INTERNATIONAL WORKSHOP ON FFAG ACCELERATORS (FFAG'10)
Kyoto University Research Reactor Institute, October 2010
FFAG SCHOOL

FFAG OPTICS

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FFAG OPTICS

SCALING FFAGs

1. Principles – scaling
2. Transverse optics
3. Longitudinal optics
4. Quasi-scaling FFAGs

NON-SCALING FFAGs

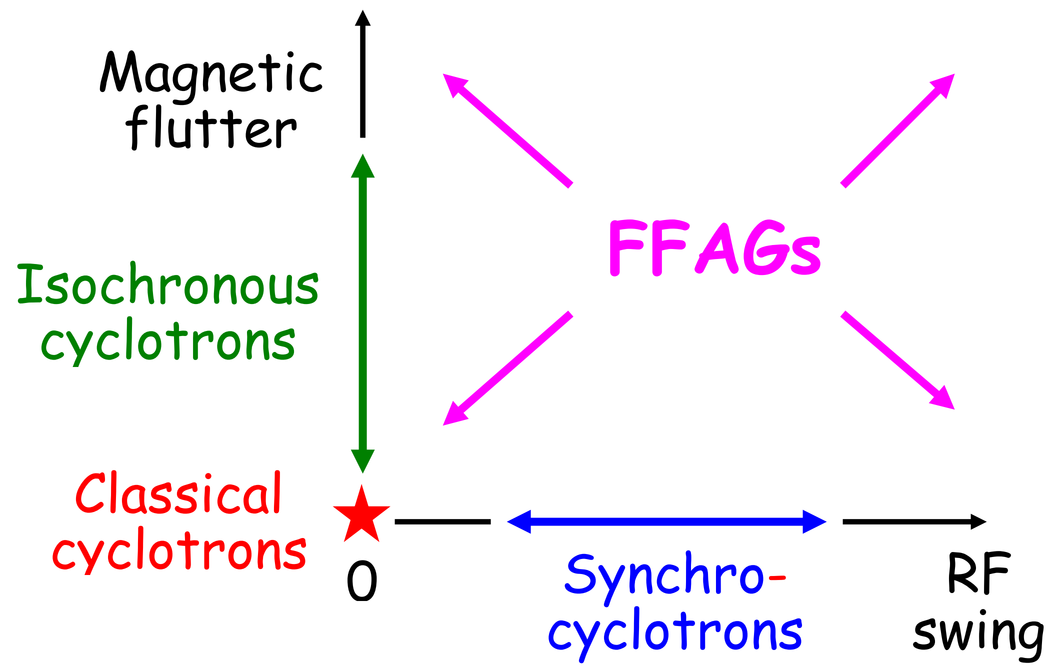
1. Linear Non-Scaling LNS-FFAGs
2. Johnstone Tune-stabilized NLNS-FFAGs
3. Rees Pumplet NLNS-FFAGs

FFAGs – Fixed Field Alternating Gradient accelerators

Fixed Magnetic Field – members of the **CYCLOTRON** family[1]

Magnetic field variation B(θ)	Fixed Frequency (CW beam)	Frequency-modulated (Pulsed beam)
Uniform	Classical	Synchro-
Alternating	Isochronous	FFAG

But FFAG enthusiasts sometimes express an alternative view:
– cyclotrons are just special cases of the FFAG!



1. E.M. McMillan, *Particle Accelerators,* in *Experimental Nuclear Physics*, **III**, 639-786 (1959)

THE FFAG IDEA

- was to introduce alternating “strong” focusing to fixed-field accelerators (enabling higher rep rates and beam currents than in synchrotrons)
- either by alternating +ve and –ve bending magnets with radial edges, creating Alternating Gradient focusing **(Ohkawa, Kolomensky, Symon, 1953-4)**
- or by using spiral sector magnets **(Kerst 1955)** - as later used in cyclotrons.

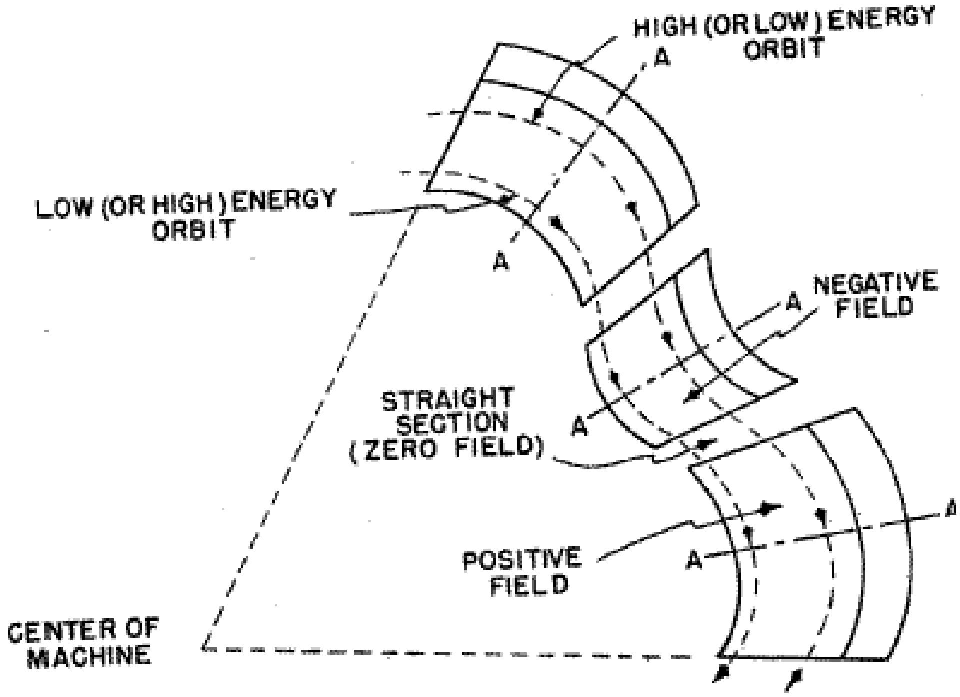


FIG. 2. Plan view of radial-sector magnets.

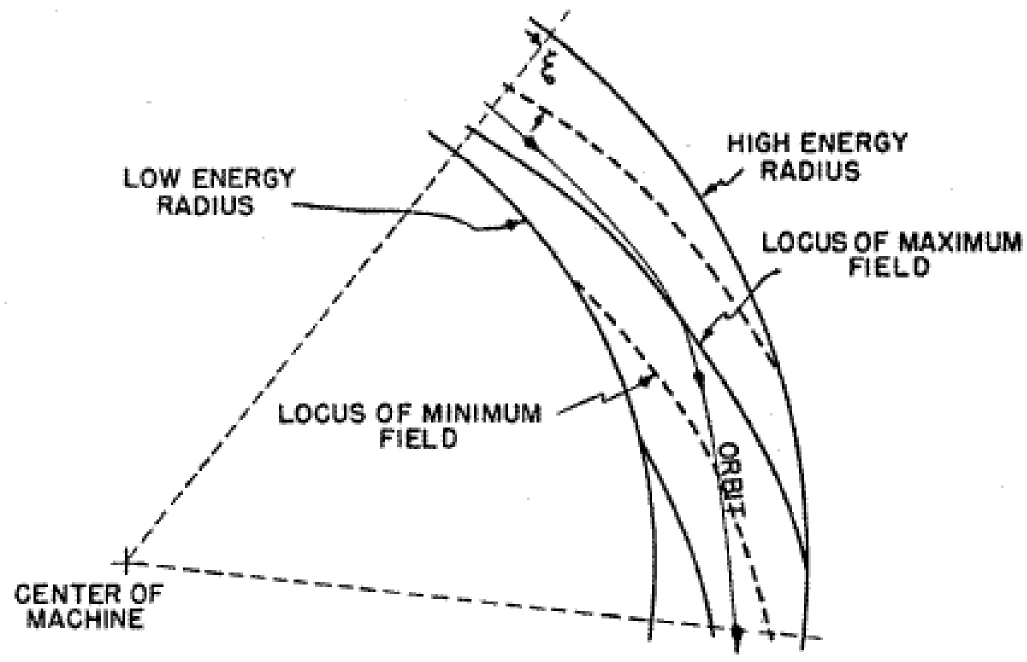


FIG. 3. Spiral-sector configuration.

K.R. Symon, D.W. Kerst, L.W. Jones, L.J. Laslett and K.M. Terwilliger, *Phys. Rev.* **103**, 1837 (1956)

BASIC CHARACTERISTICS OF FFAGs

are determined by their **FIXED MAGNETIC FIELD**

- Spiral orbits
  - needing wider magnets, rf cavities and vacuum chambers (compared to AG synchrotrons)
- Faster rep rates (up to kHz?) limited only by rf capabilities
  - not by magnet power supplies
- Large acceptances
- High beam current

The last 3 factors have fuelled the interest in FFAGs over 50 years!

The most intensive studies were carried out by Symon, Kerst, et al. at the Mid-west Universities Research Association (MURA) in the 1950s and 60s – who adopted the “scaling” principle

- and built several successful electron models.

SCALING DESIGNS

Betatron resonances were a big worry in early days, because of low ΔE/turn:

So "**Scaling**" designs were used, with:

- the same orbit shape at all energies
- the same optics " " " " "
- the <u>**same tunes**</u> " " " " " $\Rightarrow$ no crossing of resonances!

To 1st order, the tunes are given by

$$\nu_r^2 \approx 1 + k \qquad \nu_z^2 \approx -k + F^2(1 + 2\tan^2 \varepsilon)$$

So constant high tune values require:

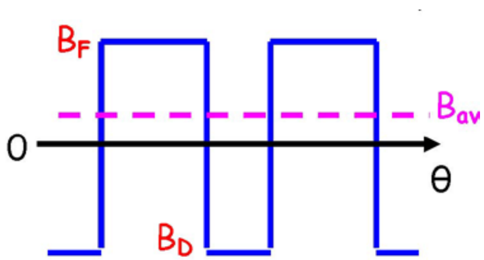
- constant average field index $k(r) \equiv \dfrac{r}{B_{av}} \dfrac{dB_{av}}{dr} >> 0$ where $B_{av} = \langle B(\Theta) \rangle$

  (and hence $B_{av} = B_0 (r/r_0)^k$ and $p = p_0 (r/r_0)^{(k+1)}$)

- constant magnetic flutter $F^2 = \left\langle (B - B_{av})^2 \right\rangle / B_{av}^2$

  i.e. constant profile $B(\Theta)/B_{av}$

  \- maximized for radial sectors by choosing $B_D = -B_F$



- constant spiral angle ε (sector axis follows $R = R_0 e^{\Theta \cot \varepsilon}$)

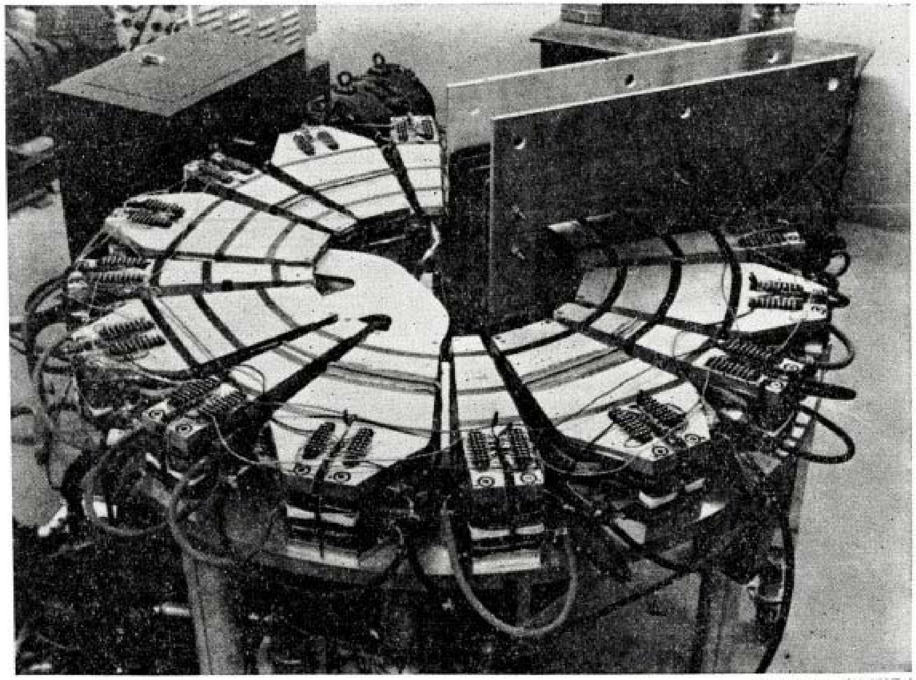
MURA Electron FFAGs

400keV radial sector →

50 MeV radial sector ↘

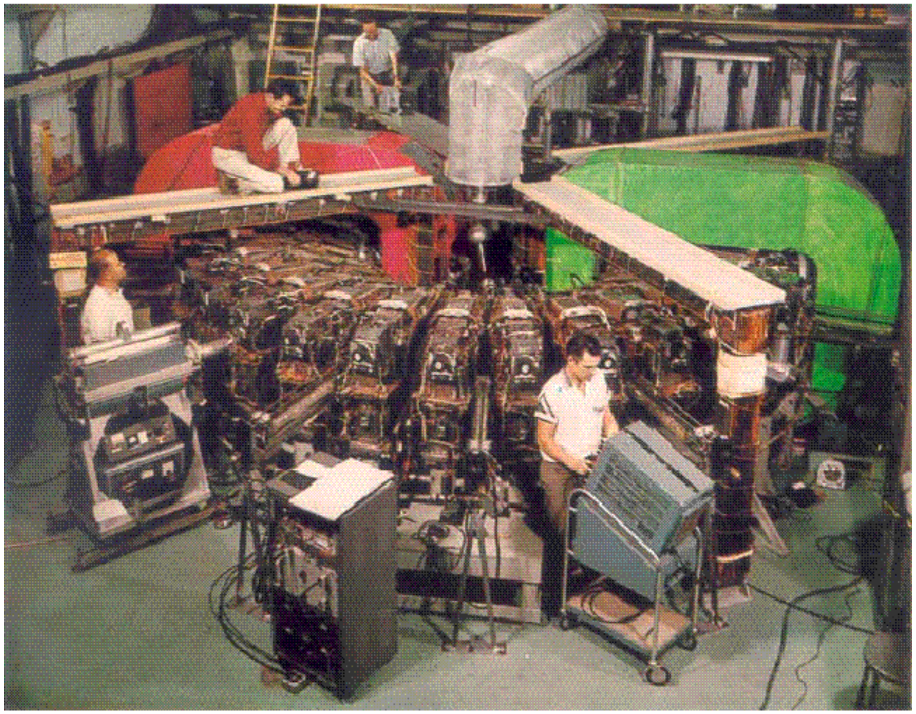
120 keV spiral sector ↓

Courtesy of MURA



Courtesy of MURA

K.R. Symon, Proc PAC03, 452 (2003)



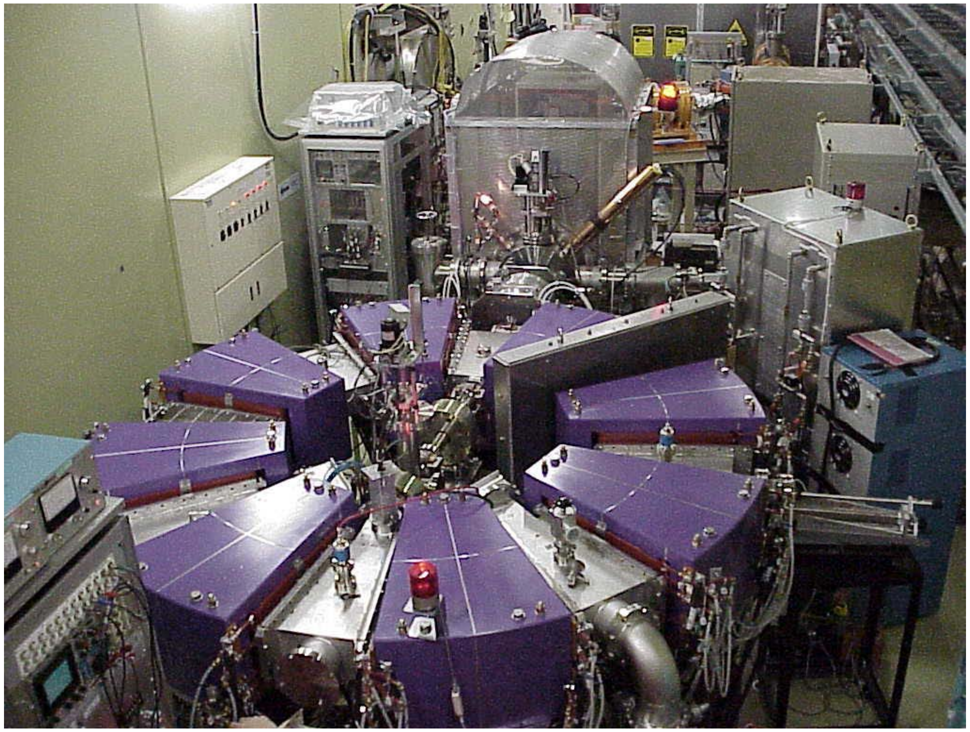
SUBSEQUENT HISTORY

In spite of the success of the **electron models**, none of MURA's proposals for **proton FFAGs** (0.5, 10, 15, and 20 GeV) were funded. Nor were proposals for 1.5-GeV x 4-mA spallation neutron sources by Argonne and Jülich in the 1980s.

The first **proton FFAGs** were **Mori's** at **KEK** (1 MeV 2000, 150 MeV 2003).

Since 2000 an explosion of interest!

