



Zero-chromatic FFAGs for future Neutrino Factories and Muon Colliders

T. Planche, J-B. Lagrange, E. Yamakawa, T. Uesugi, B. Qin,
Y. Kuriyama, K. Okabe, Y. Ishi, and Y. Mori.

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- (ii) No switchyard: larger number of passes possible.

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In addition, **zero-chromaticity** provides:

- (i) **Large transverse acceptances**, with a proper choice of the working point;
- (ii) The possibility to **avoid longitudinal emittance degradation** when accelerates beams with large transverse emittances, due to time-of-flight dependence on the transverse amplitudes*.

* see S. Berg, Nucl. Instr. and Meth. A 570, p.~15, (2007).

Aim

Zero-chromatic FFAG scheme achieving:

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Magnetic field variation $B(\theta)$	Fixed Frequency (CW beam)	Frequency-modulated (Pulsed beam)
Uniform	Classical	Synchro-
Alternating	Isochronous	FFAG

From: M. Craddock presentation at FFAG-School'10

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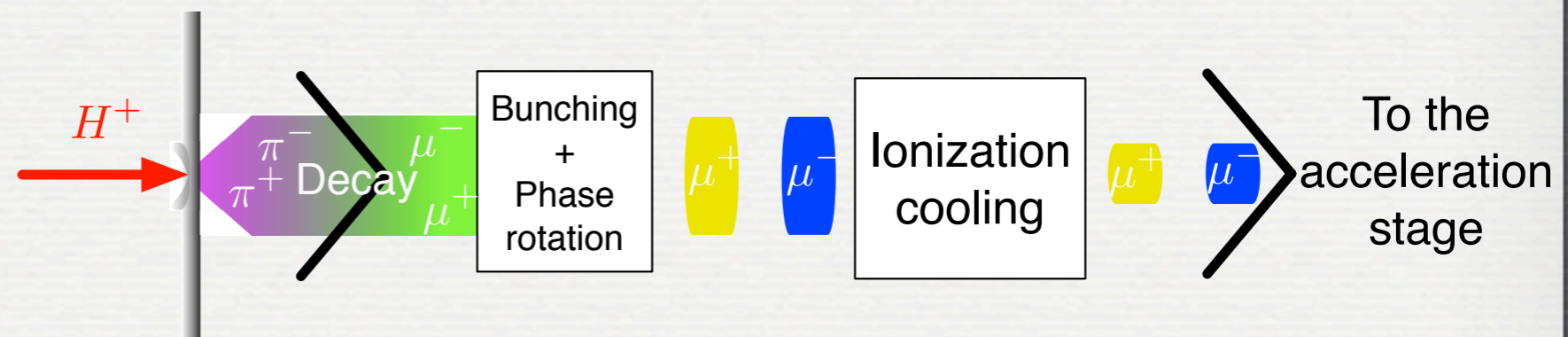
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Muon front-end, schematic view.

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- (iv) From lower energy than the non-scaling FFAG ring.

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Constant frequency acceleration in non-isochronous
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Constant frequency acceleration in non-isochronous zero-chromatic FFAGs:

- **Harmonic number jump (HNJ) acceleration in zero-chromatic FFAG.**
- **Stationary bucket (SB) acceleration of muons in scaling FFAG.**

Harmonic number jump acceleration

Maintaining the
synchronization condition:

$$f_{rf} = h \cdot f_{rev_s}, \quad h(i) \in \mathbb{N}$$

by changing the harmonic number
 h_i by an integer number every turn.

Harmonic number jump acceleration

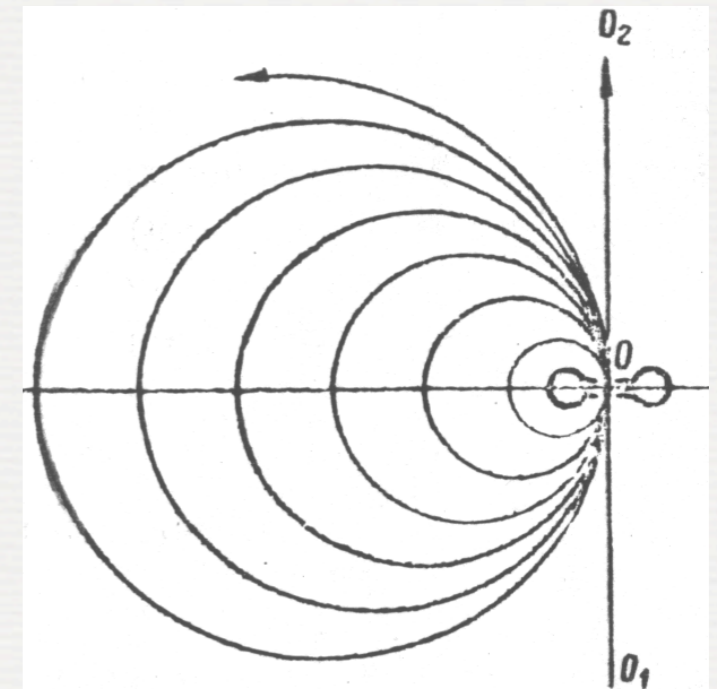
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Method used to accelerate electrons
in microtrons, and originally
proposed by Veksler (1944).

Re-considered for FFAGs: A.G. Ruggiero (2006).



Trajectories of particles
accelerated in a microtron
(from: A.A. Kolomenskii Sov. Phys.
Tech. Phys., Vol. 5, 11, 1961).

Harmonic number jump acceleration

One must give the right energy gain to change the revolution period of an integer number of rf period every turn:

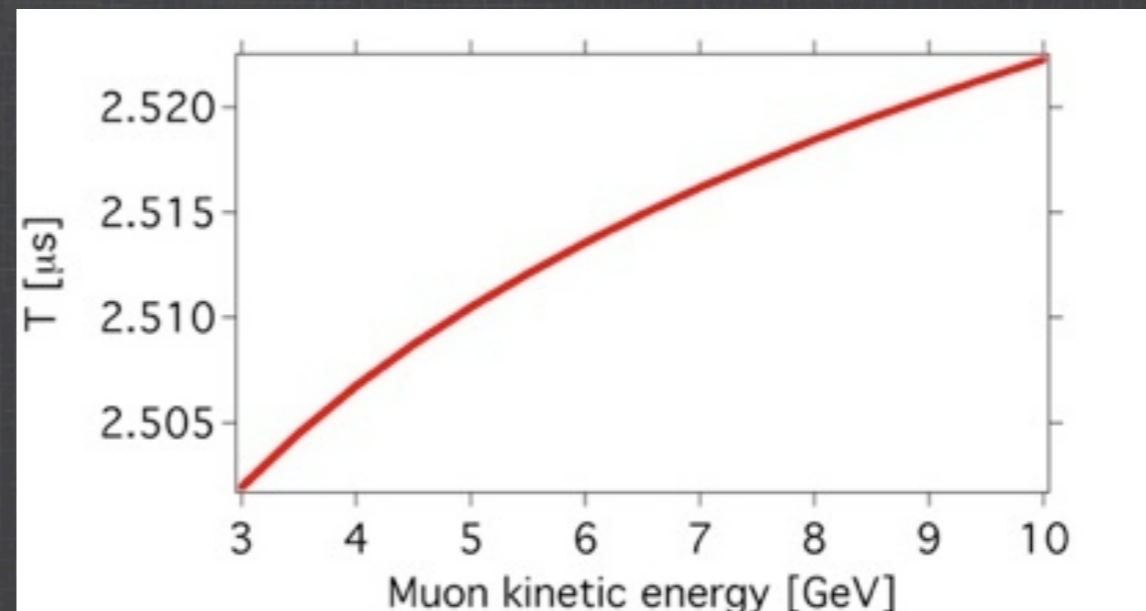
$$T_{i+1} - T_i = \frac{\Delta_i h}{f_{rf}} \Leftrightarrow E_{i+1} - E_i = \frac{\Delta_i h}{f_{rf} \cdot \left. \frac{\partial T}{\partial E} \right|_{E_i}}.$$

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Example of non-linear variation of the revolution time: scaling FFAG ring with $k = 145$ and average radius = 120 m.

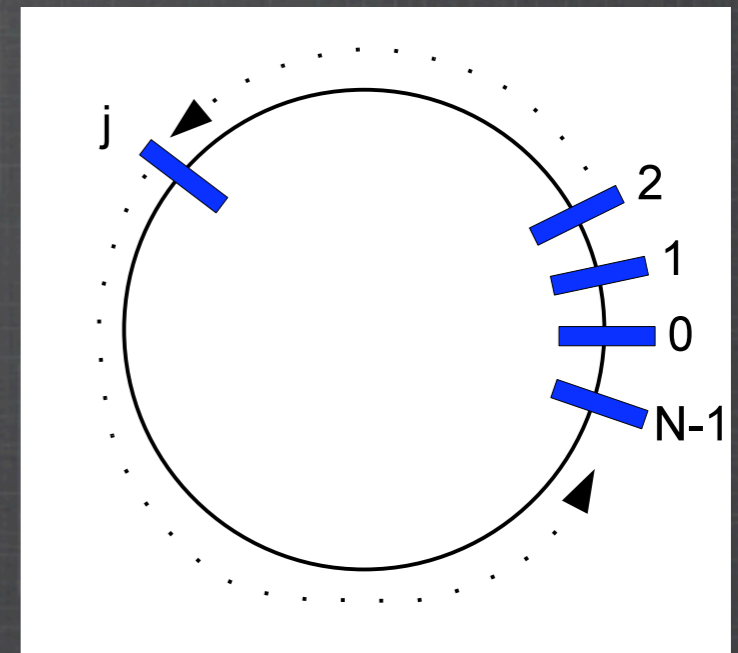


Ring with many rf cavities

Assuming that h_0 is large one gets:

$$f_j \simeq f_{ref} \left[1 - \frac{\Delta h}{h_0} \cdot \left(\frac{2j+1}{2N} + \frac{1}{2} \right) \right]$$

⇒ Each frequency must be tuned at a slightly different frequency.



