Scaling FFAG lattices for muon acceleration

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Motivations

Use the large **transverse acceptance** of scaling FFAG lattices

while using constant RF frequency acceleration to reach **high accelerating gradient.**

Possible with either:

or

Harmonic number jump acceleration

Stationary bucket acceleration!

Part I

Scaling FFAG lattices for

Harmonic number jump acceleration

HARMONIC NUMBER JUMP ACCELERATION

To jump one harmonic every turn: $$T_{i+1} - T_i = \frac{1}{f_{RF}}$$

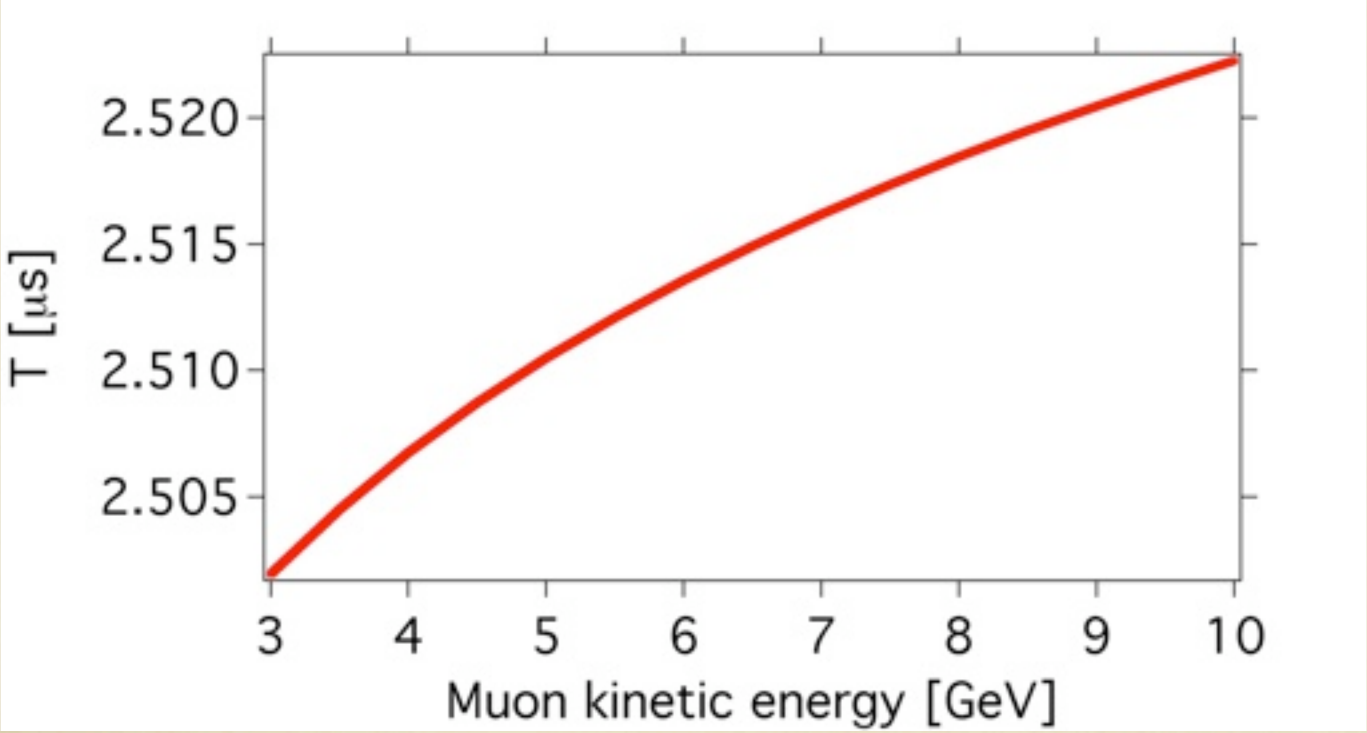


Figure 1 - Revolution time as a function of particle energy in the case of a 3 to 10 GeV scaling FFAG ring, with k = 145 and average radius = 120 m.

→ **Energy gain per turn must follow:** $$\Delta E_i = \frac{1}{f_{RF} \cdot \left[\frac{\Delta T}{\Delta E}\right]_{E_i}}$$

NEED FOR DISPERSION SUPPRESSOR INSERTIONS!

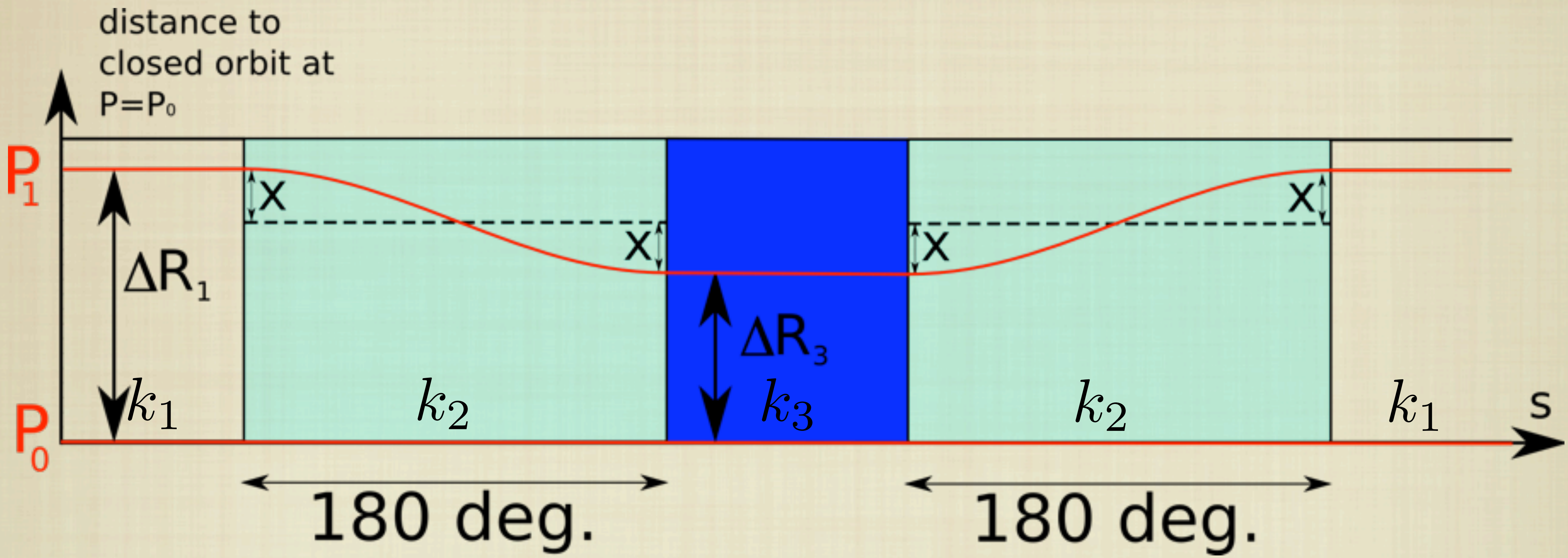
Harmonic jump condition: $$T_{i+1} - T_i = \frac{1}{f_{RF}}$$

In the same time: $$\frac{\Delta C_i}{\beta c} = T_{i+1} - T_i$$

In case of highly relativistic particles: $$\Delta R_i \approx \frac{c}{2\pi f_{RF}} = \frac{\lambda_{RF}}{2\pi}$$

$$average\ excursion = \lambda_{RF} \cdot \frac{N_{turns}}{2\pi}$$ ⟶ **Need for excursion reduced areas!**

Dispersion suppressor with FFAG magnets



$$with \ \frac{2}{k_2+1} = \frac{1}{k_1+1} + \frac{1}{k_3+1}$$

HNJ with cavities distributed around the ring

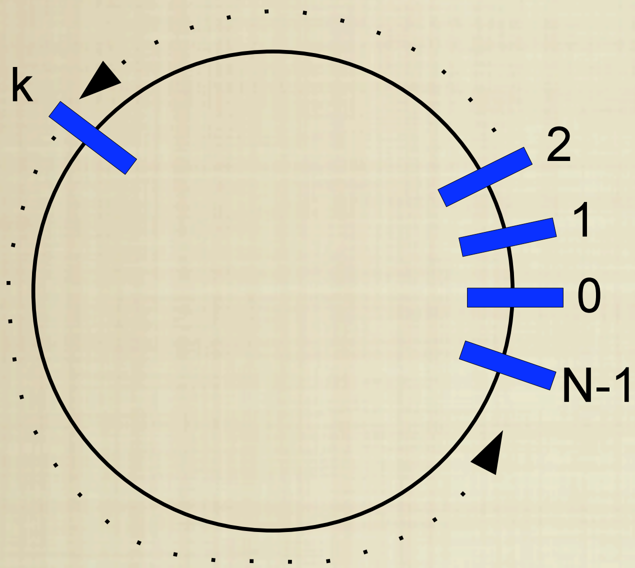


Figure 2 - N cavities homogeneously distributed around the ring.

Assuming that the initial number of harmonic h_0 is large we get(*):

$$f_k \approx f_0\left(1 - \frac{1}{h_0} \cdot \frac{k}{N}\right)$$

Every cavity working at a constant frequency f_k but the frequency has to be tuned to a slightly different value!

μ+ and μ- beams cannot be accelerated simultaneously if they circulated in opposite directions...

(*)Look at the proceedings of PAC'09 for all details.

Need for a double beam lattice

A solution to circulate a particle and its antiparticle **in the same direction** in a scaling FFAG ring is to use a FD-symmetric lattice (T. Okawa):

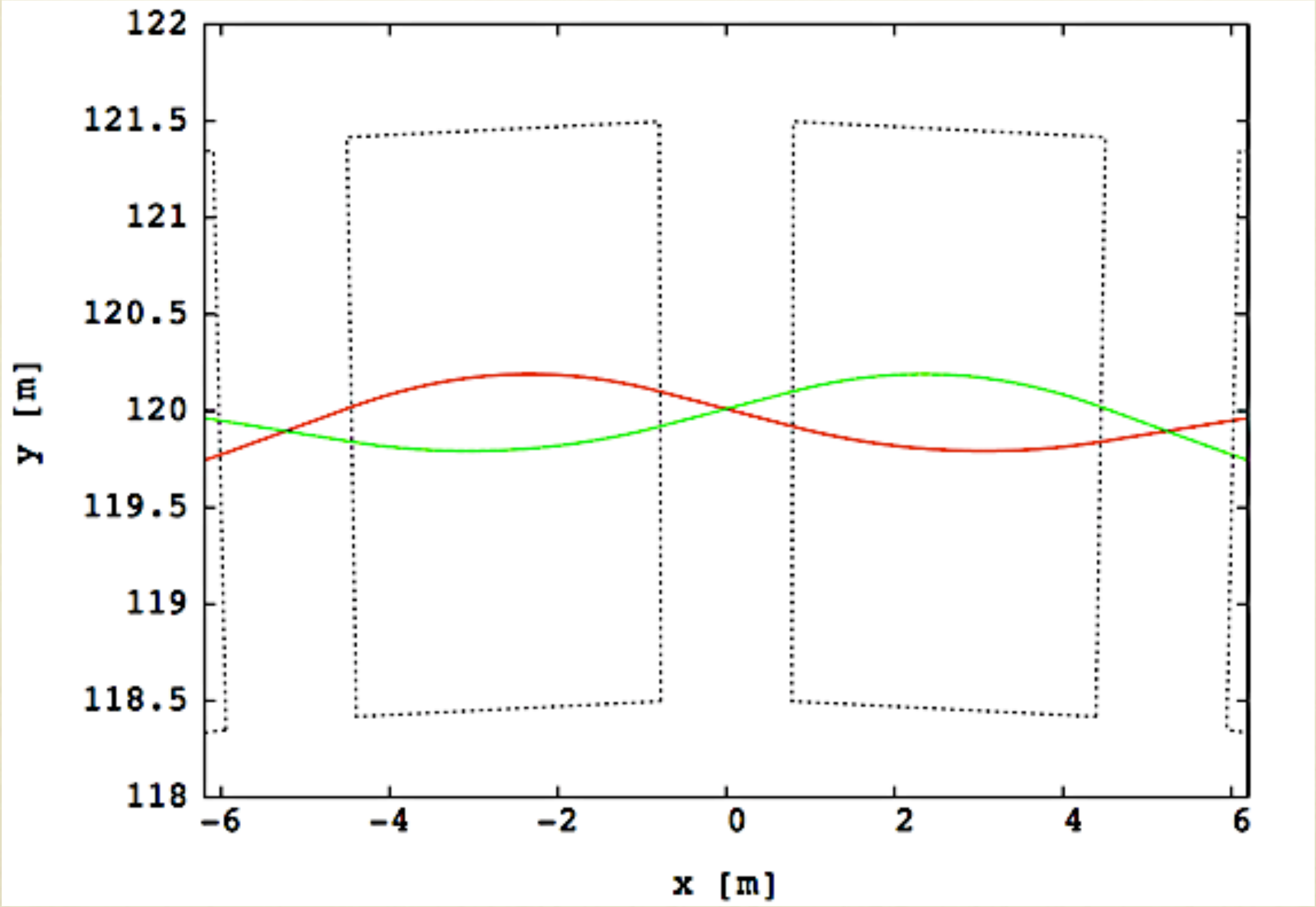


Figure 3 - Double beam FFAG lattice (k = 145). Closed orbits of μ+ and μ− circulating in the same direction. Results are obtained from Runge-Kutta stepwise tracking in hard-edge field.

Need for a double beam lattice

ANOTHER solution to circulate a particle and its antiparticle **in the same direction** in a scaling FFAG ring is to use a fDFd-symmetric quadruplet lattice:

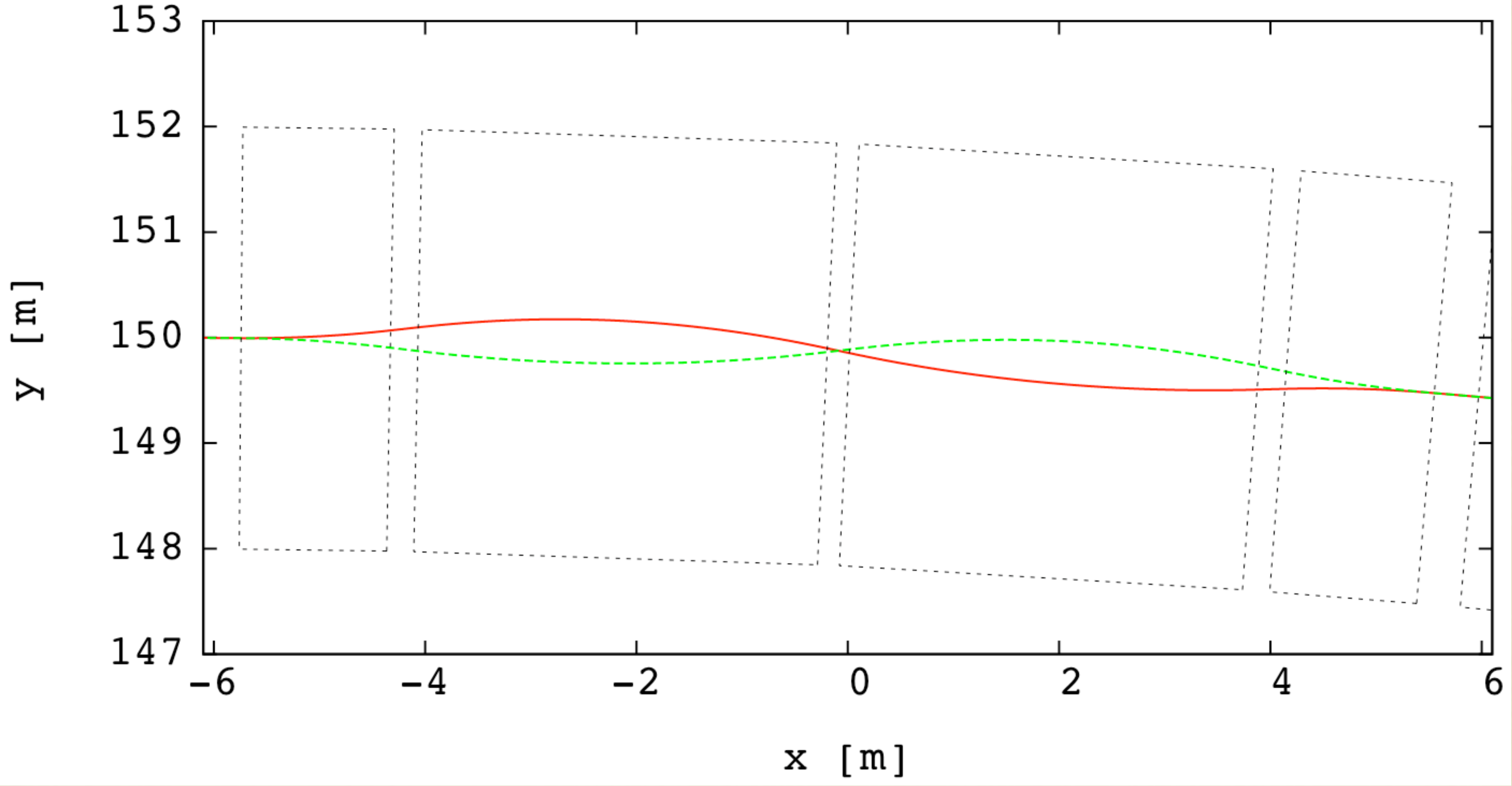


Figure 4 - Double beam FFAG quadruplet lattice (k = 145). Closed orbits of μ+ and μ– circulating in the same direction. Results are obtained from Runge-Kutta stepwise tracking in hard-edge field.