- **6 more now operating (for p, e, α)** and **3 more (e) being built**
- **~15 designs** under study:
  - for **protons**, **heavy ions**, **electrons** and **muons**
  - many of **novel "non-scaling" design**
- with **diverse applications:**
  - **cancer therapy**
  - **industrial irradiation**
  - **driving subcritical reactors**
  - **intense many-GeV proton beams**
  - **producing neutrinos.**

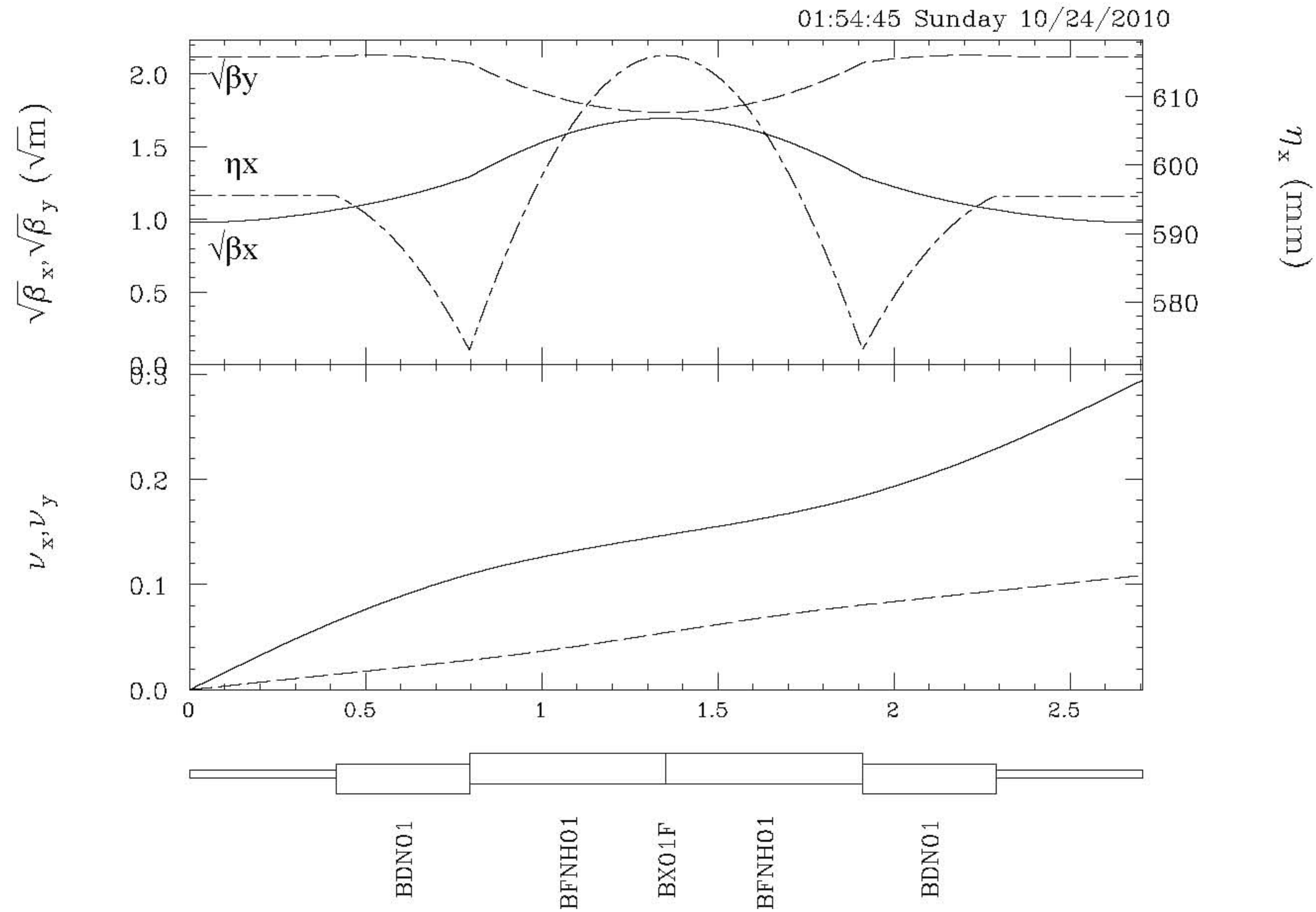


KEK Proof-of-Principle 1-MeV proton FFAG

FFAG Complex at Kyoto University Research Reactor Inst.

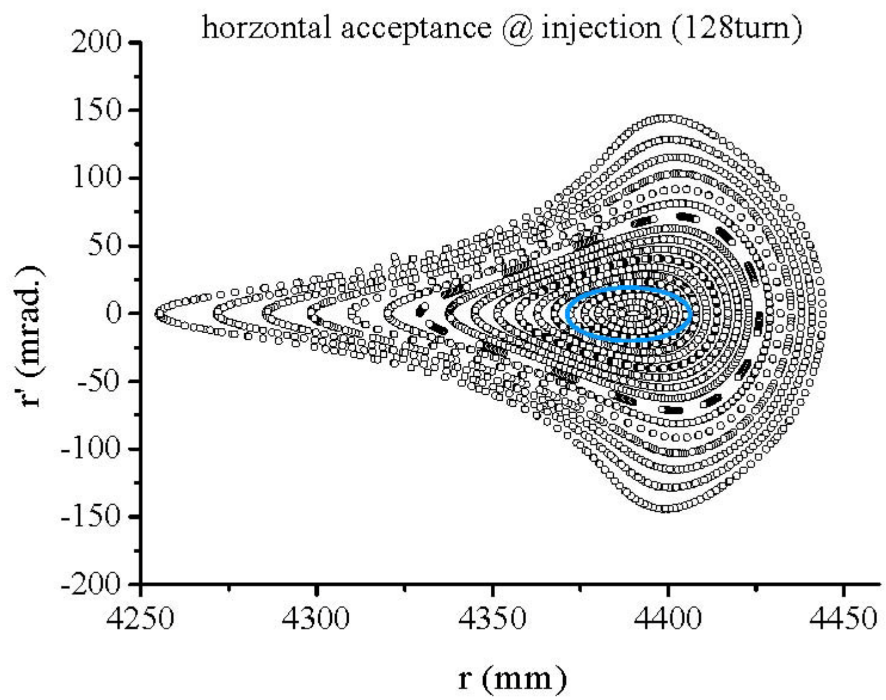
The World's first test of Accelerator-Driven Sub-critical Reactor (ADSR) operation was performed in March 2009.

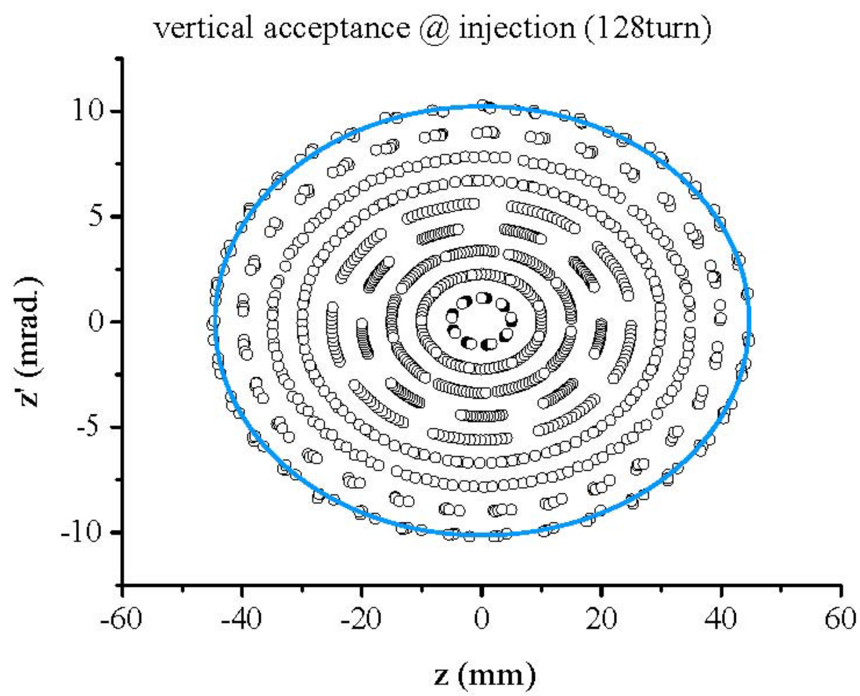
LATTICE FUNCTIONS



The tune advances (denoted by ν_x, ν_y) and beta- and dispersion functions in a DFD cell of the KEK 150-MeV radial-sector FFAG.

ACCEPTANCES





These simulation results for the 150-MeV FFAG demonstrate the large acceptances (areas of stable motion) available in phase space – in both planes.

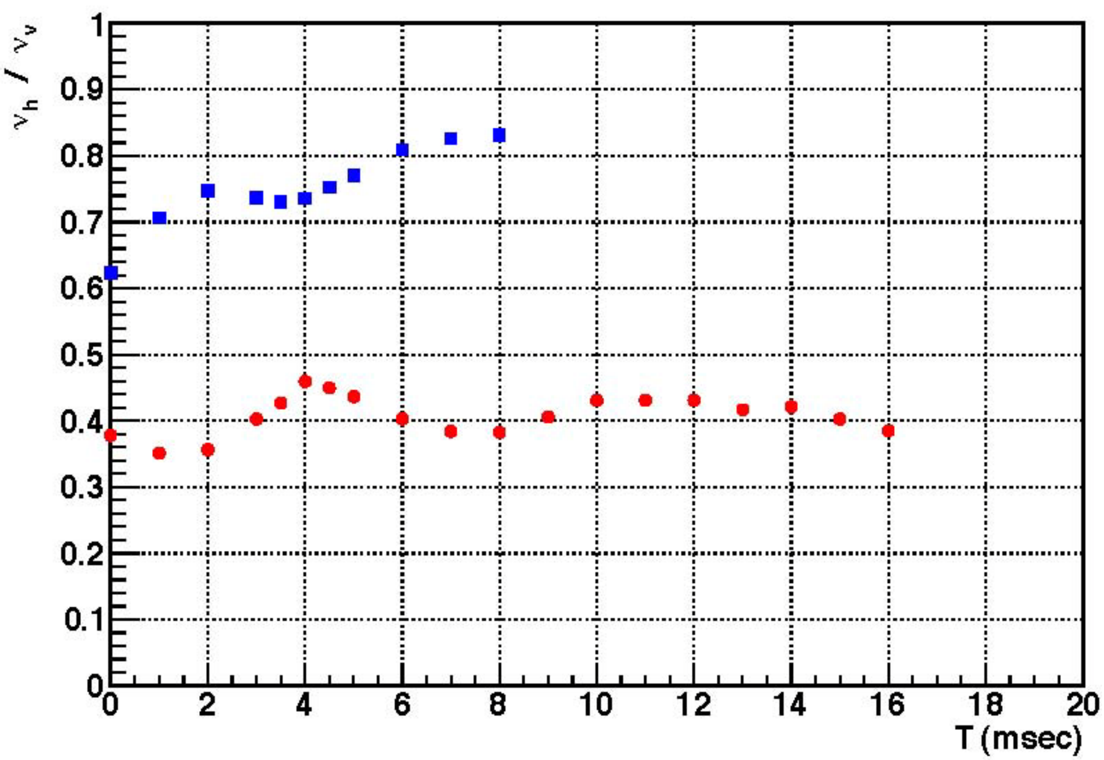
(Note the different displacement scales in the two planes.)

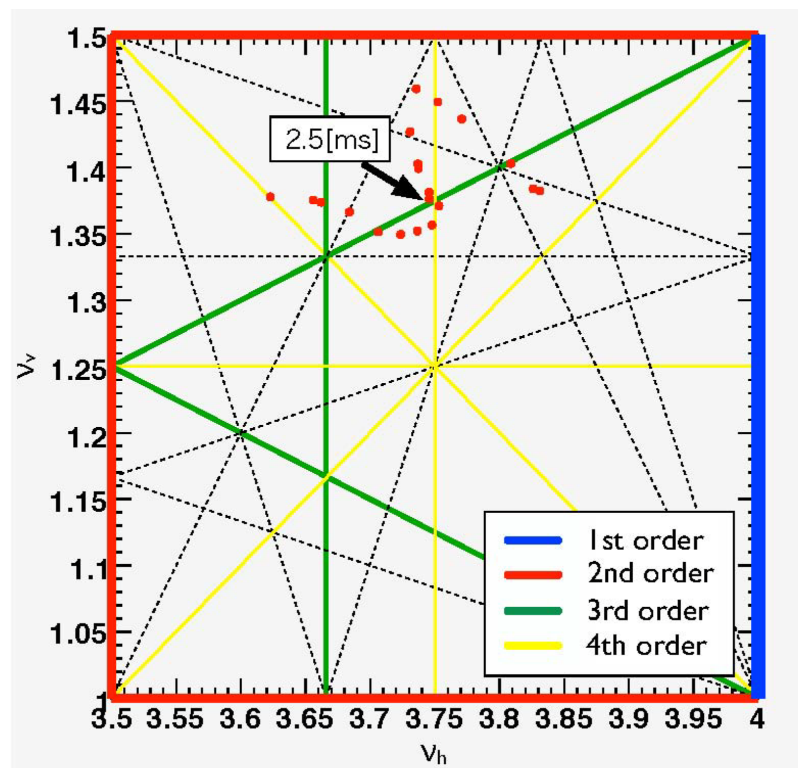
OBSERVED TUNES

In practice, it's difficult to build magnets with identical field index k at all radii, to give $B_z \propto r^k$, especially where $\boldsymbol{k} >> \mathbf{1}$.

The plots below show measured tunes for the KURRI 150-MeV FFAG, for which $k = 7.6$. (The left plot shows only the decimal part of the tune.)

The tune diagram shows that 1st and 2nd order resonances are avoided, but that two 3rd-order resonances are crossed.





RECAP – OFF-MOMENTUM ORBIT PARAMETERS

Momentum dispersion:

$$\eta_x(s) \equiv \frac{x(s)}{\delta p / p_0}.$$

Momentum compaction:

$$\alpha \equiv \frac{\delta C / C}{\delta p / p} = \frac{\overline{\eta}_x}{R} = \frac{1}{\gamma_t^2}$$

Slip factor:

$$\eta \equiv \frac{\delta\omega / \omega}{\delta p / p} = \frac{1}{\gamma^2} - \alpha = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}$$

Transition energy:

$$\gamma_t = \frac{1}{\sqrt{\alpha}} = \sqrt{\frac{R}{\overline{\eta}_x}}.$$

LONGITUDINAL PARAMETERS FOR SCALING FFAGS

In a **scaling FFAG**, like a synchrocyclotron, the field B is constant, but instead of being uniform, it increases with radius:

$$\boldsymbol{B} \propto \boldsymbol{r}^{\boldsymbol{k}} \text{ and } \boldsymbol{p} \propto \boldsymbol{r}^{\boldsymbol{k+1}} \quad (k >> 1).$$

As the momentum $p = qBr$, the momentum compaction equation gives:

$$\frac{1}{\gamma_t^2} = \alpha \equiv \frac{\delta C / C}{\delta p / p} = \frac{1}{k+1}$$

and $\gamma_t = \sqrt{(k+1)}$.

As in AG synchrotrons, the transition energy γ_t may lie within the acceleration range, and depending whether γ </> γ_t, the slip factor

$$\eta = \gamma^{-2} - (k+1)^{-1} >/< 0,$$

and the synchronous phase ϕ_s </> $\pi/2$.

The orbital and rf frequencies at first rise with energy, reaching a peak at transition, γ_t, but then fall away.

PHASE STABILITY - THE SYNCHRONOUS PHASE

The condition for synchronism between rf and revolution frequency in a magnetic field B: $\boxed{\omega_{rf}(t) = hqB/m(t)}$

can only be satisfied for ions at the particular "synchronous phase" ϕ_s for which the energy gain/turn exactly matches the frequency change.

Noting that:
$$\frac{d\omega}{\omega} = \frac{dB}{B} - \frac{dm}{m} = \left(\frac{k}{k+1}\right)\frac{dp}{p} - \frac{dE}{E} = \left[\left(\frac{k}{k+1}\right)\frac{1}{\beta^2} - 1\right]\frac{dE}{E},$$

we see that, on average, the energy gain per unit path length is:
$$\frac{qV_0 \sin\phi_s}{2\pi r} = \frac{dE}{ds} = \frac{dE}{v\,dt} = \frac{E}{\omega}\left(\frac{k/(k+1)}{\beta^2} - 1\right)^{-1}\frac{d\omega}{v\,dt}.$$

[Here V_0 denotes the total voltage/turn, and $\phi = 0$ where $V = 0$.]

The synchronous phase is therefore defined by:
$$\boxed{qV_0 \sin\phi_s = -\frac{2\pi E}{\omega^2}\left(\frac{k/(k+1)}{\beta^2} - 1\right)^{-1}\frac{d\omega}{dt}.}$$

But do ions at neighbouring phases oscillate stably about ϕ_s - or not?

Veksler and McMillan showed that only certain ranges of ϕ_s are stable.

