



Zero-chromatic FFAGs for future Neutrino Factories and Muon Colliders... Part II

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Y. Kuriyama, K. Okabe, Y. Ishi, and Y. Mori.

Motivations

FFAG rings are potentially cheaper than RLAs.

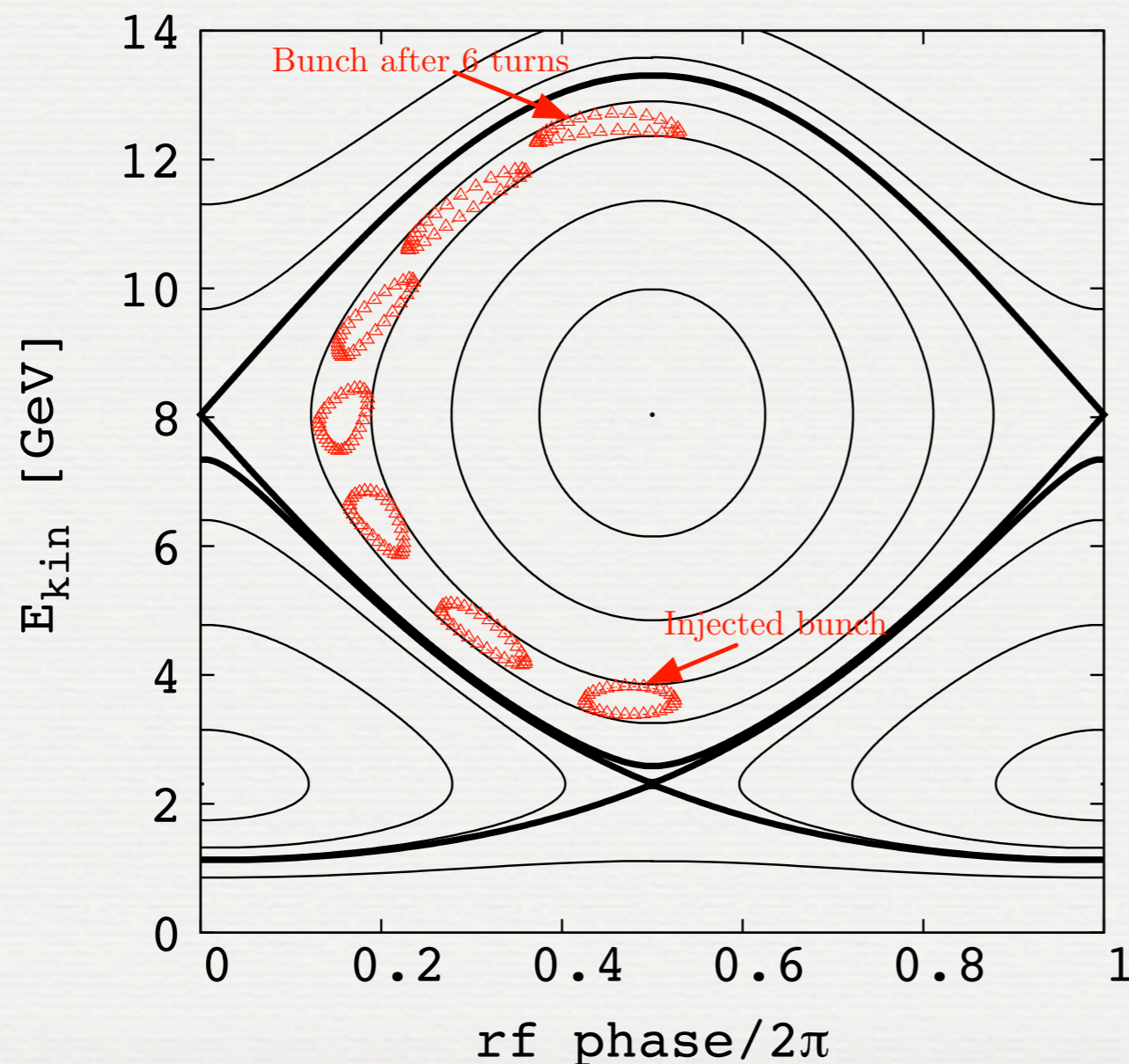
In addition, **zero-chromaticity** provides:

- (i) **Large transverse acceptances**, with a proper choice of the working point;
- (ii) The possibility to **avoid longitudinal emittance degradation** when accelerates beams with large transverse emittances, due to time-of-flight dependence on the transverse amplitudes*.

* see S. Berg, Nucl. Instr. and Meth. A 570, p.~15, (2007).

Stationary bucket acceleration

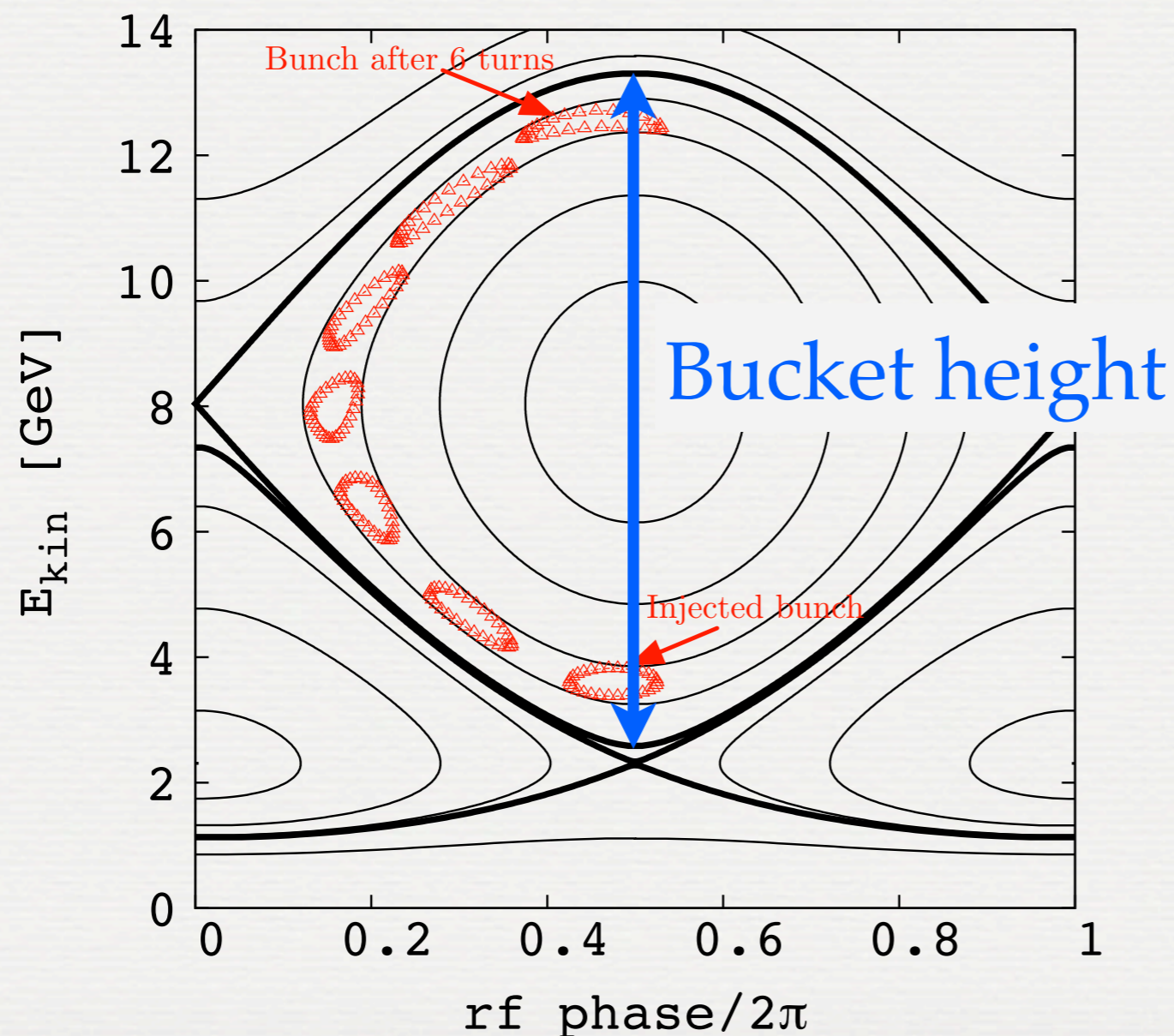
- Principle: use the synchrotron motion to accelerate beam inside a stationary rf bucket.



Longitudinal phase space showing the acceleration of a muon beam (red) inside the above transition stationary rf bucket of a scaling FFAG ring. Hamiltonian contours are shown in black.

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Achievable bucket height

In scaling FFAG, since the momentum compaction α is constant.

\Rightarrow For ultra-relativistic particles, the longitudinal motion is fully determined

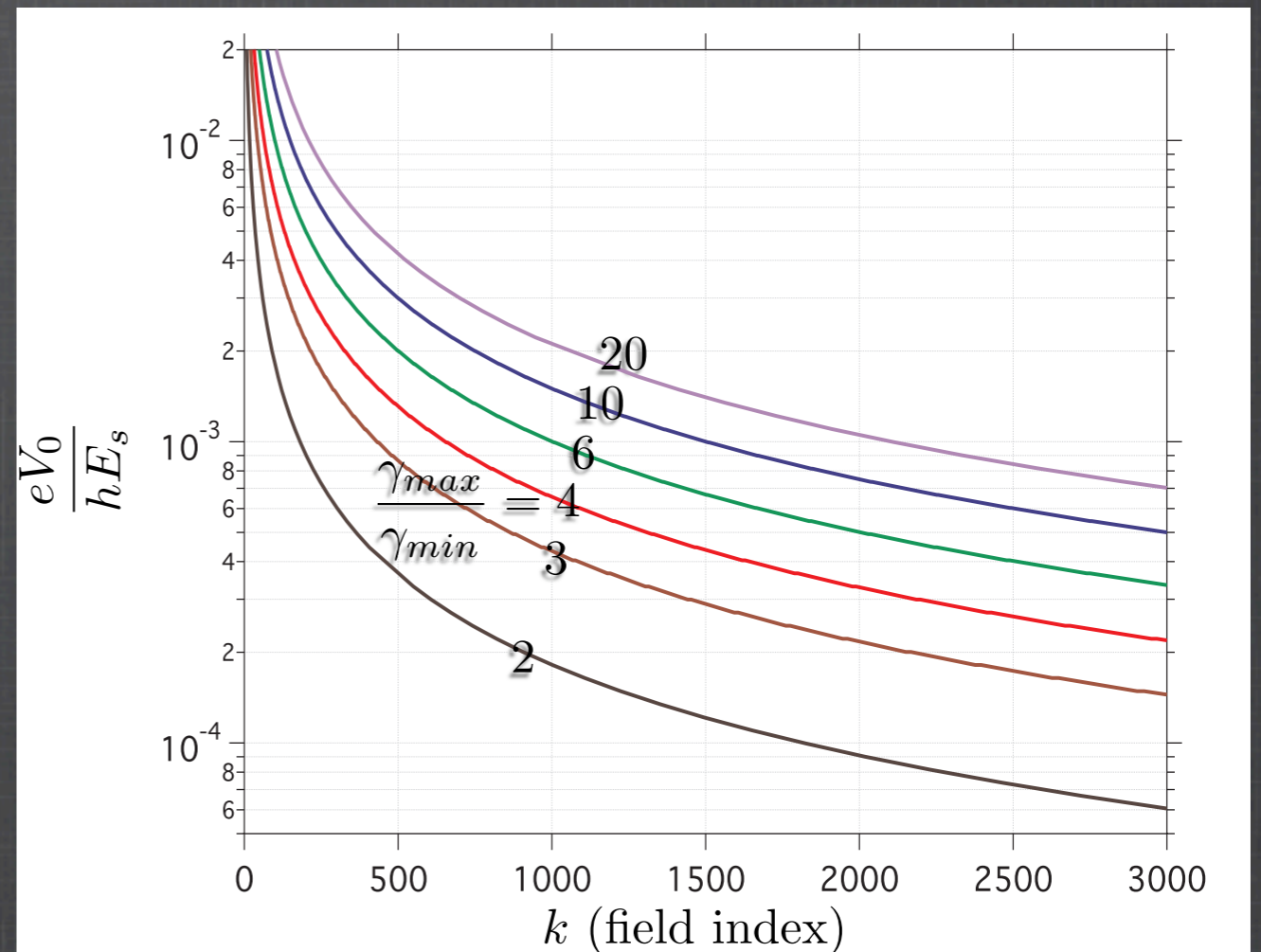
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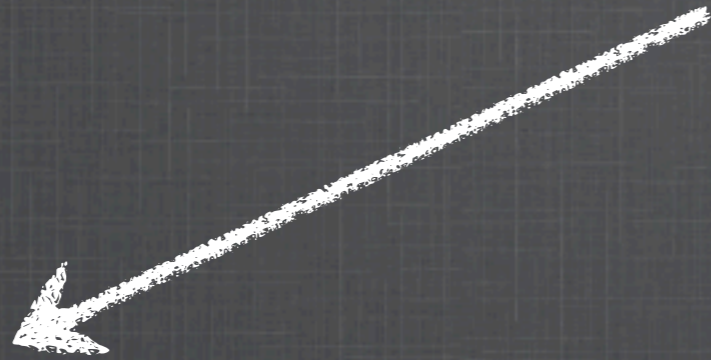