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

B_{max}	3 T
Horizontal tune	23.52
Vertical tune	7.12

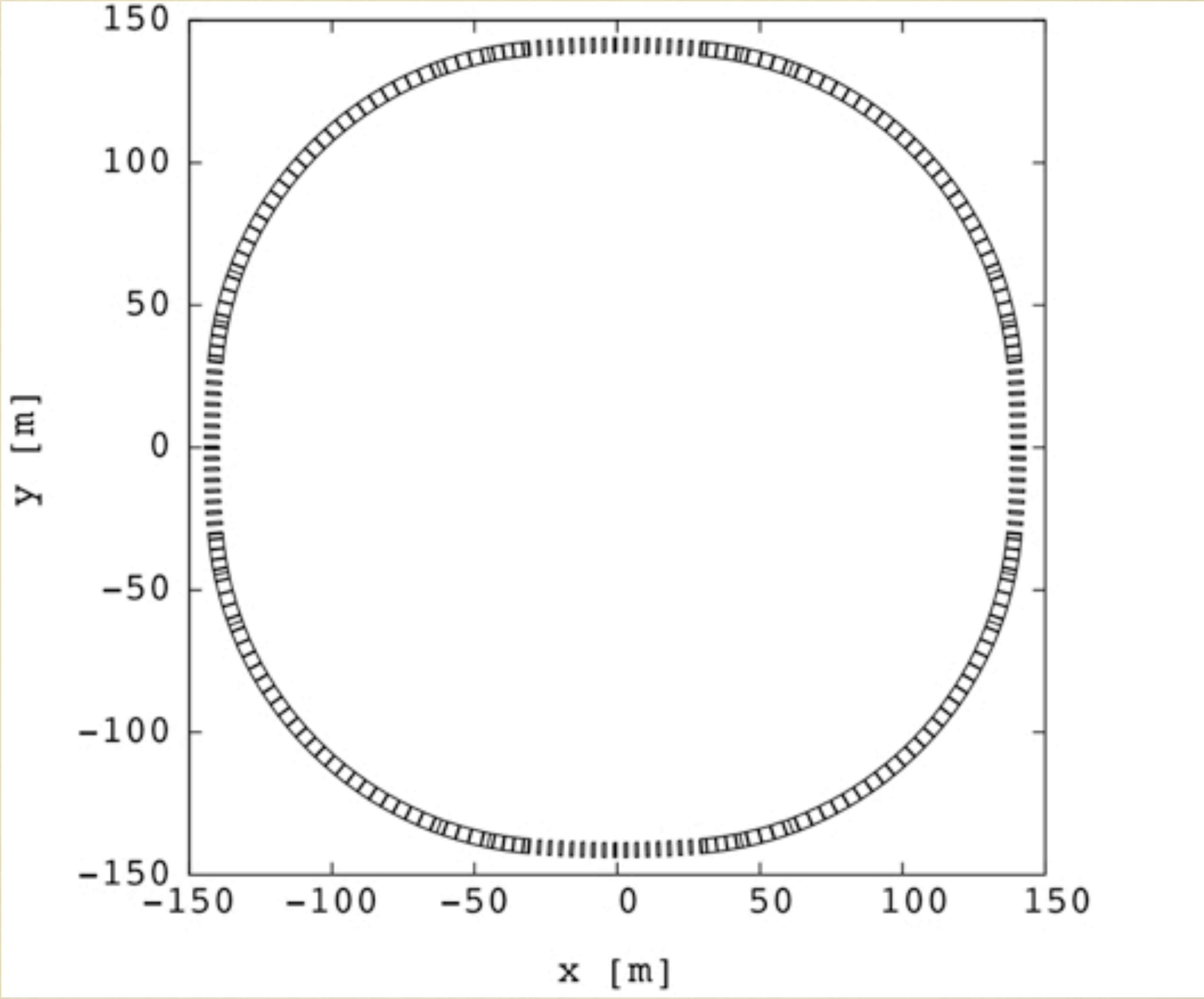


FIGURE 5 - SCHEMATIC VIEW OF A 3 TO 10 GEV DOUBLE BEAM MUON FFAG RING WITH 4 EXCURSION REDUCED INSERTIONS.

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

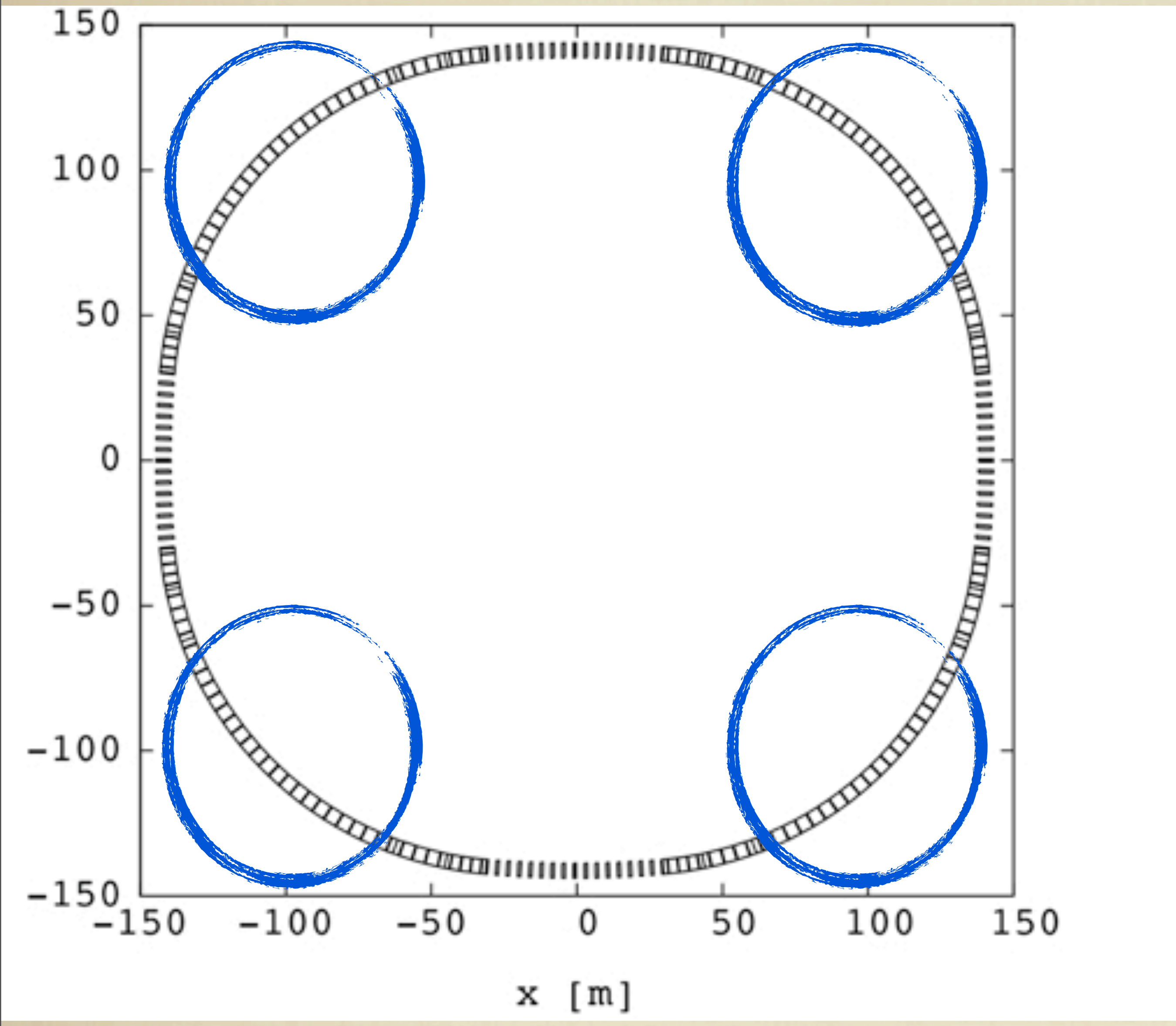


TABLE 2 - RING MAIN CELLS PARAMETERS

Mean radius	120 m
Number of cells	4×11
Cell opening angle	4.5 deg.
Field index k	145
B_{max}	3 T
Horiz. phase adv. per cell	82.1 deg.
Vert. phase adv. per cell	31.8 deg.

FIGURE 5 - SCHEMATIC VIEW OF A 3 TO 10 GEV DOUBLE BEAM MUON FFAG RING WITH 4 EXCURSION REDUCED INSERTIONS.

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

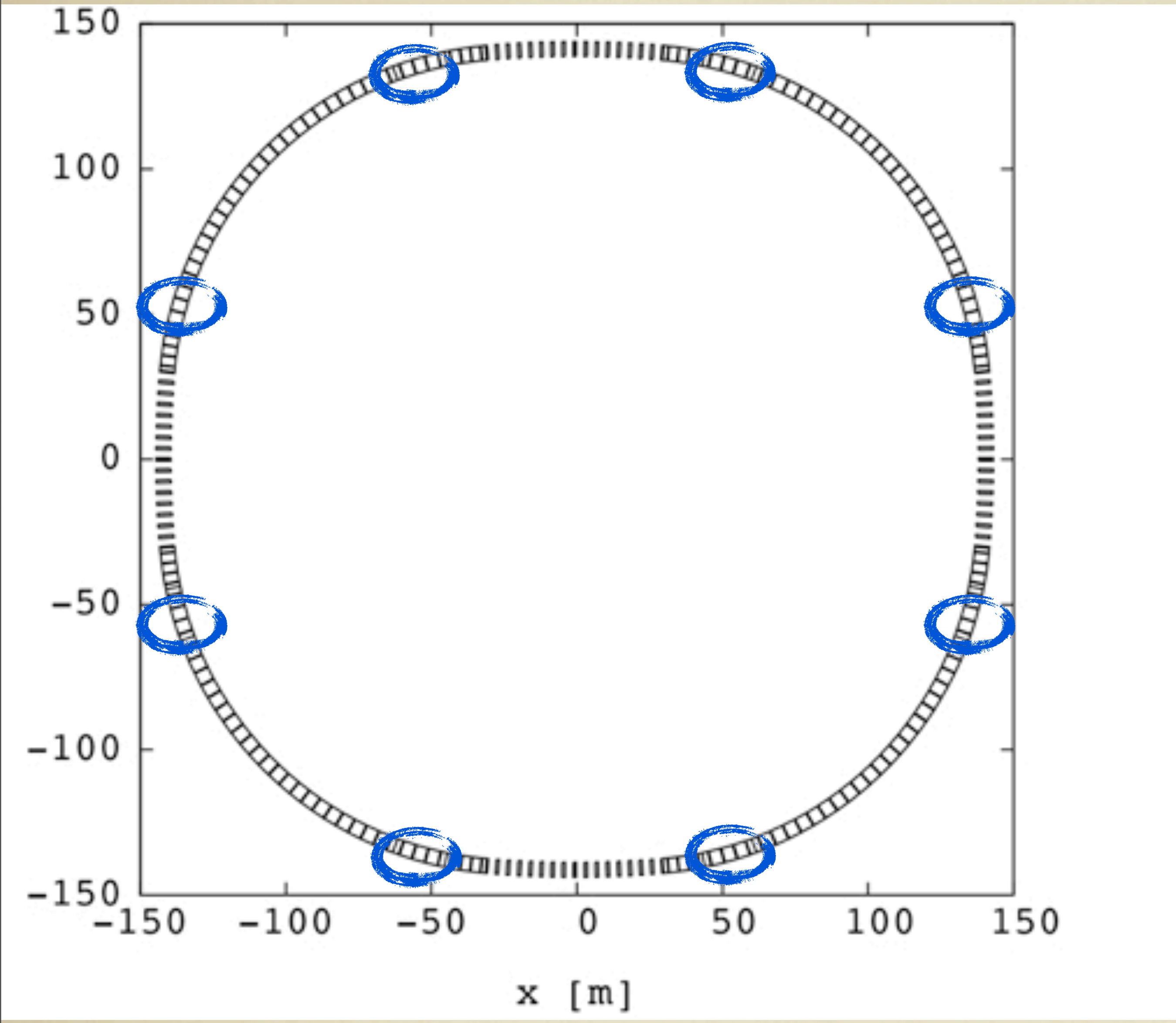


FIGURE 5 - SCHEMATIC VIEW OF A 3 TO 10 GEV DOUBLE BEAM MUON FFAG RING WITH 4 EXCURSION REDUCED INSERTIONS.

TABLE 3 - 1ST DISPERSION SUPPRESSOR

Mean radius	120 m
Number of cells	4×4
Cell opening angle	4.3 deg.
Field index k	183.6
B_{max}	3 T
Horiz. phase adv. per cell	90 deg.
Vert. phase adv. per cell	27.6 deg.

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

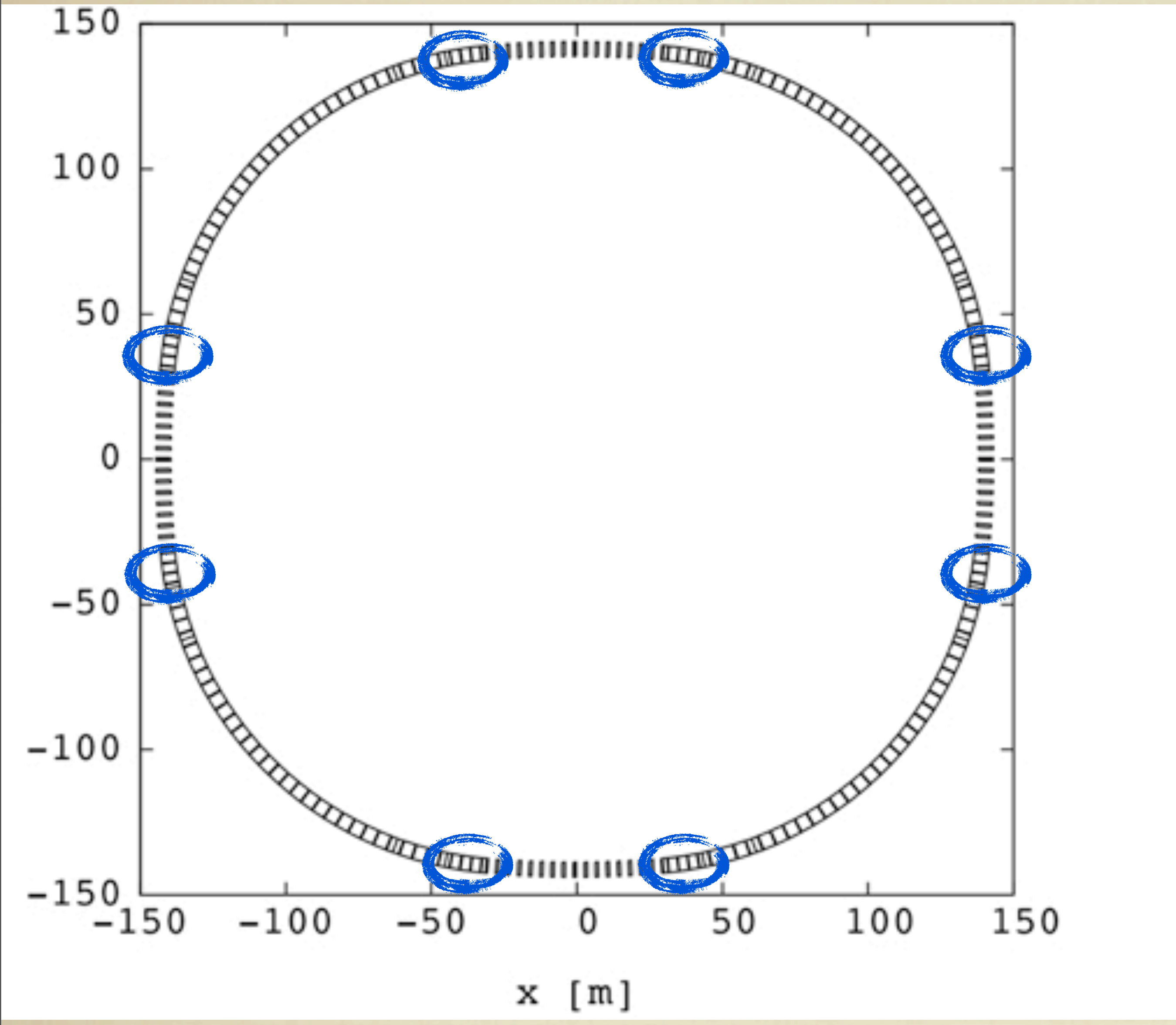


FIGURE 5 - SCHEMATIC VIEW OF A 3 TO 10 GEV DOUBLE BEAM MUON FFAG RING WITH 4 EXCURSION REDUCED INSERTIONS.