LONGITUDINAL EQUATIONS OF MOTION

For particles differing from the synchronous one by ΔT, Δp, $\Delta\omega$, etc., the rate of change of the phase:

$$\frac{d\phi}{dt} = \frac{\Delta\phi}{\tau} = \frac{-h\Delta\omega.\tau}{\tau} = -h\eta\omega_0\frac{\Delta p}{p_0} = h\eta\omega_0\frac{\Delta T}{mv^2} = \frac{-h\eta}{m_0R^2\gamma}\left(\frac{\Delta T}{\omega_0}\right).$$

Comparing the energy gain/turn $\frac{2\pi}{\omega}\frac{dT}{dt} = qV_0\sin\phi$ with that for the synchronous particle, we also find:

$$\frac{d}{dt}\left(\frac{\Delta T}{\omega_0}\right) = \frac{qV_0}{2\pi}(\sin\phi - \sin\phi_s).$$

Noting that the "angular momentum" coordinate canonically conjugate to the phase angle is $W \equiv \frac{\Delta T}{\omega_0} = R\Delta p$ we see that the equations above have the form of Hamilton's equations: $\frac{d\phi}{dt} = \frac{\partial H}{\partial W}$ and $\frac{dW}{dt} = -\frac{\partial H}{\partial\phi}$.

with the Hamiltonian:

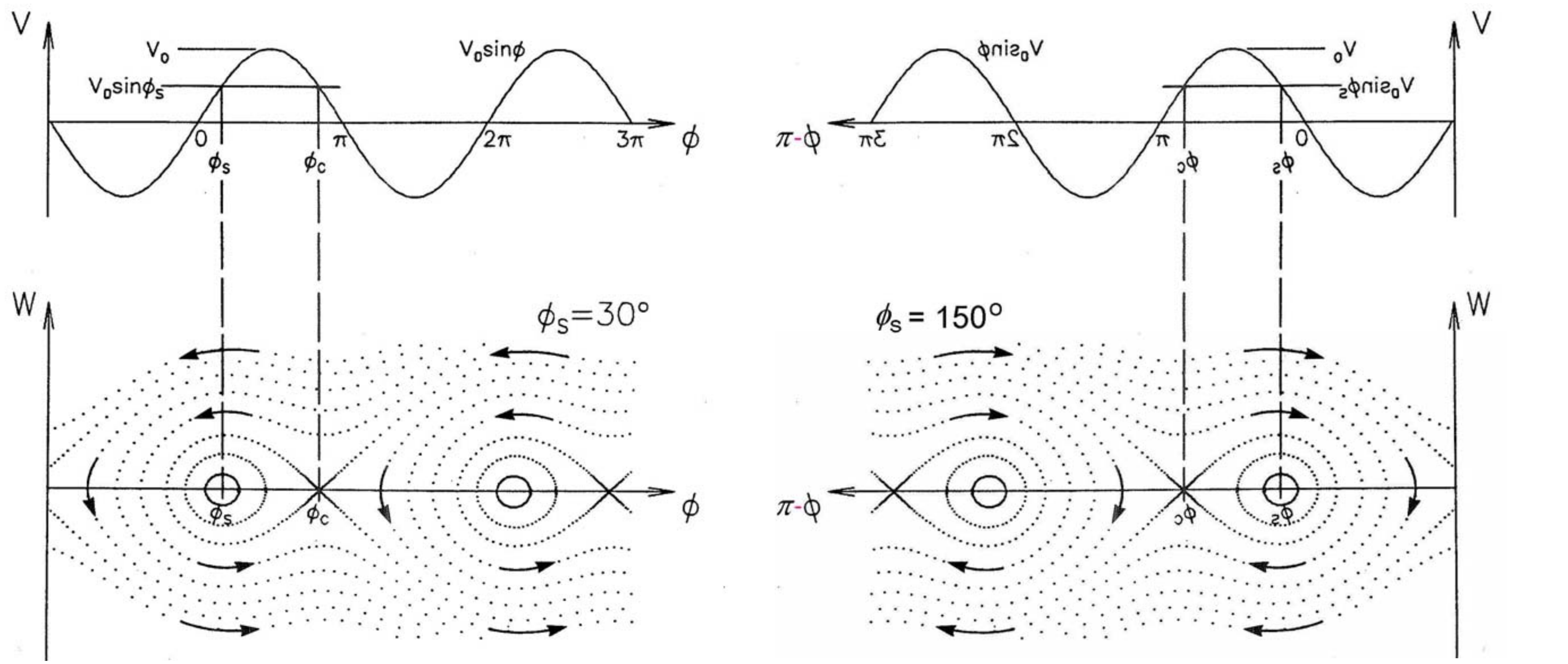
$$H(\phi, W) = -\frac{h\eta}{2m_0R_s^2\gamma}W^2 + \frac{qV_0}{2\pi}\left[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s\right]$$

$$H(\phi, W) = -\frac{h\eta}{2m_0 R_s^2 \gamma} W^2 + \frac{qV_0}{2\pi}\left[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s\right]$$

Curves of constant H in the ϕ-W plane represent particle trajectories.

$\eta > 0 \quad \gamma < \gamma_t \quad 0 < \phi_s < \pi/2$ $\quad \eta < 0 \quad \gamma > \gamma_t \quad \pi/2 < \phi_s < \pi$

Synchronous for rising voltage **Synchronous for falling voltage**



The synchronous points are surrounded by regions of stable motion

- "buckets" – bounded by pear-shaped separatrices
  - outside which the motion is unstable.

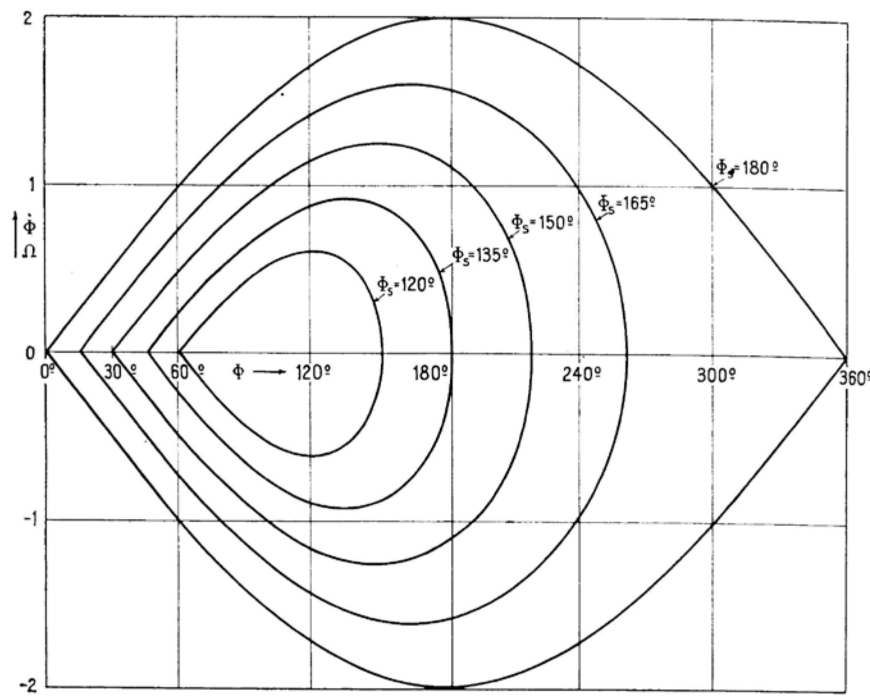
CHOICE OF SYNCHRONOUS PHASE

The area is greatest for $\phi_s = 0$ or π, and drops to zero for $\phi_s = \pi/2$.

$\therefore$ choosing ϕ_s is a design compromise between $\phi_s \approx \pi/2$ with maximum acceleration but zero acceptance, and $\phi_s \approx 0$ or π with the largest buckets but zero acceleration.

Popular choices, giving reasonable acceleration and bucket area, are:

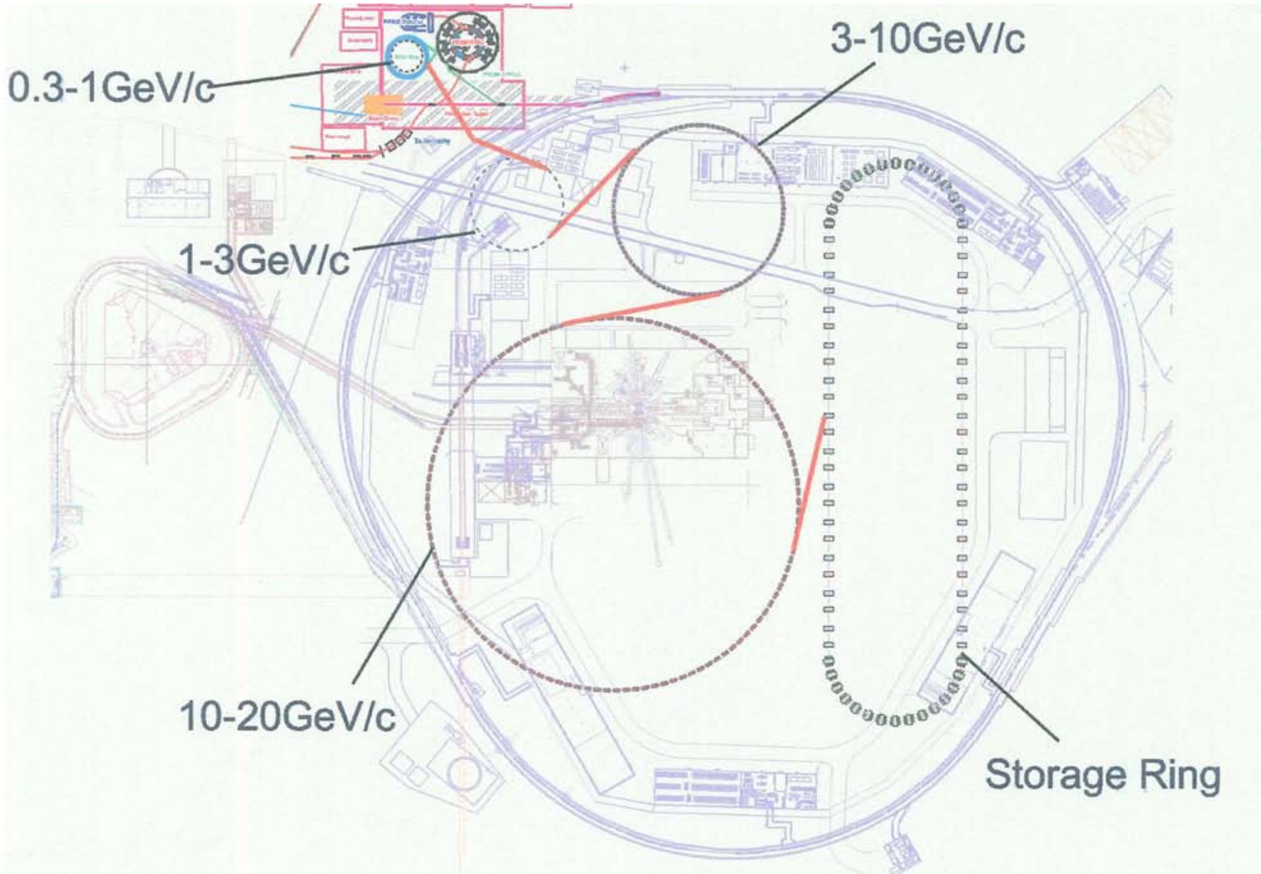
$$\phi_s \approx 30° \text{ or } 150°.$$



Note $W = \Delta T/\omega_0 \propto \Delta T$, not T – the buckets are centred on the synchronous particle – imagine them rising with T_s as it rises.

Also $W\!-\!\phi$ area is conserved by Liouville's theorem – but not $\Delta T\!-\!\phi$.

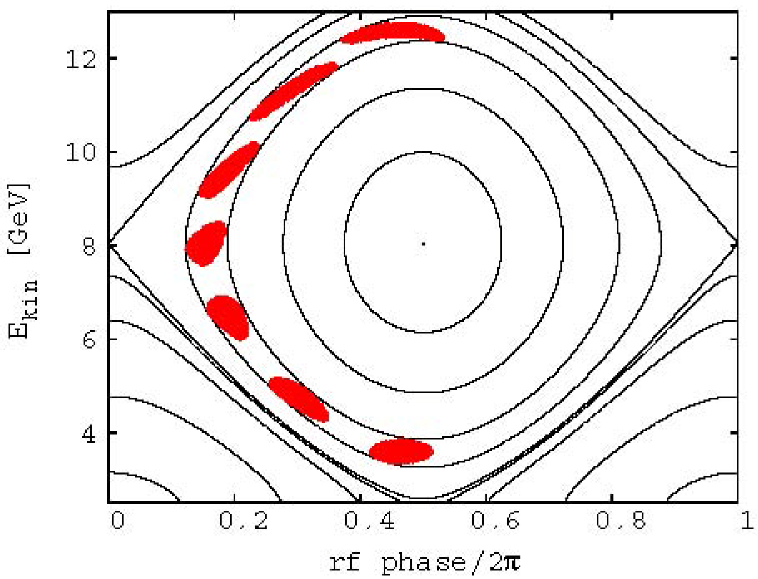
FIXED-FREQUENCY FFAG OPERATION

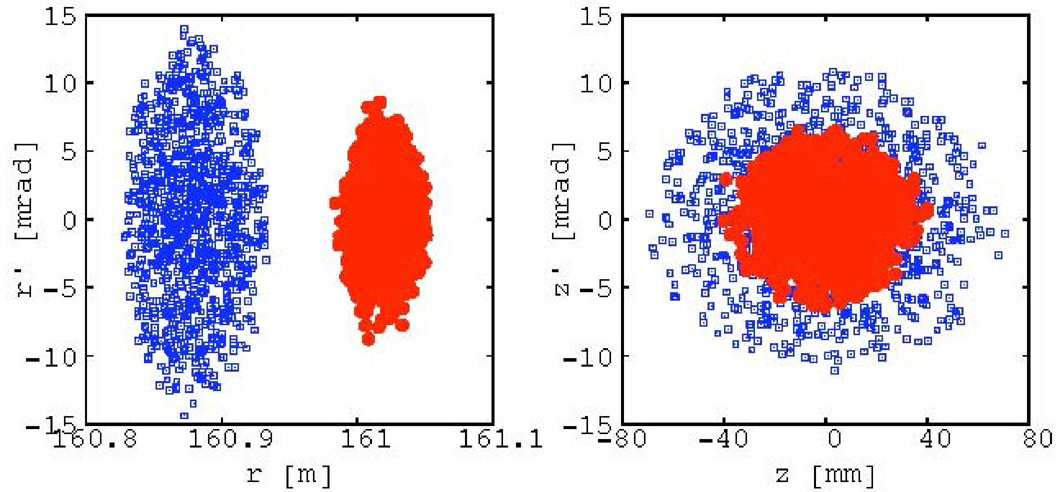


In very-high-energy FFAGs – such as those proposed as muon accelerators for a Neutrino Factory at J-PARC (shown above) – a particle's speed, radius – and therefore orbit time – change very little, so it's possible to consider operating at fixed rf frequency – a great simplification. Two methods have been proposed...........

FIXED-FREQUENCY I: STATIONARY BUCKET

Here a large rf bucket is created, spanning the whole energy range. (At fixed frequency in fixed magnetic field, it's fixed in energy.) As bucket height ∝ √(voltage gain/turn), large rf voltages are needed - but for muons (τ = 2.2 μs) these are essential anyway for rapid acceleration and survival.



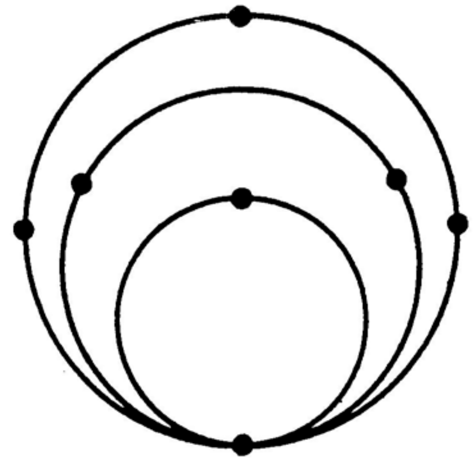


Initial and final transverse emittances

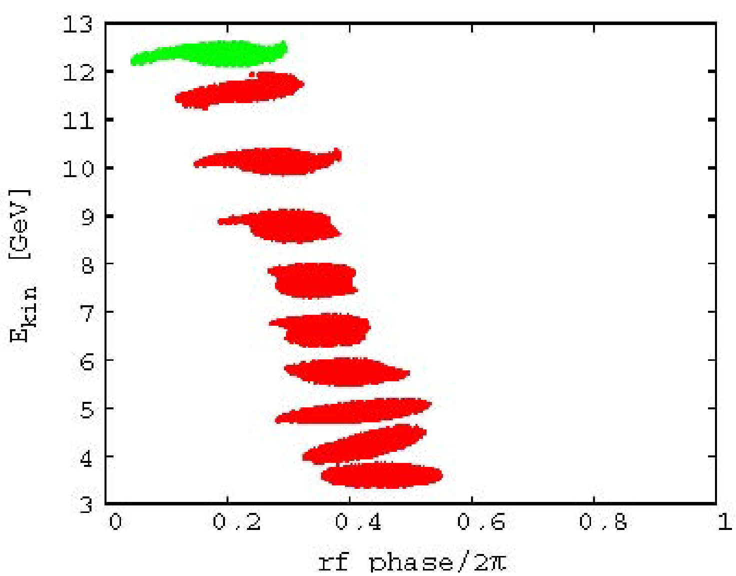
The diagrams (Planche, Mori *et al.* IPAC'10) refer to a 3.6-12.6 GeV muon FFAG with 1.8 GV/turn at 200 MHz (h=675), giving 6-turn acceleration. (Left - Right): Longitudinal, horizontal and vertical phase space.

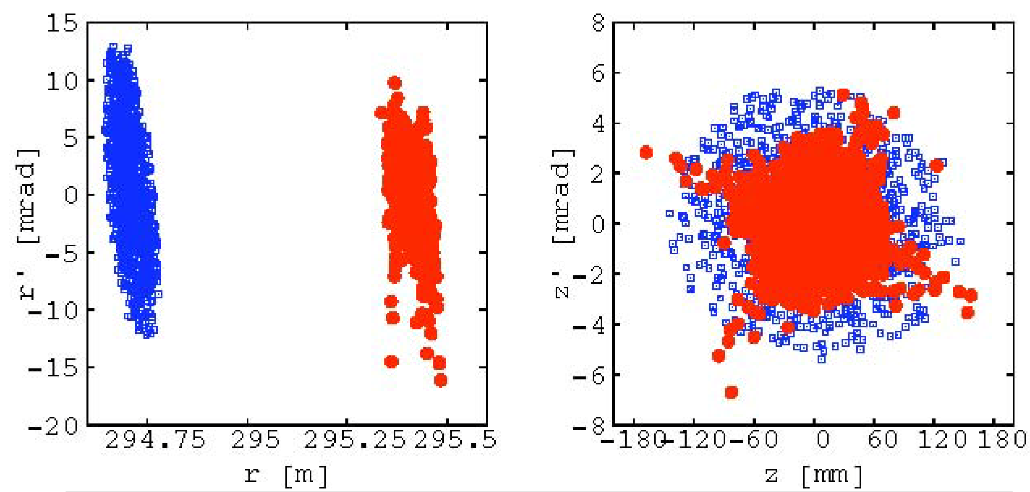
FIXED-FREQUENCY II: HARMONIC NUMBER JUMP

HNJ is a technique where the harmonic number h ($\equiv \tau_{orbit}/\tau_{rf}$) is increased by an integer on each turn (often Δh = 1) – originally devised by Veksler (1944) for electrons in a fixed-frequency microtron, so that as the orbit radius increased, τ_{orbit} remained a harmonic of the fixed τ_{rf}.