Maximum relative energy increase using acceleration inside the above-transition rf bucket of a scaling FFAG.

$$H(\phi, \gamma; \Theta) \simeq h \left[\frac{\gamma_s}{\alpha + 1} \left(\frac{\gamma}{\gamma_s} \right)^{\alpha+1} - \frac{\gamma}{\gamma_s} + \frac{1}{2\pi} \frac{eV_0}{hE_s} \cos \phi \right].$$

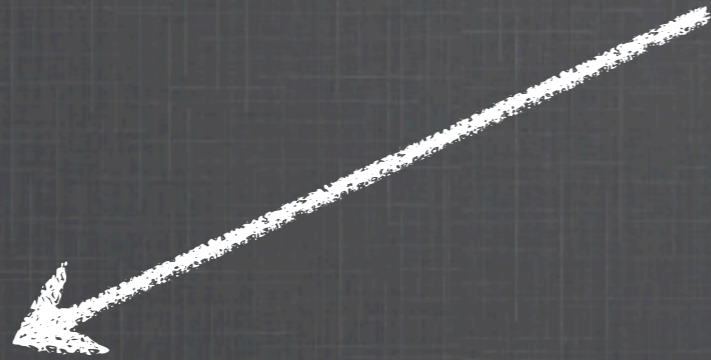
Large k value and transverse beam dynamics

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(1) Strength of the non-linear field components.

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$$\begin{aligned} B(r, \theta) &= B(r_{co}, \theta) \left(\frac{r}{r_{co}} \right)^k \\ &= B(r_{co}, \theta) \left(1 + \frac{k}{r_{co}} x + \frac{k(k-1)}{2!r_{co}^2} x^2 + \frac{k(k-1)(k-2)}{3!r_{co}^3} x^3 + \dots \right) \end{aligned}$$

Large k value and transverse beam dynamics

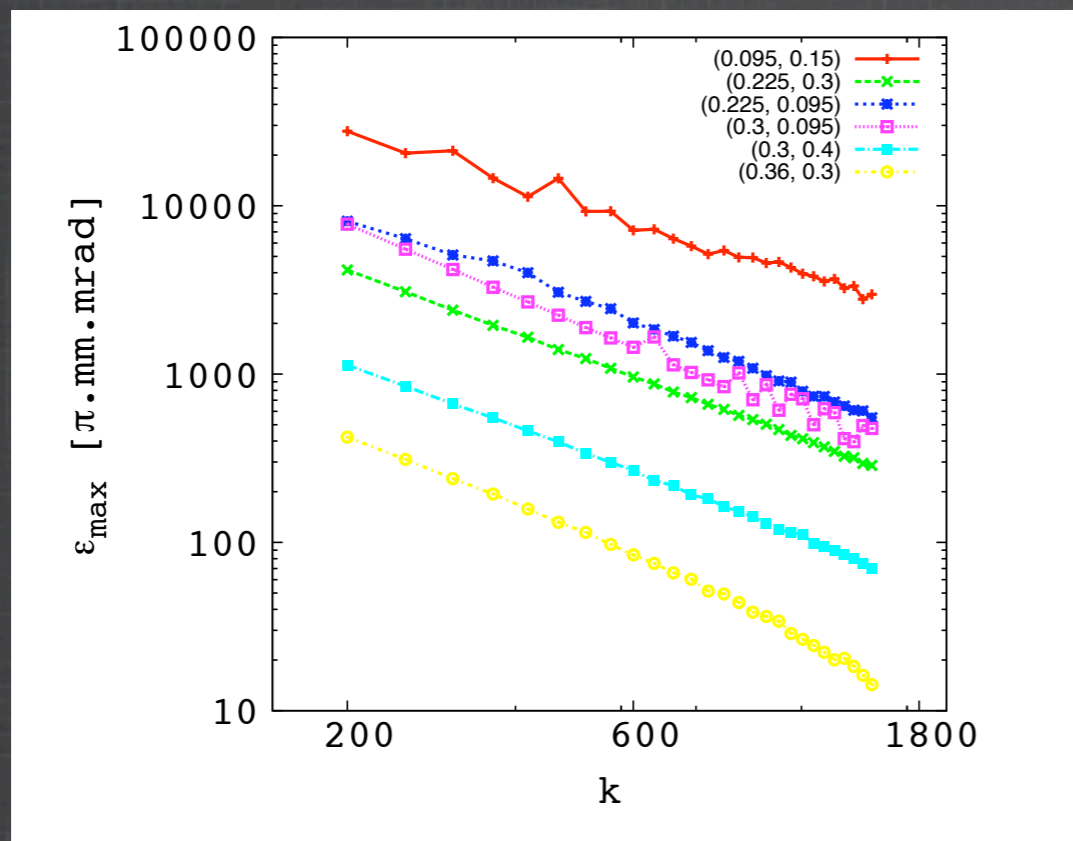


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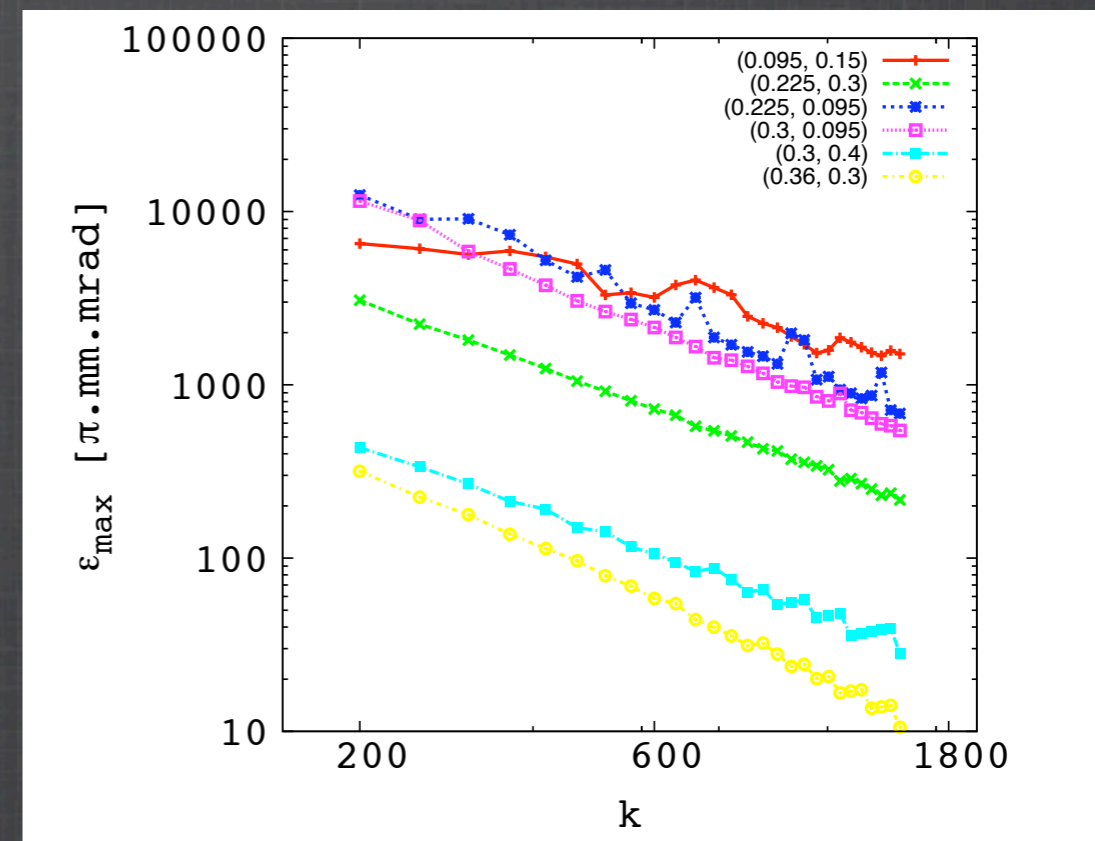
(2) Position of working point with respect to resonances.

(1) Strength of the non-linear field components:

Acceptance study at fixed tunes per cell (Log-Log scale).