TABLE 4 - 2ND DISPERSION SUPPRESSOR

Mean radius	120 m
Number of cells	4×4
Cell opening angle	3.34 deg.
Field index k	307.7
B_{max}	3 T
Horiz. phase adv. per cell	90 deg.
Vert. phase adv. per cell	20.4 deg.

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

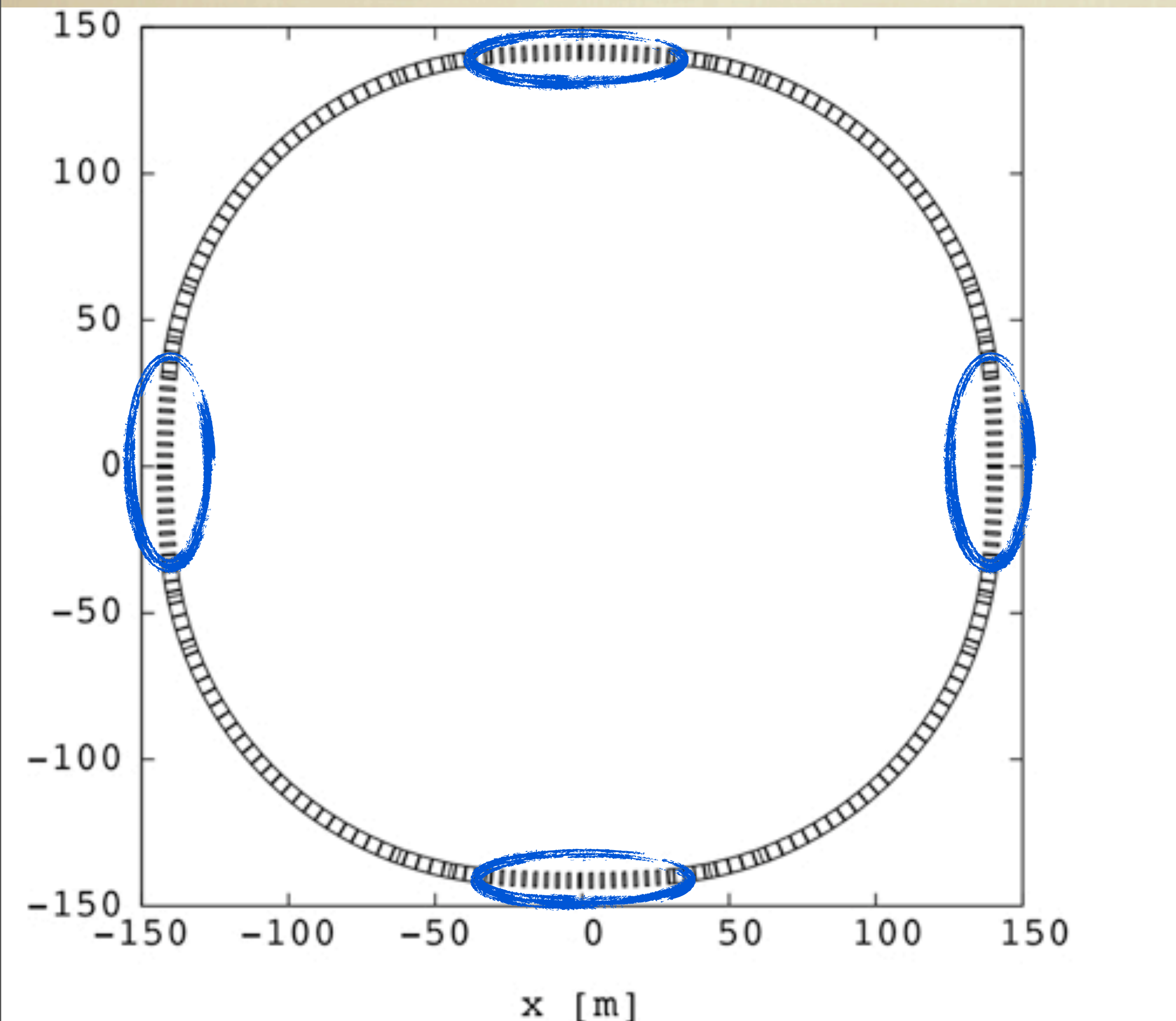


FIGURE 5 - SCHEMATIC VIEW OF A 3 TO 10 GEV DOUBLE BEAM MUON FFAG RING WITH 4 EXCURSION REDUCED INSERTIONS.

TABLE 5 - DISPERSION SUPPRESSED AREA

Mean radius	350 m
Number of cells	4×8
Cell opening angle	1.2425 deg.
Field index k	1168.6
B_{max}	3 T
Horiz. phase adv. per cell	64.6 deg.
Vert. phase adv. per cell	12.6 deg.

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

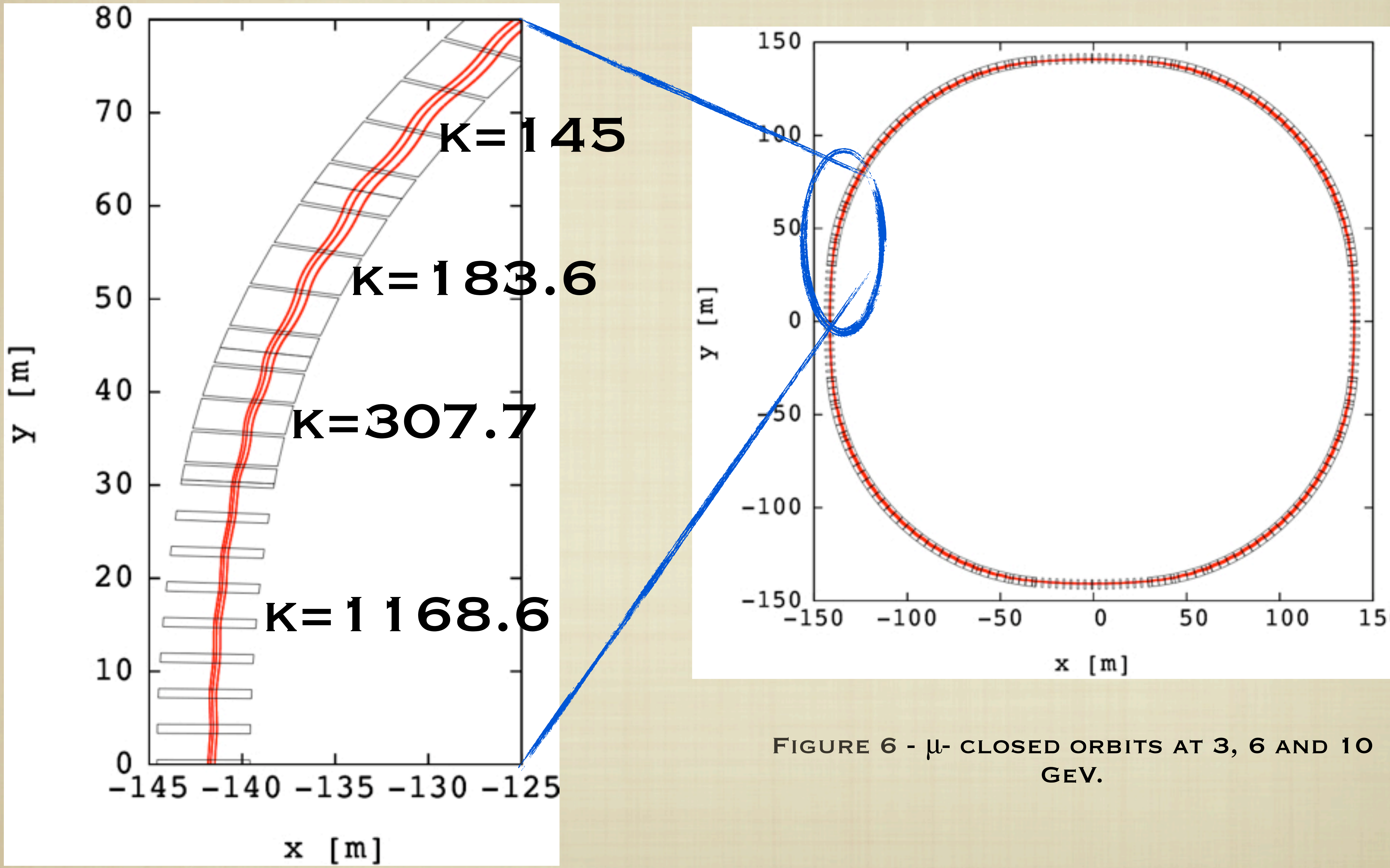


FIGURE 6 - μ- CLOSED ORBITS AT 3, 6 AND 10 GEV.

DOUBLET CELLS - 3 TO 10 GeV MUON DOUBLE BEAM FFAG

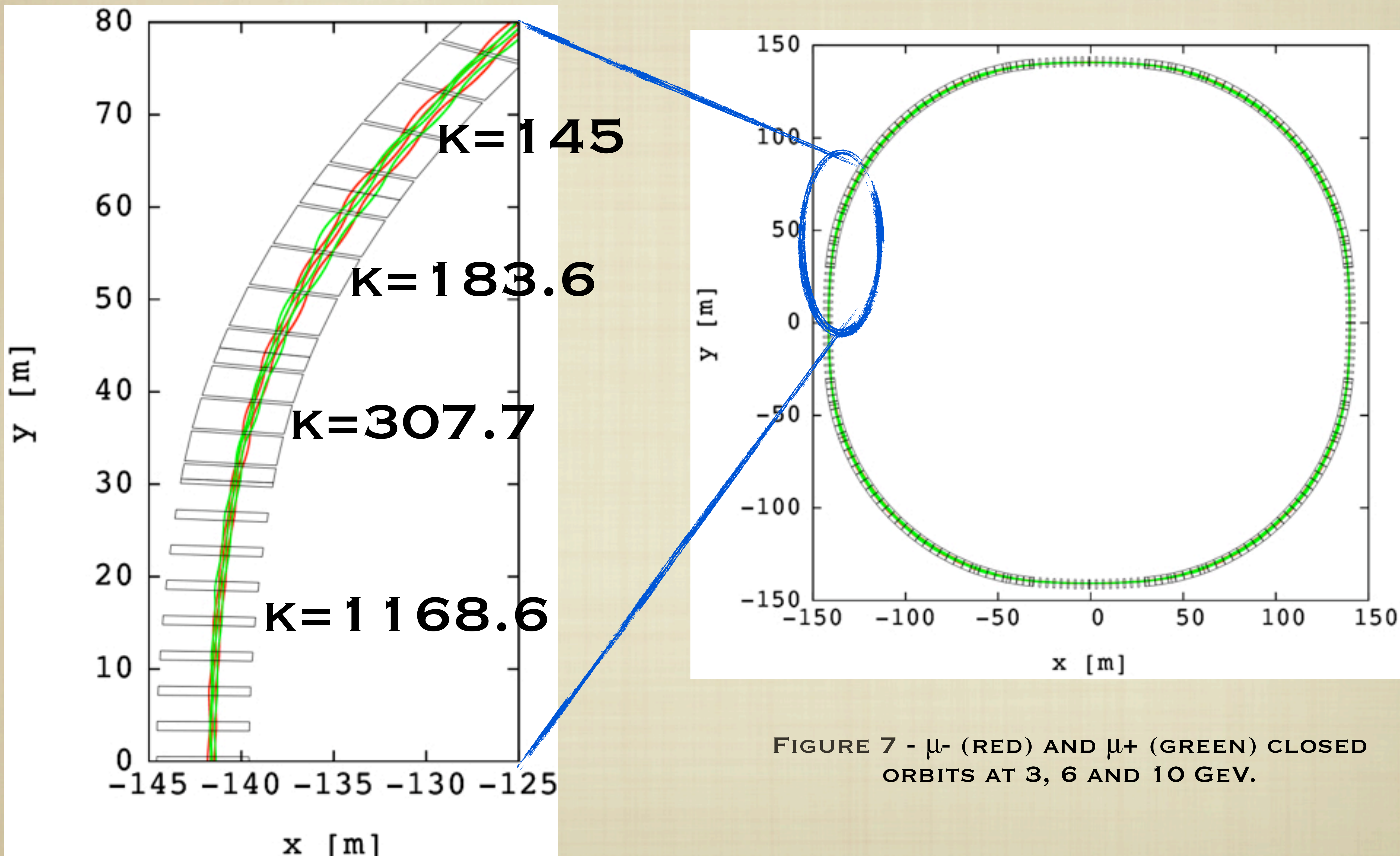
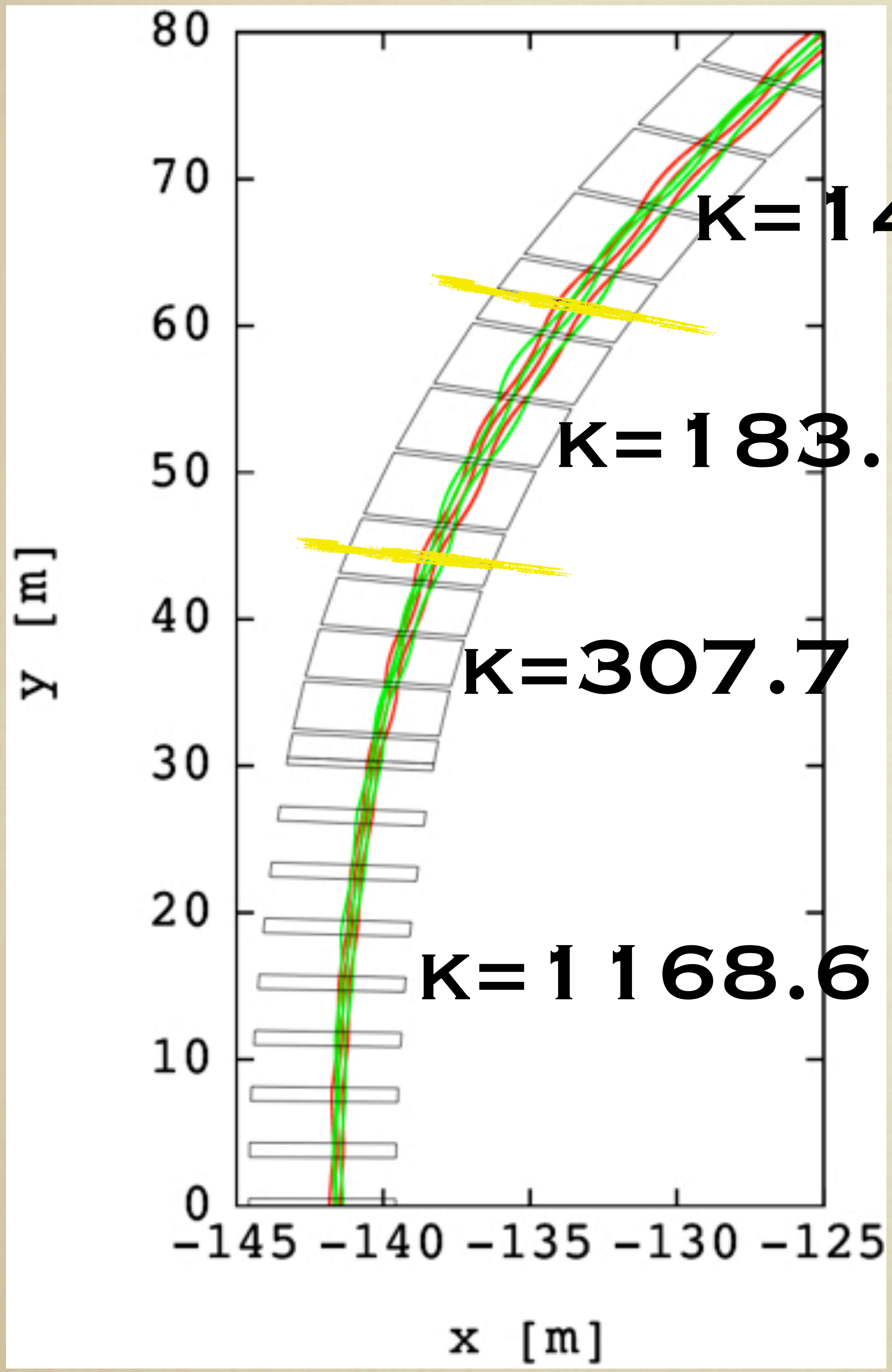
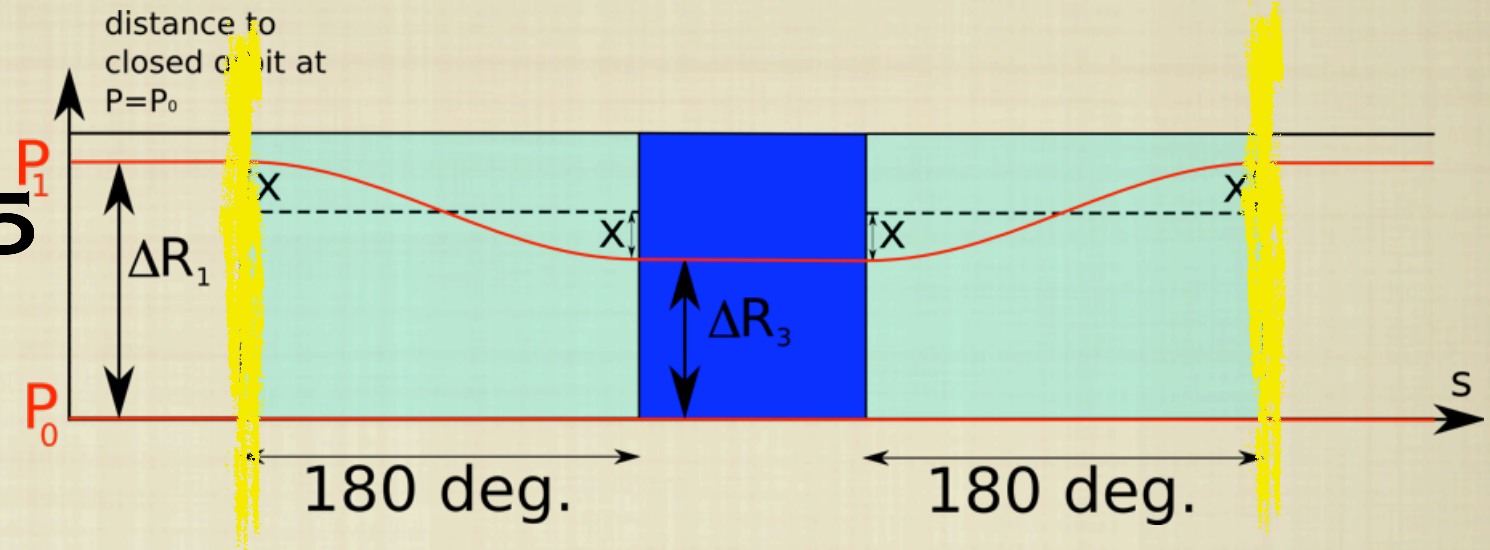