N cavities (numbered 0 to N-1) uniformly distributed around a circular ring.

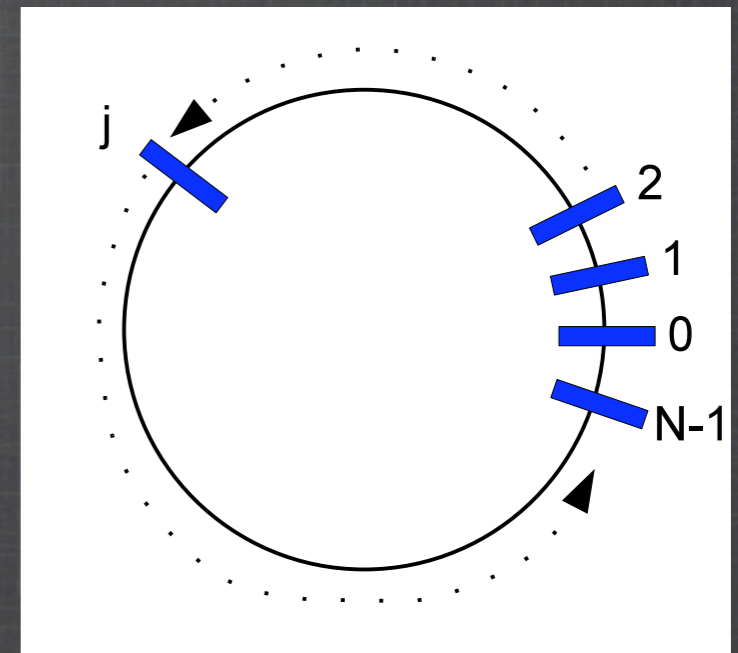
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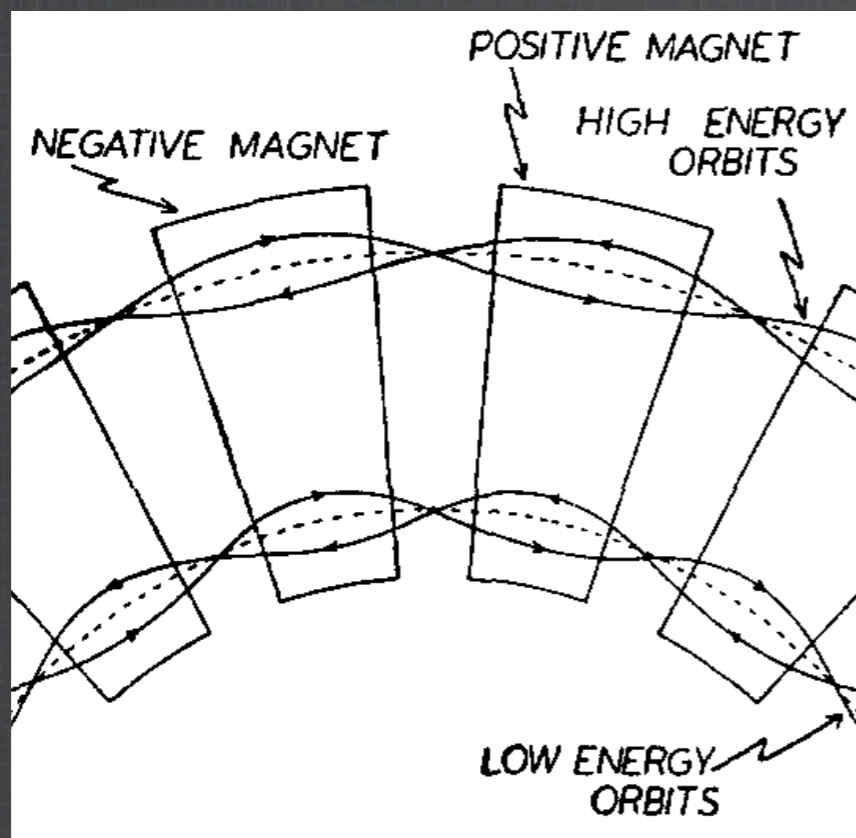
⇒ If one wants to accelerate charged particles and their antiparticles simultaneously in the same ring, they must all circulate in the same direction.



N cavities (numbered 0 to N-1) uniformly distributed around a circular ring.

Two-beam FFAG lattice

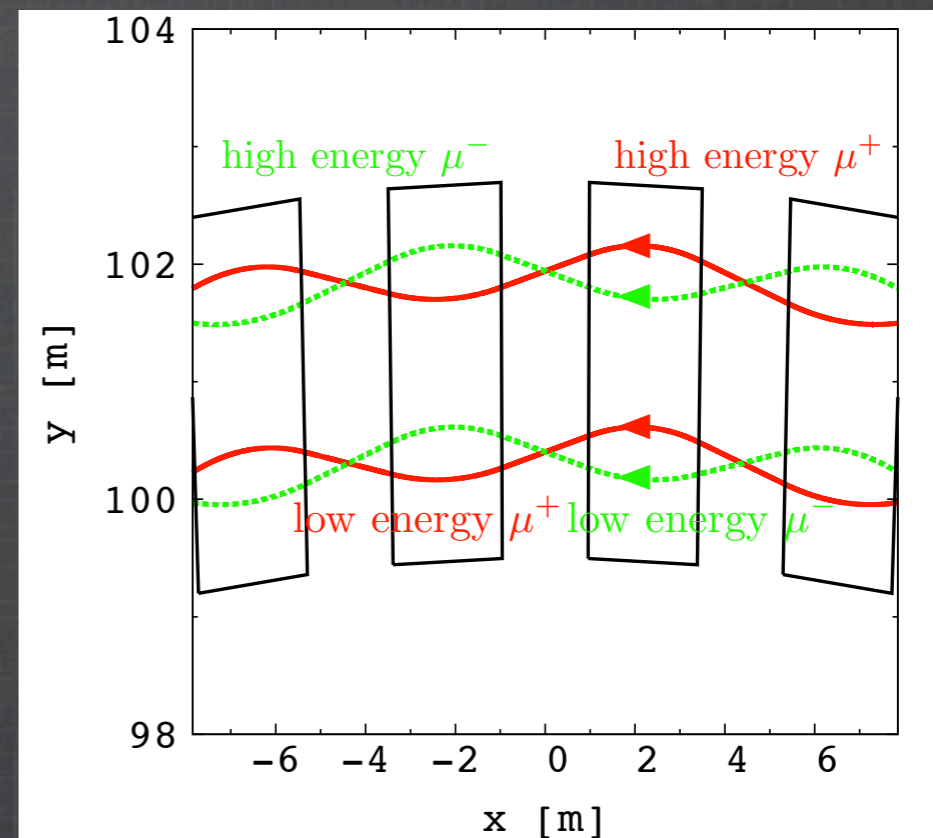
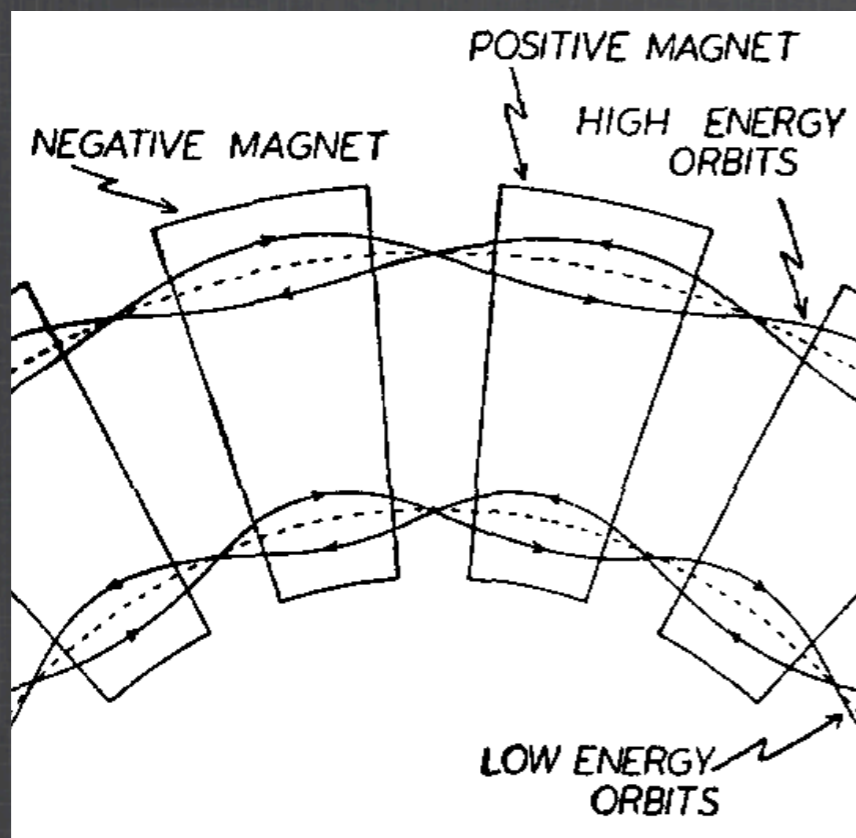
It is possible by means of two-beam FFAG lattices:



Principle of the two-beam lattice as described in the original paper [T. Okawa, Rev. Sci. Instru. 29 (2) (1957), p. 108~117].

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This type of lattice can also be used to circulate particles and anti-particles in the same direction of rotation

Required excursion

when accelerating ultra-relativistic particles

$$T(E_{i+1}) - T(E_i) = \frac{2\pi\Delta_i R}{c}.$$

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Combining this Eq. with the required change in time of flight for harmonic number jump, one gets:

$$\Delta_i R = \Delta_i h \cdot \frac{\lambda_{rf}}{2\pi}, \quad \text{with} \quad \lambda_{rf} = \frac{c}{f_{rf}}.$$

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The rf wavelength \leftrightarrow cavity size.

- For instance, in the simple case of a pillbox type rf cavity, its diameter d is approximately given by $d = 0.77 \lambda_{rf}$.

