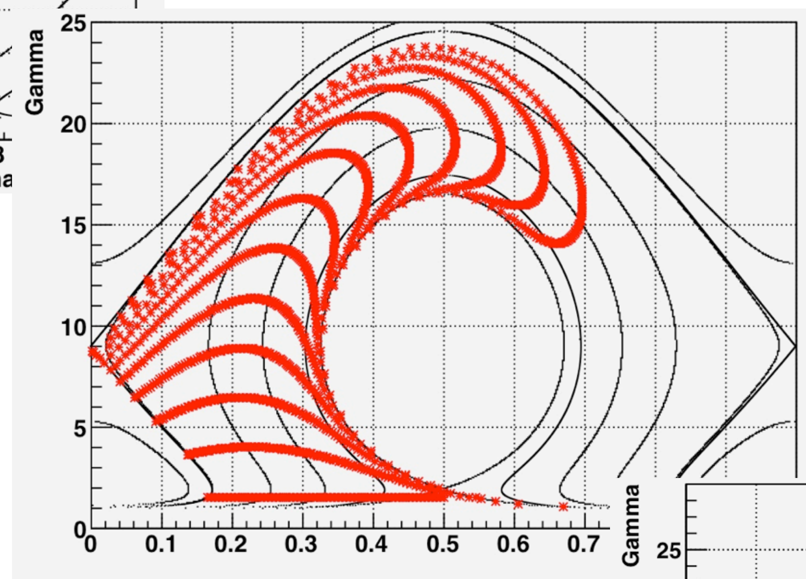
Future

- Try to study about the beam behavior depending on the injection condition.
  - Injection phase
  - Injection energy

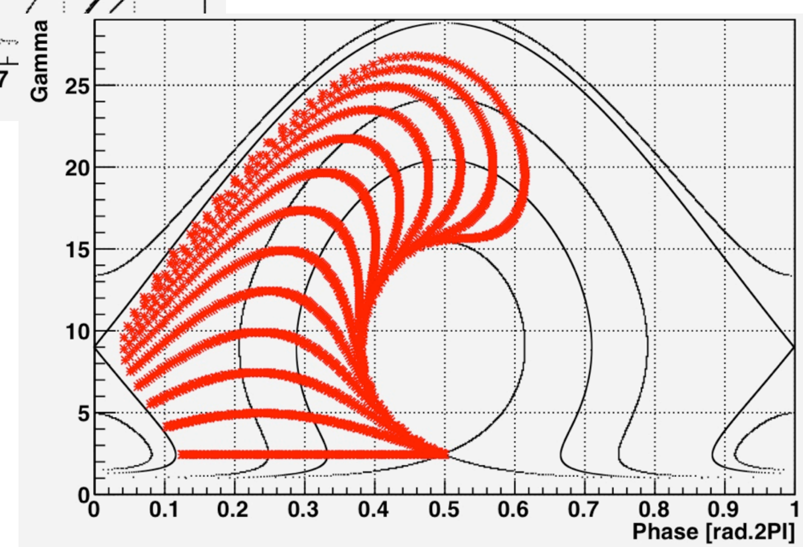
Thank you!



k=6



k=10



k=12

Detail of the shape (I)

$$H = 2\pi h\left[\frac{1}{\alpha+1}\frac{(E^2-m^2)^{\frac{\alpha+1}{2}}}{E_0 P_0^{\alpha-1}} - E\right] + V_0\cos\phi$$

E_0 : stationary energy
V_0 : RF voltage

At some point (E,ϕ) , the $dE/d\phi$ is

$$\frac{\partial H}{\partial \phi} = 2\pi h\left[\frac{1}{\alpha+1}\frac{1}{E_0 P_0^{\alpha-1}}\frac{d}{dE}[(E^2-m^2)^{\frac{\alpha+1}{2}}] - \frac{dE}{dE}\right]\frac{dE}{d\phi} - V_0\sin\phi$$