Horizontal acceptance



Vertical acceptance

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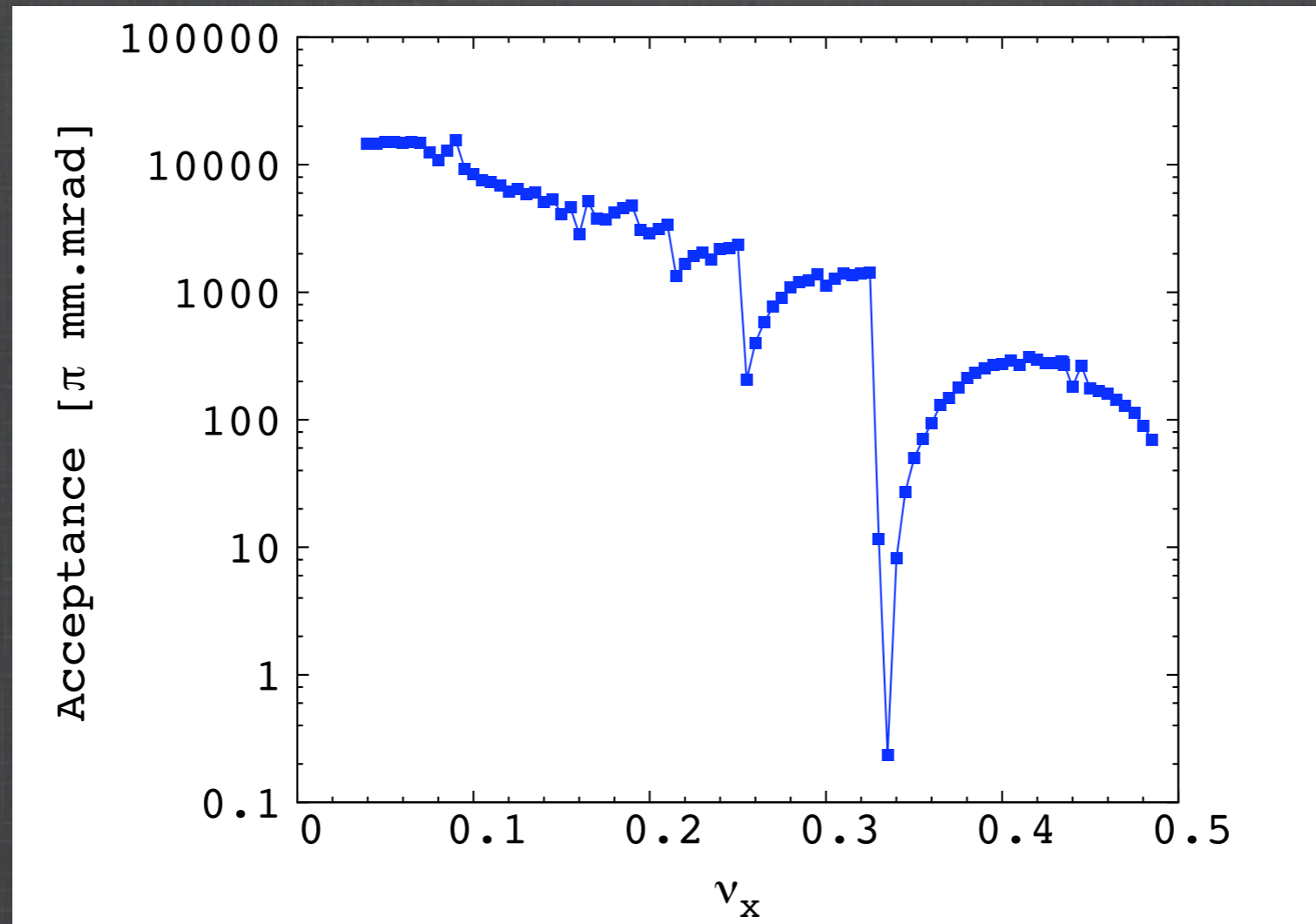
Acceptance study at fixed tunes per cell (Log-Log scale).

When amplitude detuning from the sextupole component dominates transverse acceptance:

$$\propto k^{-3/2}$$

(2) Position of working point with respect to resonances:

Horizontal acceptance vs ν_x , k kept fixed:



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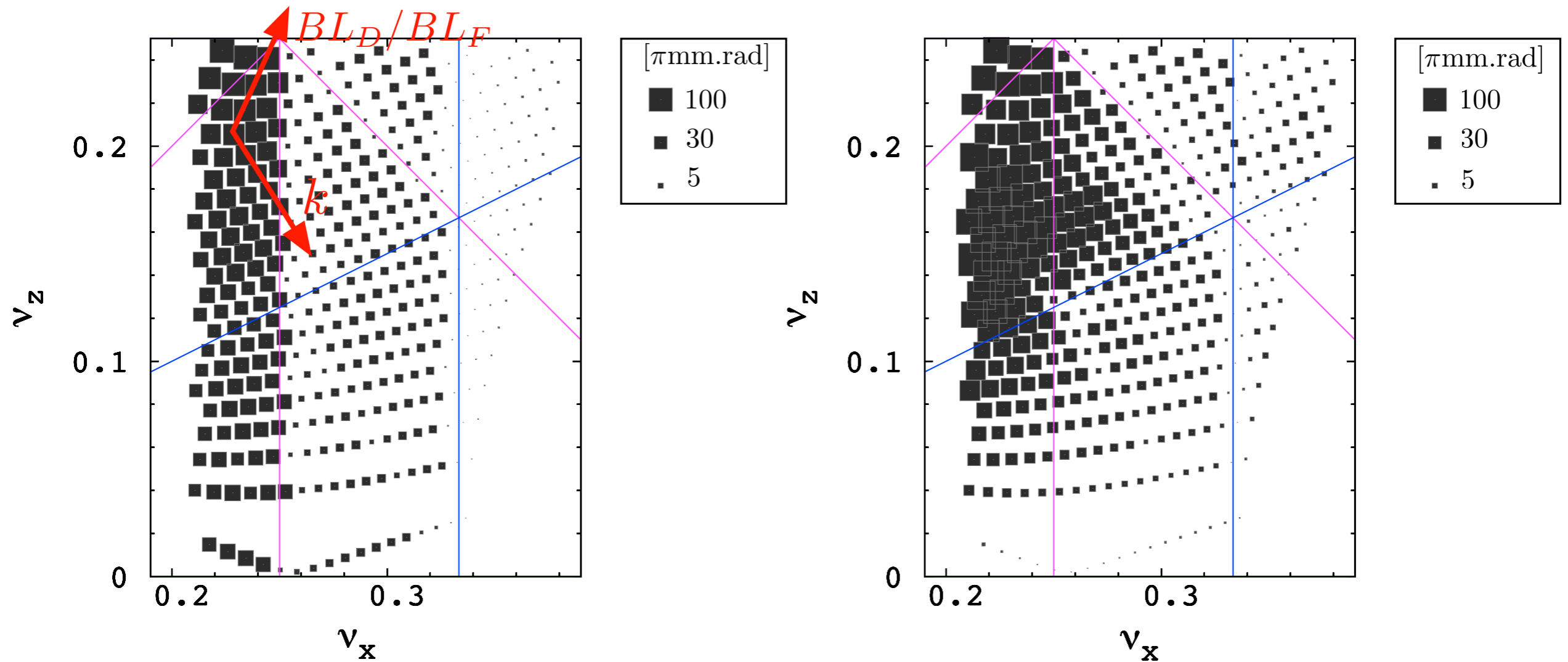
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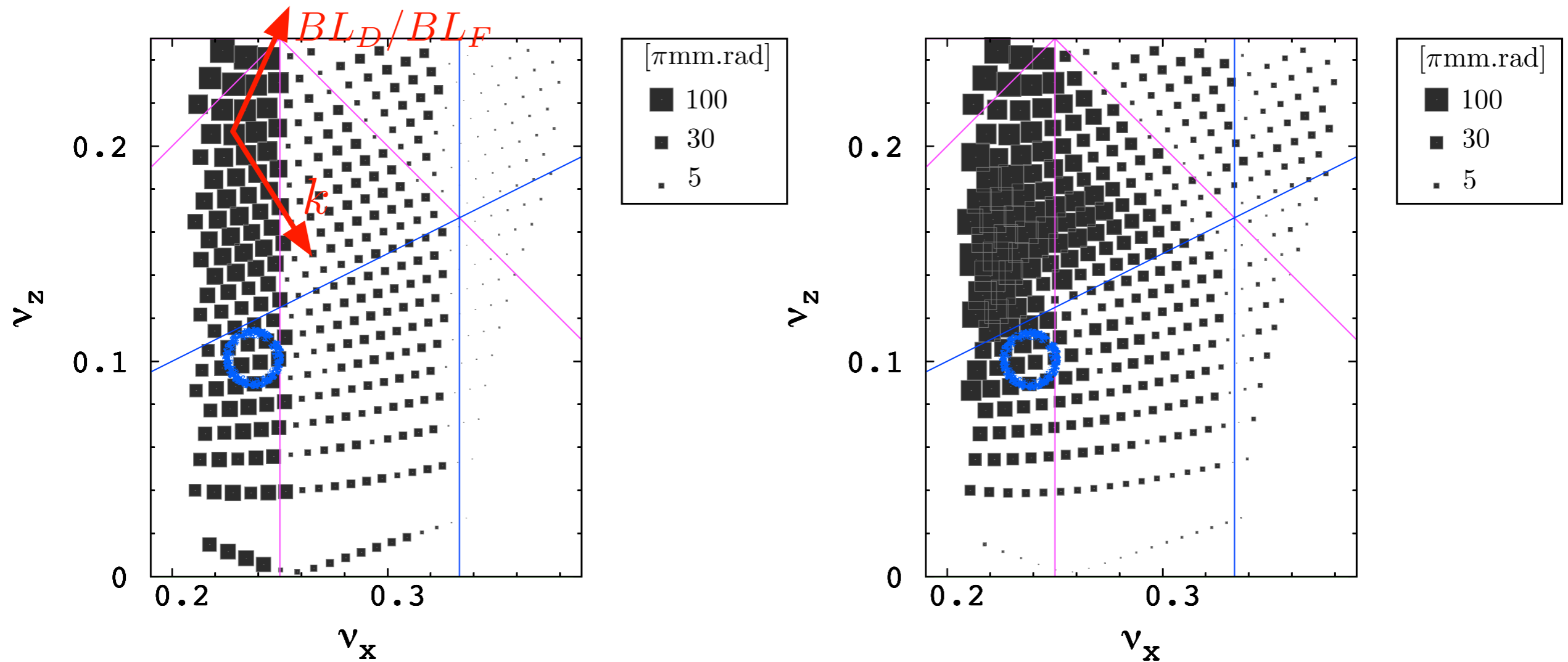
- Depends on the resonance type: sum / difference, order...
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- Very large amplitude motion: limit of validity of the canonical perturbation theory.
- Guiding principle established, but a more precise study requires numerical simulations.

Transverse acceptance: choice of the working point



Looking for a 3.6 to 12.6 GeV muon ring design. Horizontal (left diagram) and vertical (right) acceptances scan using KUT-code. The area of each square is proportional to the normalized acceptance. Normal structure resonances lines, plotted up to the octupole, are superimposed.