Planche, Mori *et al.* have also simulated this method for a 3.6-12.6 GeV muon FFAG with 2.1 GV/turn at 400 MHz, giving 8.5-turn acceleration. The practical challenge is to provide the correct energy gain each turn.



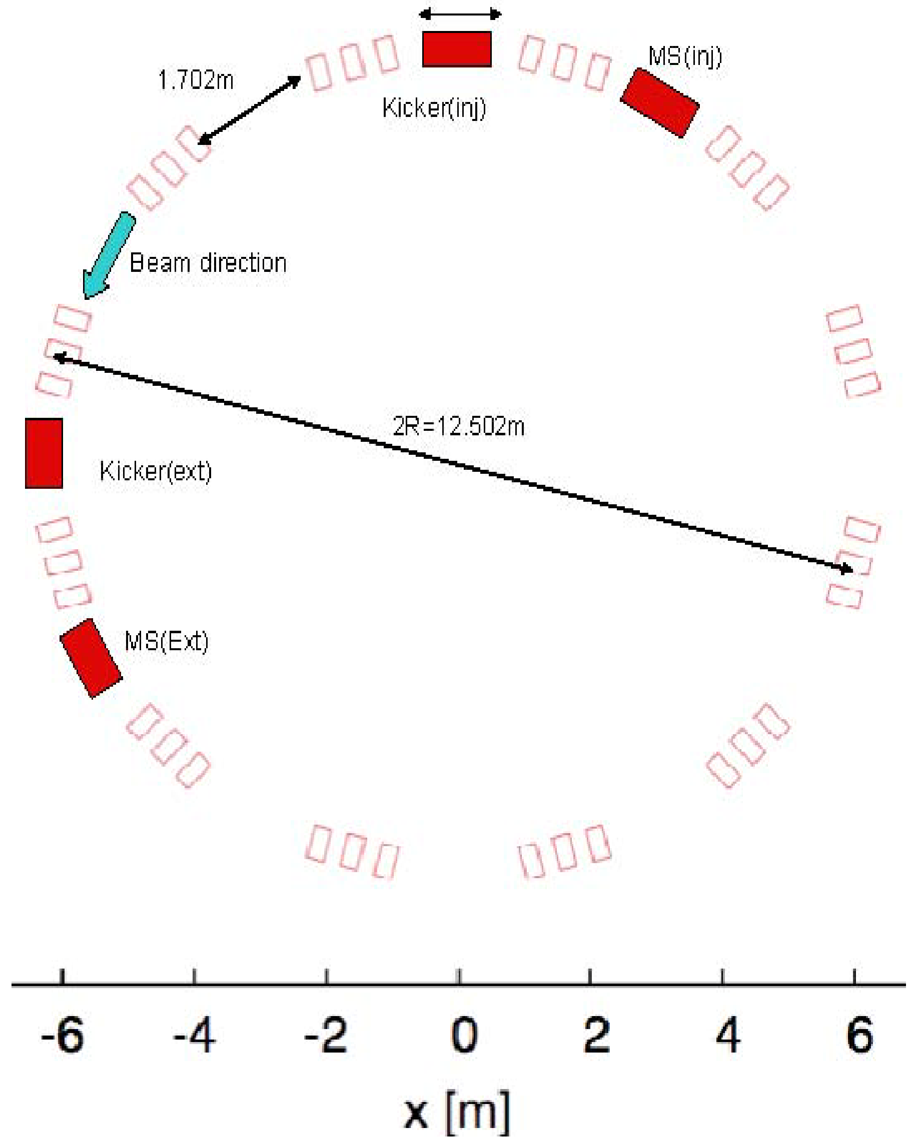


Initial and final transverse emittances

QUASI-SCALING FFAG: PAMELA (Adams Inst., Oxford)

31-250 MeV protons

- for cancer therapy
- 12-cell FDF
- radius ≈ 6.25 m
- 4-T magnets

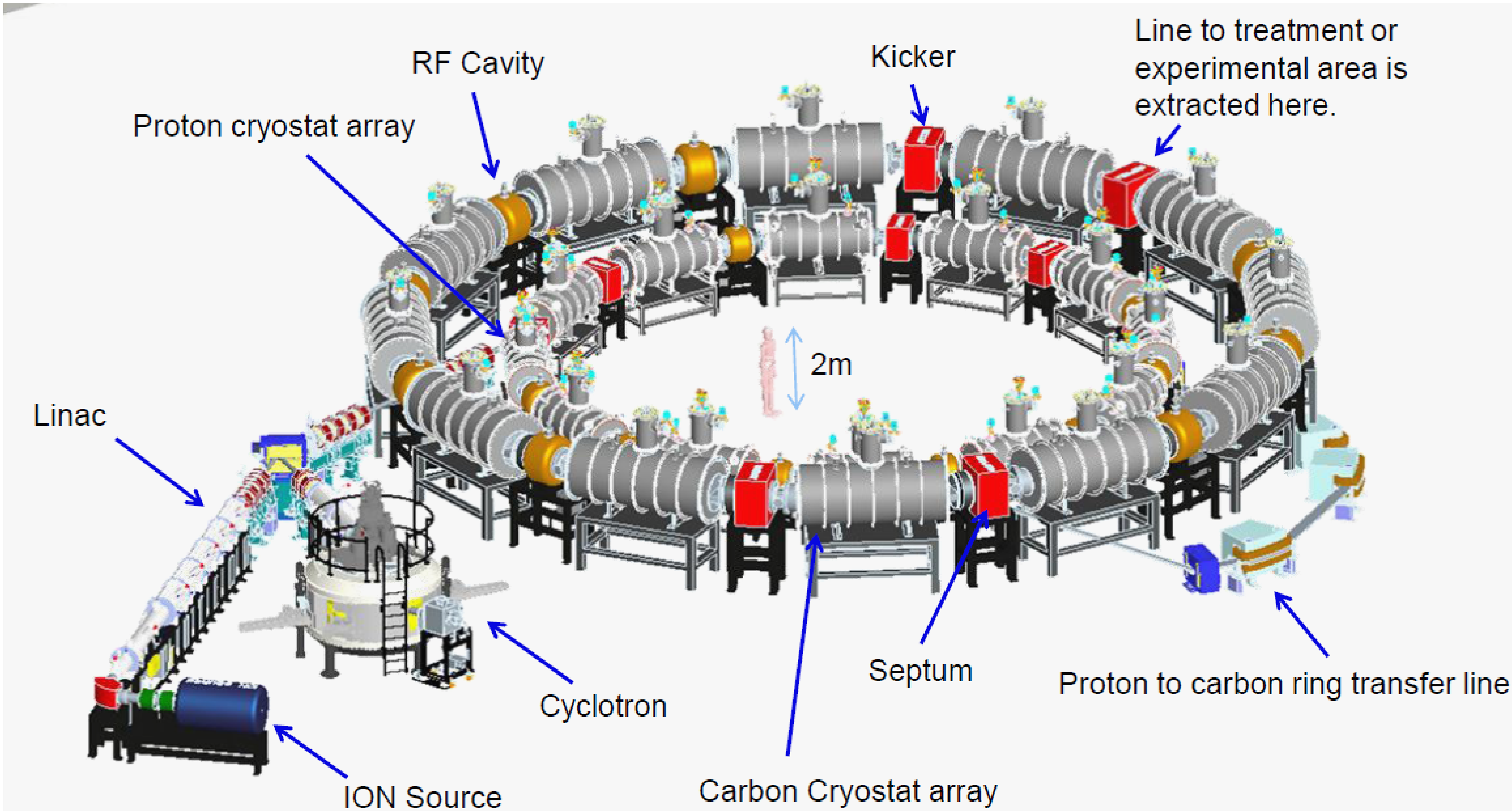


Machida "semi-scaling" lattice

- High field index k (i.e. $B \sim r^k$) for small orbit excursions
- approximate r^k locally by $\Sigma b_n x^n$ with n = 0, 1, 2, 3 only
- flat tunes, good dynamic aperture

400-MeV/u C^+ ions require a similar 2^{nd} stage (radius 9.2 m).

PAMELA CANCER THERAPY FFAGs (Adams Inst. Oxford)



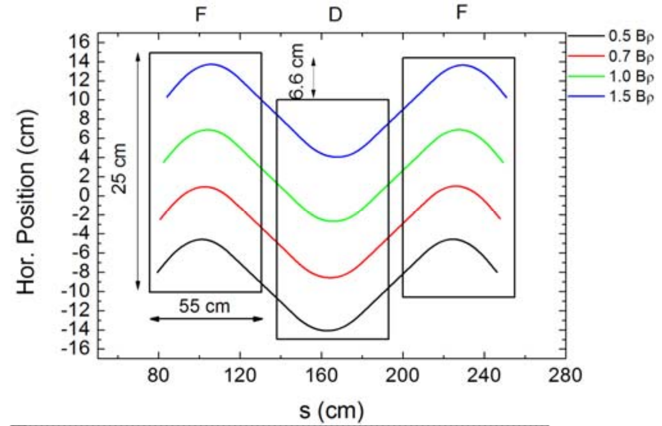
The small ring delivers 31-250 MeV protons

The large ring delivers 68-400 MeV/u C^{6+} ions

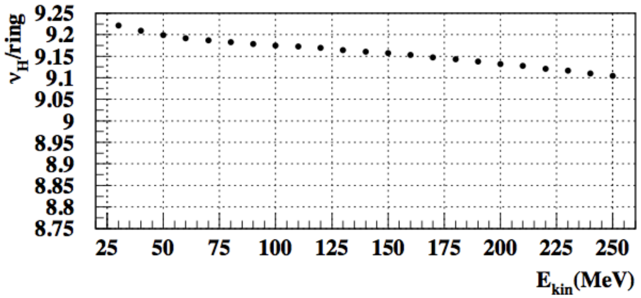
Superconducting 2-, 4-, 6- & 8-pole magnets keep the tunes constant.

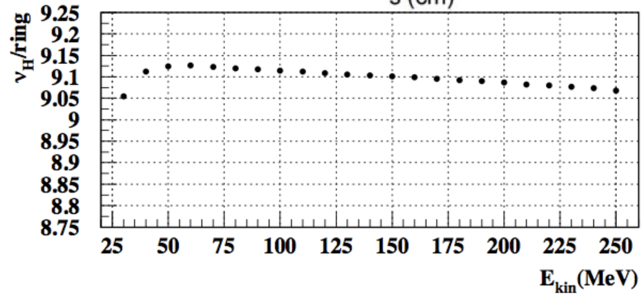
Lattice modeling

$$B = B_0\left(\frac{r}{r_0}\right)^k = B_0\left(1+\frac{k}{r_0}x+\frac{k(k-1)}{2!r_0^{\ 2}}x^2+\cdots\right)$$

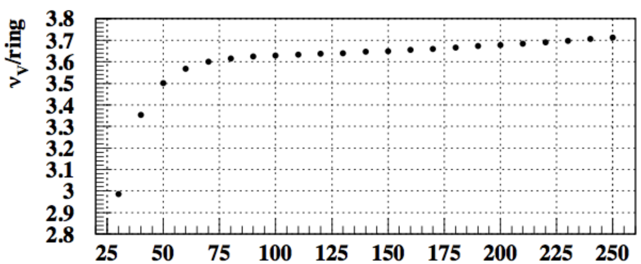


⇒ r_0 should be the center of magnet

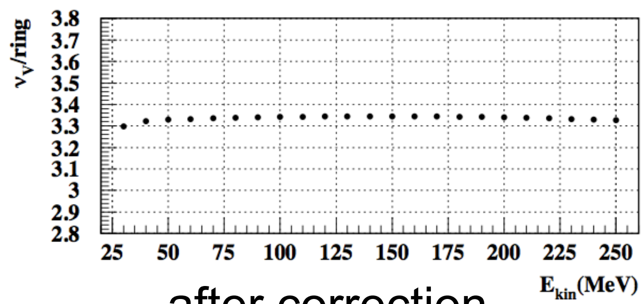




Tracked by ZGOUBI



Before correction(FFAG08)

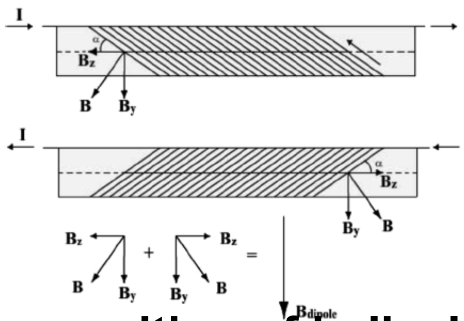


after correction

$\Delta\nu_H$<0.1, $\Delta\nu_H$<0.05
⇒ Now, accelerating speed is determined by clinical requirements, not by beam dynamics

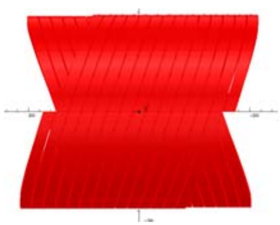
Magnet

Challenges: Large aperture, short length, strong field

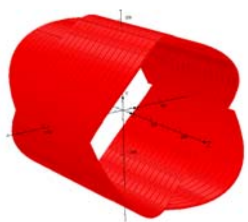


Superposition of helical field can form multipole field

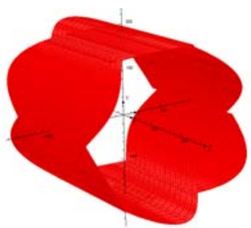
- Applicable to superconducting magnet
- Each multipole can be varied independently
  $\Rightarrow$ Operational flexibility
- Present lattice parameters are within engineering limit



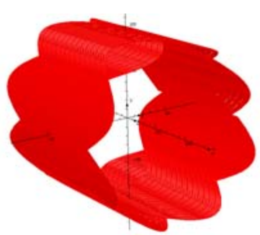
Dipole



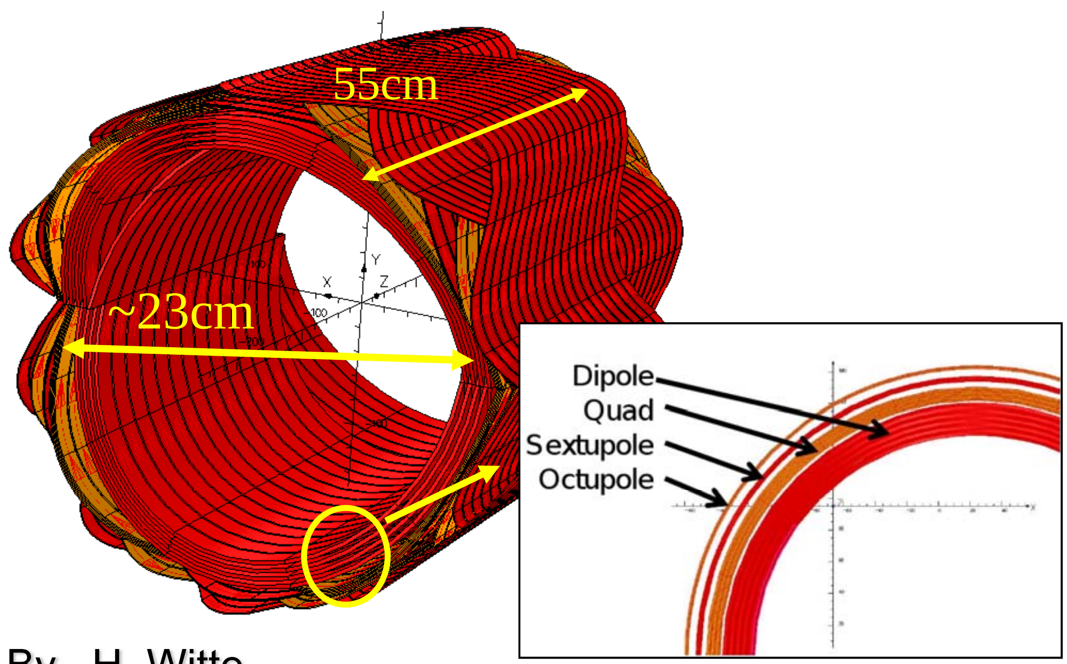
Quadrupole



Sectapole



Octapole



By H. Witte

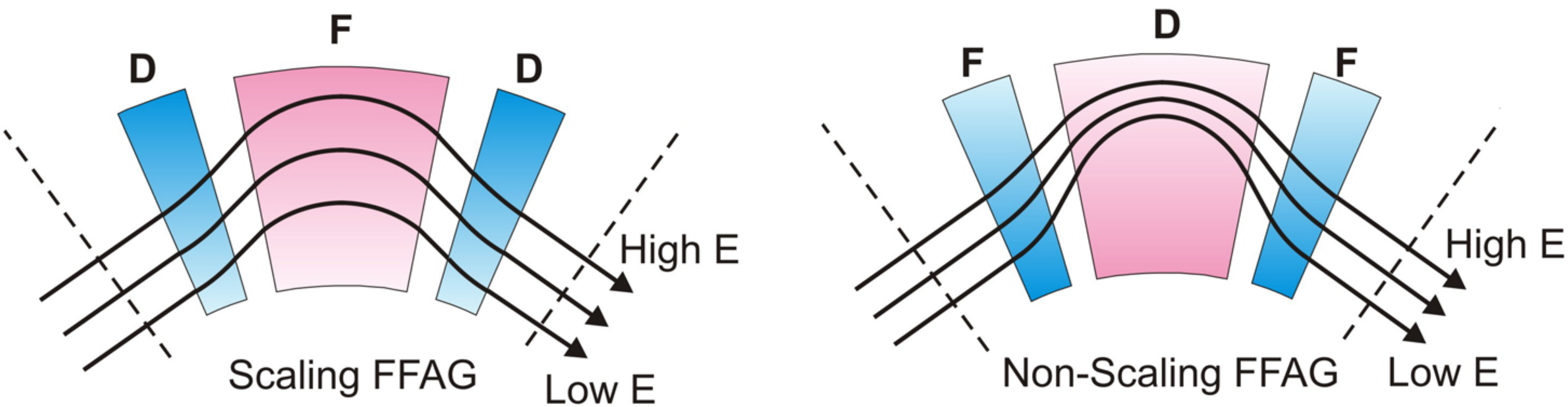
LINEAR NON-SCALING (LNS) FFAGs

FFAGs look attractive for accelerating muons in μ Colliders or ν Factories

- Large acceptance (in *r* & *p*) eliminates cooling & phase rotation stages
- Rapid acceleration (<20 turns) makes resonance crossing ignorable (Mills '97)
- Less expensive than recirculating linacs.