FIGURE 7 - μ- (RED) AND μ+ (GREEN) CLOSED ORBITS AT 3, 6 AND 10 GeV.

DOUBLET CELLS - 3 TO 10 GeV MUON DOUBLE BEAM FFAG





PERFECTLY SYMMETRICAL BEHAVIOR BROKEN BY THE CHOICE OF THE MATCHING POINT!

DOUBLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM FFAG

STUDY OF LINEAR PARAMETERS USING RUNGE-KUTTA STEPWISE TRACKING IN SOFT EDGE FIELD MODEL:

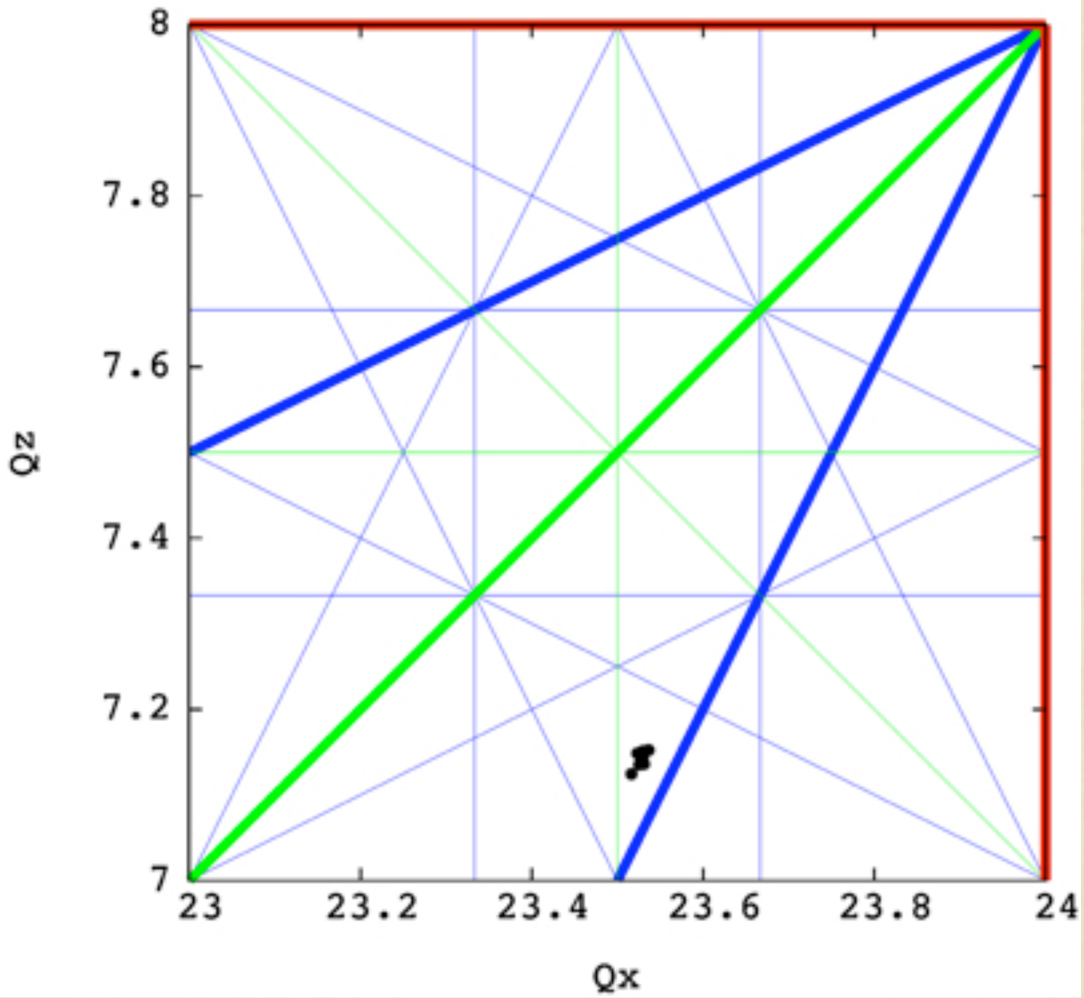


FIGURE 8 - μ- TUNE VARIATION BETWEEN 3 AND 10 GEV IN THE LATTICE WITH INSERTIONS (FROM STEPWISE TRACKING IN A SOFT EDGE FIELD MODEL).

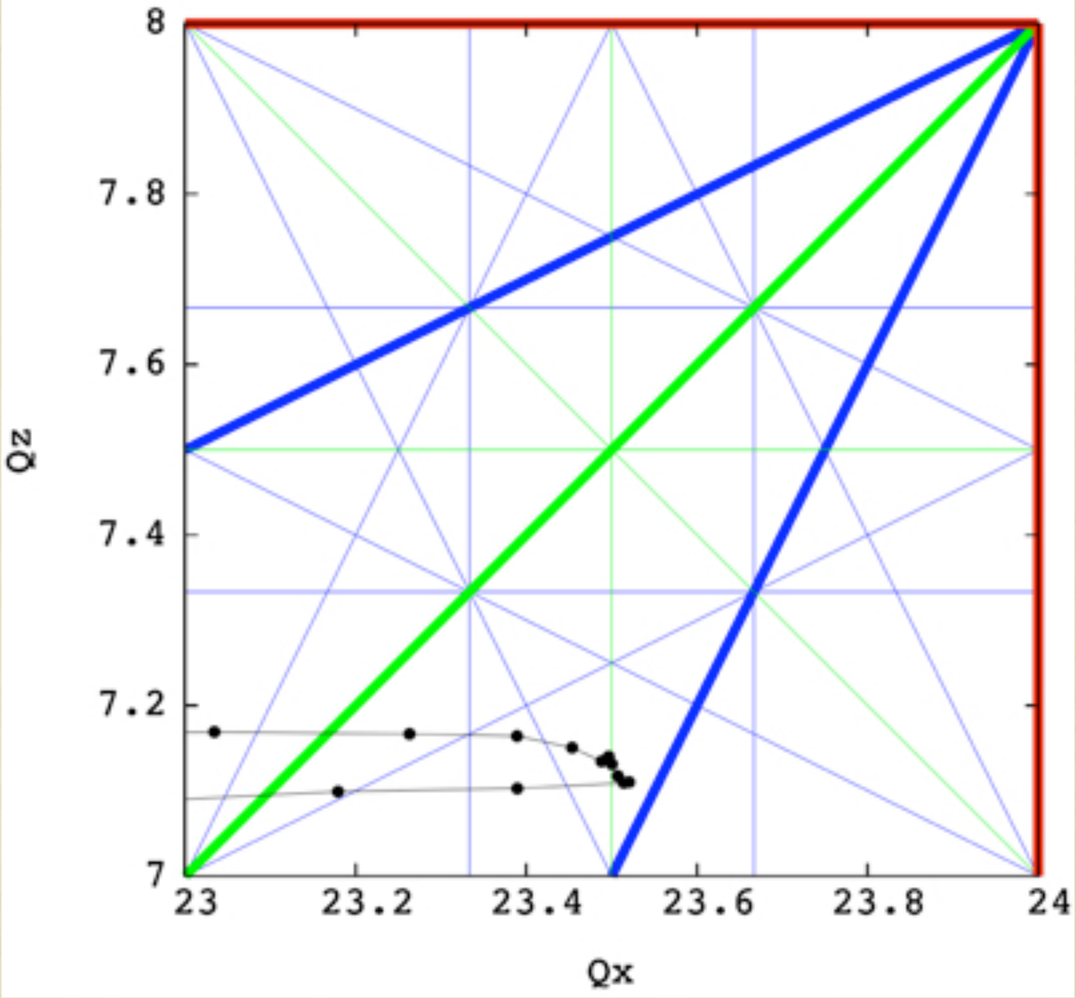


FIGURE 9 - μ+ TUNE VARIATION BETWEEN 3 AND 10 GEV IN THE LATTICE WITH INSERTIONS (FROM STEPWISE TRACKING IN A SOFT EDGE FIELD MODEL).

DOUBLET CELLS - μ-: MATCHING DONE AT THE CENTER OF F

4D TRACKING RESULTS: RF FREQUENCY = **400 MHz**, PEAK VOLTAGE **2GV/TURN**.

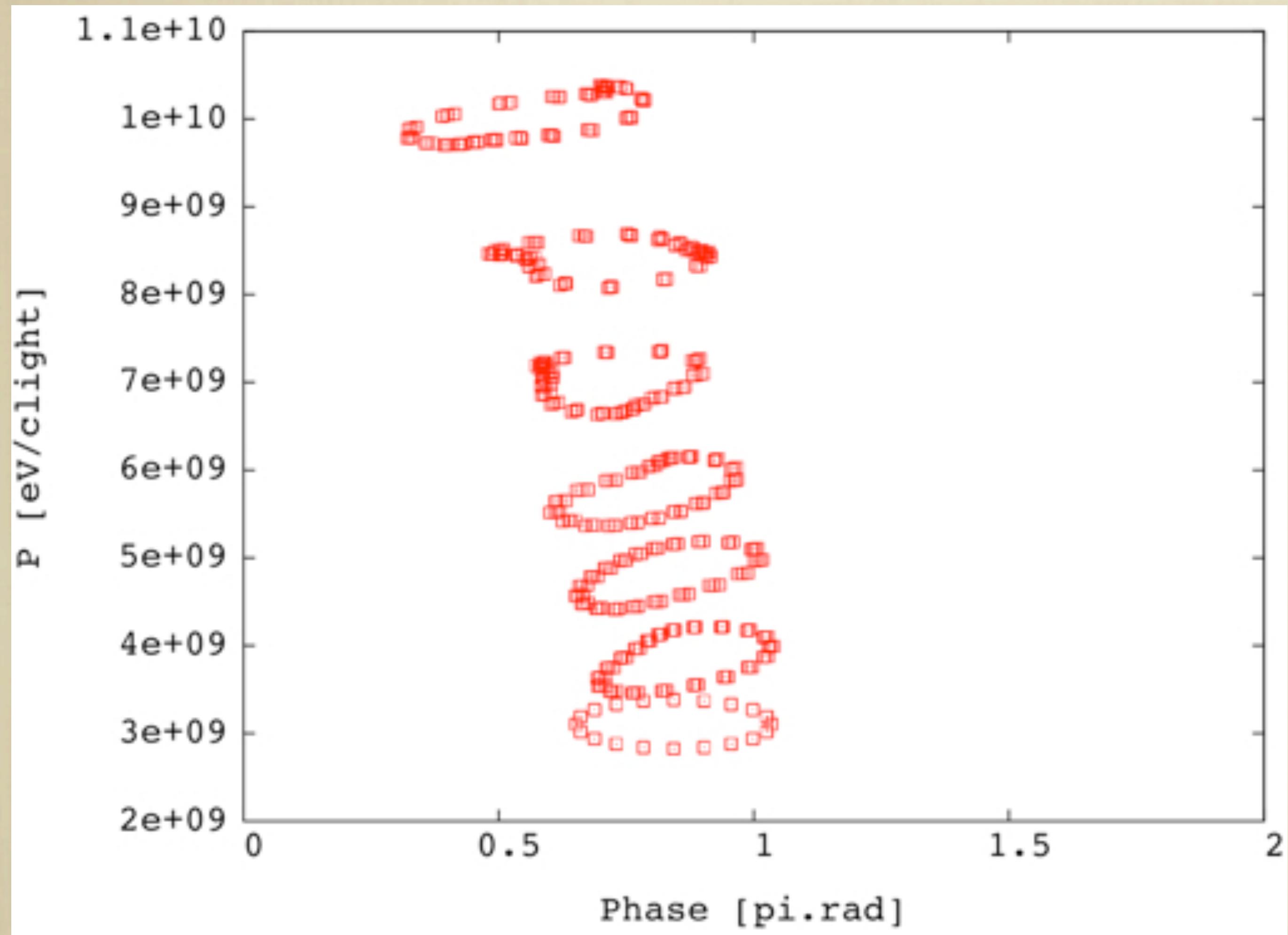


FIGURE 10 - μ- : LONGITUDINAL PHASE SPACE SHOWING A 6 TURNS ACCELERATION CYCLE FROM 3 TO 10 GEV WITH AN INITIAL BEAM 4D EMITTANCE OF 0.2 eV.sec × 30 000 π.mm.mrad.