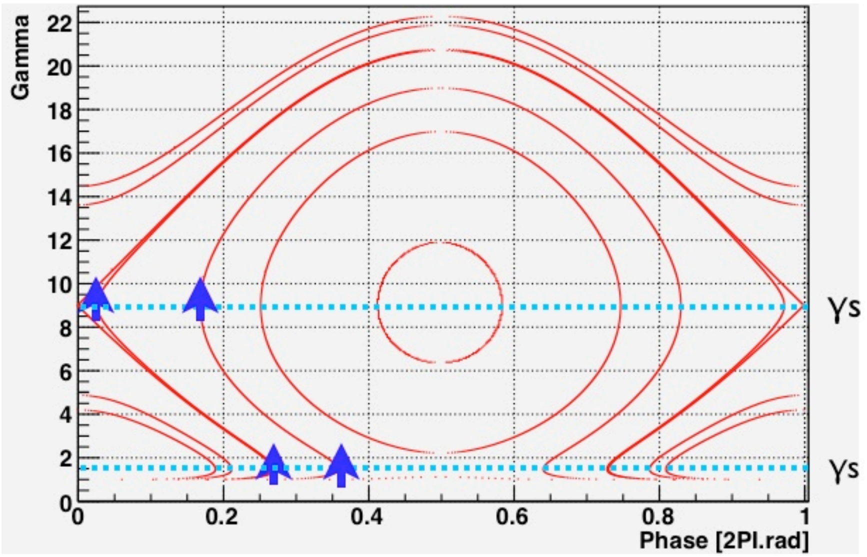
$$\left[\frac{2\pi h}{\alpha+1}\frac{1}{E_0 P_0^{\alpha-1}}\frac{\alpha+1}{2}2E(E^2-m^2)^{\frac{\alpha-1}{2}} - 2\pi h\right]\frac{dE}{d\phi} - V_0\sin\phi = 0$$

$$2\pi h\left[\frac{EP^{\alpha-1}}{E_0 P_0^{\alpha-1}} - 1\right]\frac{dE}{d\phi} = V_0\sin\phi$$

$$\therefore \frac{dE}{d\phi} = \frac{V_0\sin\phi}{2\pi h\left[\frac{EP^{\alpha-1}}{E_0 P_0^{\alpha-1}} - 1\right]}$$

From above equation, when $E = E_0$, $dE/d\phi = \infty$

- When the particle energy reaches E0, the direction of particle change.



Detail of the shape (2)

To look for inflection point, differentiate dE/dΦ with Φ again.

$$\frac{dE}{d\phi} = \frac{V_0 \sin\phi}{2\pi h\left[\frac{EP^{\alpha-1}}{E_0 P_0^{\alpha-1}} - 1\right]}$$

$$\frac{d^2E}{d\phi^2} = \frac{\frac{V_0\cos\phi}{2\pi h} - \frac{P^{\alpha-1}}{E_0P_0^{\alpha-1}}\left[1-\beta_t^2\frac{1}{\beta^2}\right]\left(\frac{V\sin\phi}{2\pi h\left[\frac{EP^{\alpha-1}}{E_0P_0^{\alpha-1}}-1\right]}\right)^2}{\frac{EP^{\alpha-1}}{E_0P_0^{\alpha-1}}-1} \qquad (1)$$

When $d^2E/d\phi^2 = 0$ (inflection condition), (1) is

$$\therefore \cos\phi = \frac{-\frac{\pi h E_0 P_0^{\alpha-1}}{V_0}\left(\frac{EP^{\alpha-1}}{E_0P_0^{\alpha-1}}-1\right)^2 \pm \sqrt{\frac{(\pi h E_0 P_0^{\alpha-1})^2}{V^2}\left(\frac{EP^{\alpha-1}}{E_0P_0^{\alpha-1}}-1\right)^4 + \left(1-\frac{\beta_t^2}{\beta^2}\right)^2 P^{2(\alpha-1)}}}{\left(1-\frac{\beta_t^2}{\beta^2}\right)P^{\alpha-1}} \qquad (2)$$

- Plot the each energy and Φ based on (2), we can know inflection point.

The red contour is inflection point.

The blue contour is Hamiltonian contour which trough (Et, π/2)