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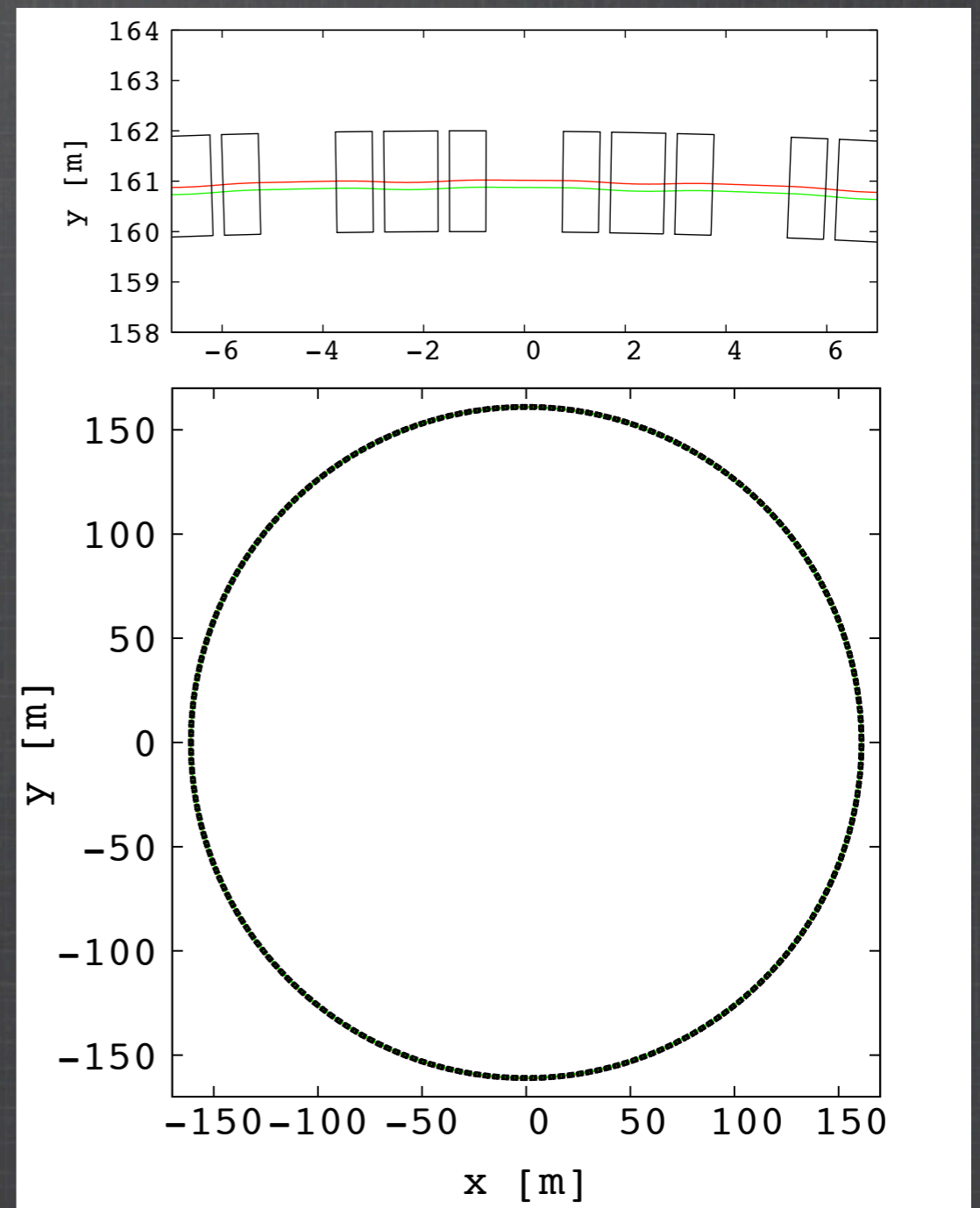


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Example of a 3.6 to 12.6 GeV muon ring

Table 1 - Scaling FFAG muon rings parameters

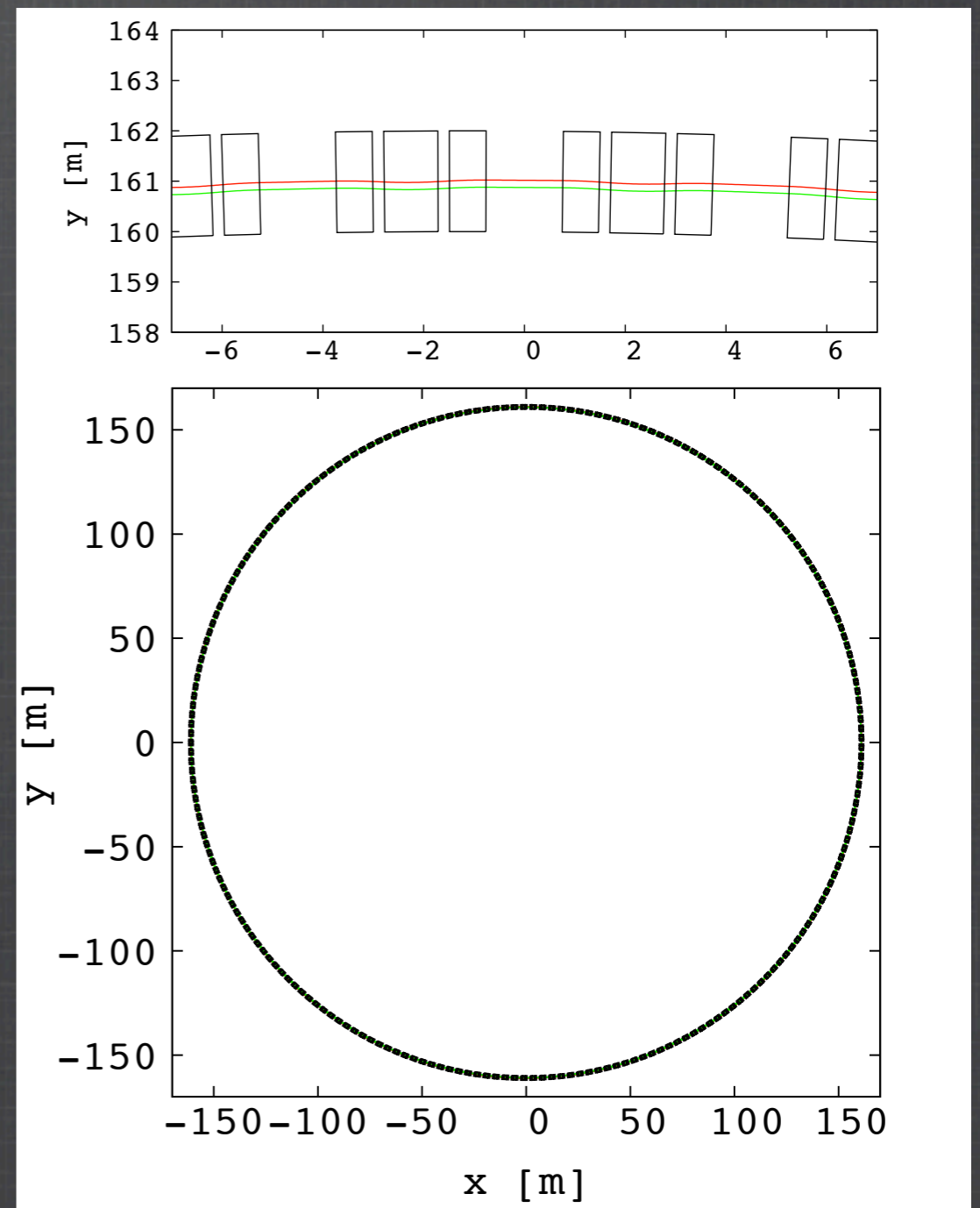
Lattice type	FDF triplet
Injection (kin) energy	3.6 GeV
Extraction energy	12.6 GeV
rf frequency	200 MHz
Mean radius	~ 161 m
Synchronous kinetic energy	8.04 GeV
Harmonic number h	675
Number of cells	225
Field index k	1390
Peak rf voltage (per turn)	1.8 GV
Number of turns	6
B_{max} (@ 12.6 GeV)	3.9 T
Drift length	~ 1.5 m
Horiz. phase adv./cell	85.86 deg.
Vert. phase adv./cell	33.81 deg.
Excursion	14.3 cm