NON-SCALING approach first tried by Carol Johnstone (arc 1997, ring 1999)

- strong positive-bending Ds + negative Fs – i.e. negative field gradients!
- "LINEAR" constant-gradient magnets (i.e. quadrupoles).



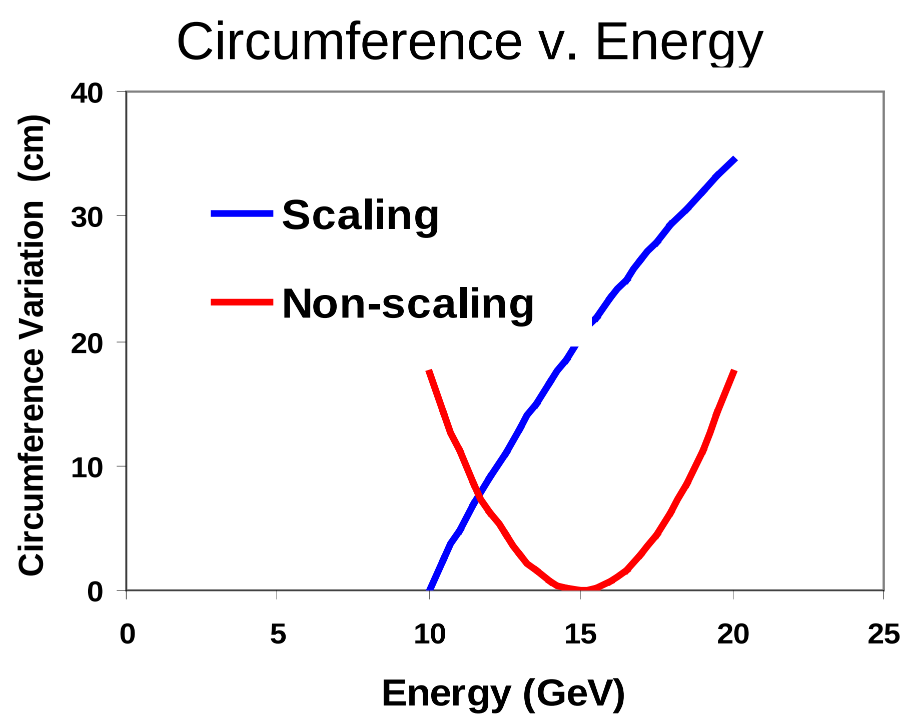
- Greater momentum compaction (& hence narrower radial apertures);
- Less orbit-time variation → fixed rf frequency & cw operation;
- No multipole field components to drive betatron resonances >1st order;
- Simpler construction (B′ = constant, rather than $B' \propto r^{k-1}$).

LINEAR NON-SCALING FFAGs (cont'd)

Note that for LNS-FFAGs, orbit circumference C varies quadratically with energy rather than rising monotonically:

$$C(p) = C(p_m) + \frac{12\pi^2}{e^2 S^2 N L_{fd}}(p - p_m)^2$$

So, compared to a scaling FFAG, there's less variation in C and orbit period
- enabling fixed rf frequency operation when v ≈ c.



Thus in a high-energy muon accelerator:
- the muons oscillate in phase across the rf voltage peak (3 crossings)
  - just as in a real, imperfectly isochronous, cyclotron (see below)!

The International Design Study for a Neutrino Factory chose LNS-FFAGs of 12.6-25 GeV and 25-50 GeV for the final stages of muon acceleration
- with designs developed by a consortium led by Johnstone (FNAL), Berg (BNL), and Koscielniak (TRIUMF).

LNS-FFAG ORBITS I

We assume FDF triplet cells with thin quadrupoles (lengths $L_{f/d} \ll L_0$, the cell length), and define a central orbit - an N-sided polygon - which is on axis in the F quad for momentum p_c, and for which all bending occurs at D:

$$p_c = qB\rho_c = qB_c \frac{L_d/2}{\pi/N}.$$

Assuming equal quad strengths:

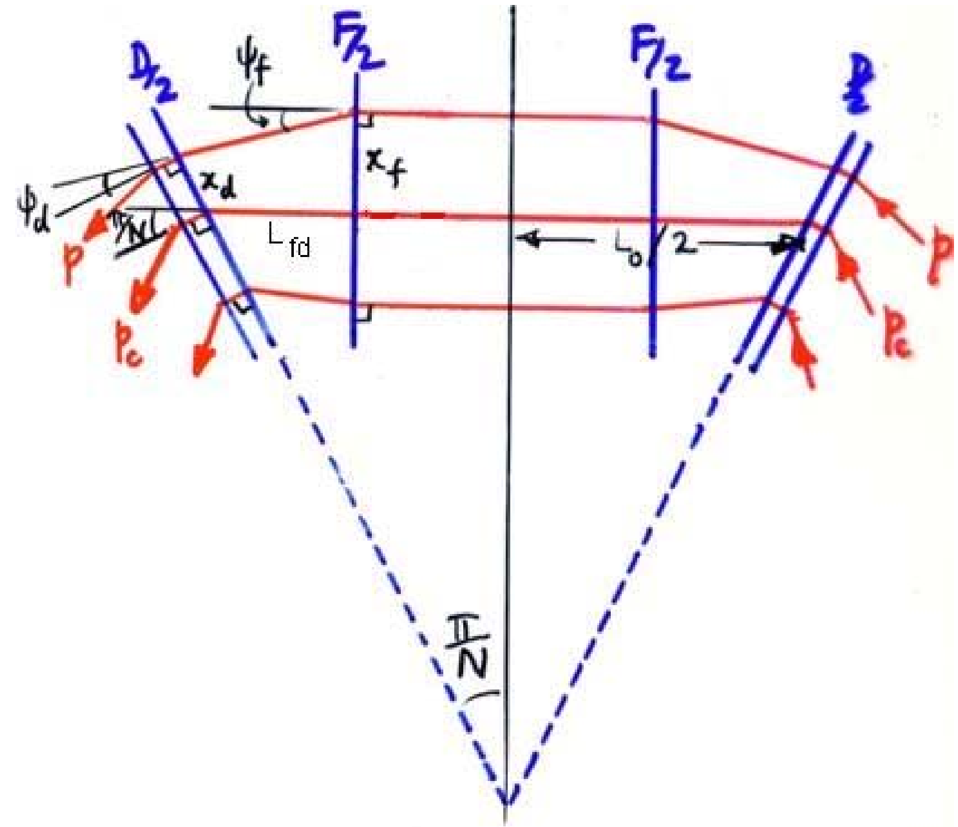
$$S \equiv B_f' L_f = -B_d' L_d$$

and writing $\sigma \equiv qS/2$,

then for other momenta p:

$$B_f = B_f' x_f \qquad \psi_f = \left(\frac{qB_f'}{p}\right)\frac{L_f}{2} x_f = \frac{\sigma}{p} x_f$$

$$B_d = B_c + B_d' x_d \qquad \psi_d = \frac{\pi}{N}\frac{p_c}{p} - \frac{\sigma}{p} x_d$$

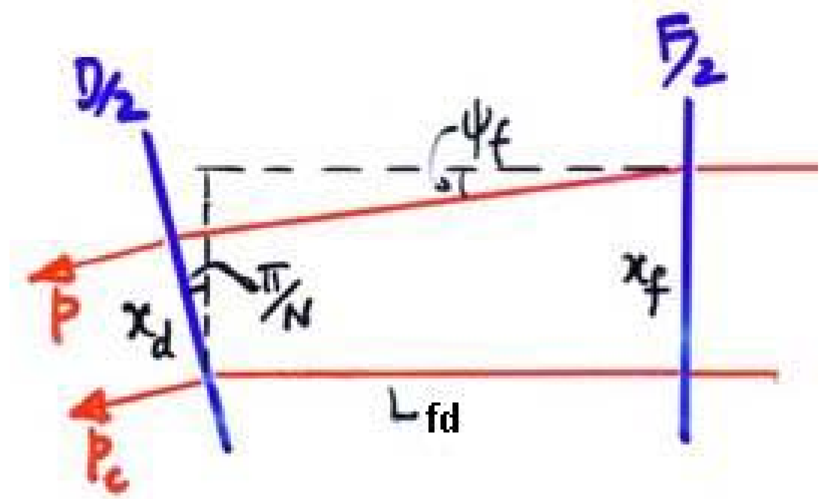


Note that as $L_{fd} \to L_0/2$ the FDF cell approaches FODO form.

LNS-FFAG ORBITS II - OFFSETS AT THE QUADS

As the total bend $\psi_f + \psi_d = \pi/N$,

then: $x_f - x_d = \dfrac{\pi}{N\sigma}(p - p_c)$.



But for large N, $\pi/N \ll 1$, $\psi_f \ll 1$, so we also have:

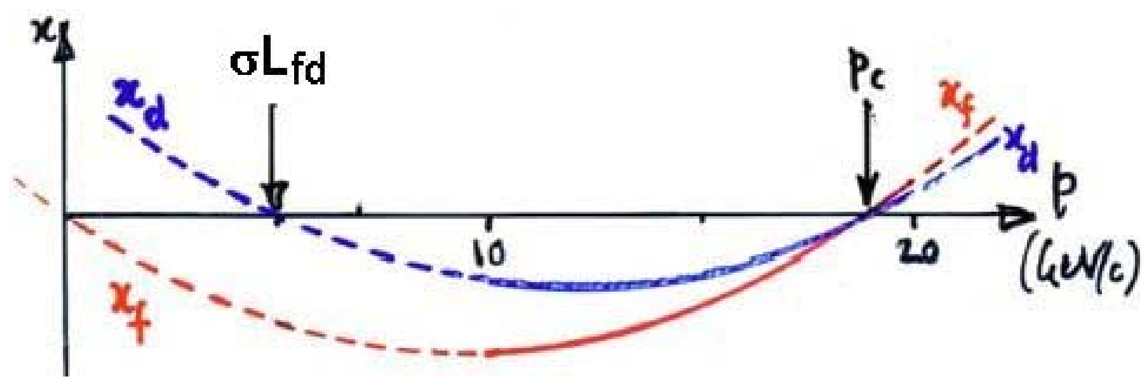
$$x_f - x_d \approx L_{fd}\psi_f = \frac{\sigma L_{fd}}{p} x_f$$

Thus:

$$x_f = \frac{\pi}{\sigma^2 N L_{fd}} p(p - p_c)$$

$$x_d = \frac{\pi}{\sigma^2 N L_{fd}} (p - p_c)(p - \sigma L_{fd}).$$

i.e. the orbit offsets vary quadratically with momentum.

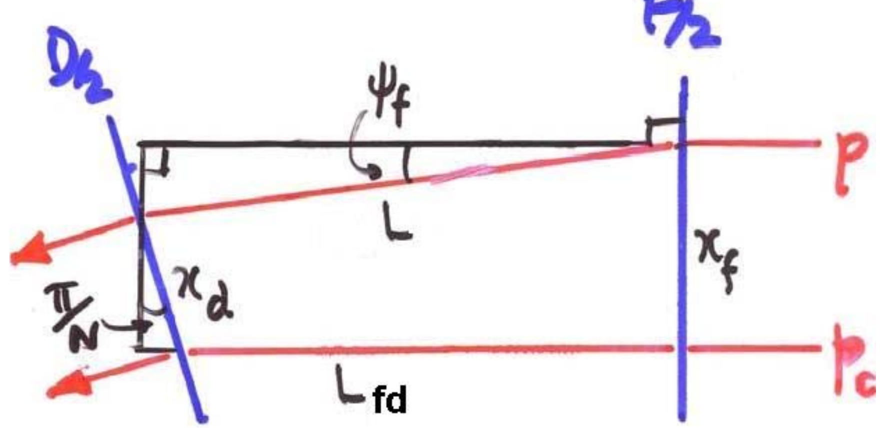


LNS-FFAG ORBITS II - PATH LENGTH

We have $L\cos\psi_f \approx L_{fd} + x_d \sin(\pi/N)$.

$$\therefore L \approx \left(L_{fd} + \frac{\pi}{N}x_d\right)\left[1+\frac{1}{2}\left(\frac{\sigma x_f}{p}\right)^2\right]$$

and $\Delta L \equiv L - L_{fd} \approx \frac{\pi}{N}x_d + \frac{1}{2}\sigma^2 L_{fd}\left(\frac{x_f}{p}\right)^2$



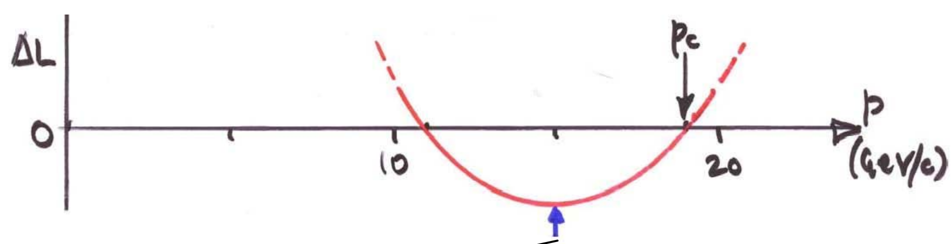
Substituting for x_d and x_f :

$$\Delta L(p) = \frac{3\pi^2}{2\sigma^2 N^2 L_{fd}}(p - p_c)\left[p - \frac{1}{3}(p_c + 2\sigma L_{fd})\right]$$

Around the whole circumference:

$$\Delta C(p) = \frac{12\pi^2}{q^2 S^2 N L_{fd}}(p - p_m)^2$$

where $\Delta C = 0$ for $p_m = \frac{1}{3}(2p_c + \sigma L_{fd})$.



LONGITUDINAL MOTION

The parabolic momentum dependence of the circumference $C(p) = C_m + c_0(p - p_m)^2$ makes the momentum compaction a bit complex:

$$\alpha \equiv \frac{\delta C / C}{\delta p / p} = \frac{\delta r / r}{\delta p / p} = \frac{2c_0 p(p - p_m)}{C_m + c_0(p - p_m)^2}$$

- and the slip factor $\eta = \gamma^{-2} - \alpha$ even more so.

For <u>fixed frequency operation</u> there is no synchronous phase, but similar equations of motion apply (the energy difference ΔT is taken w.r.t. the energy T_r at which the orbit period $\tau = \tau_{rf} \times \text{integer}$):

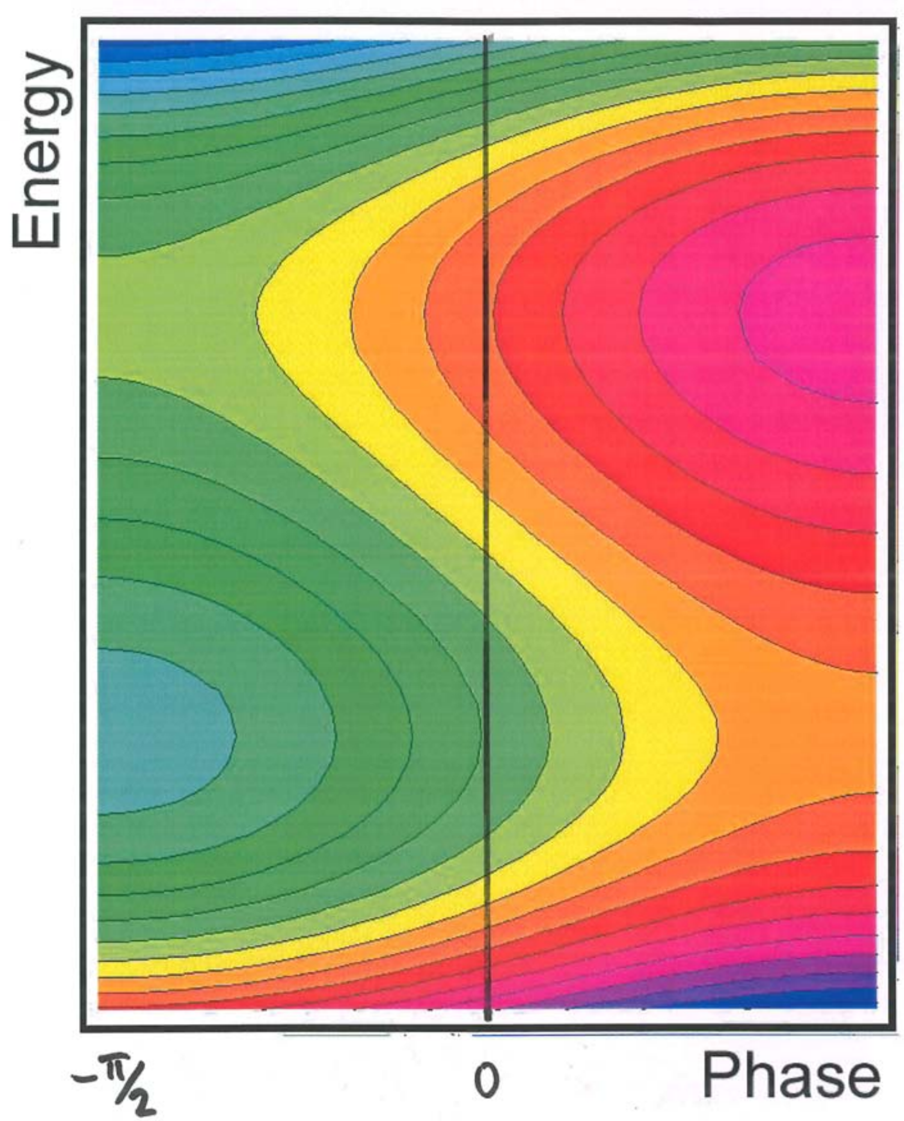
$$\frac{d\phi}{dt} = \frac{-h\eta}{m_0 R^2 \gamma}\left(\frac{\Delta T}{\omega_0}\right) \qquad \frac{d}{dt}\left(\frac{\Delta T}{\omega_0}\right) = \frac{qV_0}{2\pi}\sin\phi.$$