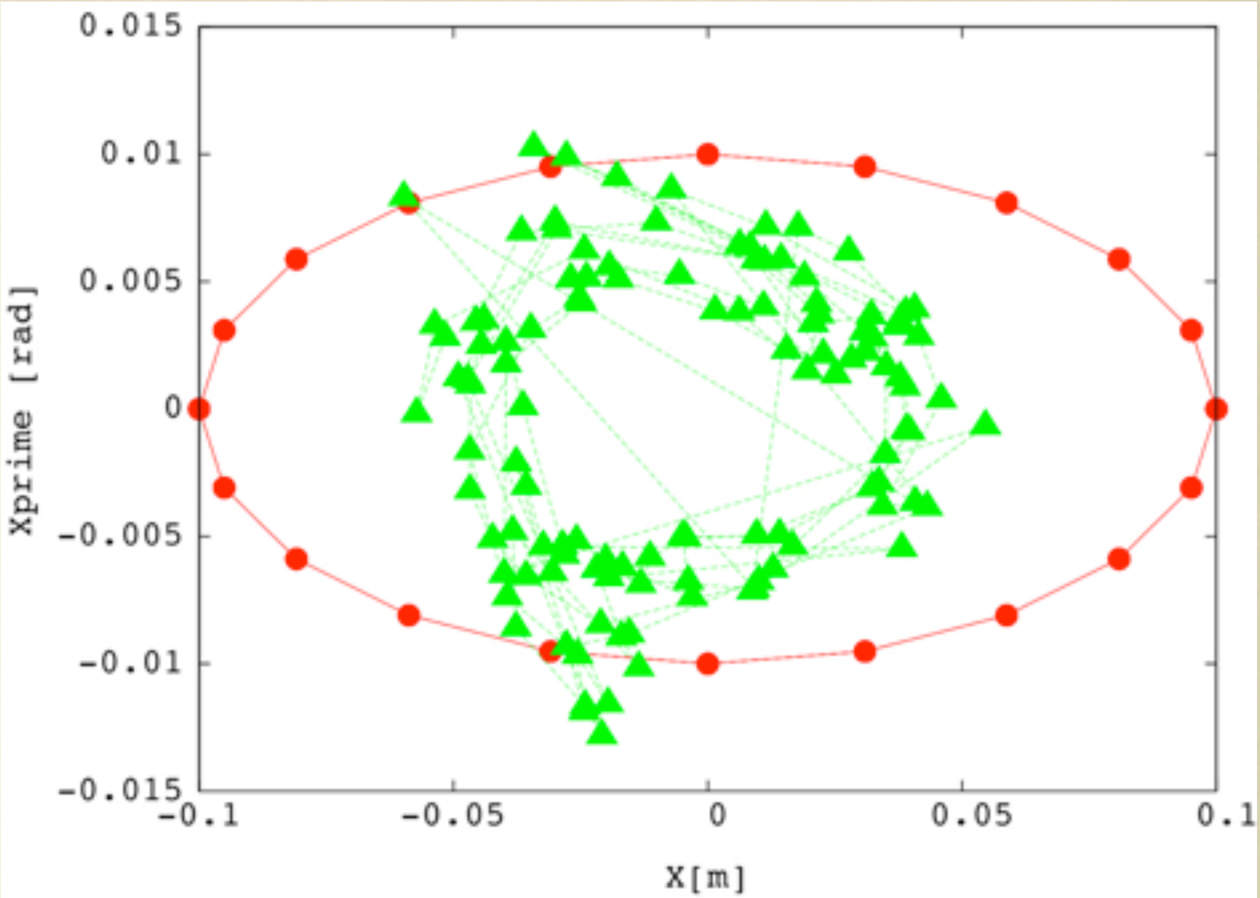


FIGURE 11 - μ- : HORIZONTAL PHASE SPACE SHOWING THE INJECTED BEAM PROFILE (RED) AND THE SAME BEAM AFTER A 6 TURNS ACCELERATION CYCLE (GREEN) WITH (4D EMITTANCE OF 0.2 eV.sec × 30 000 π.mm.mrad).

Doublet cells - $\mu+$: matching done at the center of D

4D tracking results:

Tried 3 to 10 GeV acceleration cycle
(with RF frequency = 400 Hz, peak voltage 2GV/turn)

Particle lost on collimator even for small transverse emittance...

DOUBLET VS QUADRUPLET

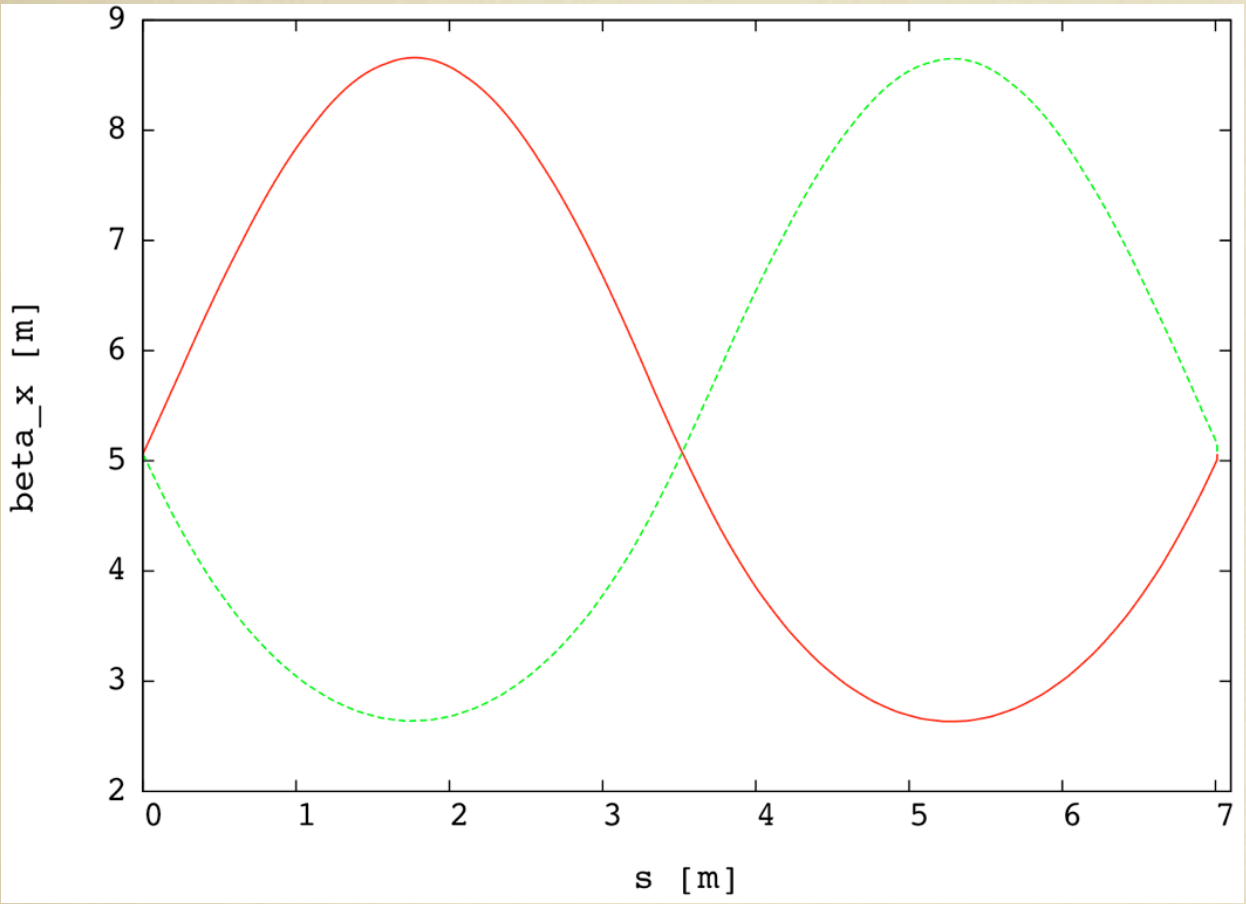


FIGURE 10 - HORIZONTAL BETA FUNCTION FOR μ+ (REG) AND μ– (GREEN) IN A DOUBLE BEAM **DOUBLET** CELL.

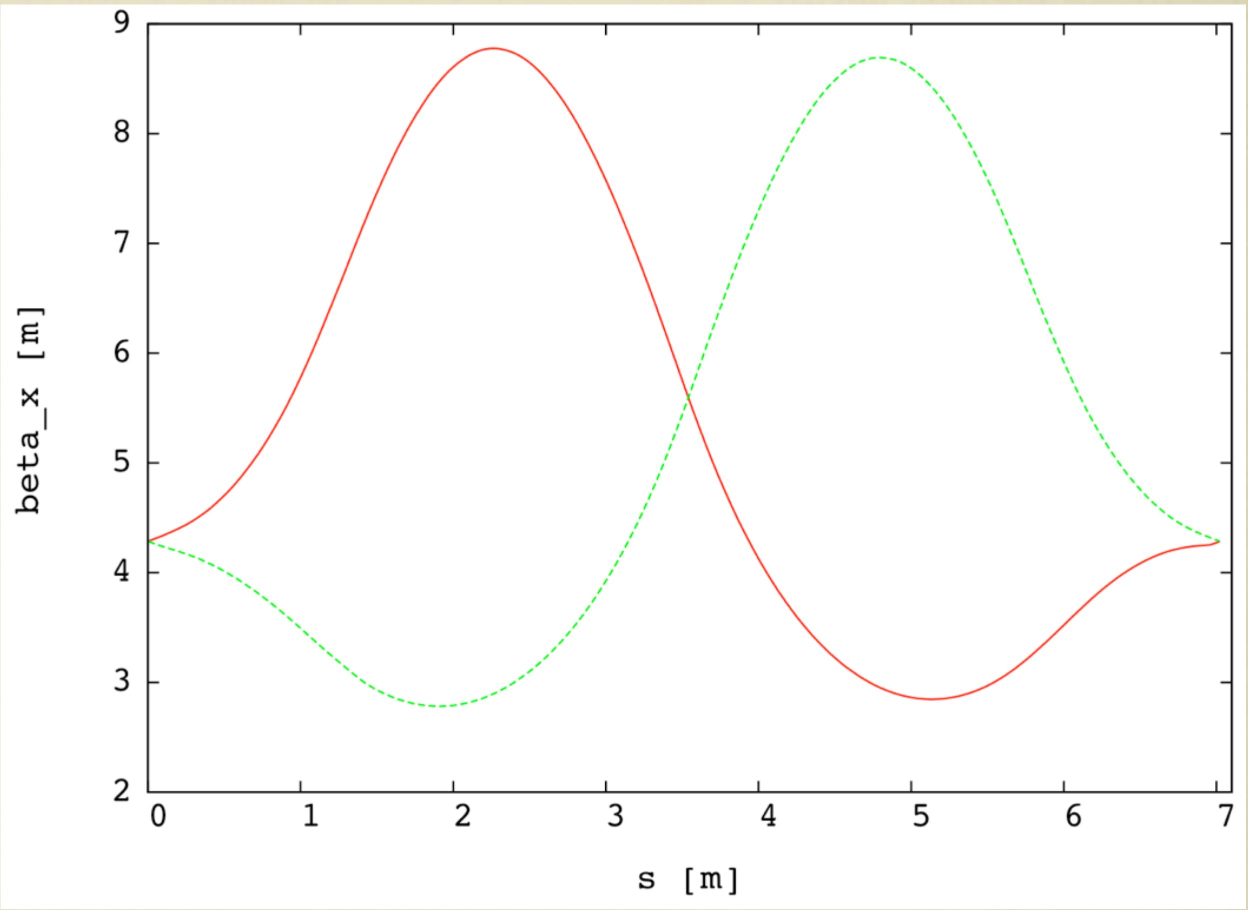


FIGURE 11 - HORIZONTAL BETA FUNCTION FOR μ+ (REG) AND μ– (GREEN) IN A DOUBLE BEAM **QUADRUPLET** CELL.

Quadruplet cells - 3 to 10 GeV muon double beam

Ring all made of quadruplet cells. Here dispersion is reduced by a factor 2 (not 3 as before):

B_{max}	3 T
Horizontal tune	21.6
Vertical tune	11.4

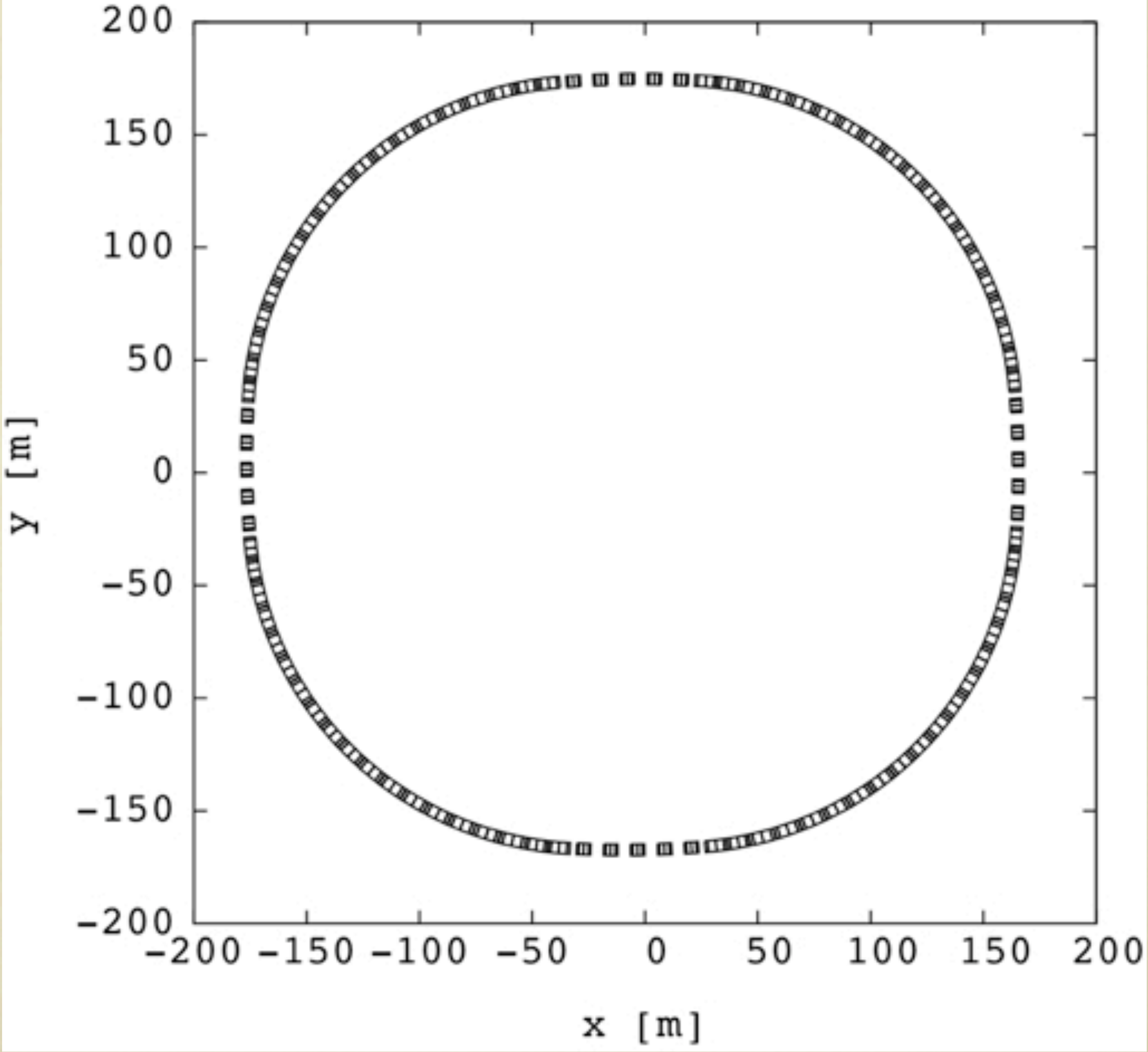


Figure 12 - Schematic view of a 3 to 10 GeV double beam (quadruplets) muon FFAG ring with 4 excursion reduced insertions.

QUADRUPLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM

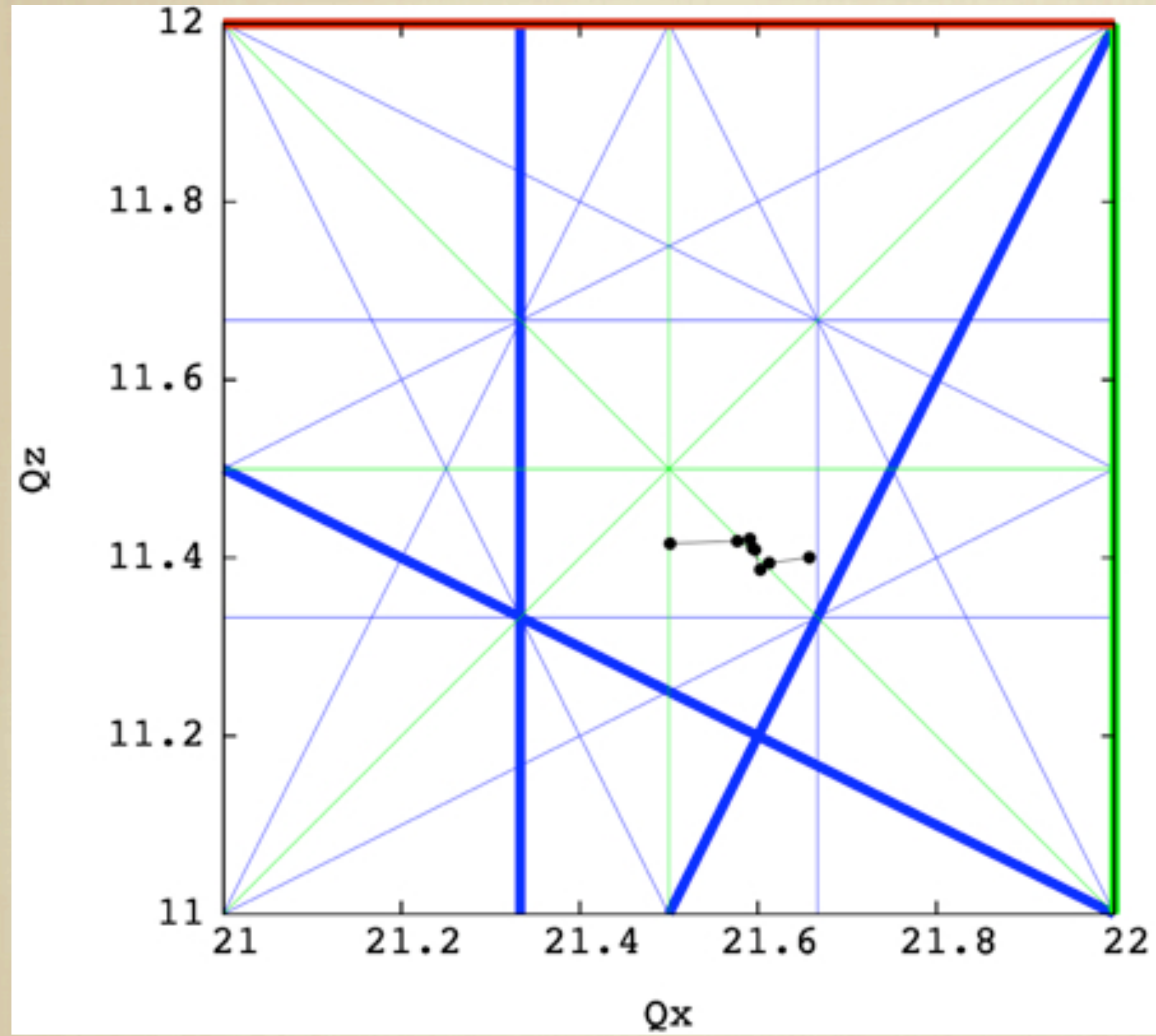


FIGURE 13 - μ+ TUNE VARIATION BETWEEN 3 AND 10 GEV (FROM STEPWISE TRACKING IN A SOFT EDGE FIELD MODEL).

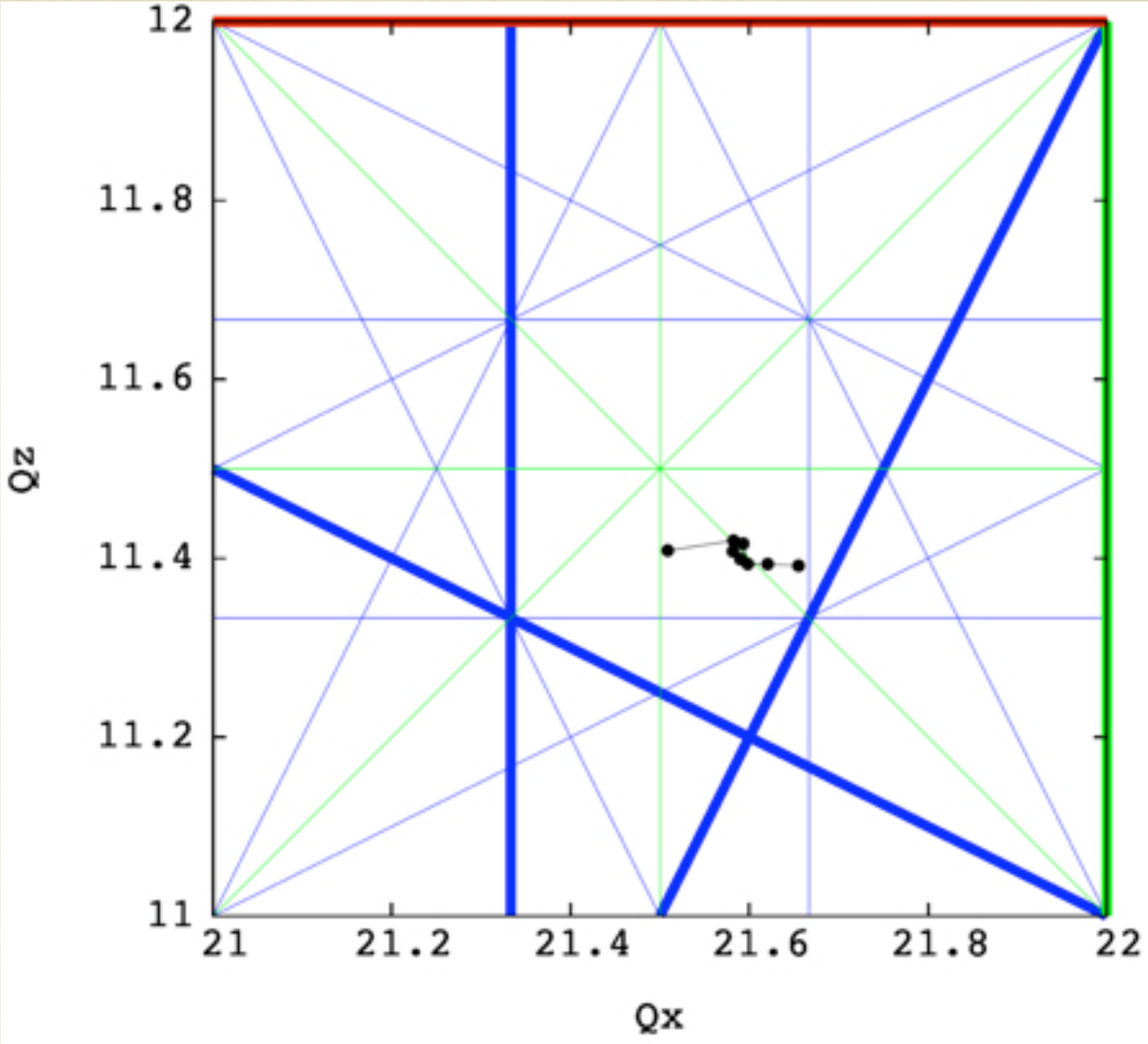
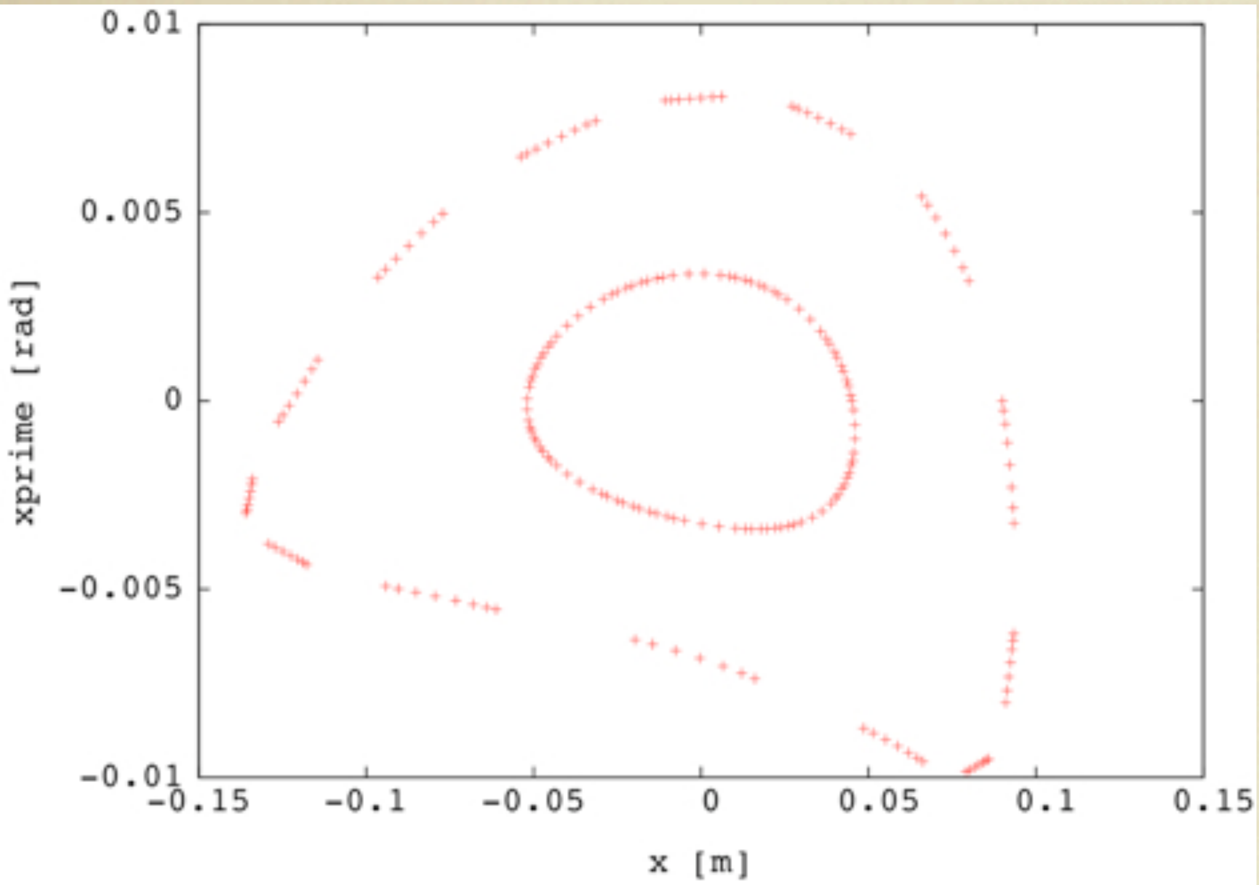


FIGURE 14 - μ- TUNE VARIATION BETWEEN 3 AND 10 GEV (FROM STEPWISE TRACKING IN A SOFT EDGE FIELD MODEL).

QUADRUPLET CELLS - 3 TO 10 GEV MUON DOUBLE BEAM



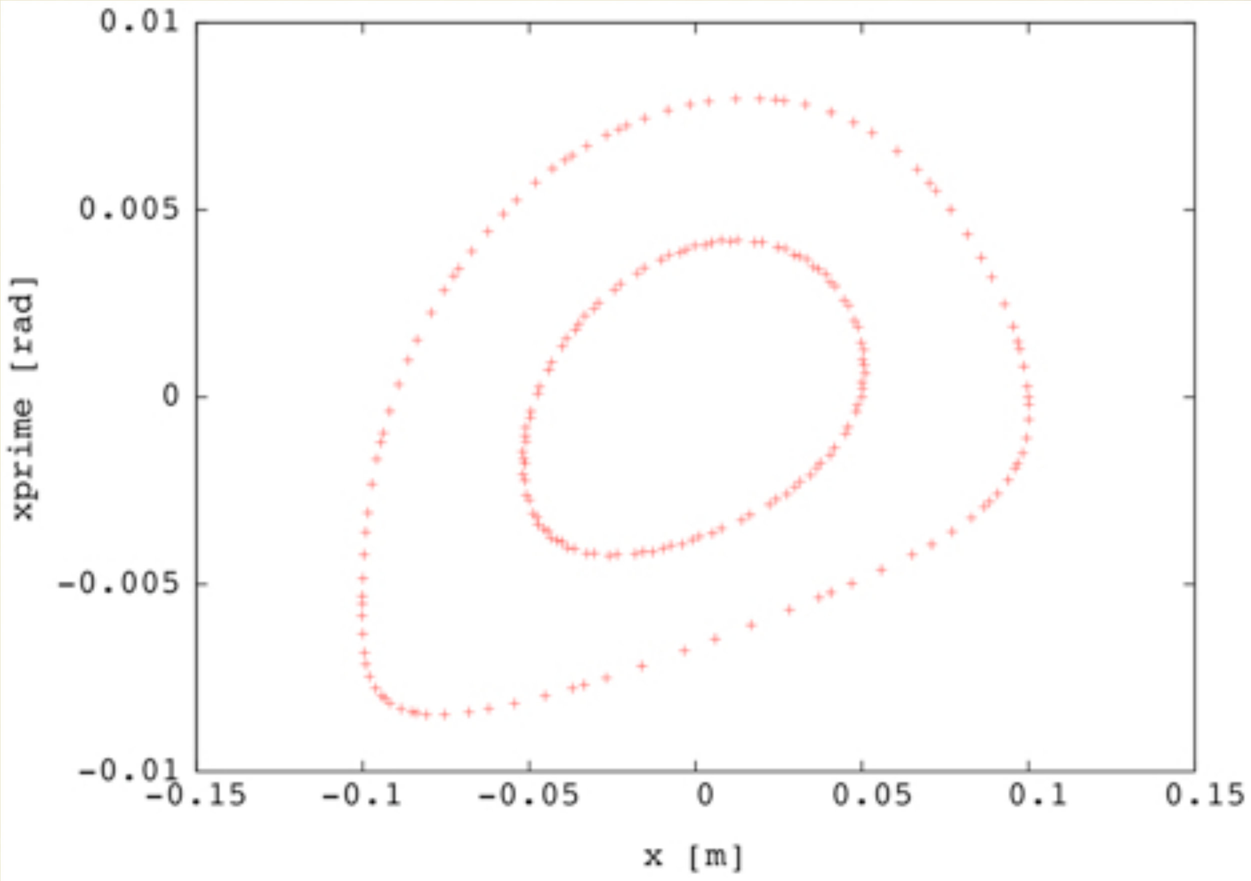


FIGURE 15 - μ+ - TRANSVERSE PHASE SPACE SHOWING A 25 000 π.mm.mrad NORMALIZED TRANSVERSE ACCEPTANCE AT 3 GEV (TRACKING DONE WITH SMALL VERTICAL MOTION).

FIGURE 16 - μ- - TRANSVERSE PHASE SPACE SHOWING A 25 000 π.mm.mrad NORMALIZED TRANSVERSE ACCEPTANCE AT 3 GEV (TRACKING DONE WITH SMALL VERTICAL MOTION).

Summary on harmonic number jump

Works well

* Large transverse acceptance.
* Large longitudinal acceptance, and no emittance degradation during acceleration.
* Excursion reduction of a factor 3 has been shown with doublet lattice.
* Possible with RF frequency in the 200 MHz to 400 MHz range.

Possible to accelerate μ+ and μ- in the same time!... Although it is a very challenging scheme.

* More work has to be done on the quadruplet cells, better lattice design, 6D tracking...

Part II

Scaling FFAG lattices for

Stationary bucket acceleration

STATIONARY BUCKET ACCELERATION

Stationary bucket acceleration for 3.6 to 12.6 GeV muon

RF frequency = **200 MHz**

STATIONARY BUCKET ACCELERATION

Lattice example for 3.6 to 12.6 GeV muon acceleration:

Table 6 - lattice parameters.