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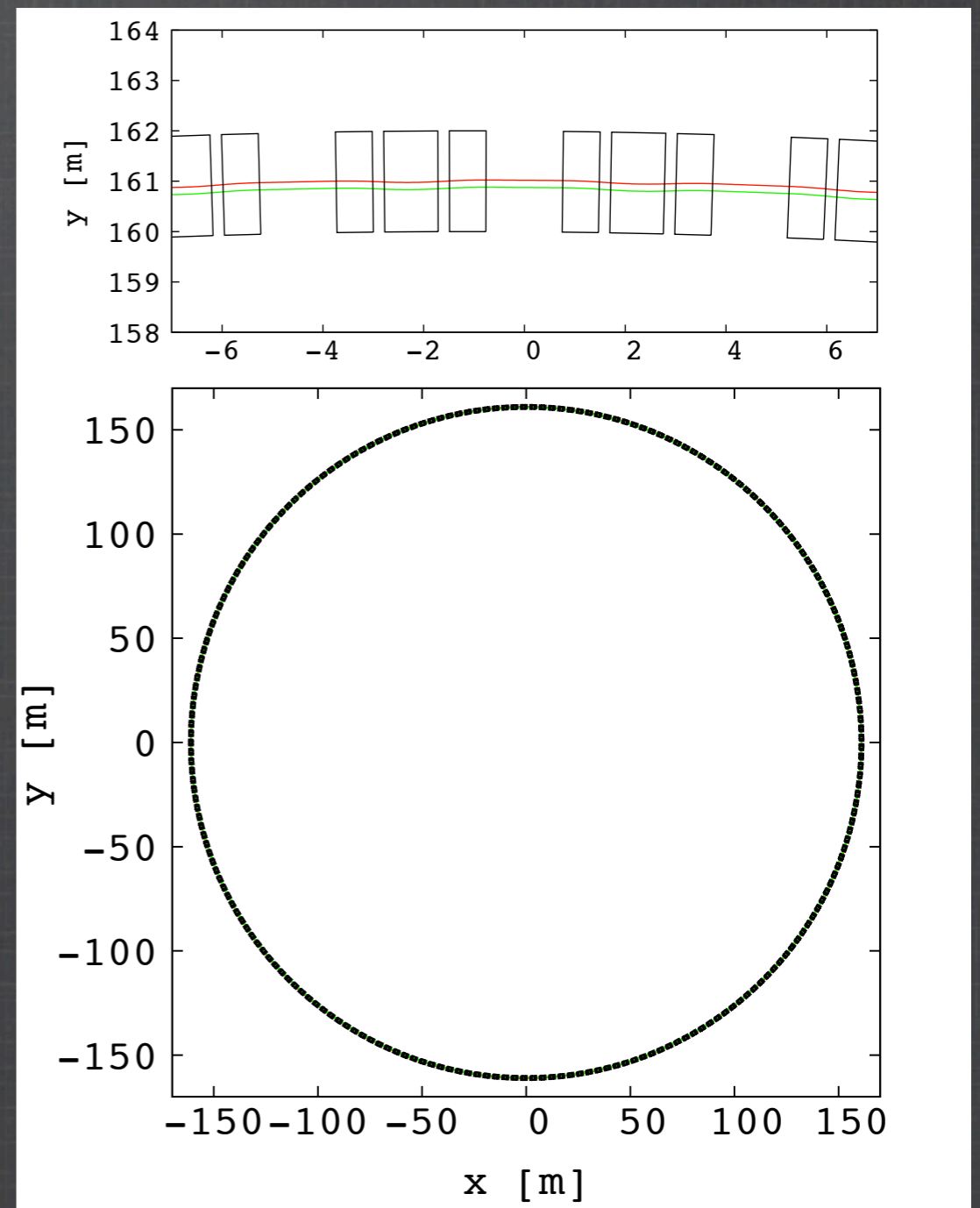
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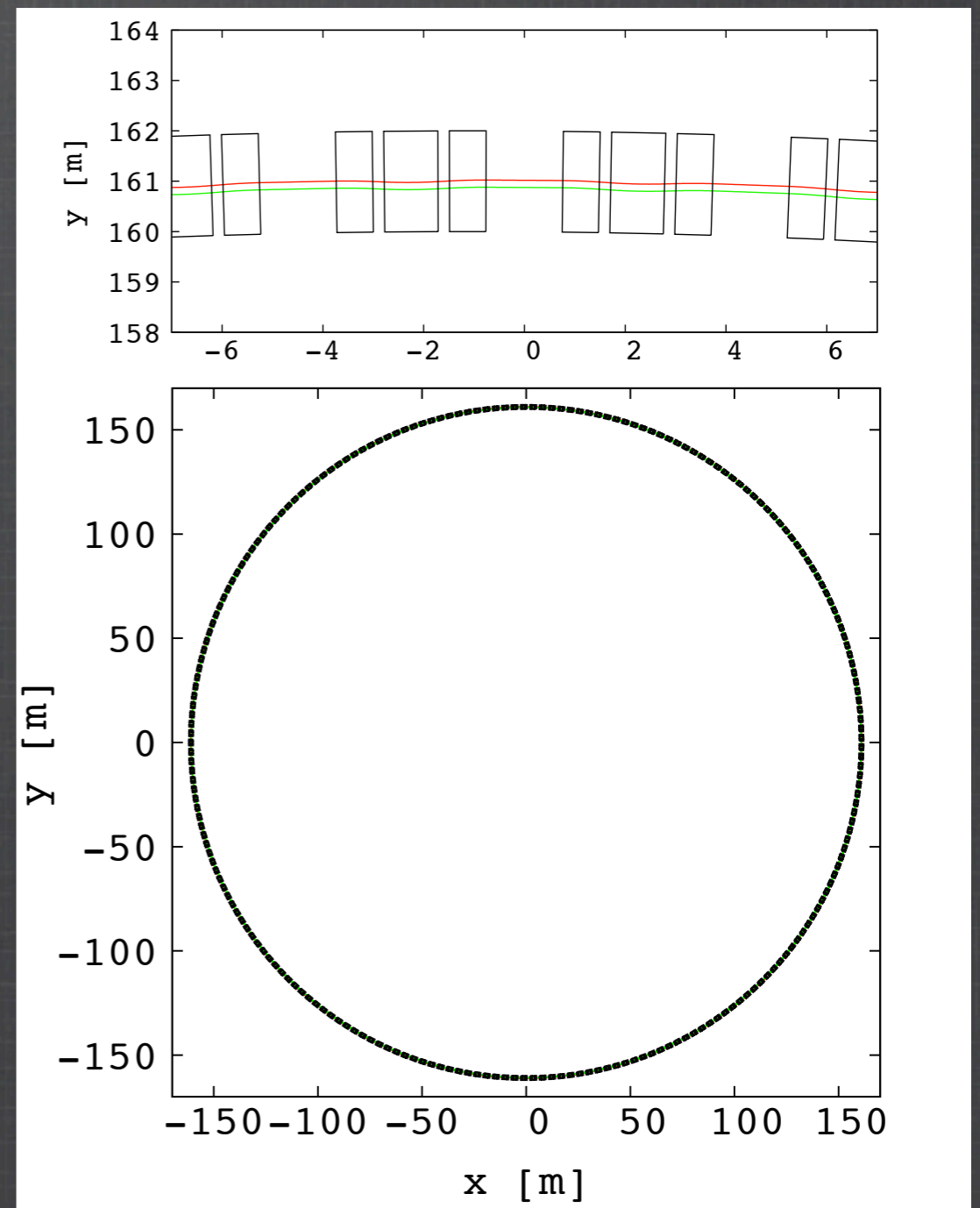
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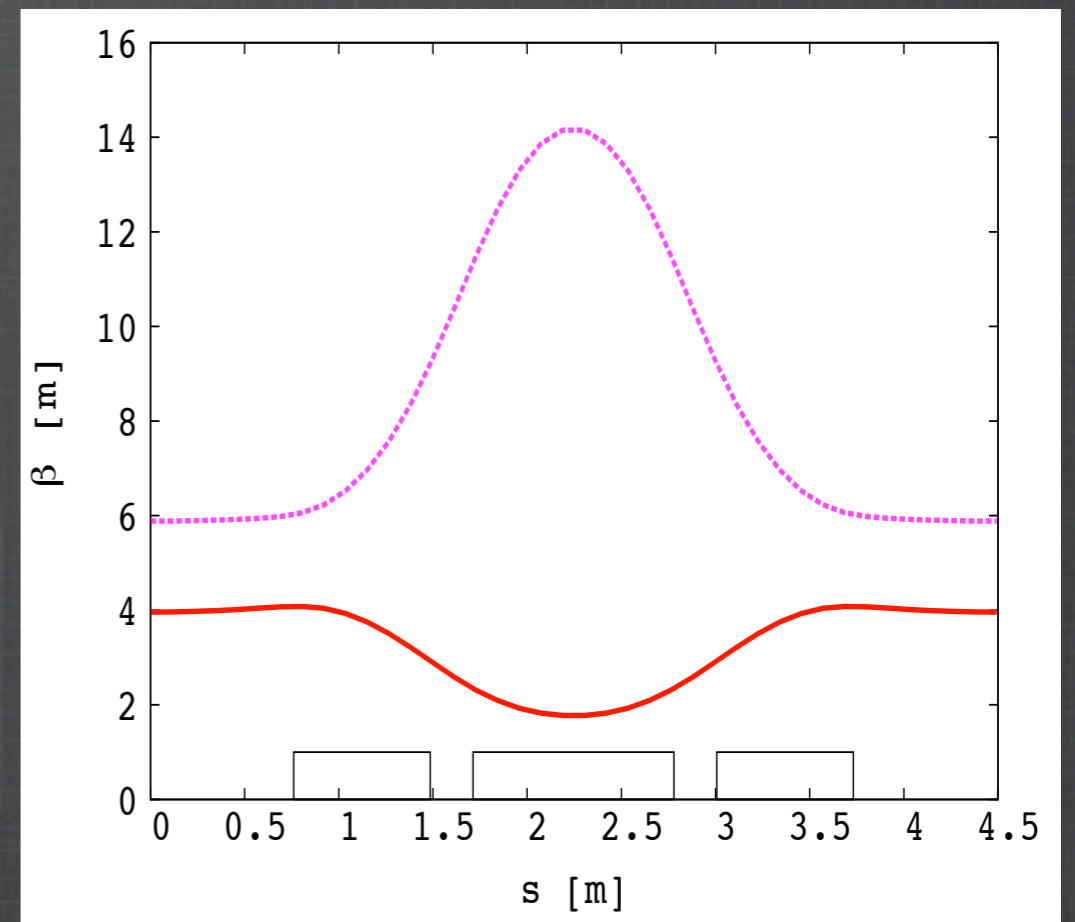
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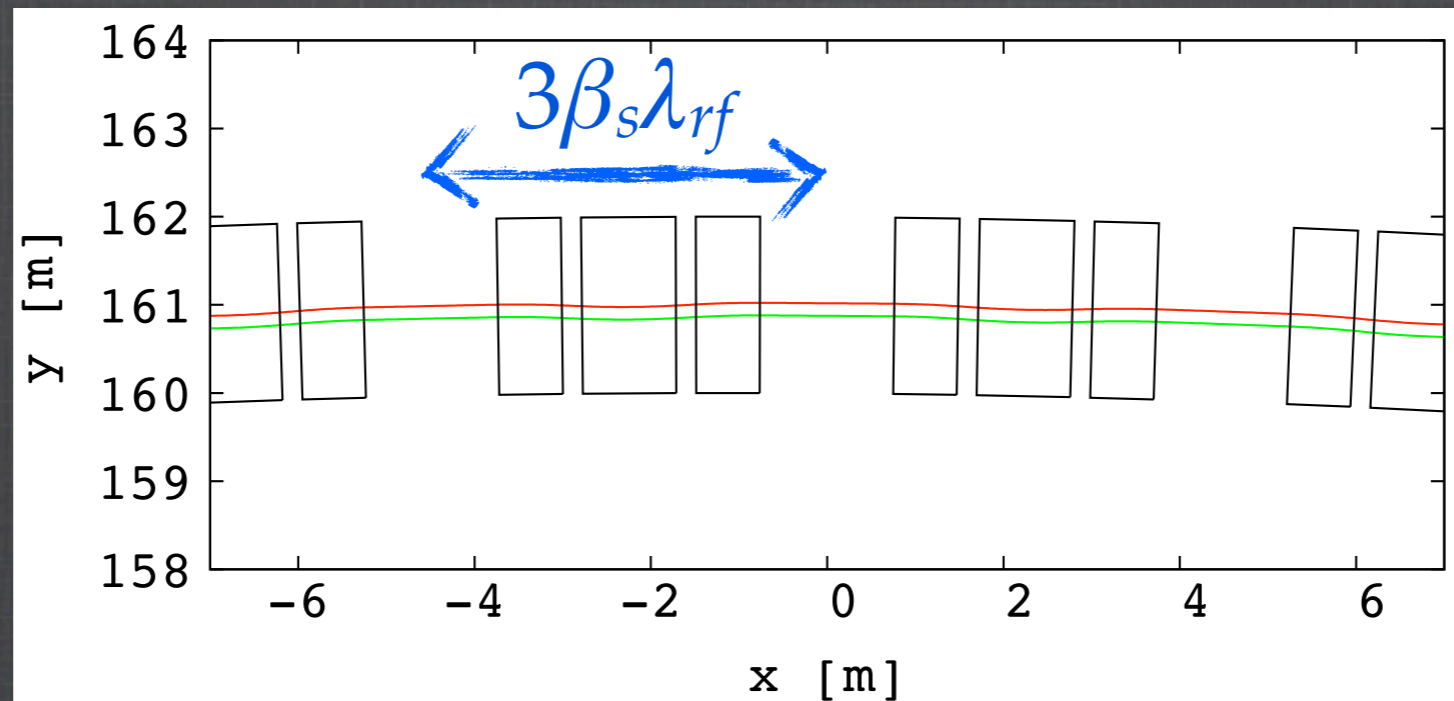
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Horizontal (red) and vertical (purple) beta function at 3.6 GeV, calculated using set-wise tracking in soft-edge field model from small amplitude motion around the closed orbit. Position of the magnets effective field boundaries are shown with rectangles.