The complex form of η leads to a Hamiltonian that is cubic in $(\Delta T/\omega_0)$ with critical parameters:

$$w \equiv \frac{qV_0/(\hat{T} - \breve{T})}{\omega_0(\hat{\tau} - \breve{\tau})}, \quad z \equiv \frac{\tau_r}{\hat{\tau} - \breve{\tau}},$$

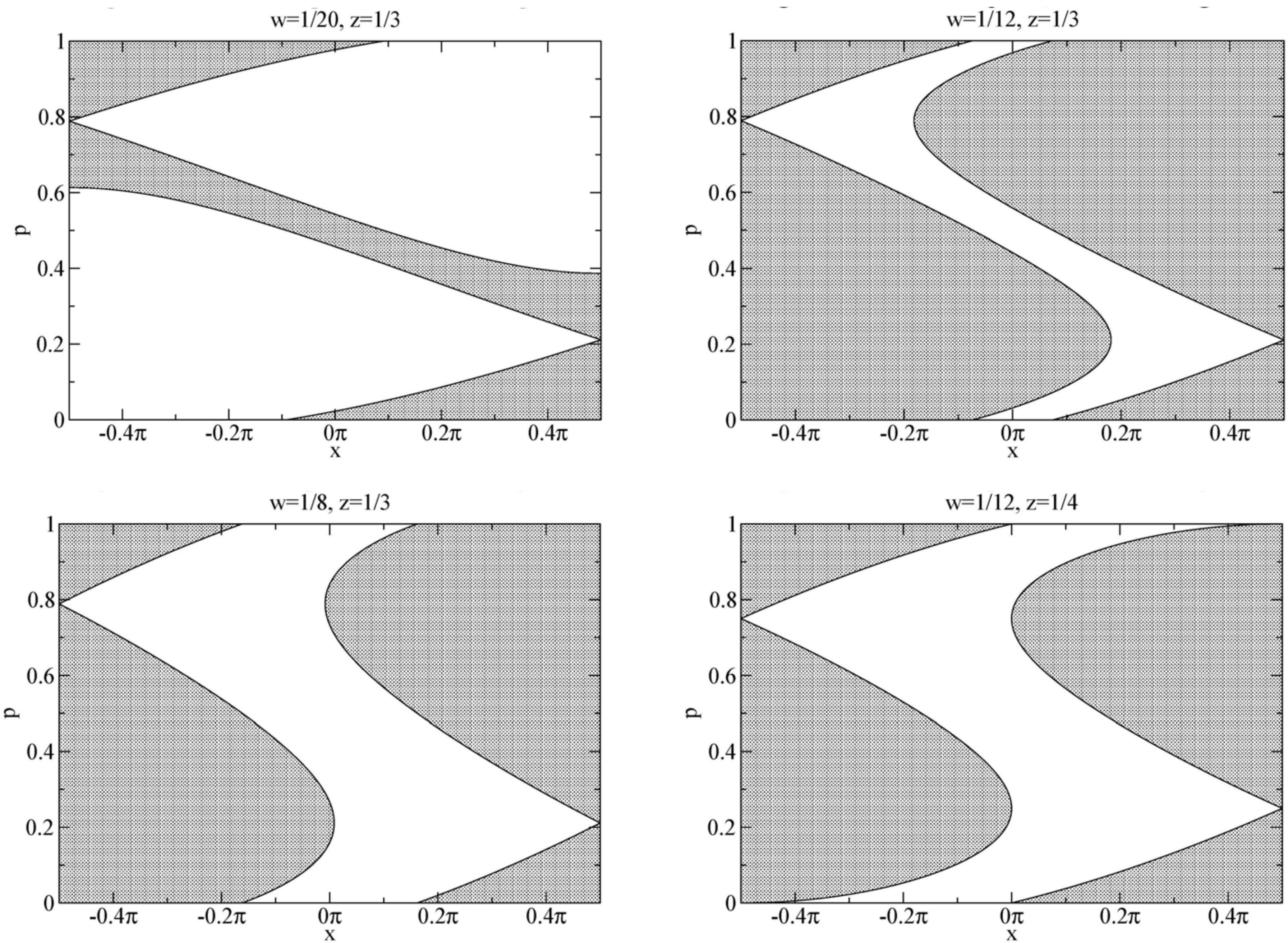
where τ_r is a time offset to take account of T_r differing from $\langle T \rangle$.

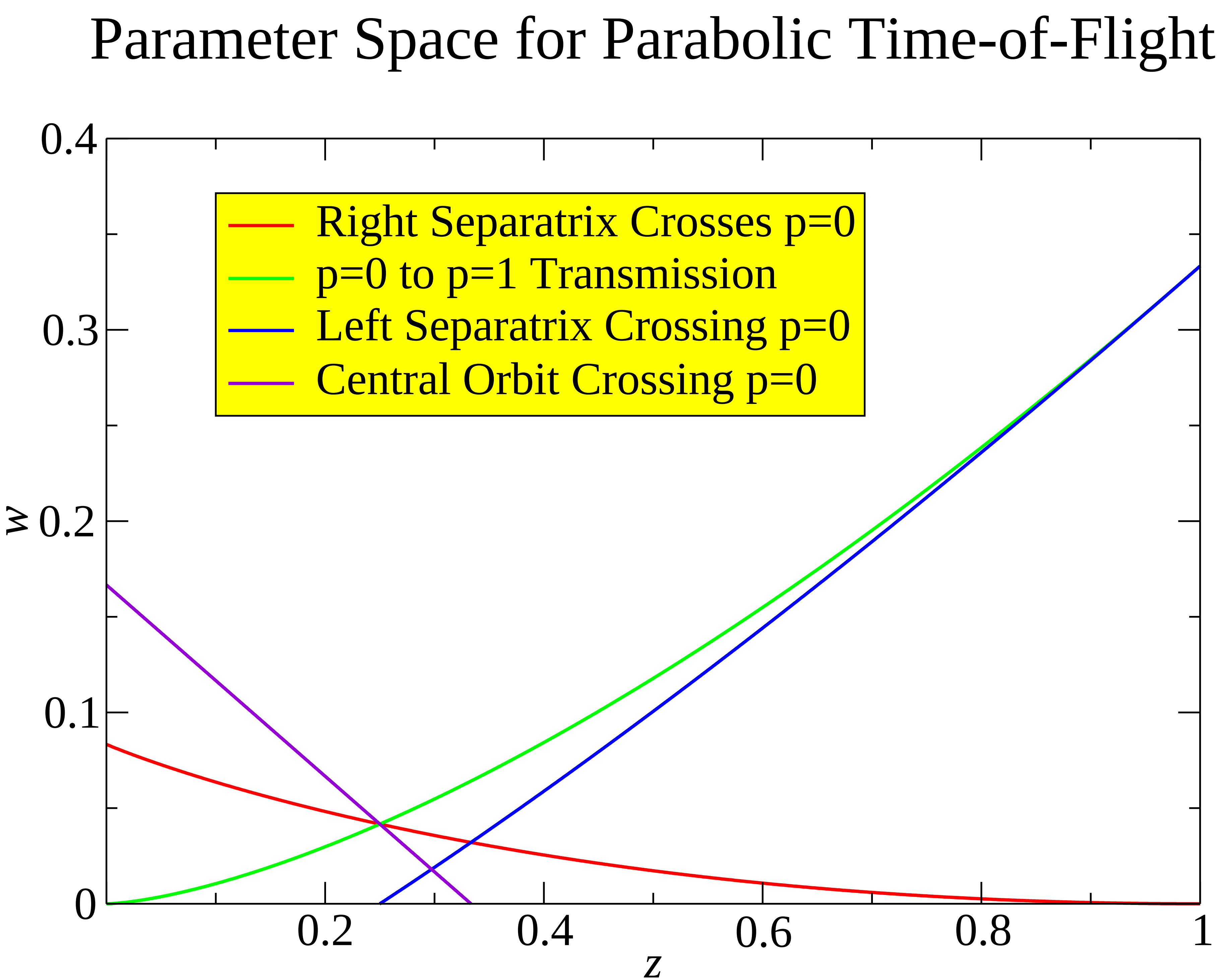
SERPENTINE ACCELERATION IN LNS-FFAGs



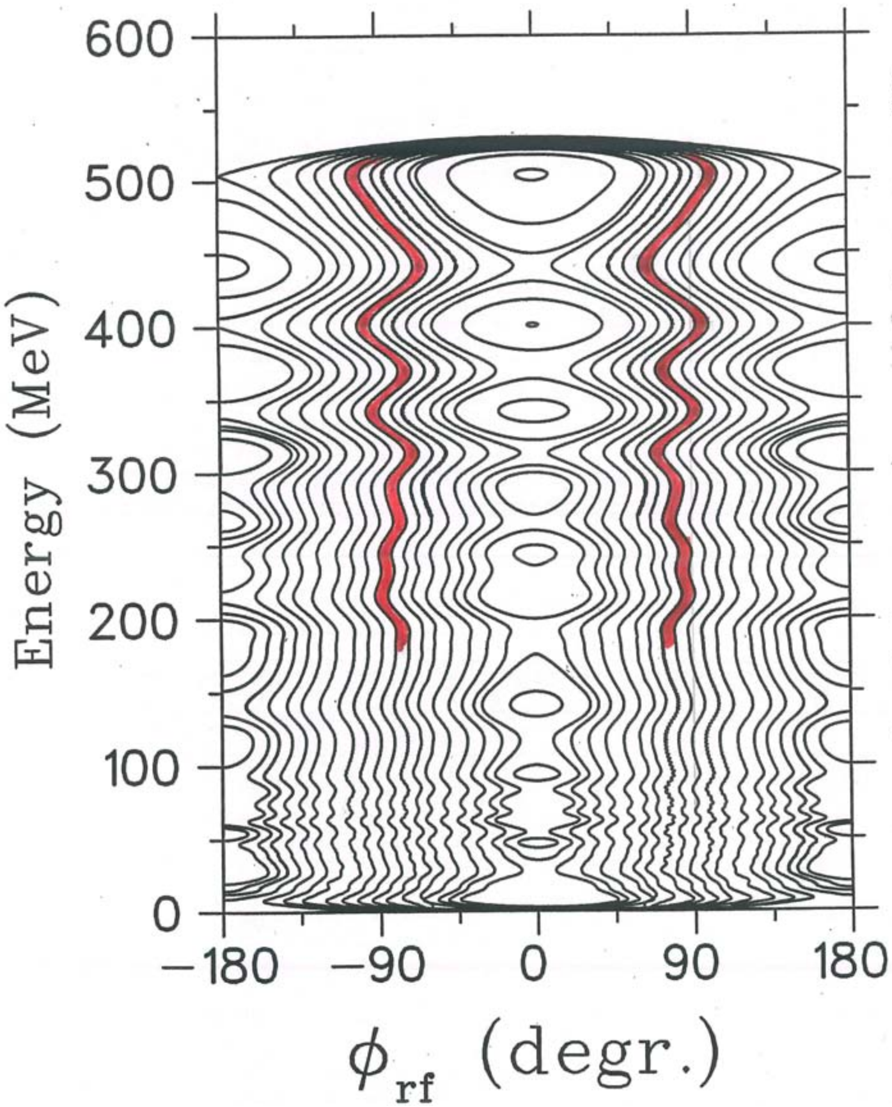
- Not within the buckets – but between them
- Follow the golden trail!

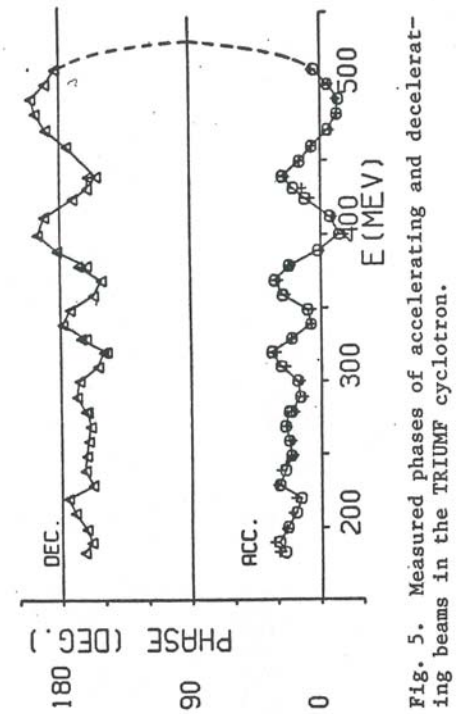
SEPARATRICES IN THE PHASE-ENERGY PLANE




Parameter Space for Parabolic Time-of-Flight
Right Separatrix Crosses p=0
p=0 to p=1 Transmission
Left Separatrix Crossing p=0
Central Orbit Crossing p=0
0.4
0.3
0.2
0.1
0
w
0.2
0.4
0.6
0.8
1
z

SERPENTINE ACCELERATION IN CYCLOTRONS

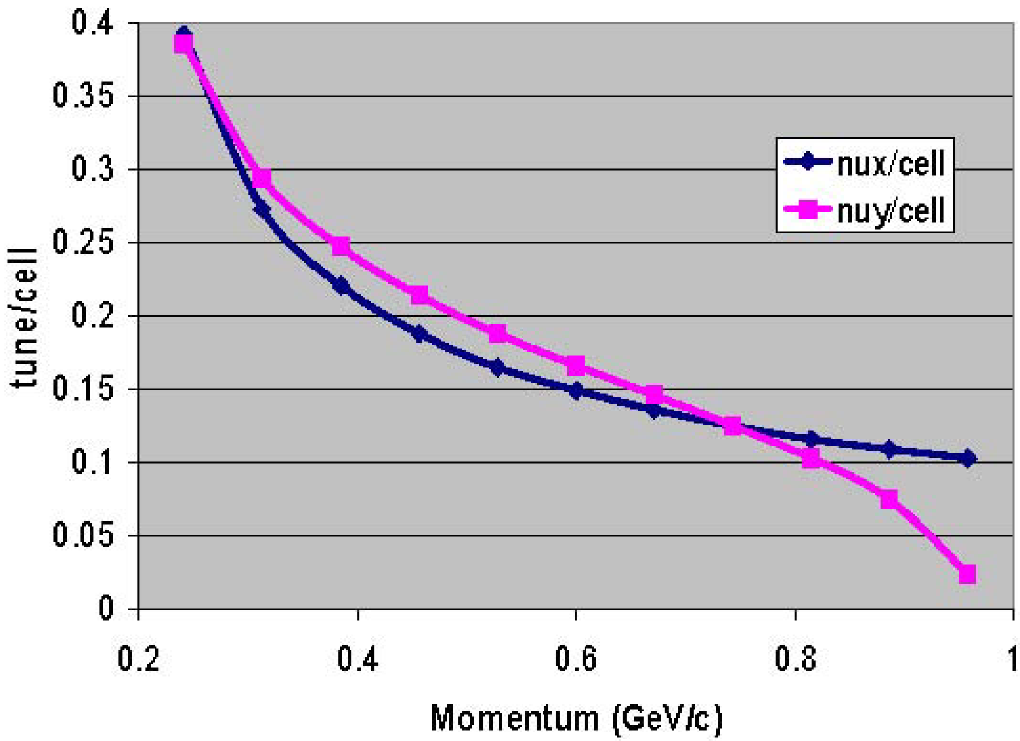




Measured phase history
in the TRIUMF cyclotron

- Real cyclotrons are only imperfectly isochronous
- Acceleration occurs along a serpentine path

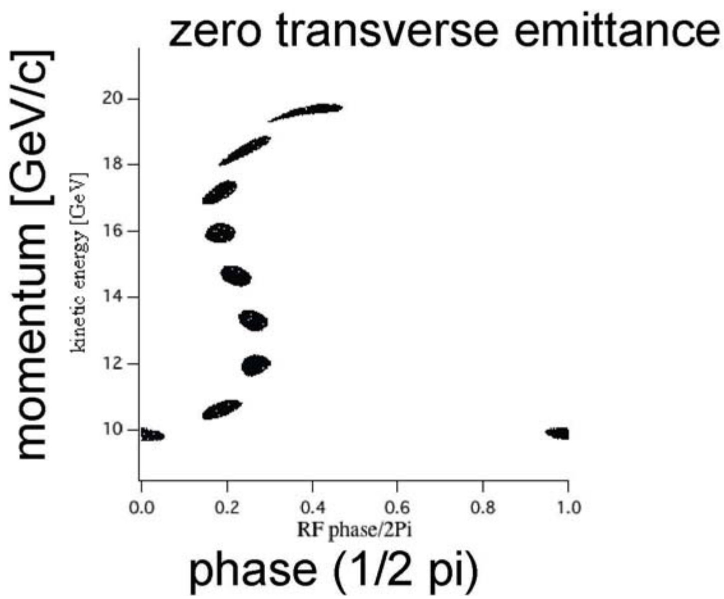
TUNES IN LNS-FFAGs

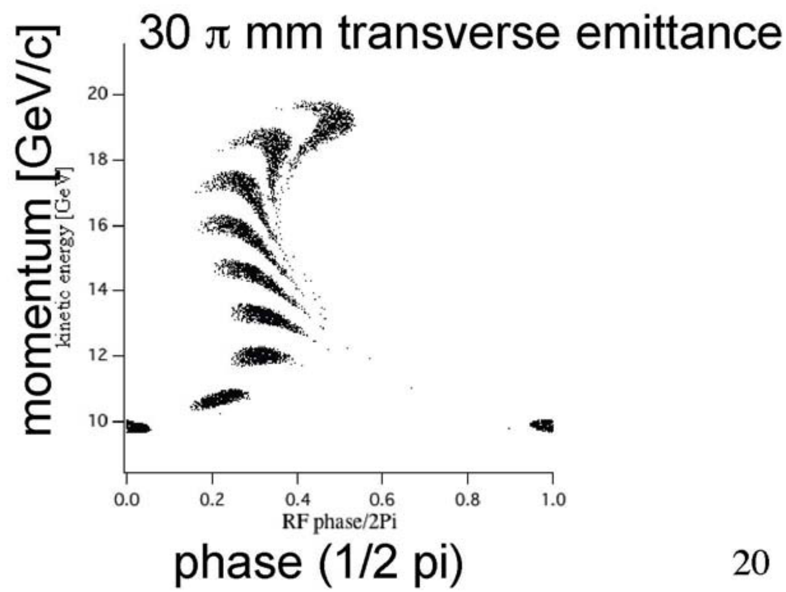


If the orbits cross the magnet ends perpendicularly:

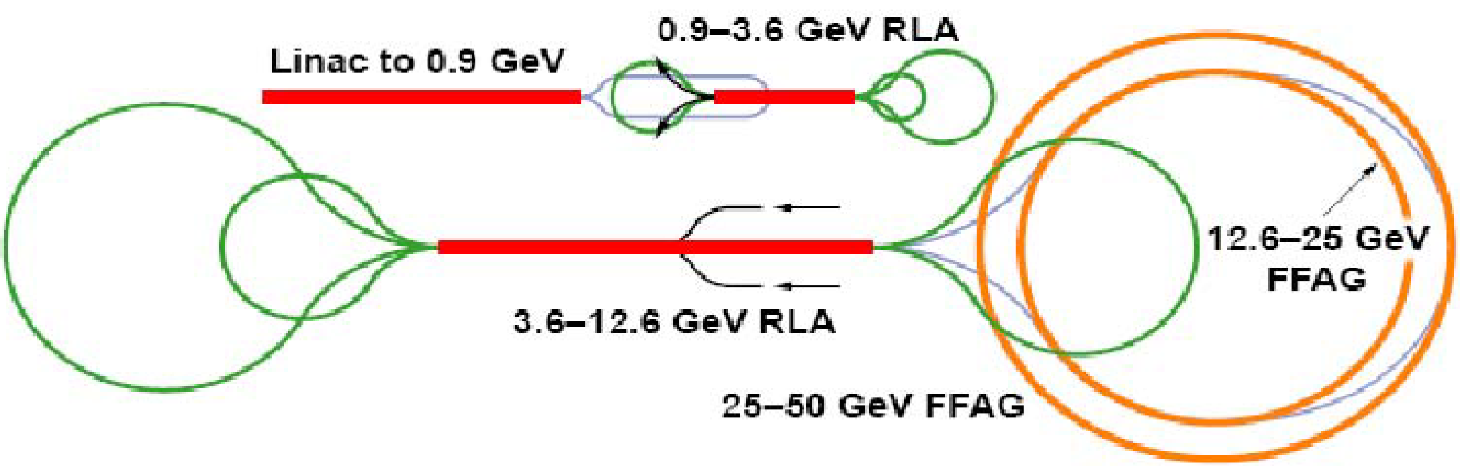
- the tunes fall sharply with energy, crossing betatron resonances
- possibly leading to loss of beam quality/quantity
- danger lessened by rapid energy gain, but very expensive
- for muons (τ = 2 μs): expensive but essential anyhow
- for ions: just expensive

MATCHING LNS-FFAGs





Unfortunately, for large-emittance beams, the radial longitudinal coupling in LNS-FFAGs makes transfer matching difficult. Mitigation techniques exist, but the ν Factory ISS concluded that >2 LNS-FFAGs would not be practical – and opted for the more costly recirculating linacs below 12.6 GeV.



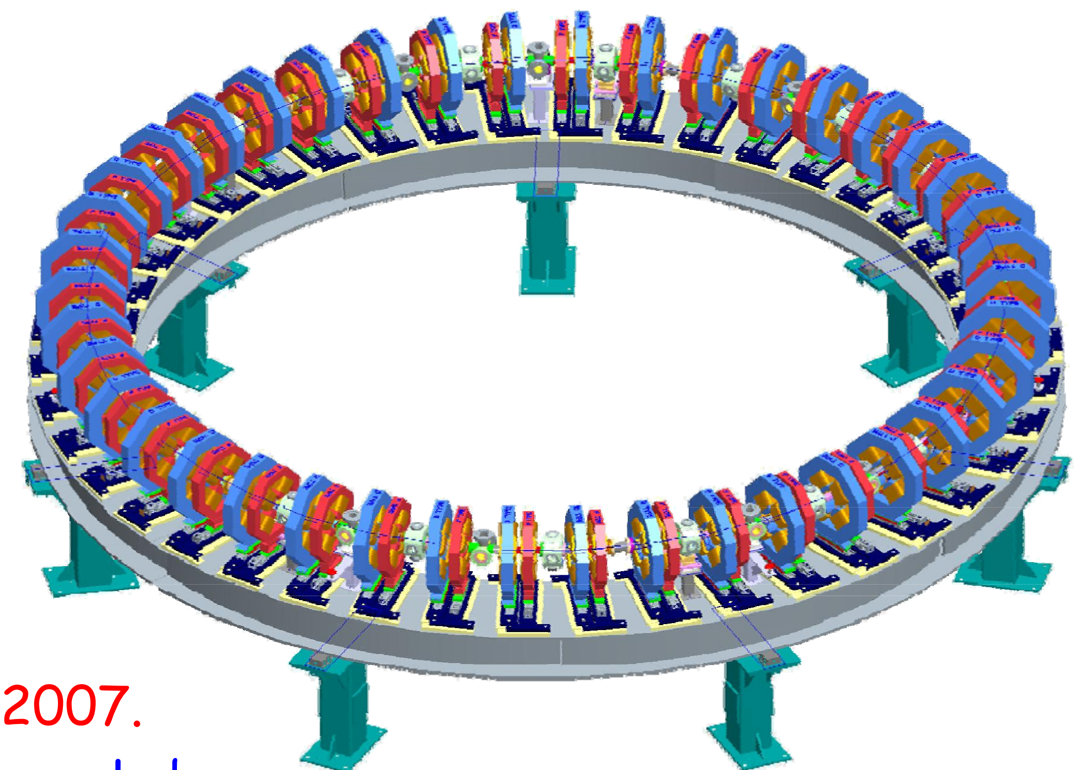
ELECTRON MODEL LNS-FFAG "EMMA"

A Proof of Principle machine for linear non-scaling FFAGs
to demonstrate their two novel features:

- safe passage through many low-order structural resonances
- acceleration outside buckets.

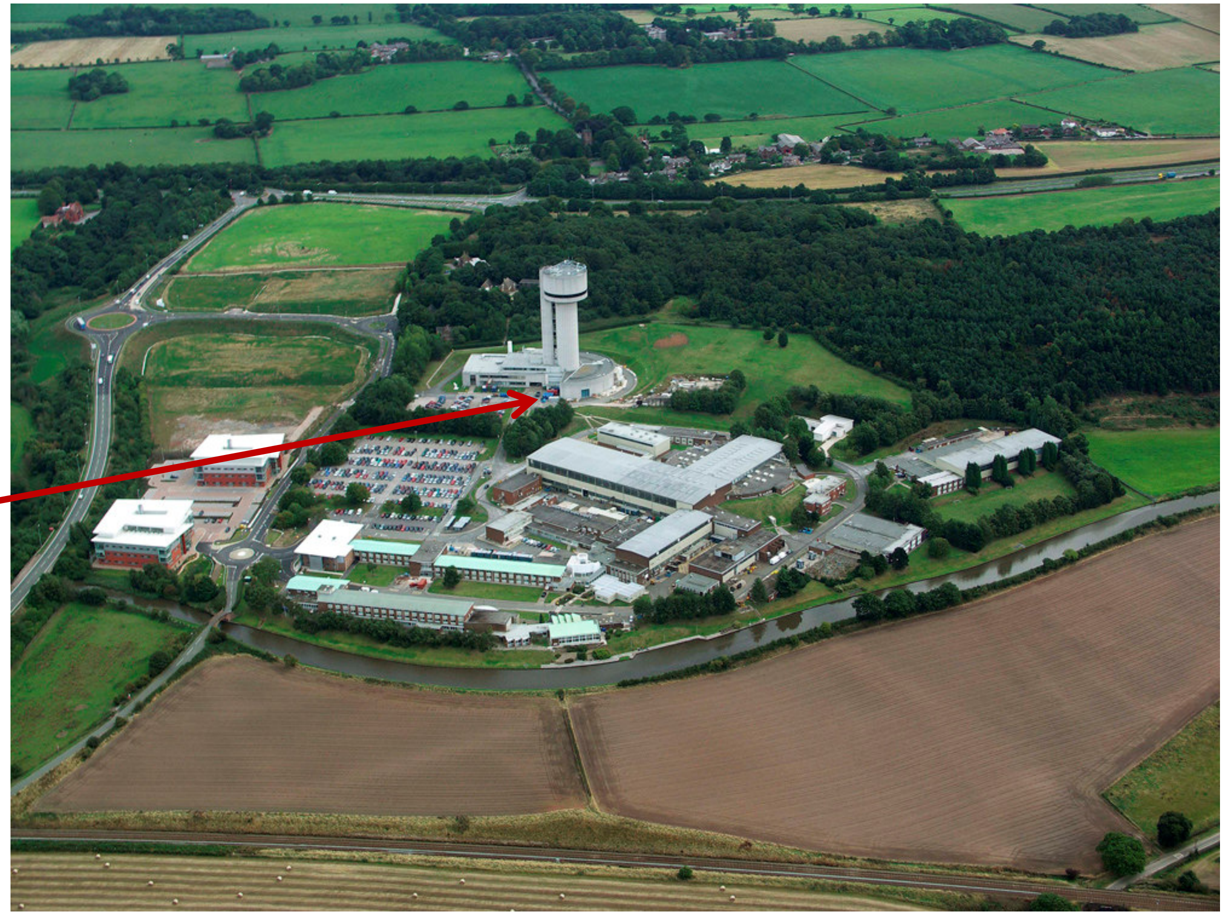
EMMA has relativistic parameters similar to those of a 10-20 GeV muon FFAG, with a doublet lattice based on offset quadrupoles:

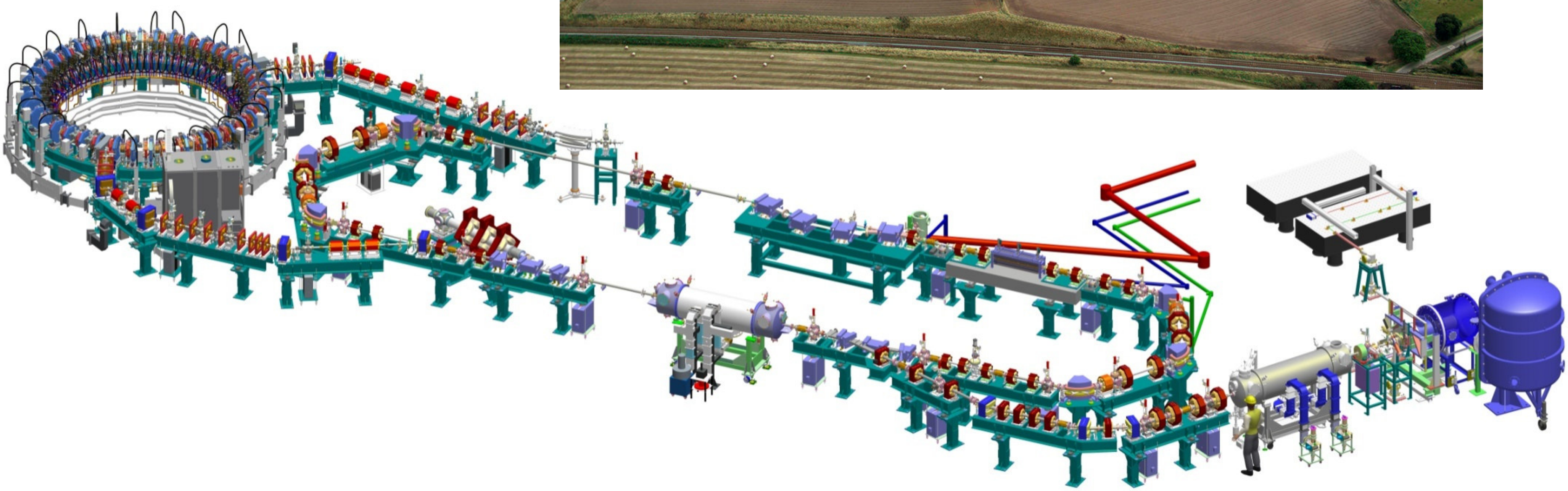
Energy	10-20 MeV
Circumference	16.57 m
Cells	42
N.T. Acceptance	3 mm
F quad length	5.88 cm
D quad length	7.57 cm
RF frequency	1.3 GHz
Cavities	19 x 120 kV
Injector	ALICE (7-35 MeV)



UK funding ($16M) started April 2007.
Construction under way at Daresbury Lab.

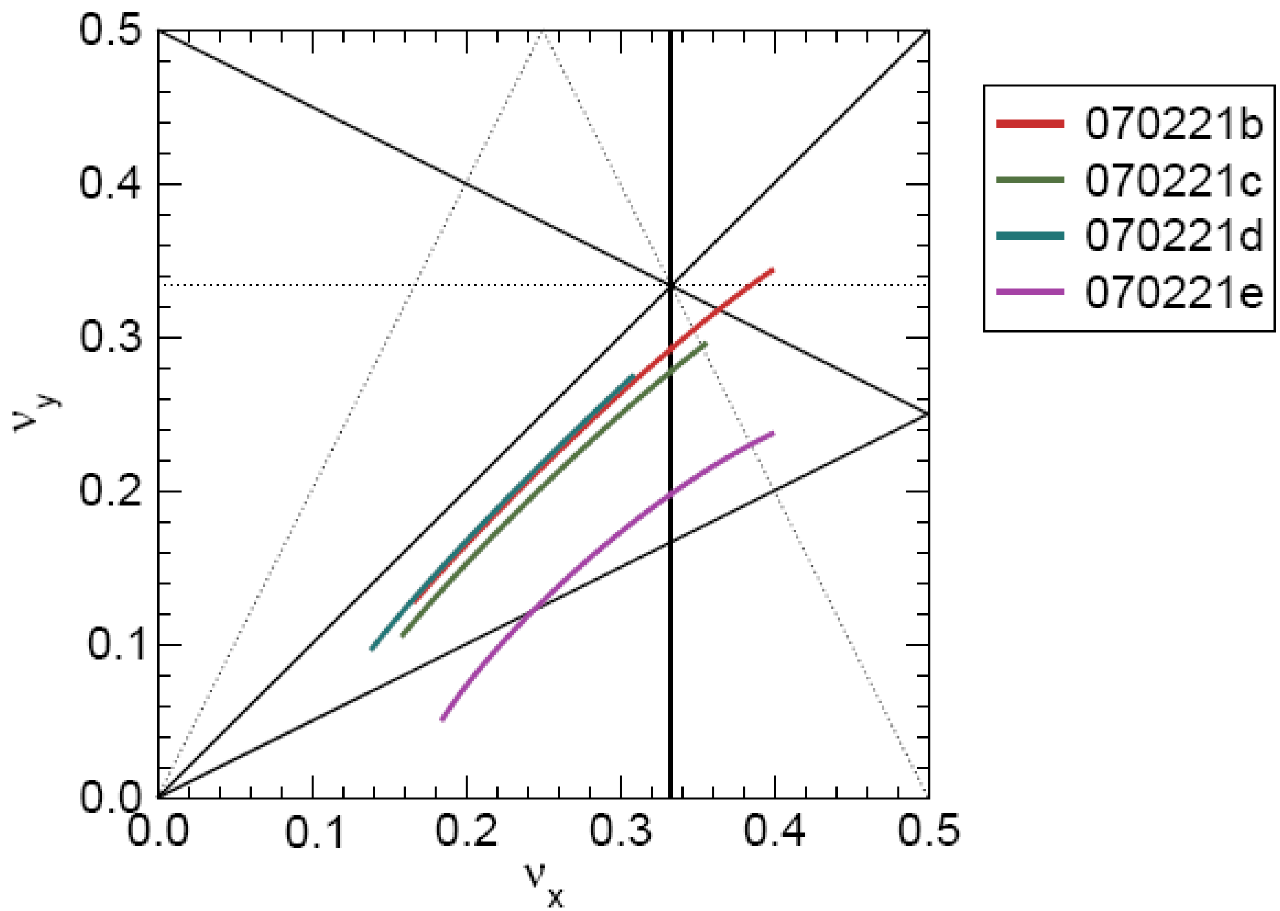
RAL 1957 2007
Celebrating 50 years of world class science