Lattice type	scaling FFAG FDFDF quintuplet
Mean radius	170 m
B_{max}(@ 12.6 GeV)	3 T
Number of cells	120
Field index k	1500
Horiz. phase adv. per quintuplet	166.8 deg.
Vert. phase adv. per quintuplet	56.26 deg.
Available drift space per cell	2 m

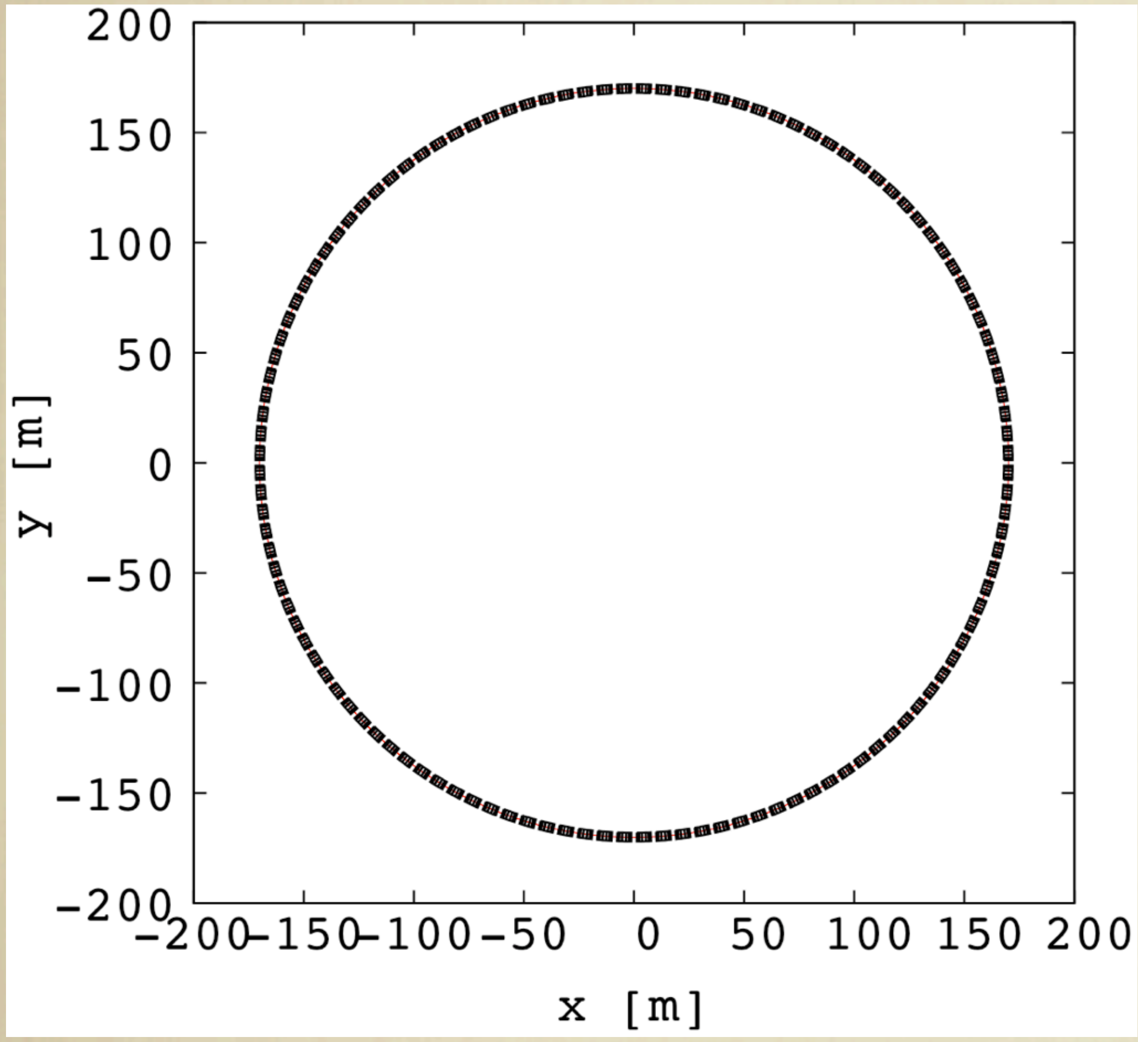
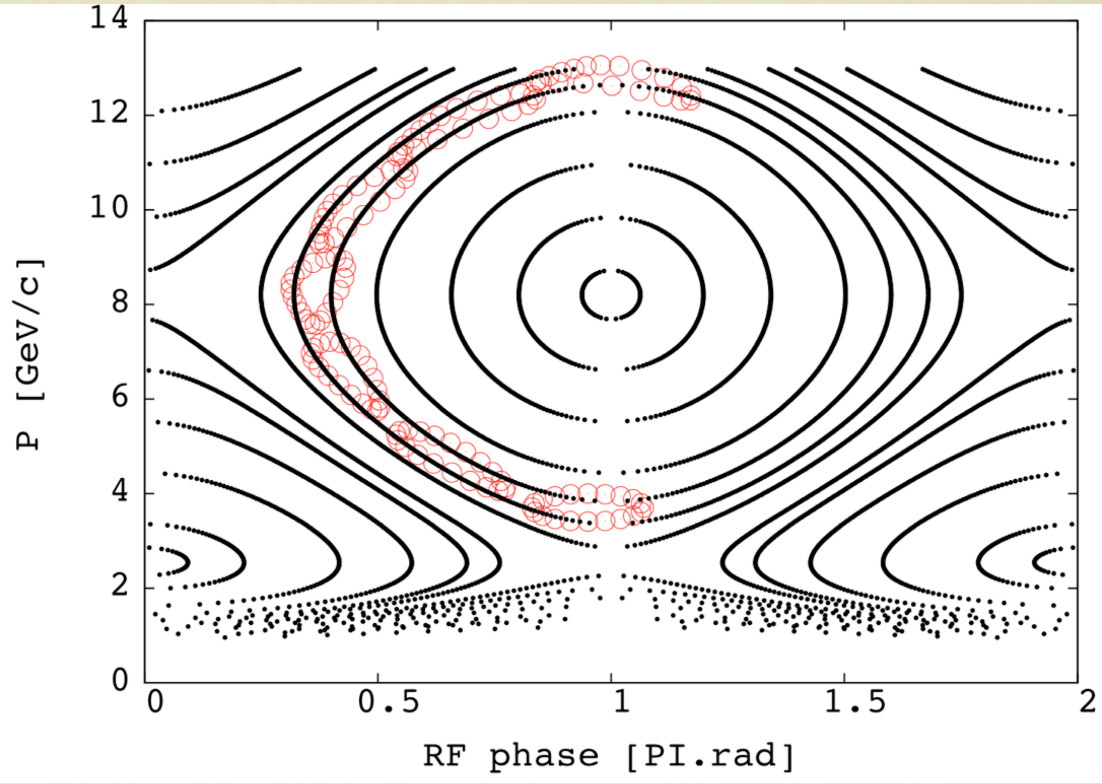


Figure 17 - Scaling FFAG lattice for 3.6 to 12.6 GeV muon acceleration.



Triplet vs quintuplet

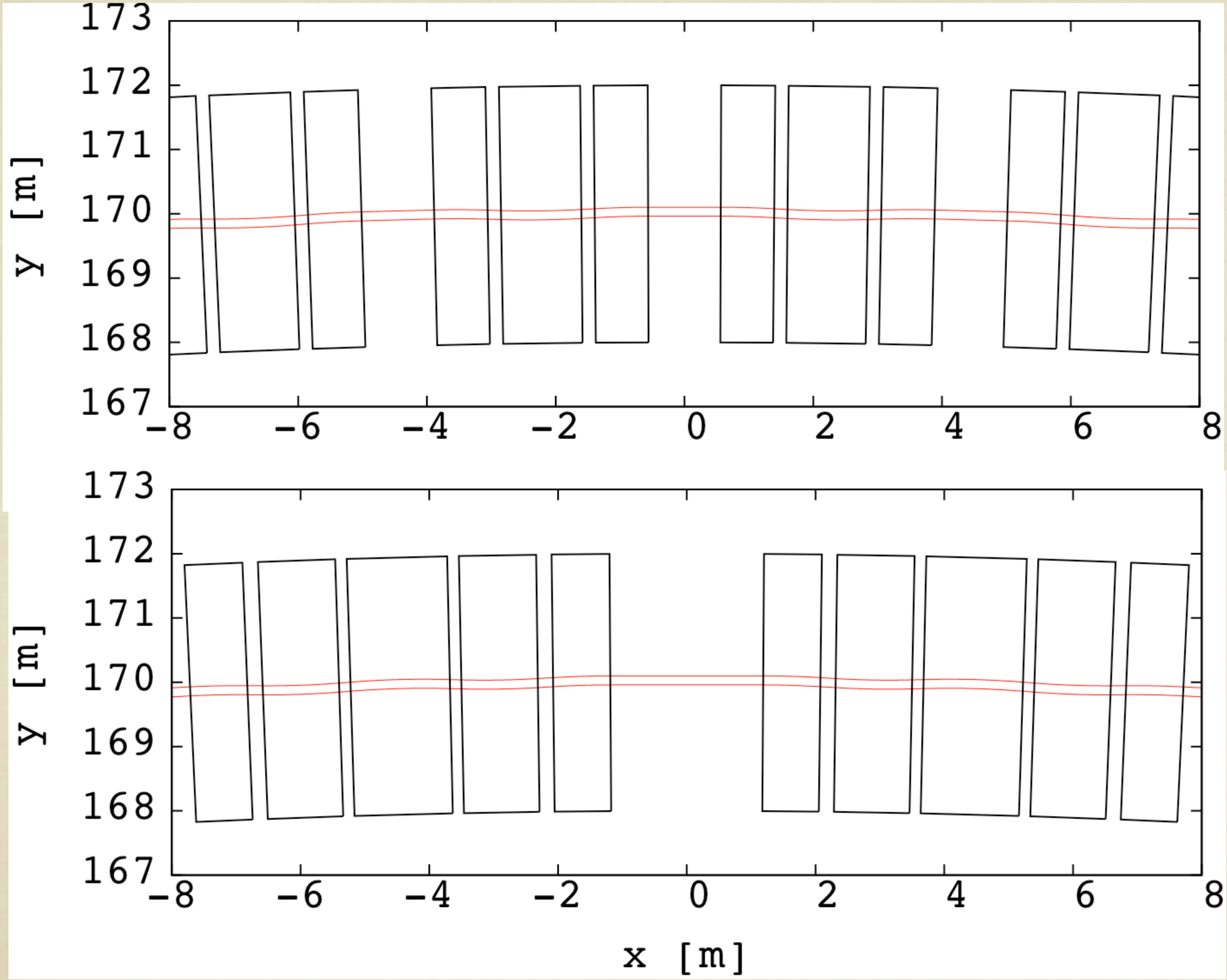


Figure 18 - A quintuplet cell is made of two triplet cells merged.

STATIONARY BUCKET ACCELERATION

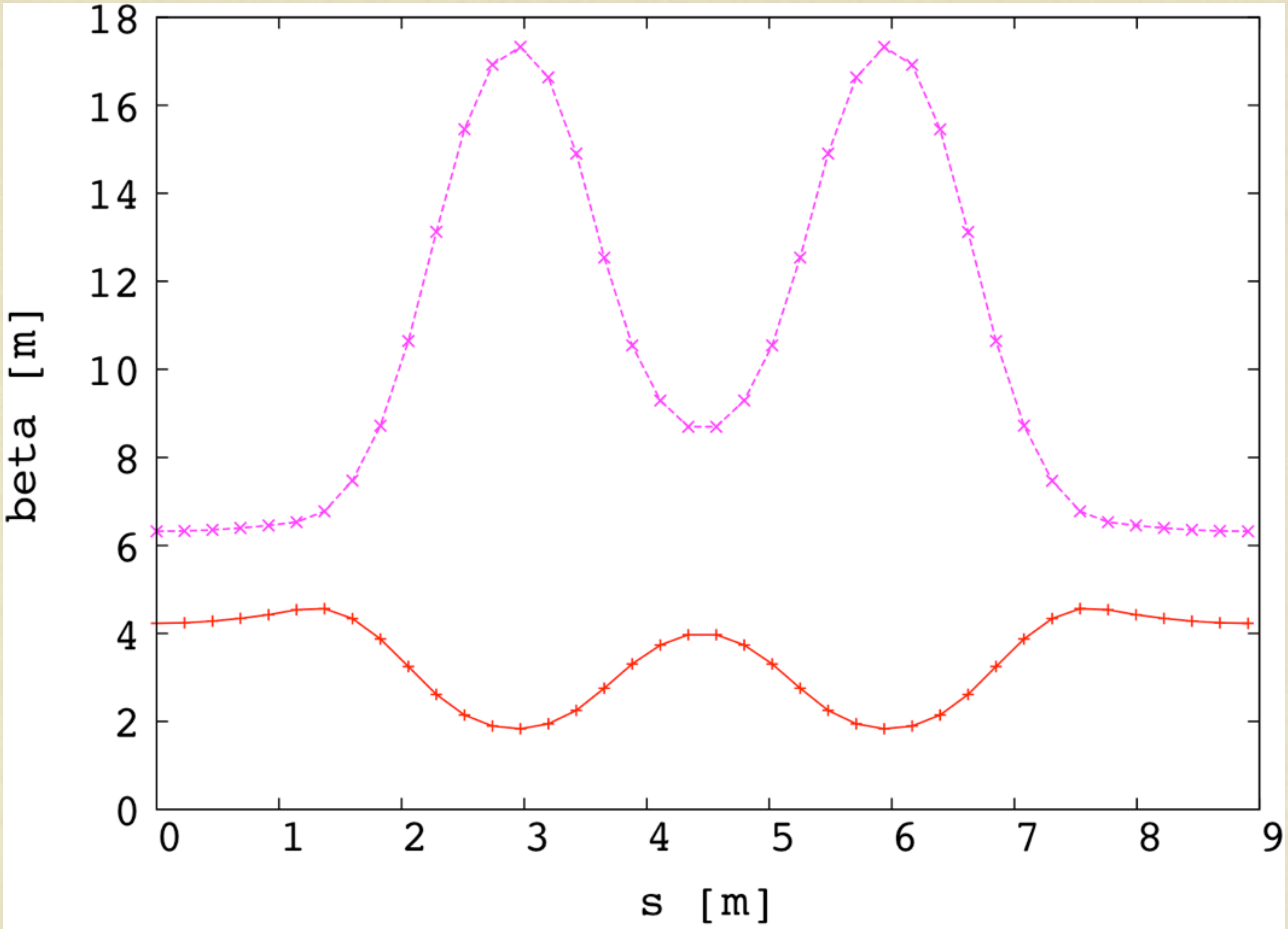


FIGURE 19 - QUINTUPLET CELL, HORIZONTAL (RED) AND VERTICAL BETA FUNCTIONS (PURPLE).

STATIONARY BUCKET ACCELERATION

Transverse tracking(*) results (4D, no acceleration):