Example of a 3.6 to 12.6 GeV muon ring

Simultaneous acceleration of μ^+ and μ^- beams:

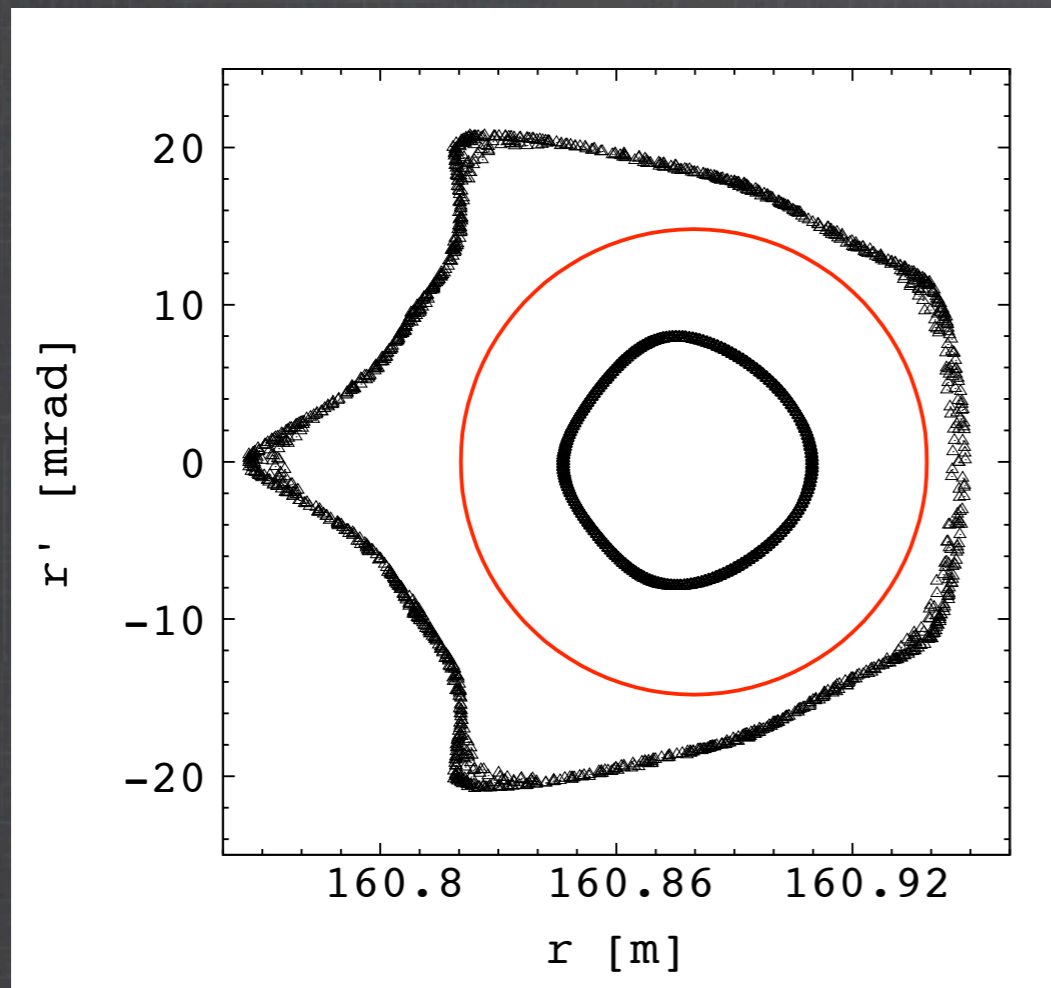


In order to allow the simultaneous acceleration of μ^+ and μ^- beams, the synchronous particle orbit length is adjusted to a multiple of $\frac{1}{2}\beta_s\lambda_{rf}$.

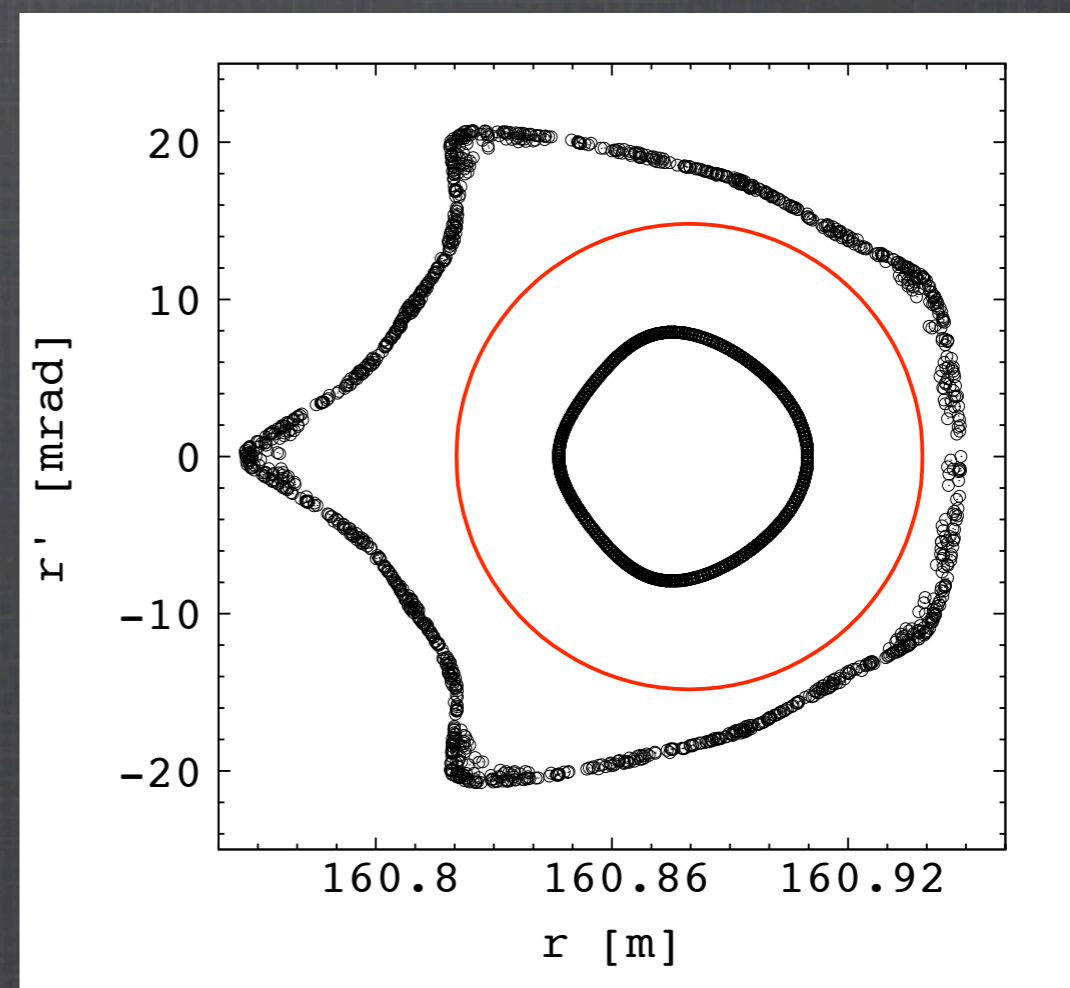
Transverse acceptance at fixed energy

Horizontal acceptance $> 30,000 \pi \cdot \text{mm} \cdot \text{mrad}$ normalized:

KUT-code



'FFAG' procedure of Zgoubi

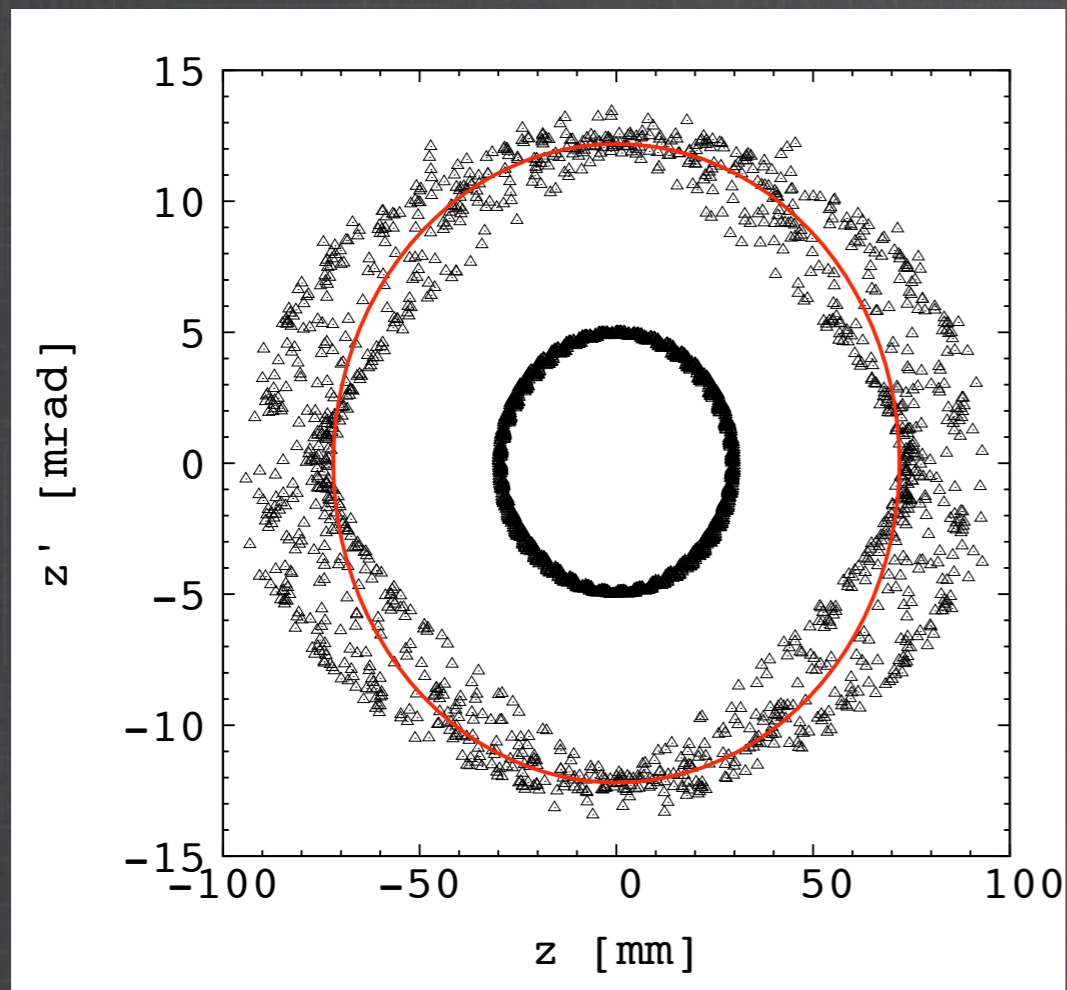


(r, r') plane showing a multi-turn tracking of 2 particles with different initial horizontal amplitudes, with an initial vertical displacement = 1 mm.