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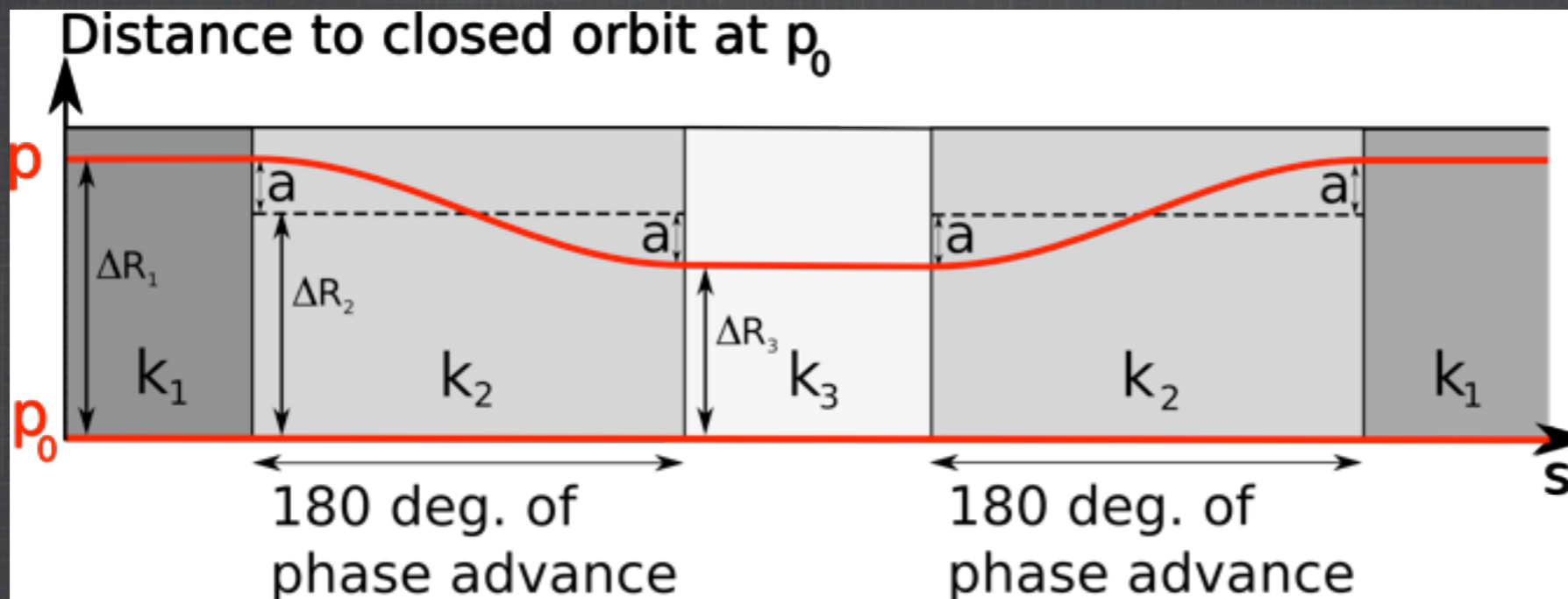
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To avoid cavity design problems, one can vary the excursion along the ring, introducing reduced excursion insertions in which cavities could be installed.

Zero-chromatic FFAG insertions

Dispersion suppressor insertion made of scaling FFAG type of magnets developed:



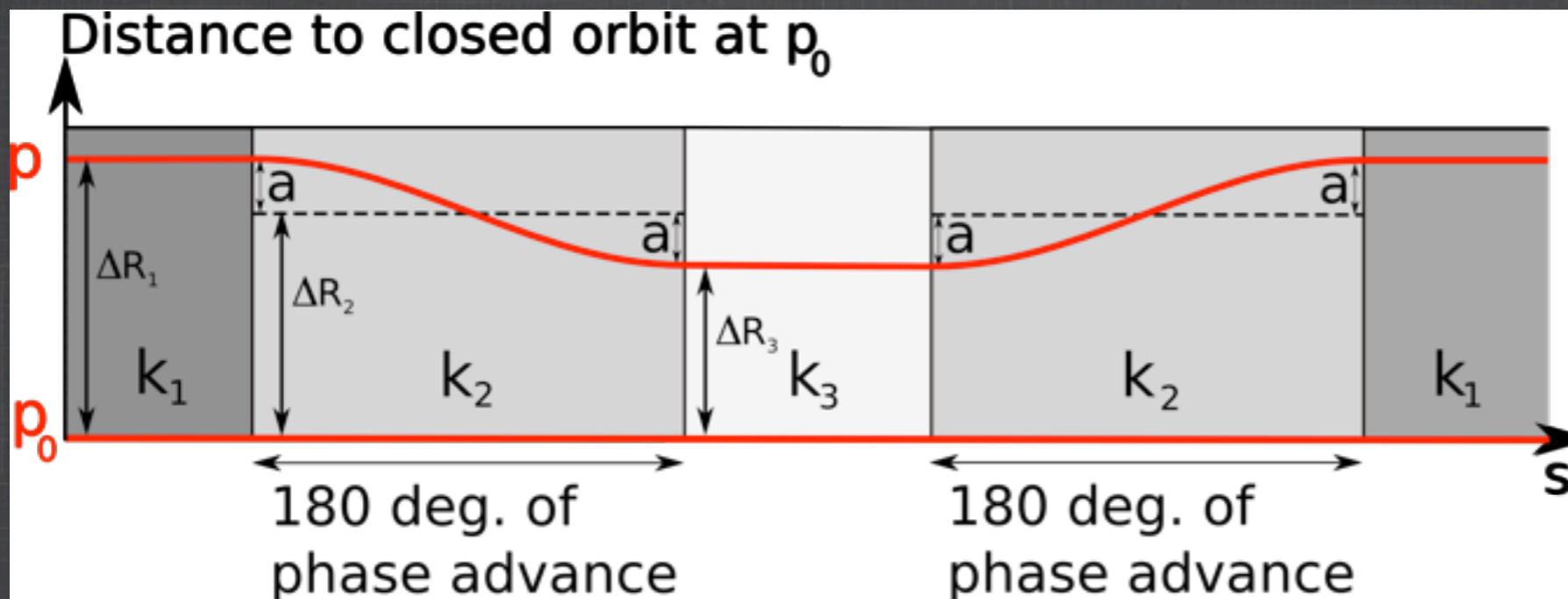
Schematic view of an excursion reduced insertion made of scaling FFAG cells.

Condition:

$$\frac{2}{k_2 + 1} = \frac{1}{k_1 + 1} + \frac{1}{k_3 + 1}.$$

Zero-chromatic FFAG insertions

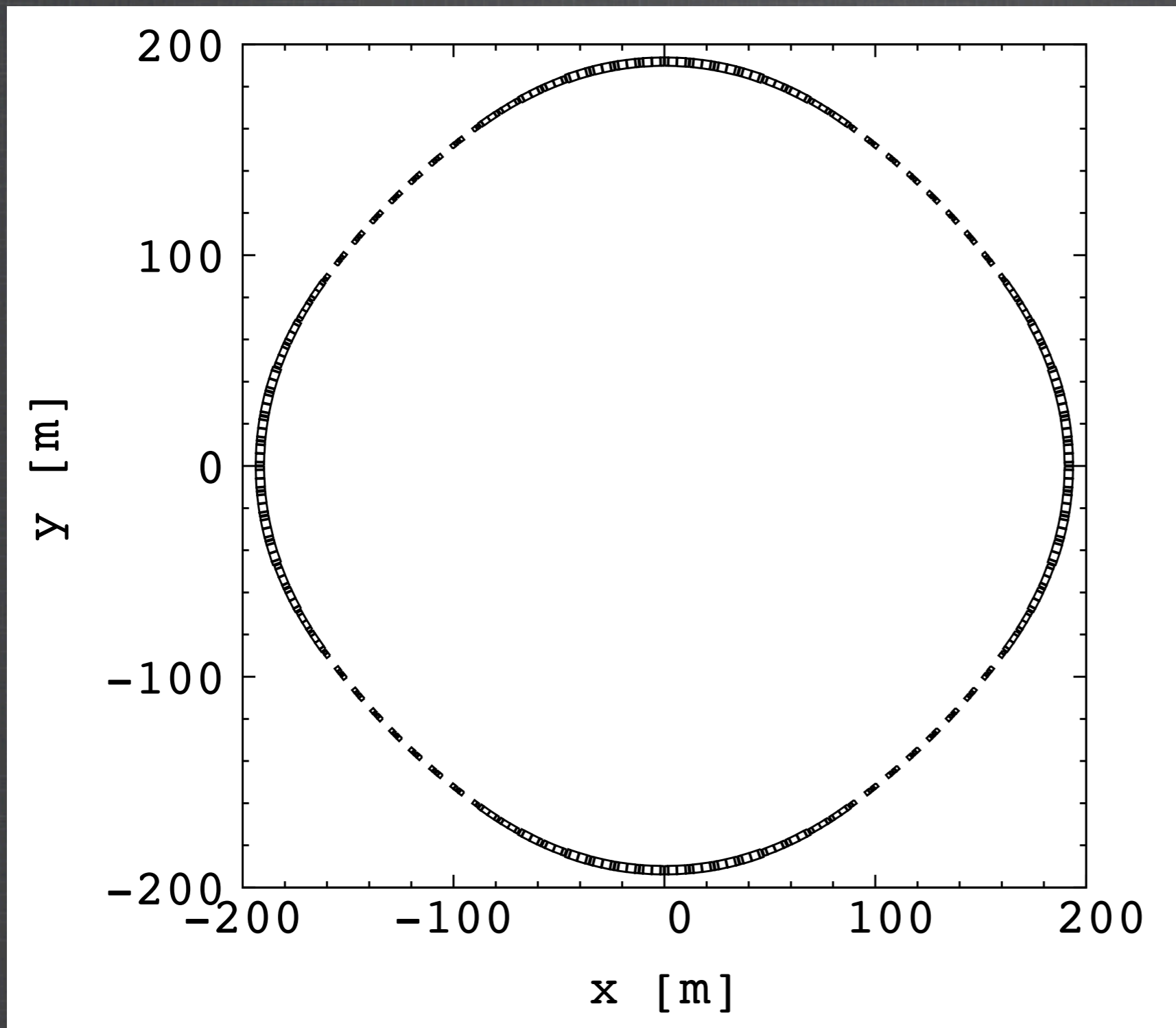
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Zero-chromatic as long as the effect of non-linear field components is negligible.