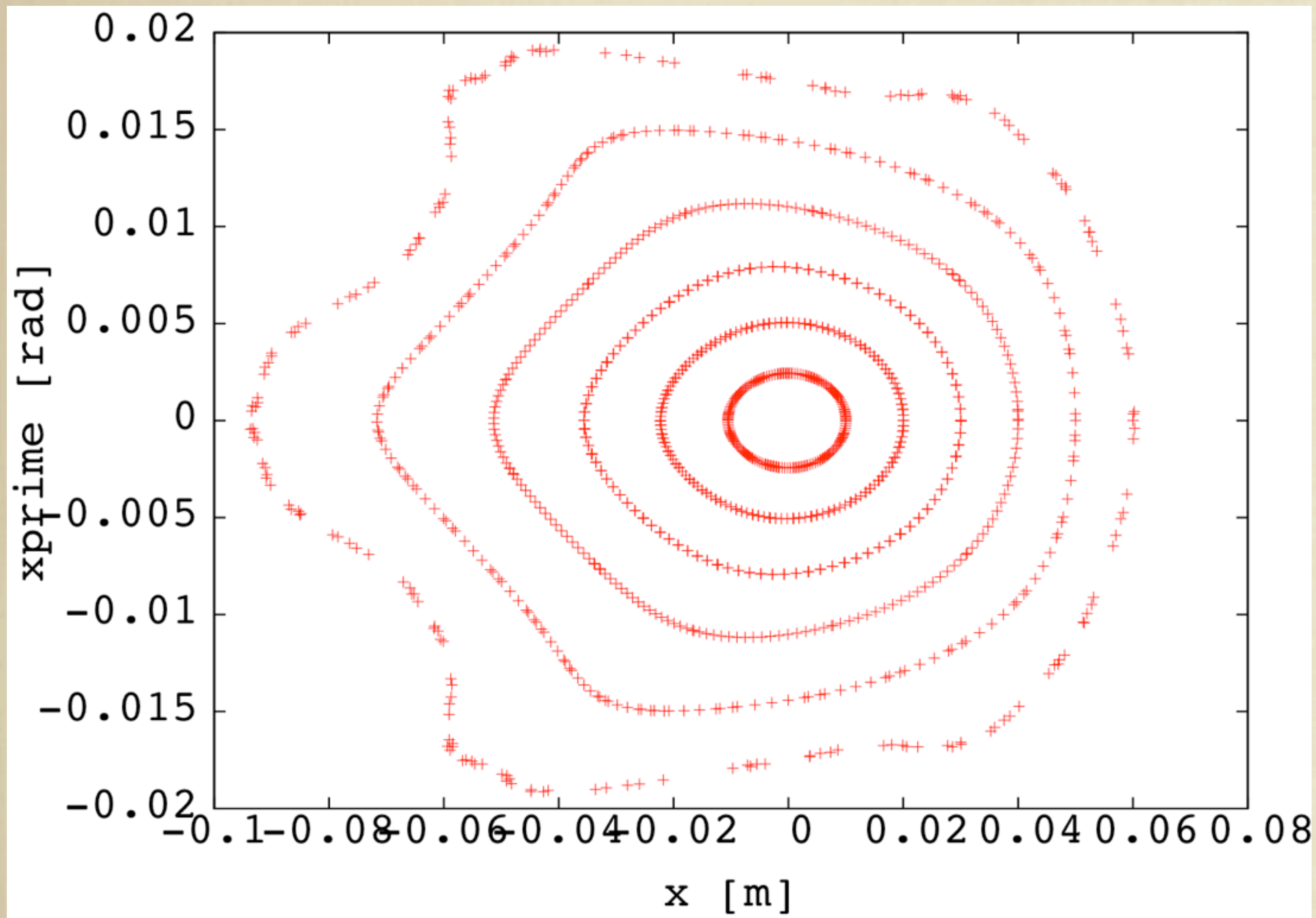


Figure 20 - Quintuplet cell, search for **Horizontal** dynamical aperture

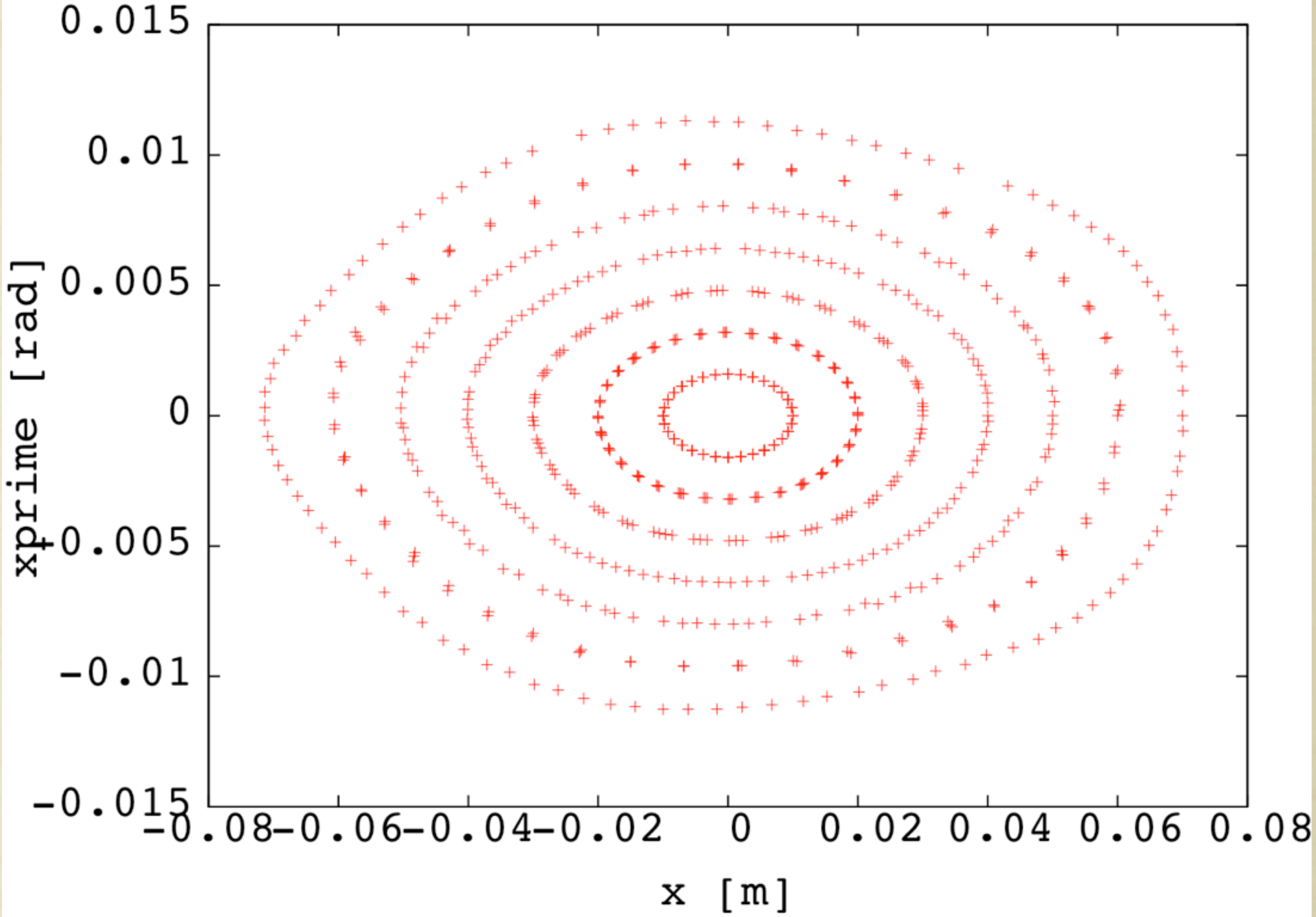


Figure 21 - Quintuplet cell, search for **vertical** dynamical aperture

(*) Runge-Kutta integration in soft edge field model.

STATIONARY BUCKET ACCELERATION

4D HORIZONTAL+LONGITUDINAL TRACKING (*) RESULTS.
ACCELERATING VOLTAGE ASSUME: 1.85 GV/TURN.

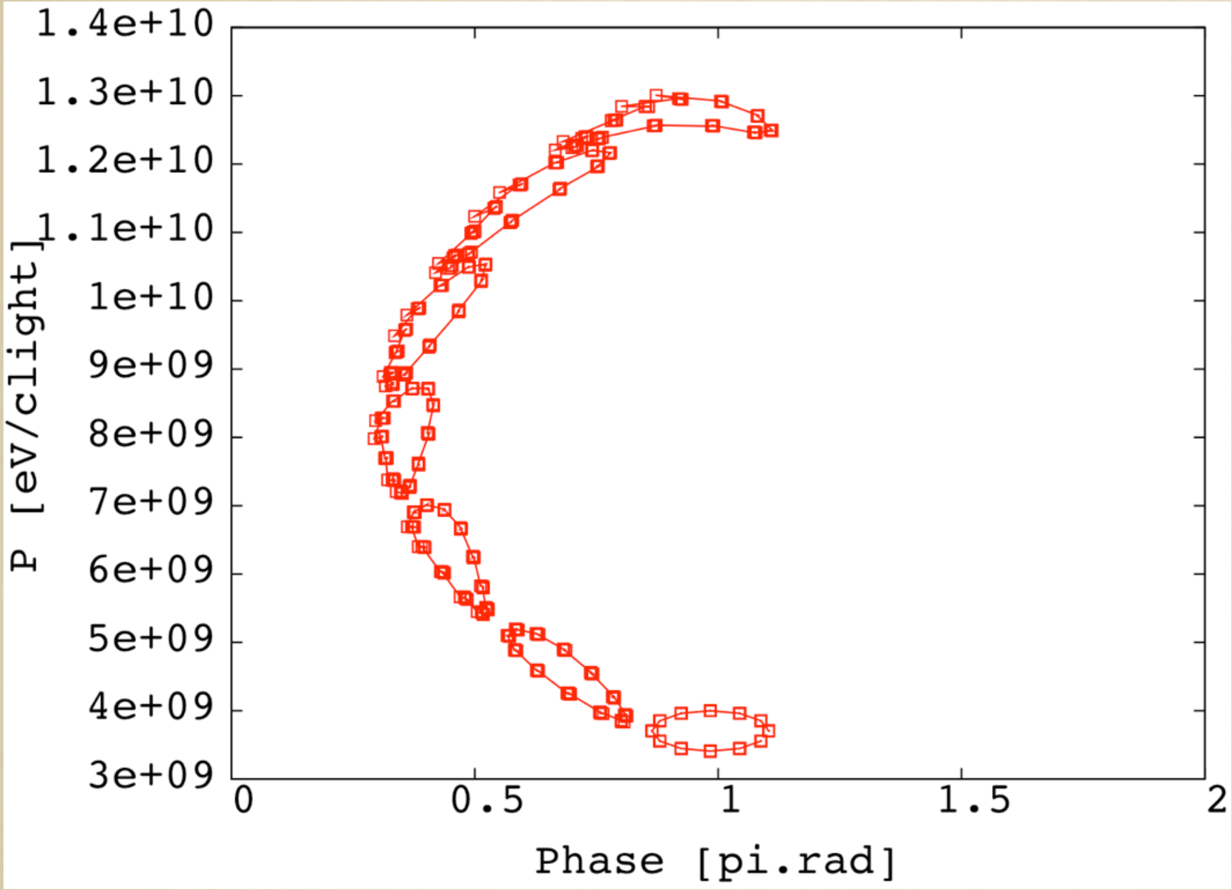


FIGURE 22 - LONGITUDINAL PHASE SPACE SHOWING THE 6 TURNS ACCELERATION CYCLE. INITIAL BEAM 4D EMITTANCE : 25 000 π.mm.mrad × 0.27 eV.sec.

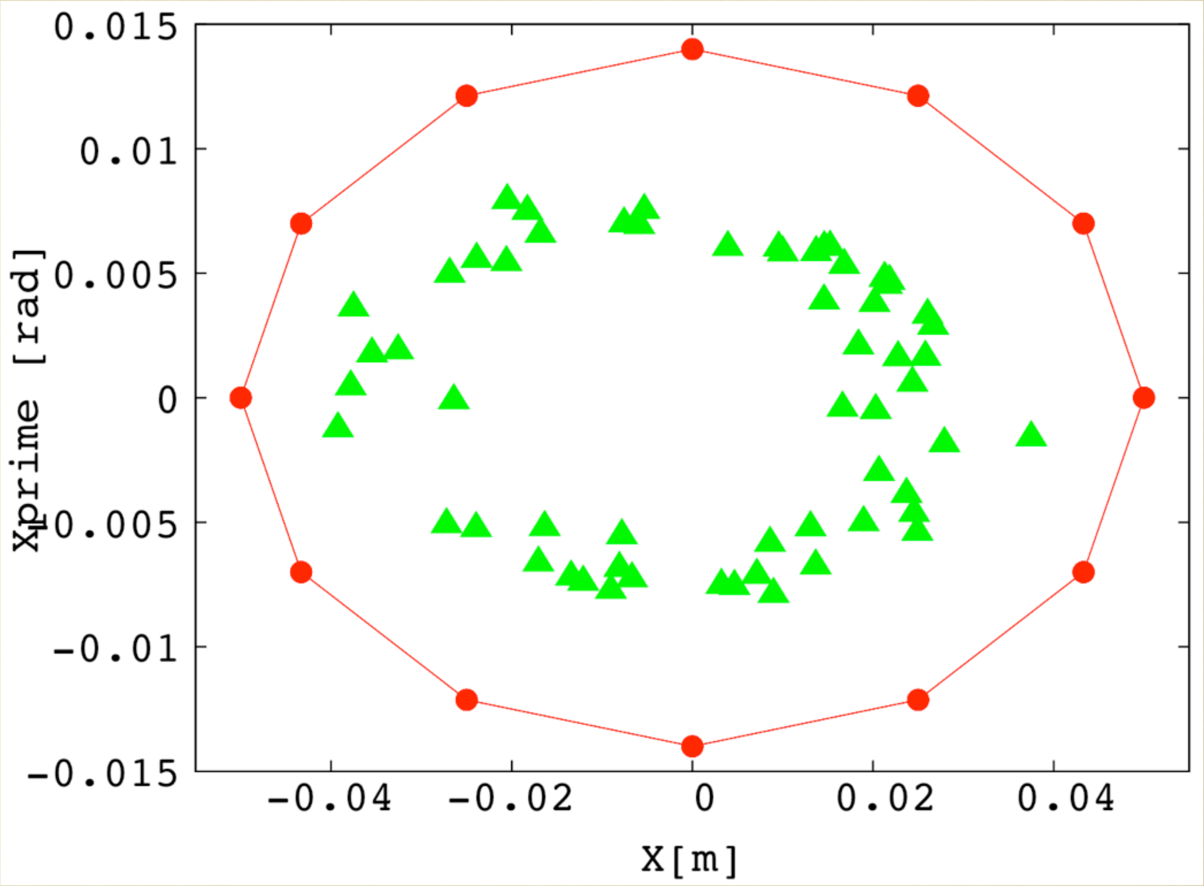
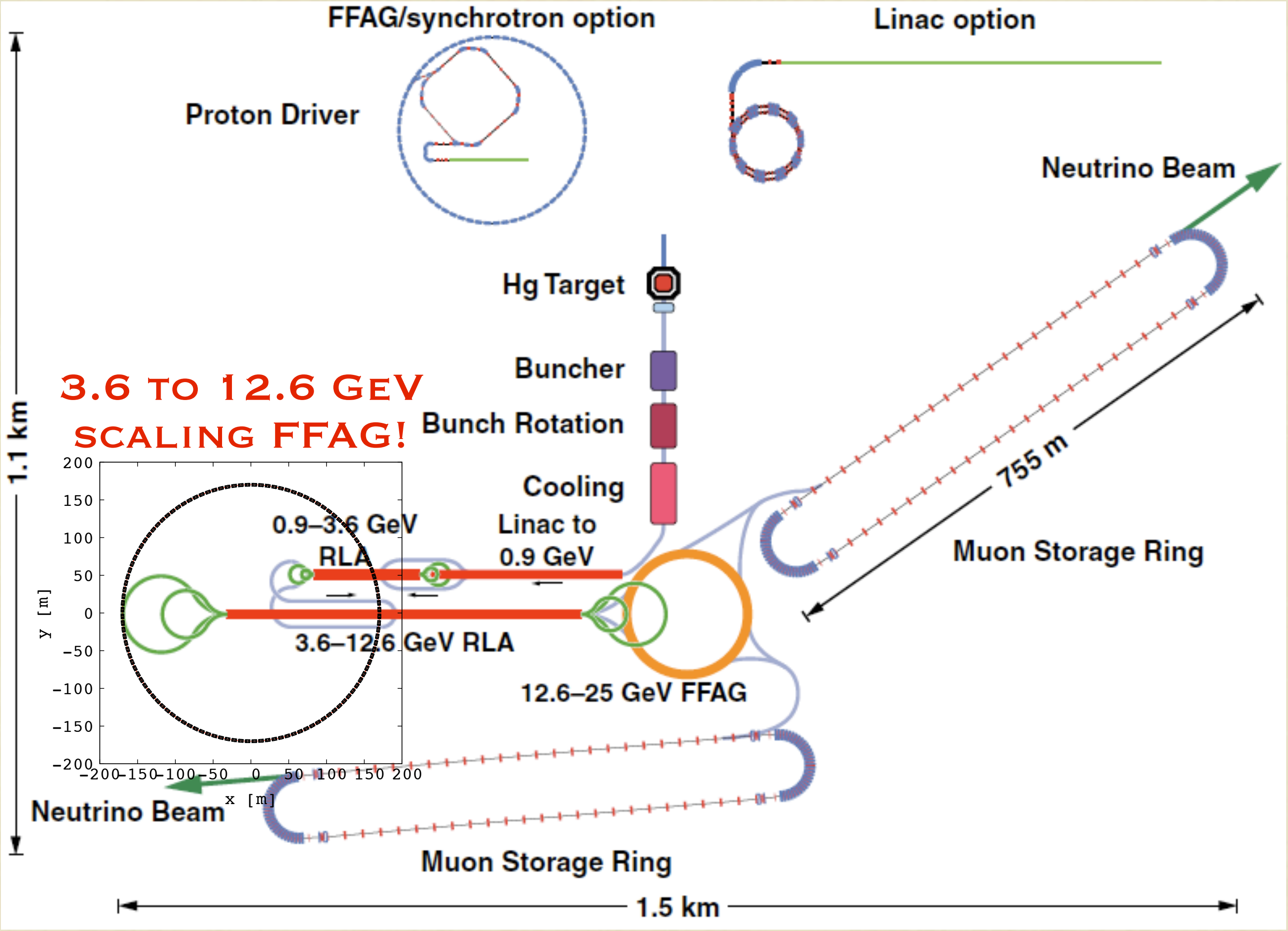


FIGURE 23 - HORIZONTAL PHASE SPACE AFTER (GREEN) AND BEFORE THE 6 TURNS ACCELERATION CYCLE. INITIAL BEAM 4D EMITTANCE : 25 000 π.mm.mrad × 0.27 eV.sec.

(*) RUNGE-KUTTA INTEGRATION IN SOFT EDGE FIELD MODEL.

Summary on stationary bucket acceleration

Looks like a good and robust alternative to RLA:



Further study are needed, full 6D tracking with errors, injection/extraction issues...

Thank you!