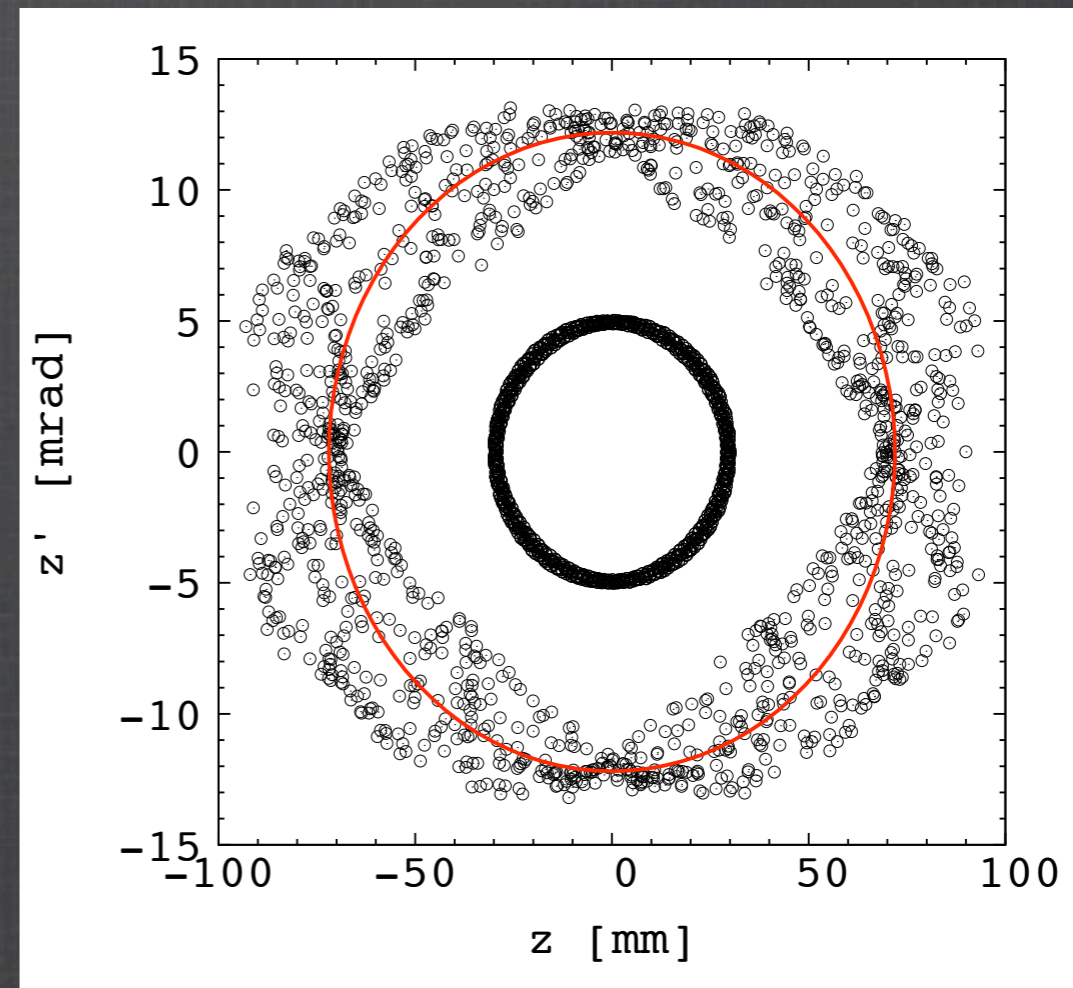
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Vertical acceptance $\sim 30,000 \pi \cdot \text{mm} \cdot \text{rad}$ normalized:

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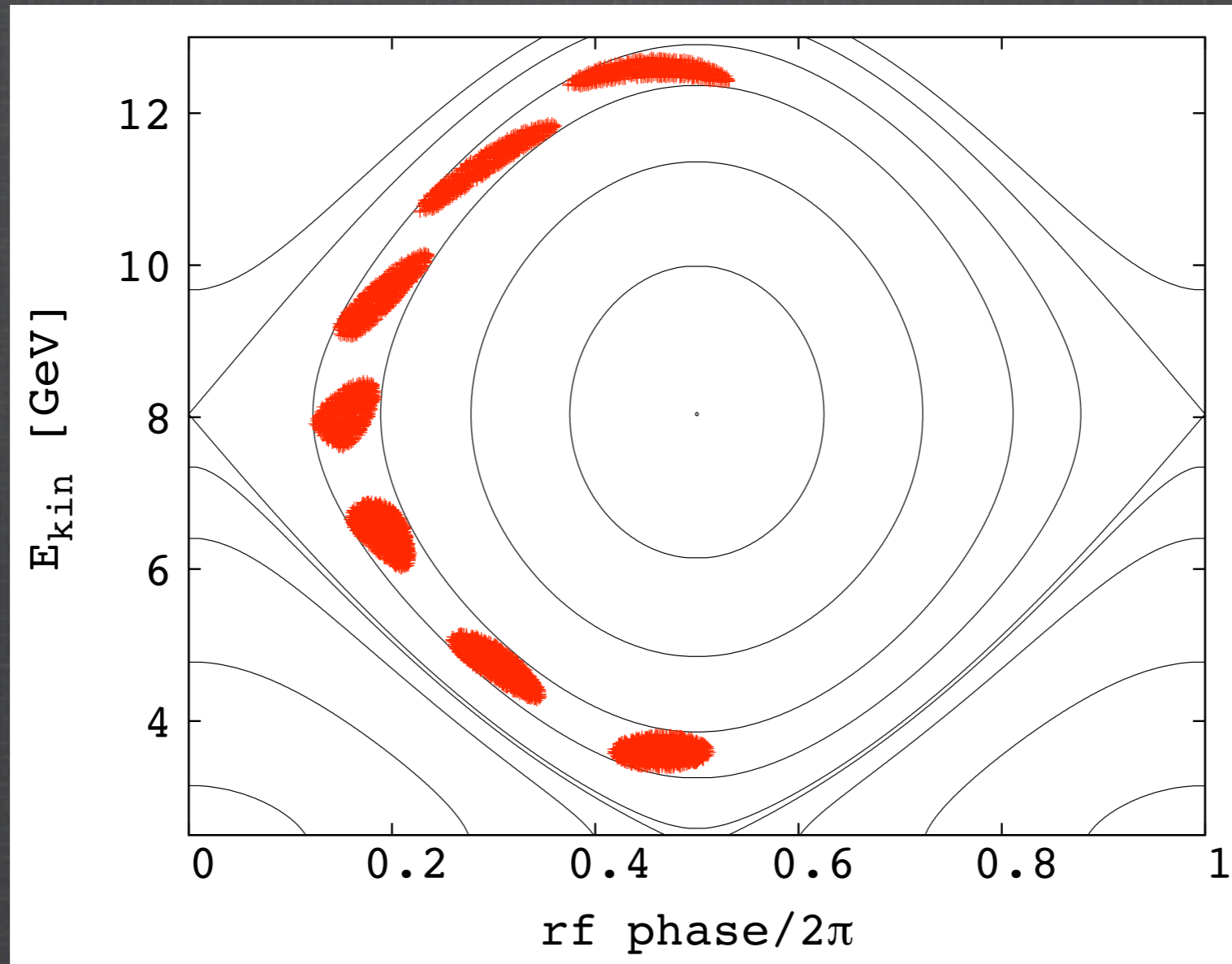


(z, z') plane showing a multi-turn tracking of 2 particles with different initial vertical amplitudes, with an initial horizontal displacement = 1 mm.

Full acceleration cycle - 6D tracking -

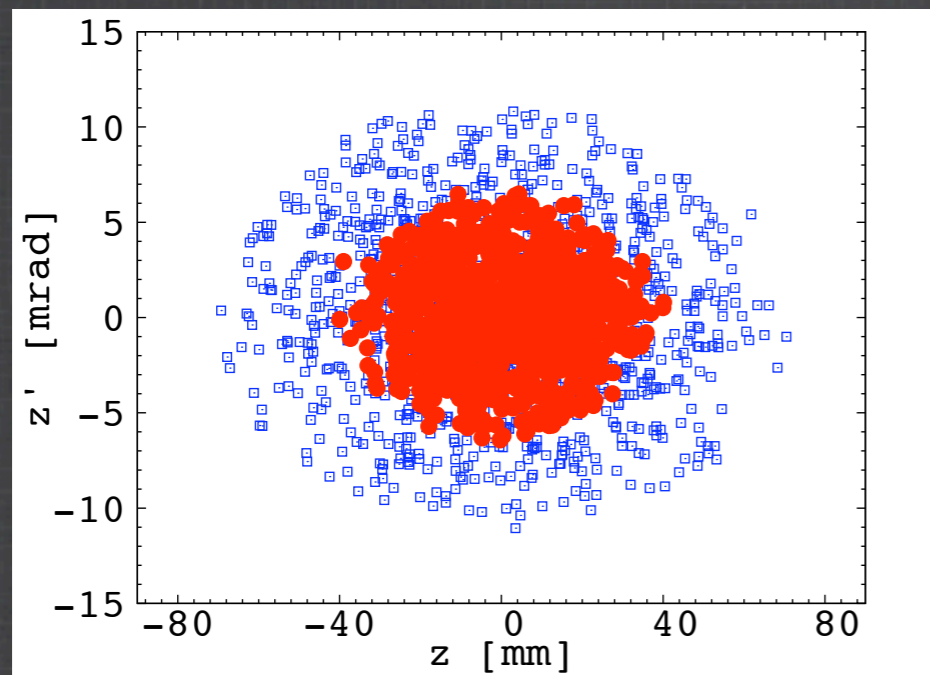
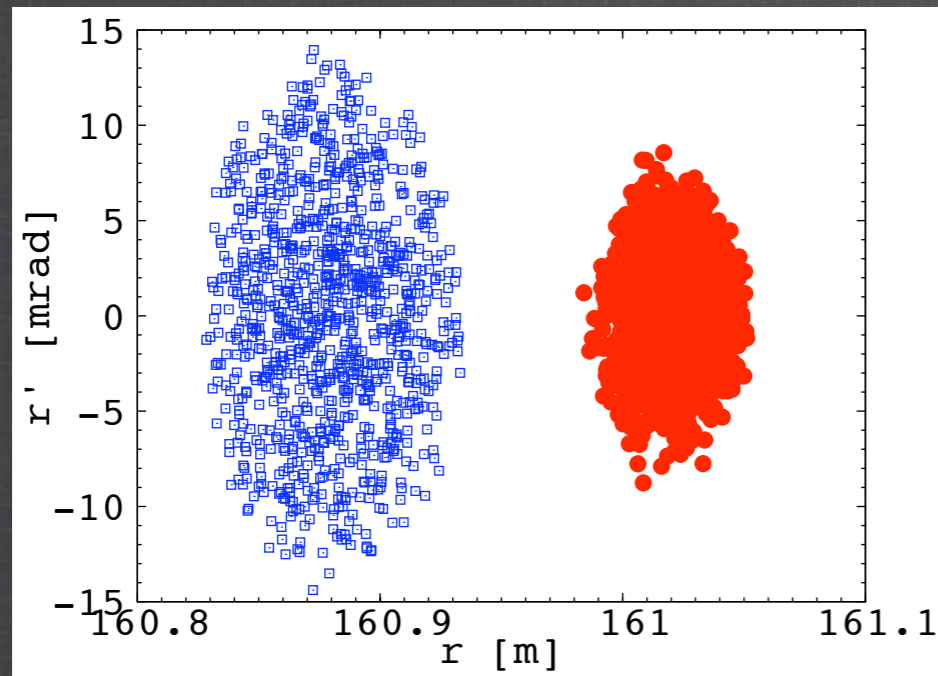
- 1000 particles are uniformly distributed inside a transverse 4D ellipsoid (Waterbag distribution).
- These particles are then independently distributed uniformly inside an ellipse in the longitudinal plane.
- Initial normalized bunch emittances are 30,000 π .mm.mrad in both horizontal and vertical planes and 150 mm in the longitudinal plane.
- RF kicks used to simulate the effect of this rf gaps distributed around the ring