Example of a 3.6 to 12.6 GeV ring



Schematic view of a 3.6 to 12.6 GeV muon ring made of quadruplet two-beam cells.

Example of a 3.6 to 12.6 GeV ring

Table 2 -Parameters of each type of cells constituting the accelerator ring

	Ring main part	Reduced excursion area	First dispersion suppressor	Second dispersion suppressor
Cell opening angle [deg.]	5.	2.25	4.5	3.5
Mean radius [m]	136.2	332.3	155.5	184.9
Field index k	130	638	169.9	283.5
Horiz. phase adv./cell [deg.]	87.4	86.4	90.0	90.0
Vert. phase adv./cell [deg.]	50.6	32.0	44.4	34.6
Number of times used in the ring	8×4	8×4	4×4	4×4

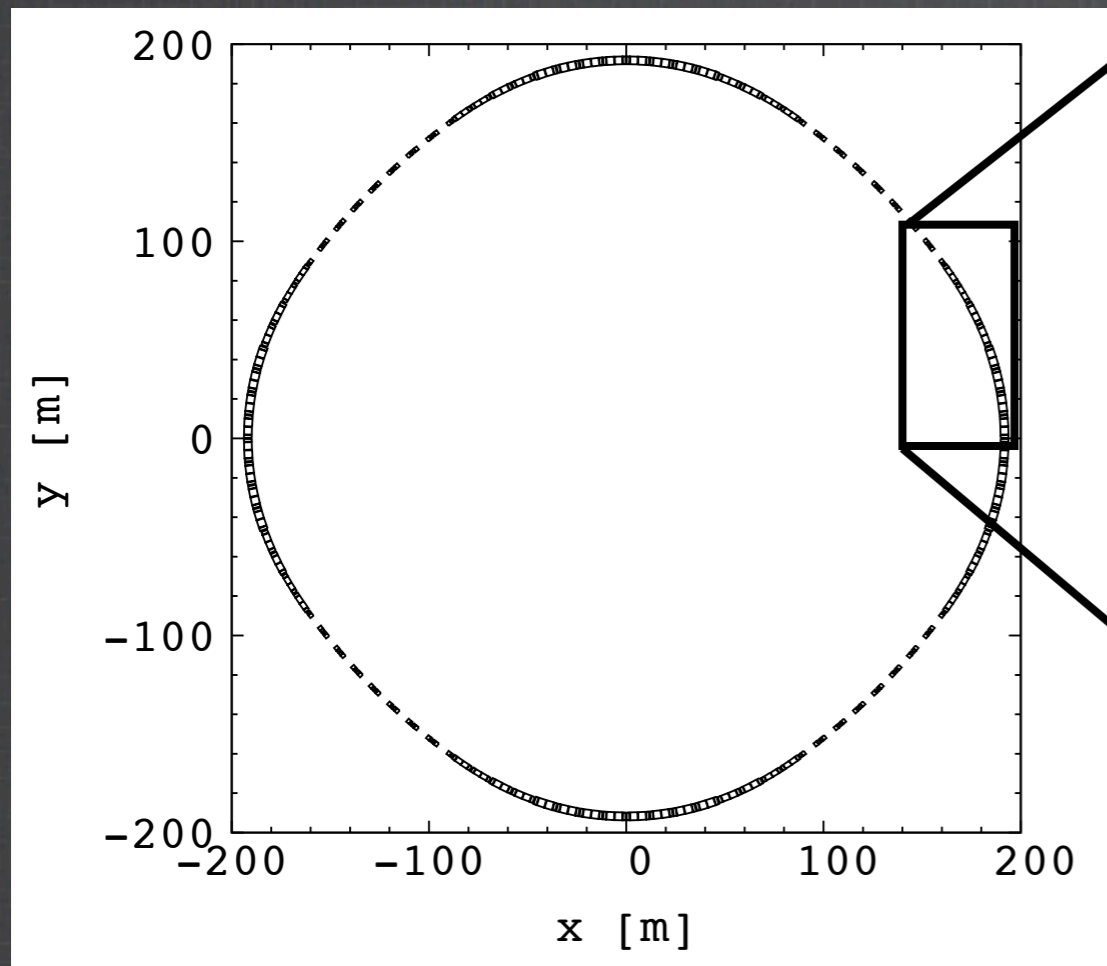
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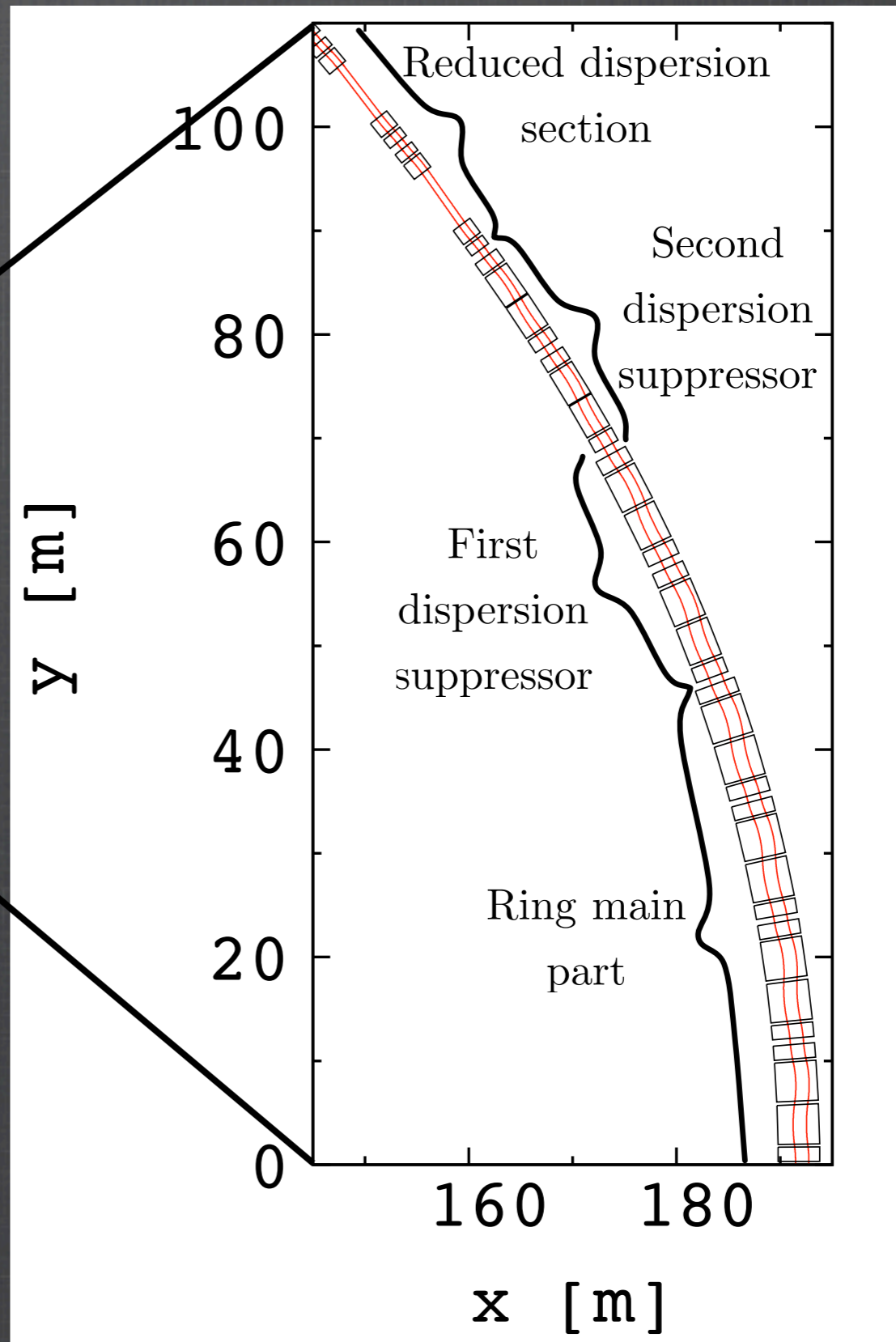
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B_{\max} @ 12.6 GeV: 4 T