6D tracking results



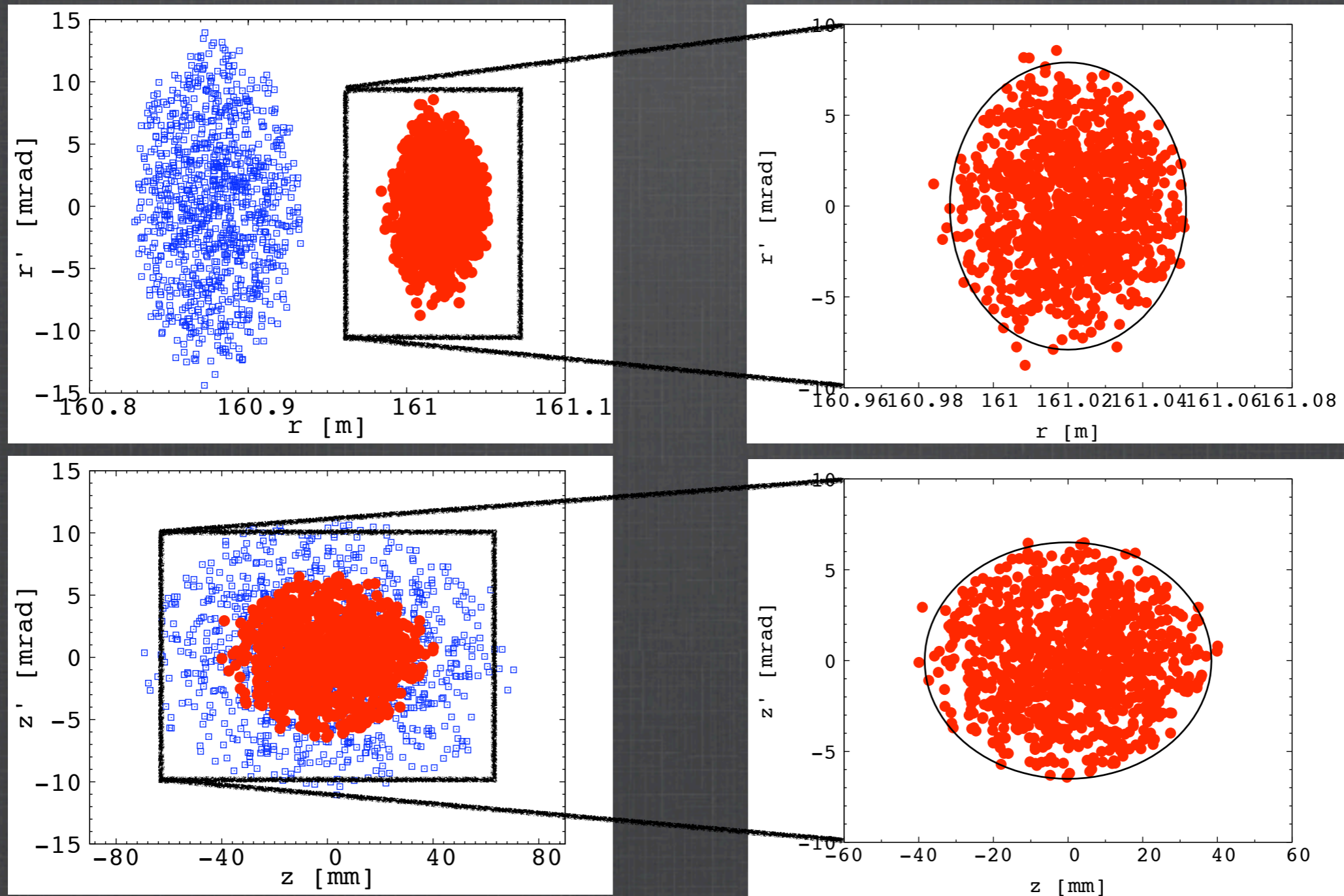
longitudinal phase space plot showing a 6-turn acceleration cycle. Hamiltonian contours are superimposed.

6D tracking results



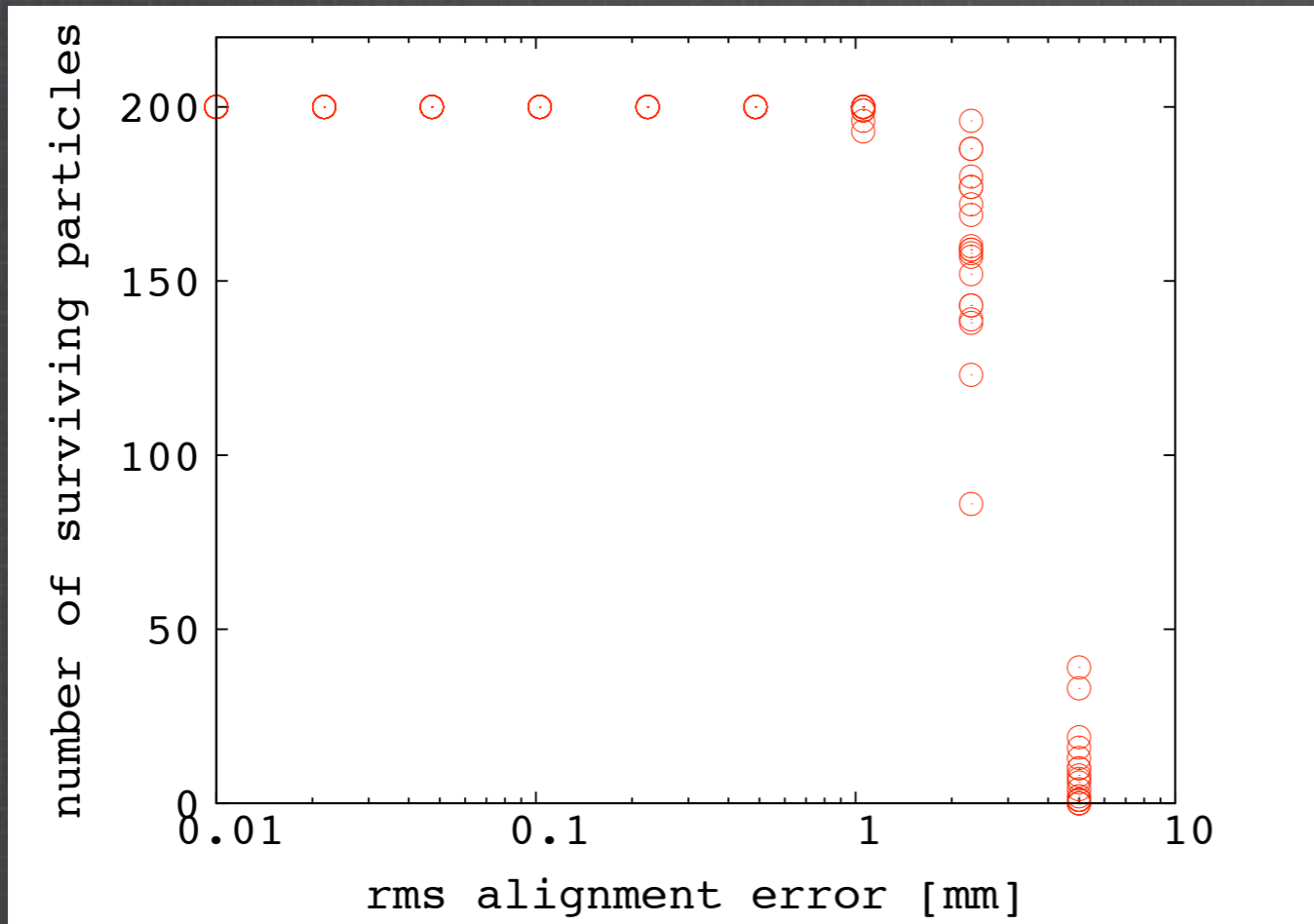
Initial (blue) and final (red) particles distribution in the horizontal (top), and vertical (bottom) phase space.

6D tracking results



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Study with errors



Error in translation: number of surviving particles depending on the rms error. 20 different lattices have been generated and tested for each value of rms error.

Compared to the linear non-scaling FFAG: one order of magnitude better tolerance to errors!

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 - ➔ Scaling FFAG can be used as an injector to the linear non-scaling FFAG.

Summary

Table 4 - Comparison between the different acceleration schemes.

3.6 to 12.6 GeV muons	RLA	SB-FFAG	HNJ-FFAG
Transverse acceptance [π mm.mrad]	30	30	30
Longitudinal acceptance [mm]	150	150	150
Max. B field on closed orbit [T]	4	4	4
Number of pass	4.5	6	8.25
Total path length [km]	2.7	6.0	9.9
Rate of muon decaying	5.5%	12%	19%
Total length of the system [km]	1.8	1.0	1.2
Installed rf peak voltage [GV]	2.*	1.8	2.1
Total available drift space [m]	\sim 200	340	200
Min. accelerating gradient [MV/m]	10.	5.3	10.
“Lattice simplicity”	-	+	-

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Conclusion

📌 Improvement of the baseline using SB acceleration in scaling FFAG:

☑ more turns \Rightarrow cost-efficient use of the rf,

☑ 45% shorter,

☑ 70% more available drift space.

➡ Significant potential of cost reduction!!

Further steps...

SB acceleration in advance scaling FFAGs

Very small ($\rightarrow 0$) momentum compaction while using relatively large k values:

Isochronous zero-chromatic FFAG muon ring?...

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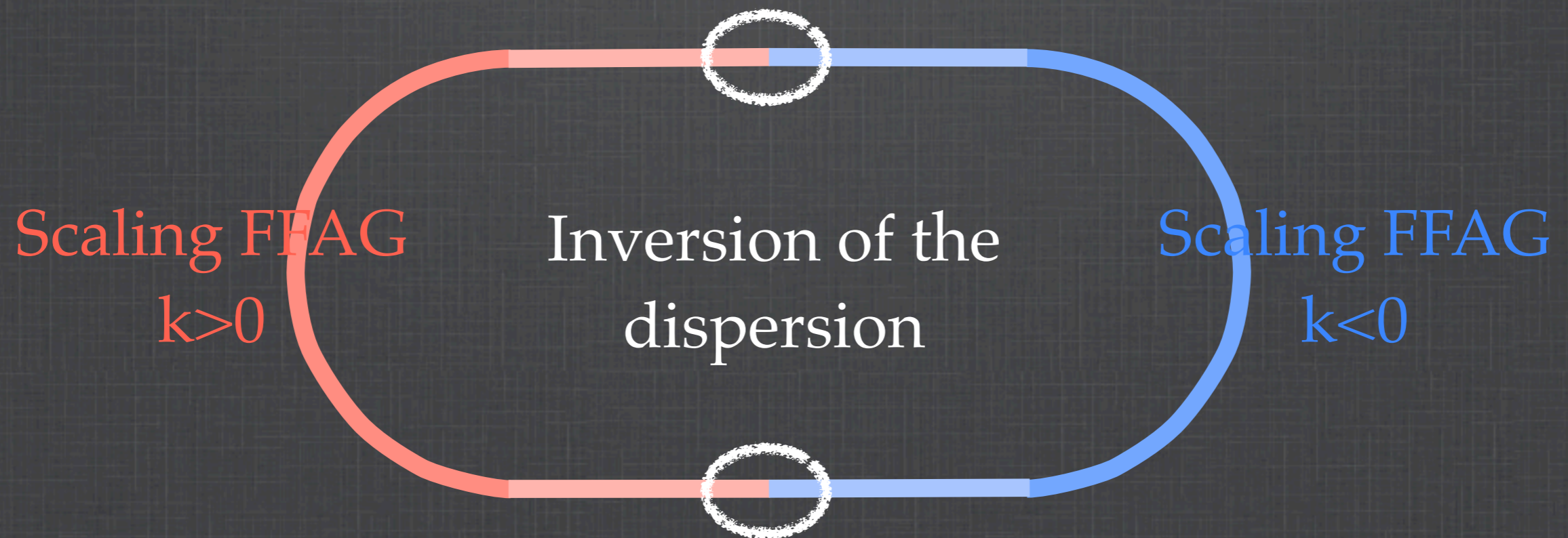


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Thank you for your attention