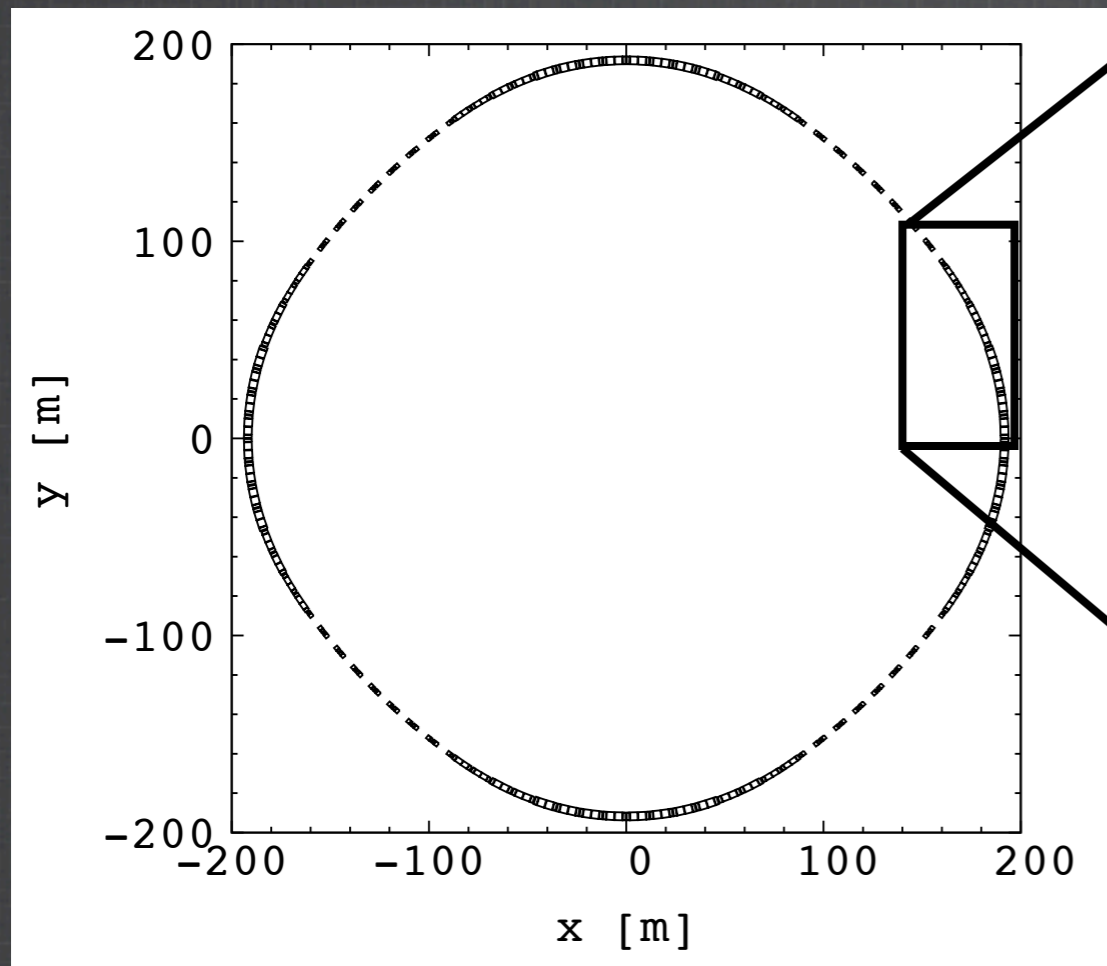
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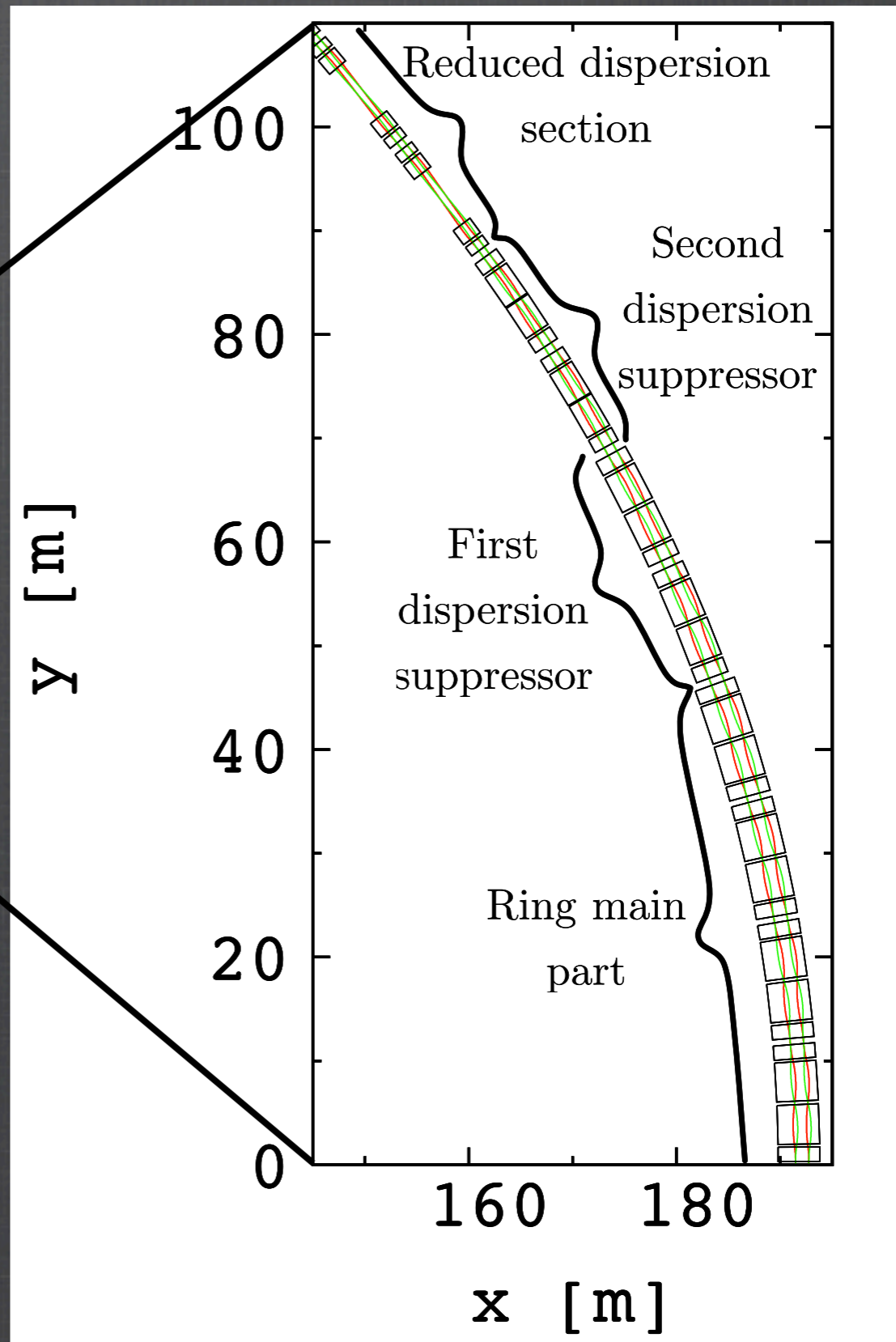
Schematic view of the ring made of quadruplet two-beam cells.



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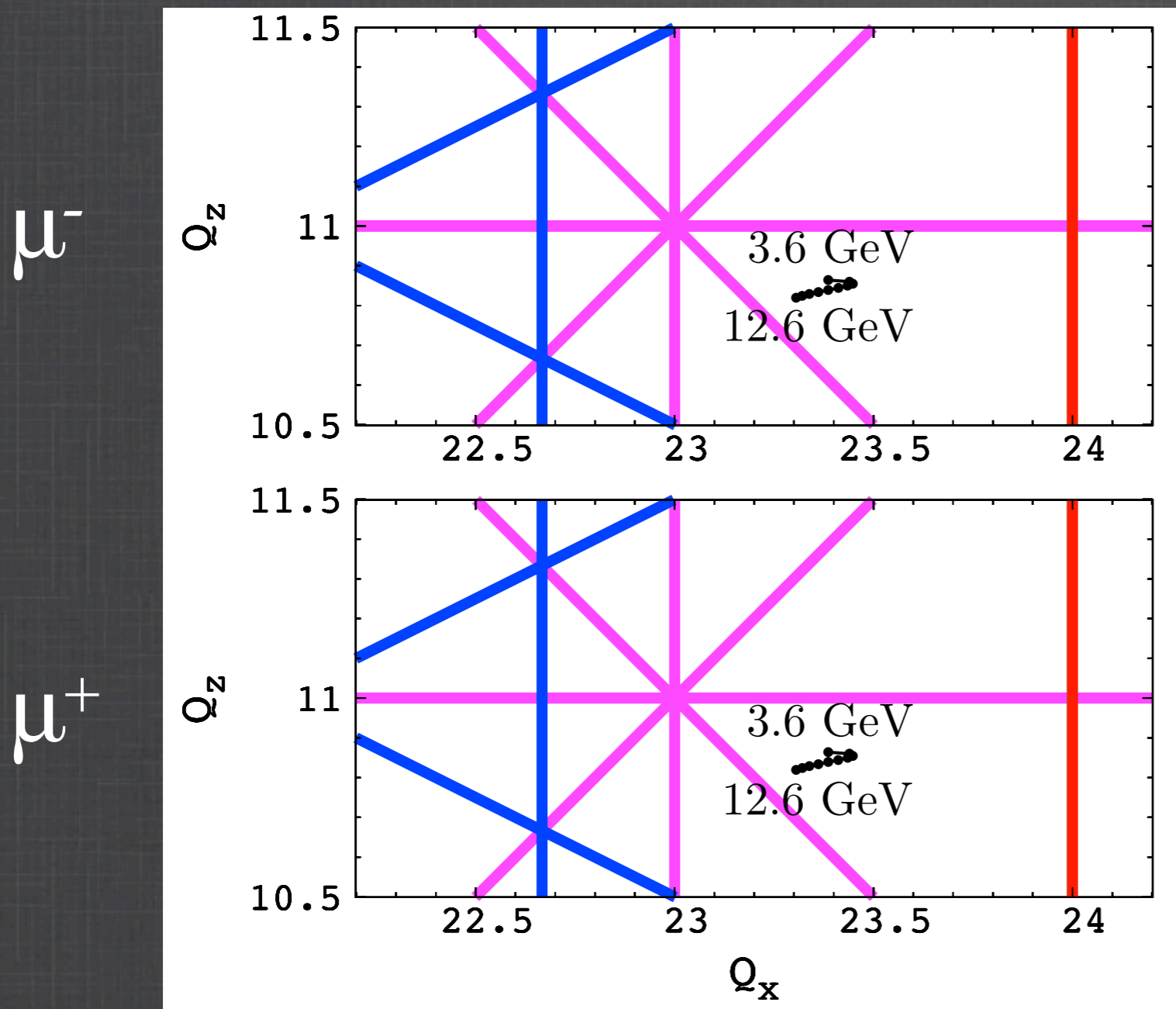


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Linear parameters

Betatron tunes:

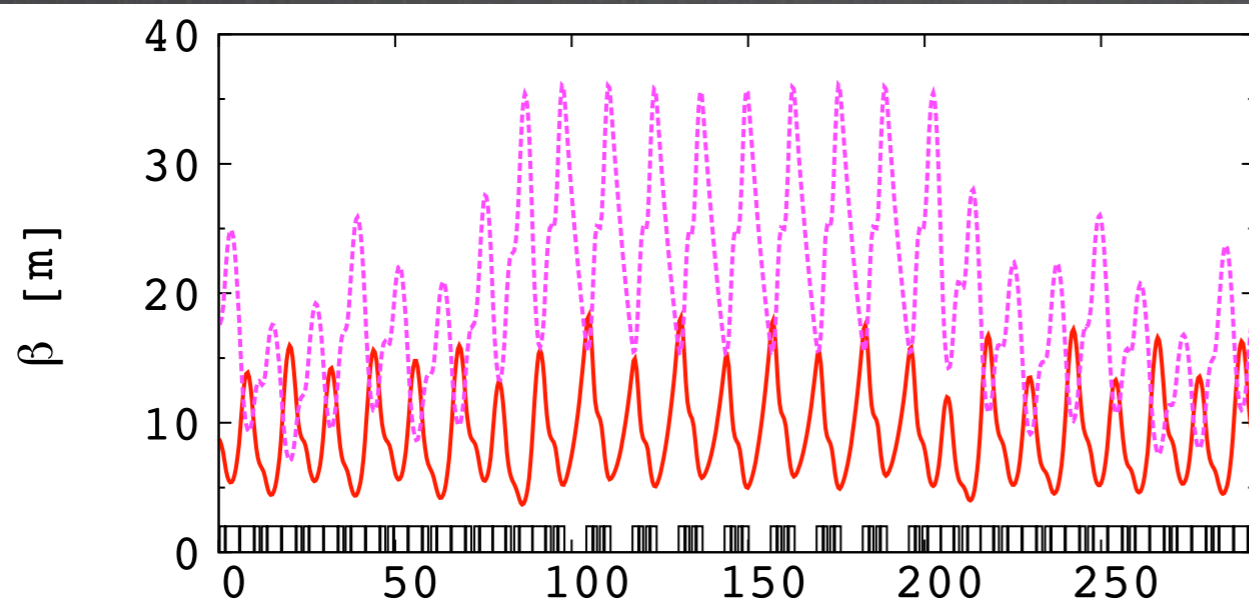


- Variation of the betatron tunes, plotted every 1 GeV.
- Structure normal resonance lines up to octupole are superimposed.

Linear parameters

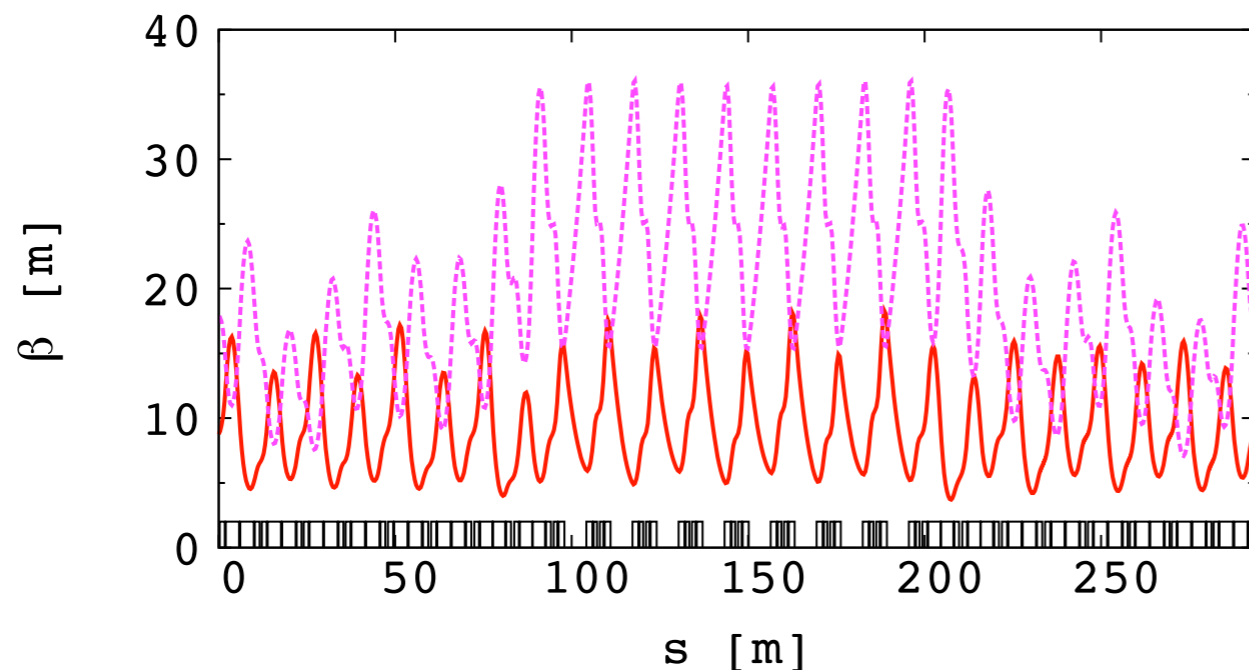
Periodic beta-functions:

μ^-



Horizontal (red) and vertical (dotted purple) beta-functions in the case of 5 GeV.

μ^+



Remark: beta-functions are mirror symmetric of each other.

Full acceleration cycle - 6D tracking -

- 1000 particles are uniformly distributed inside a transverse 4D ellipsoid (Waterbag distribution).
- These particles are then independently distributed uniformly inside an ellipse in the longitudinal plane.
- Initial normalized bunch emittances are 30,000 π .mm.mrad in both horizontal and vertical planes and 150 mm in the longitudinal plane.
- RF kicks used to simulate the effect of this rf gaps distributed around the ring

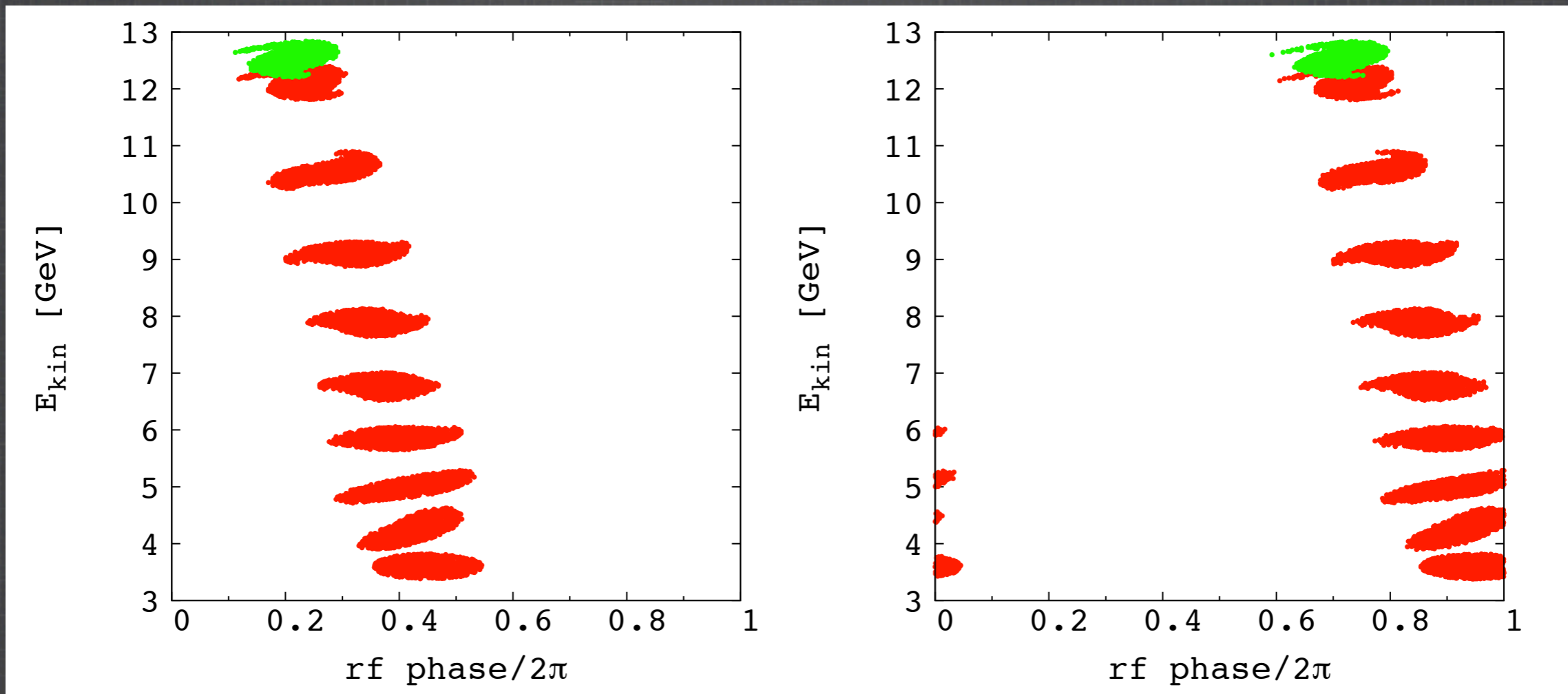
Full acceleration cycle - 6D tracking -

- RF kicks are given in every cell of the reduced excursion areas. Four different rf frequencies are used, one in each reduced excursion section.

Parameters of the rf scheme assumed for the acceleration tracking simulation.

Assumed peak rf voltage per kick	65 MV
Sum of the rf peak voltages over one turn	2.08 GV
Initial rf phase for μ^+	$0.45 \cdot 2\pi$
Initial rf phase for μ^-	$0.95 \cdot 2\pi$
Initial harmonic number	1568
Frequency of cavities in:	
the 1 st reduced excursion section	399.6752 MHz
the 2 nd reduced excursion section	399.6115 MHz
the 3 rd reduced excursion section	399.5477 MHz
the 4 th reduced excursion section	399.4840 MHz

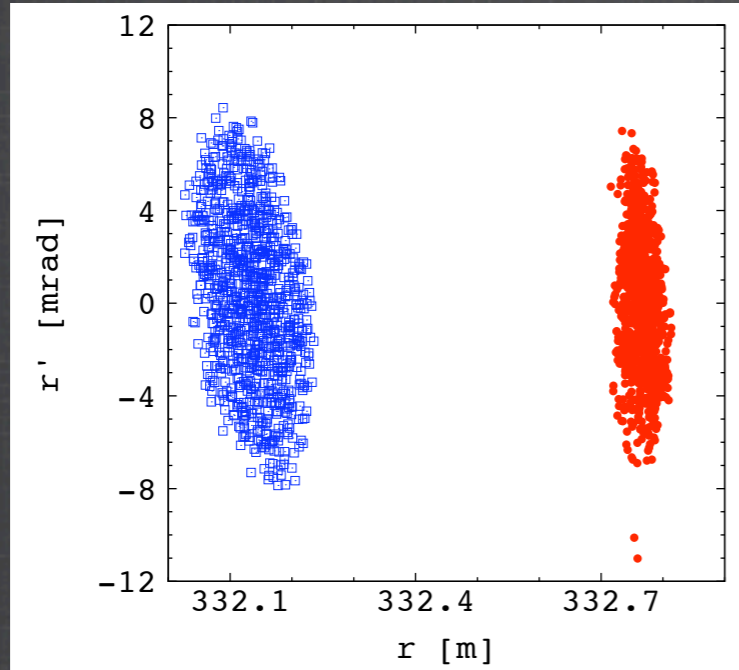
6D tracking results



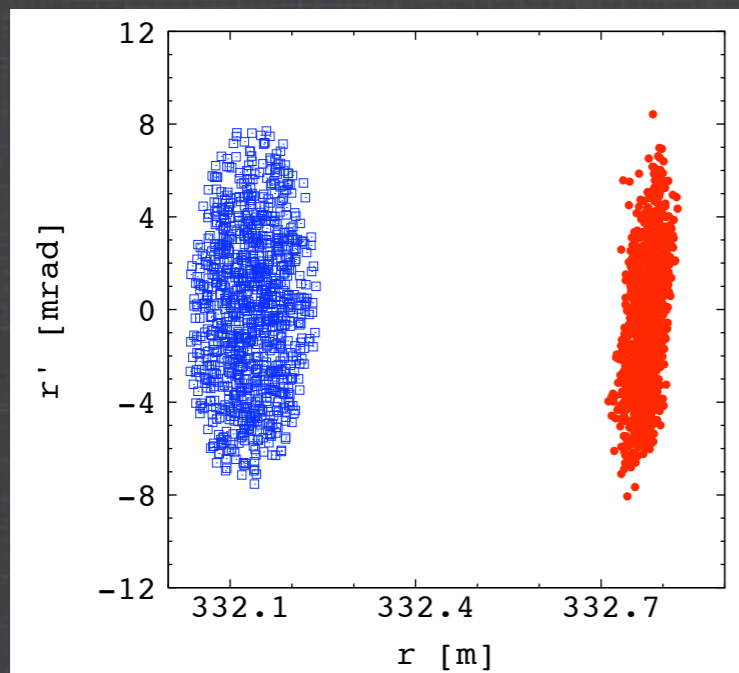
Longitudinal phase space plot showing the 8.25-turn acceleration cycle. The left plot shows the μ^+ bunch, while the right plot shows the μ^- bunch.

6D tracking results

μ^-



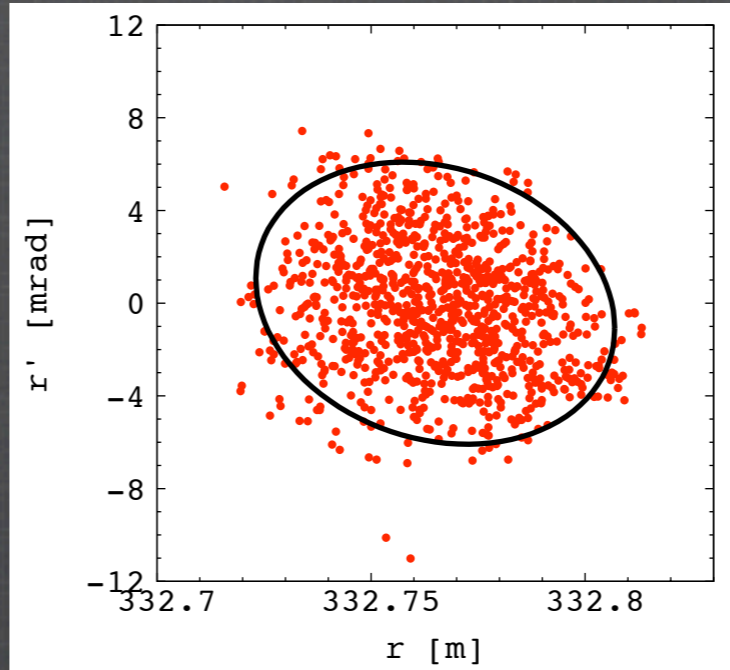
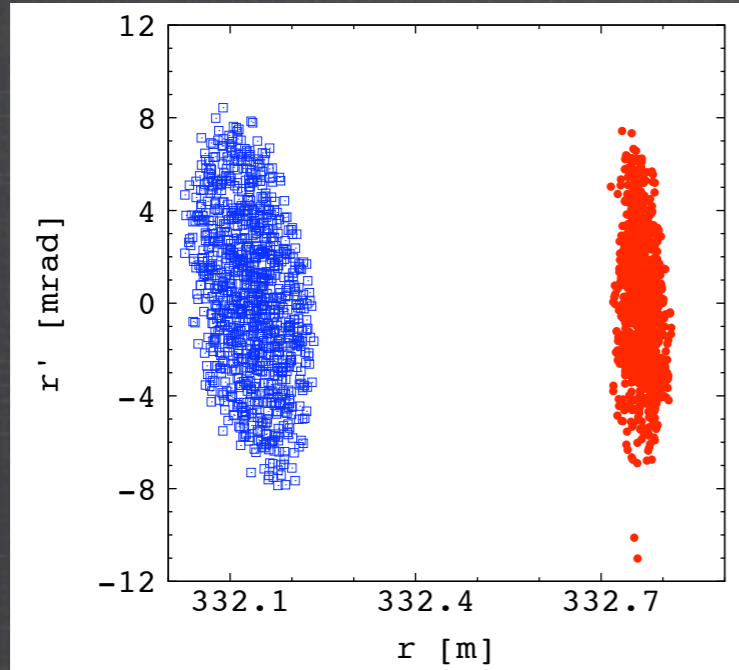
μ^+



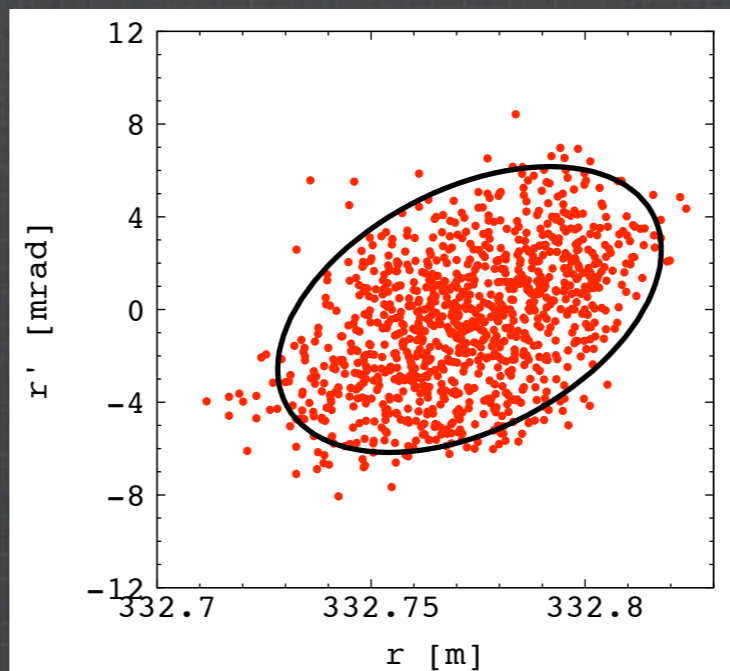
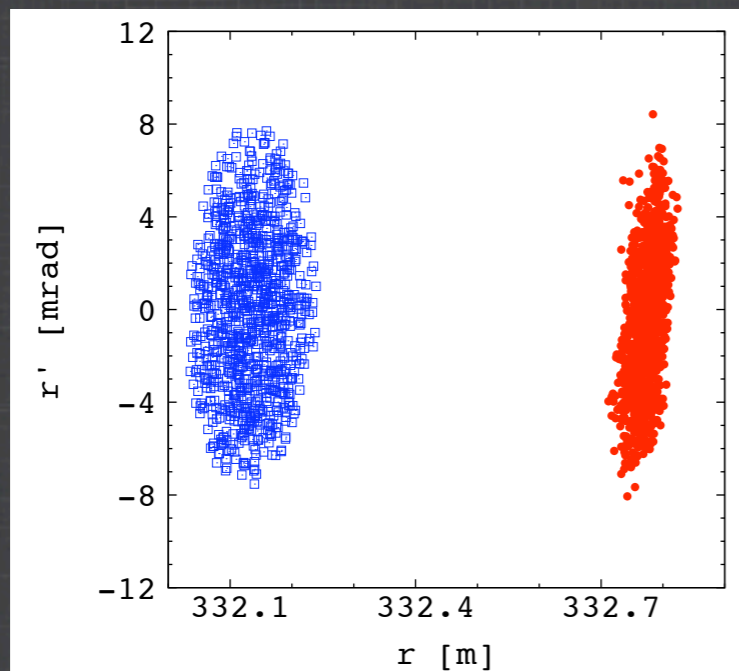
Horizontal (r,r') phase space showing the position at the beginning (blue squares) and at the end (red dots) of acceleration. The area of black ellipses correspond to $30,000 \pi \cdot \text{mm} \cdot \text{mrad}$.

6D tracking results

μ^-



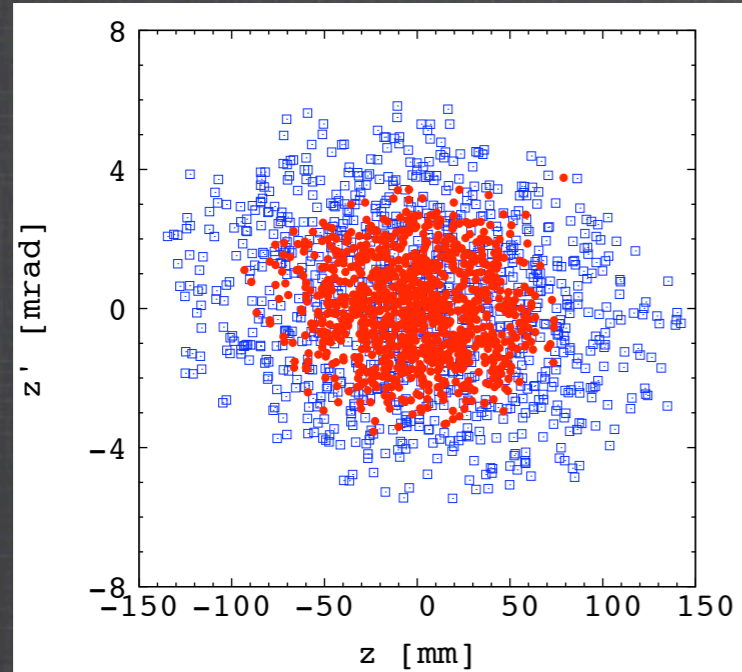
μ^+



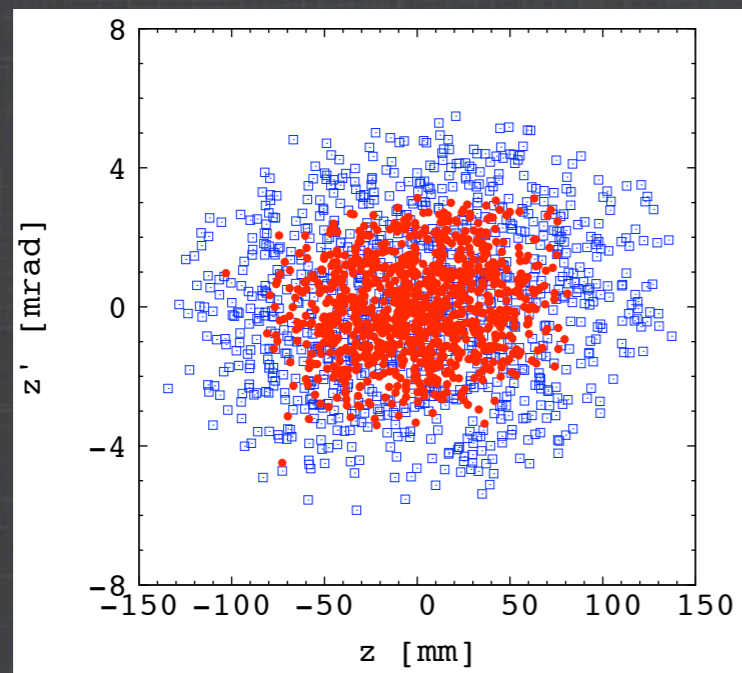
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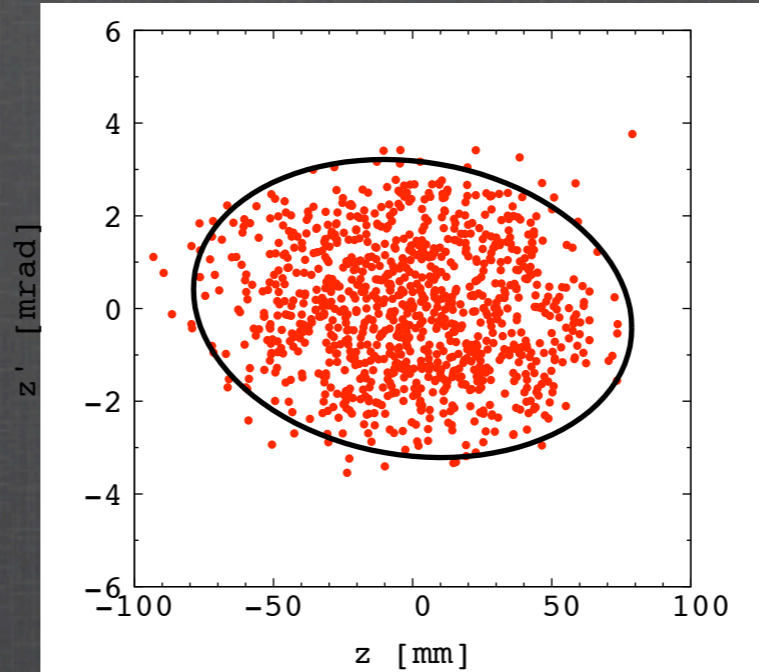
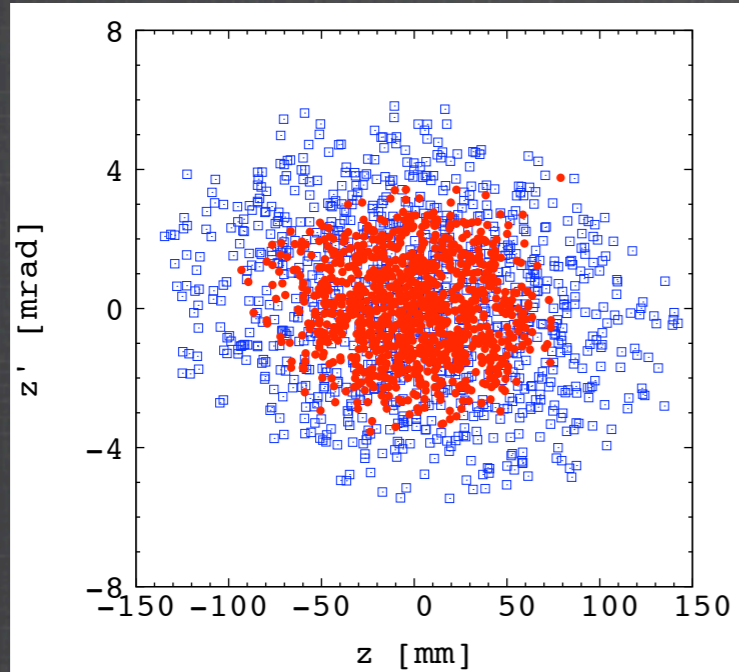
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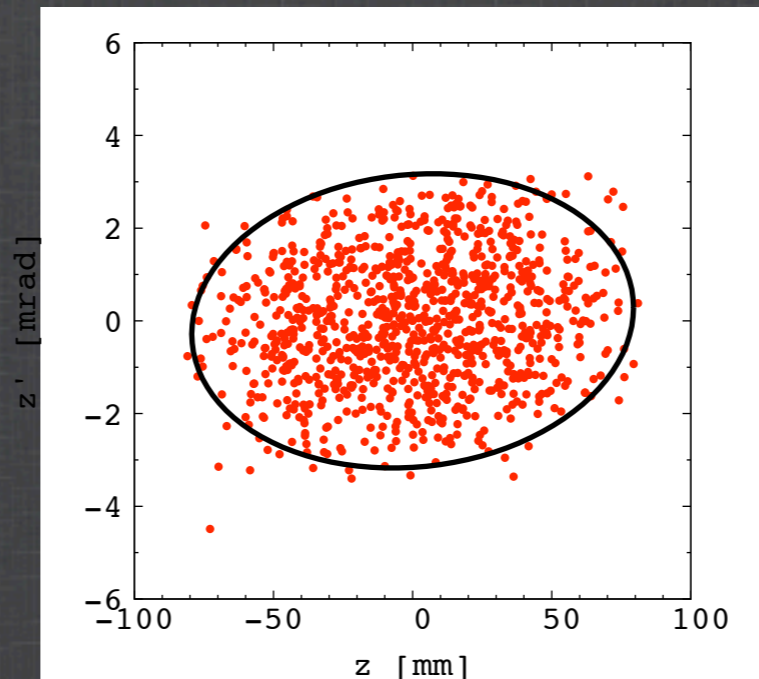
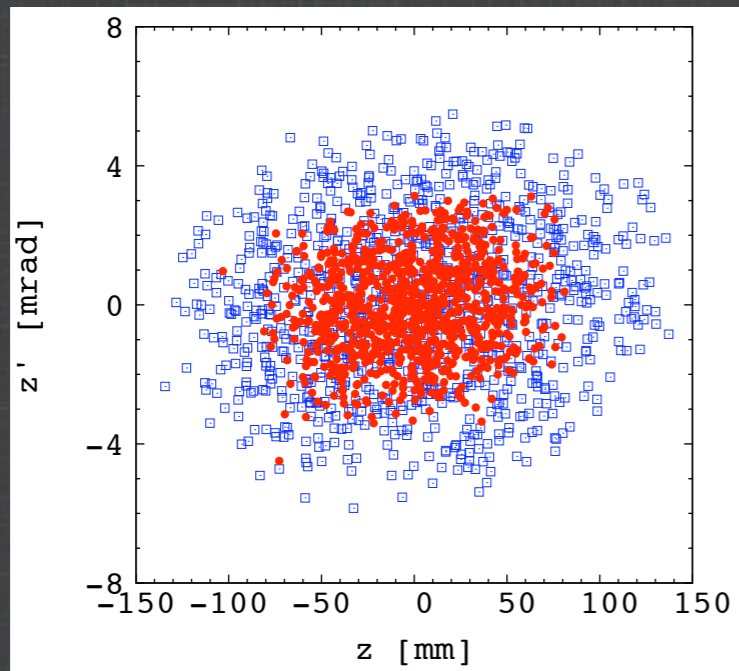
Same plot in the vertical (z, z') phase space .

6D tracking results

μ^-



μ^+



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 - ➔ More flexibility than in usual scaling FFAG lattices.
- Detailed design of a 3.6 to 12.6 GeV muon ring.
- Satisfies all requirements: acceptance, RF frequency, simultaneous acceleration of μ^+ and μ^- .



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