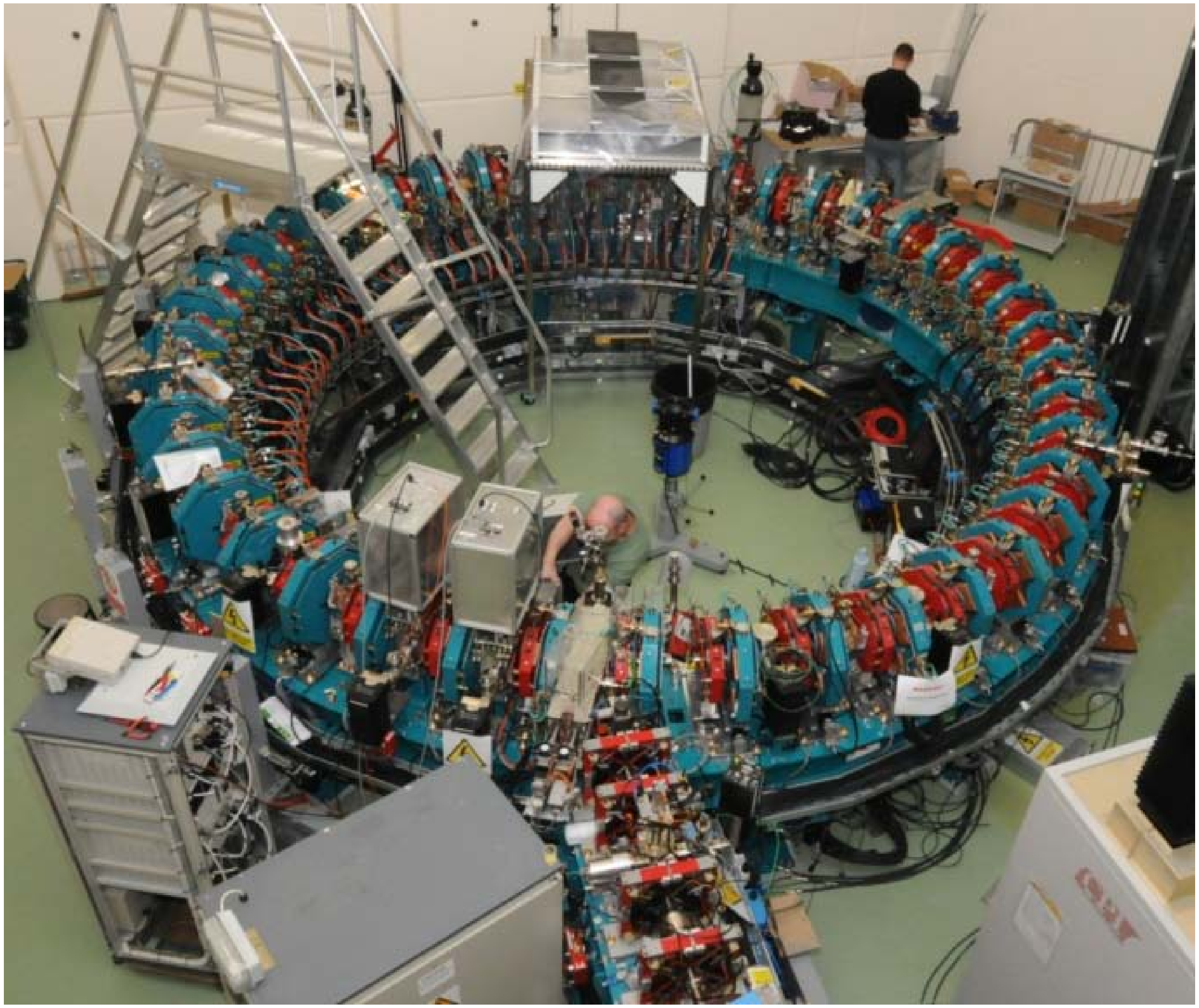


EMMA - Design

- Demonstrate features of linear, non-scaling FFAG
- Possible upgrade to non-linear in future
- Main parameters:
  - electrons, 10-20I
  - linear magnets, c
  - 42 cells, doublet
- In addition
  - very flexible
  - injection into full
  - lots of diagnostic
  - need flexible (10
  - small
  - not too expensive!

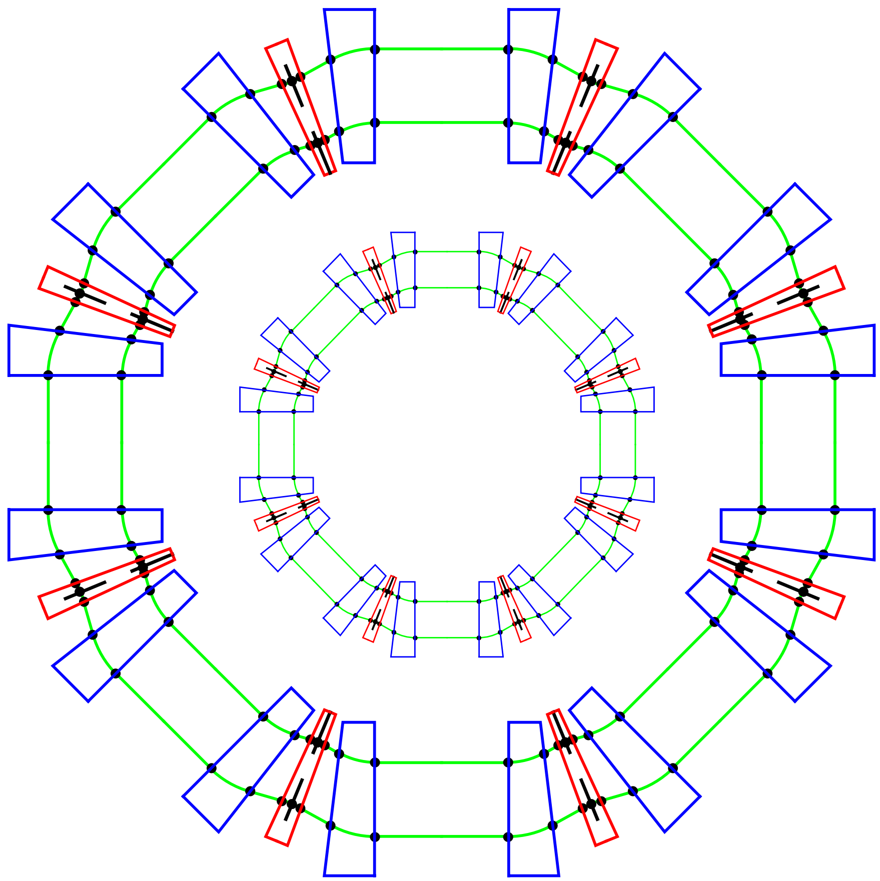


EMMA – THE FIRST NON-SCALING FFAG



EMMA is a 10-20 MeV electron LNS-FFAG model
for a 10-20 GeV muon accelerator for a neutrino factory
- currently undergoing beam commissioning at Daresbury, UK.

Johnstone-Koscielniak "Tune-Stabilized" NLNS-FFAGs

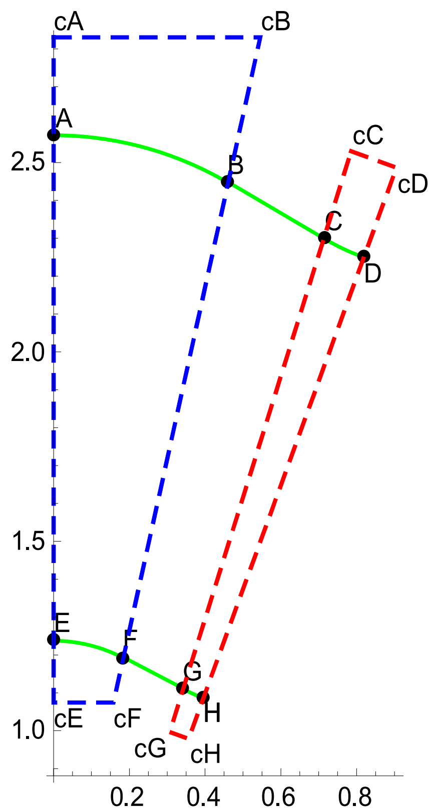


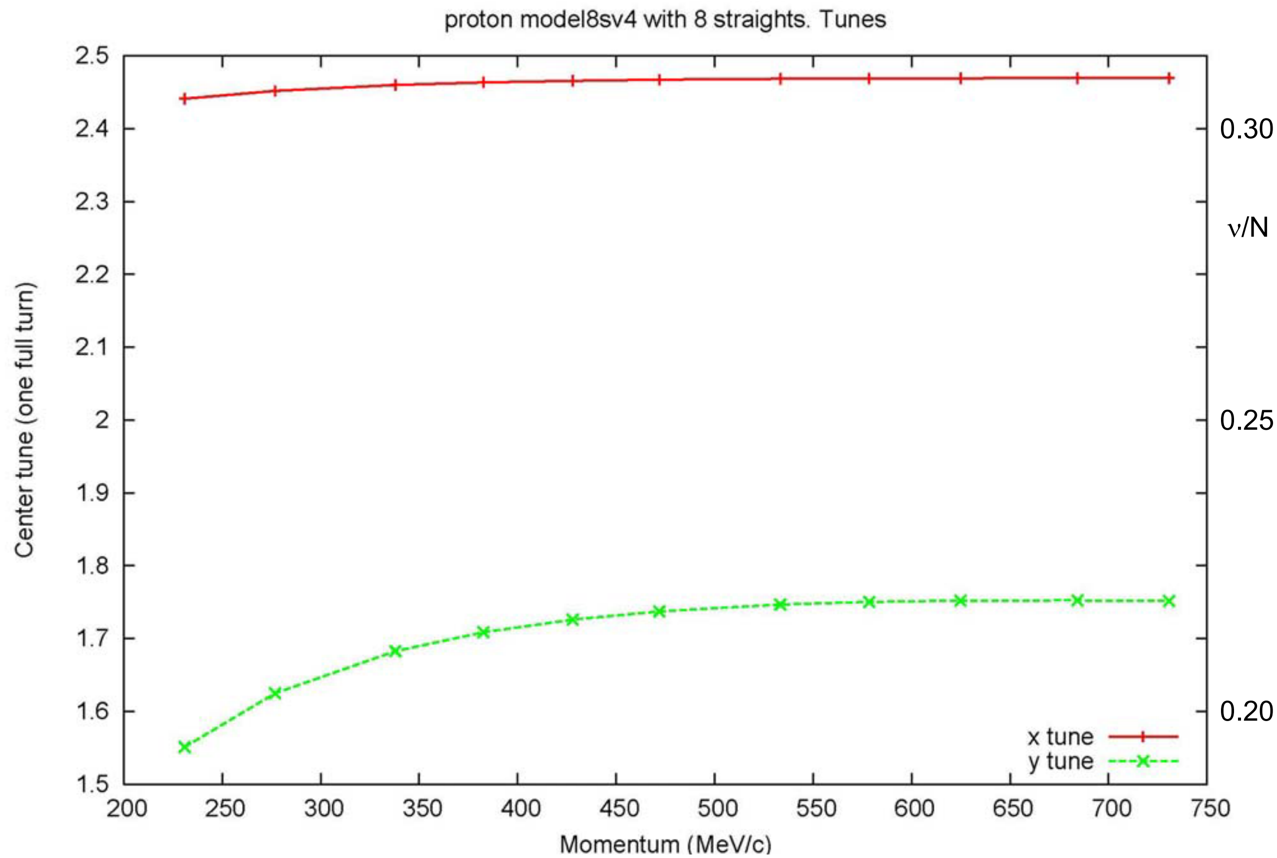
A pair of nested 8-cell-FDF rings form a multi-ion cancer treatment facility. The inner ring (orbit radii 2.75-3.39 m) takes protons from 30-250 MeV carbon ions, and acts as injector to the outer ring (5.5-6.9 m) for carbon ions.

Tune Stabilized NLNS-FFAGs (2)

Tune drop-off with energy is avoided by:

- employing the "edge focusing" that occurs for non-perpendicular magnet entry/exit
- allowing a non-linear B(r) field variation

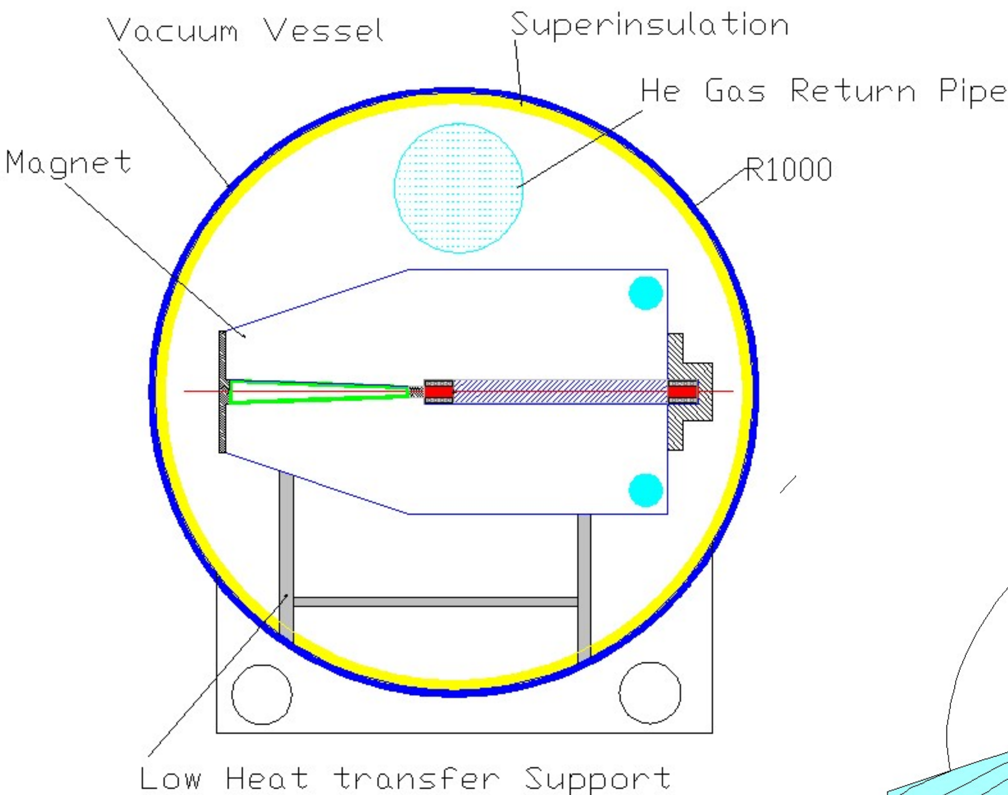


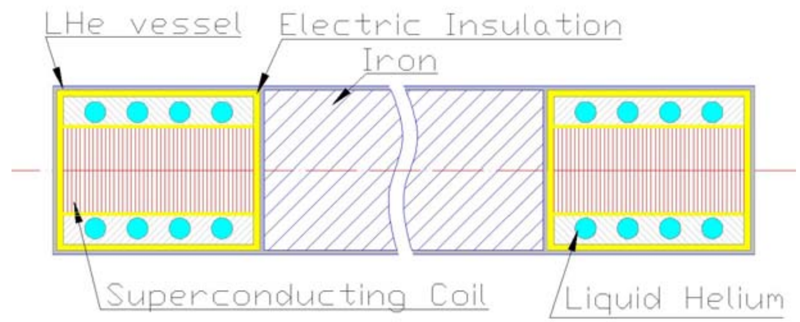


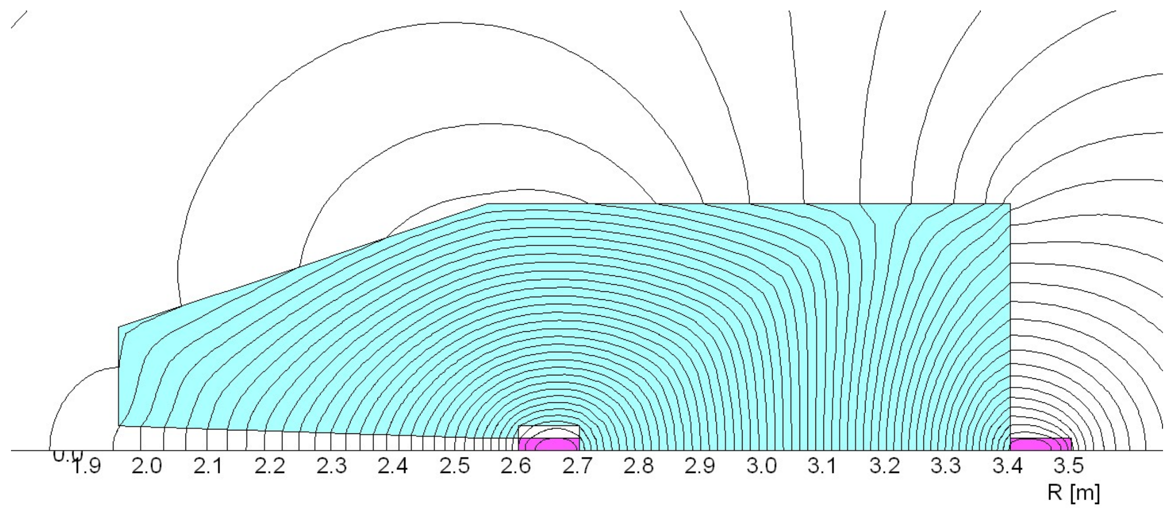
Nearly flat tunes are obtained, with large dynamic apertures.

Tune Stabilized NLNS-FFAGs (3)

4-T superconducting magnet designs have been prepared



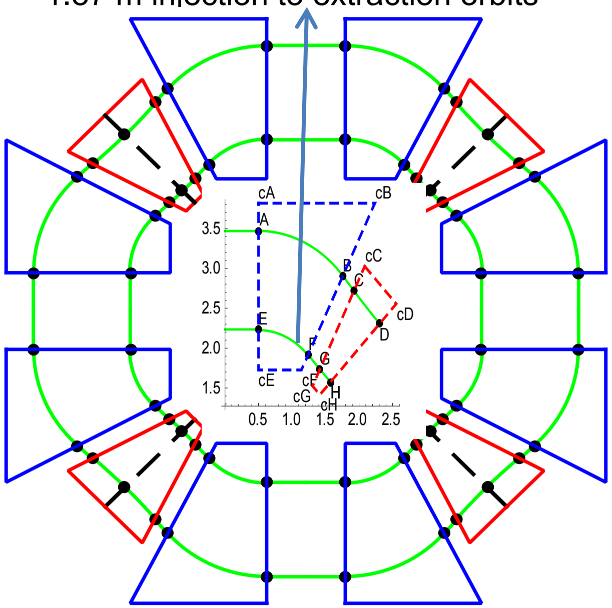




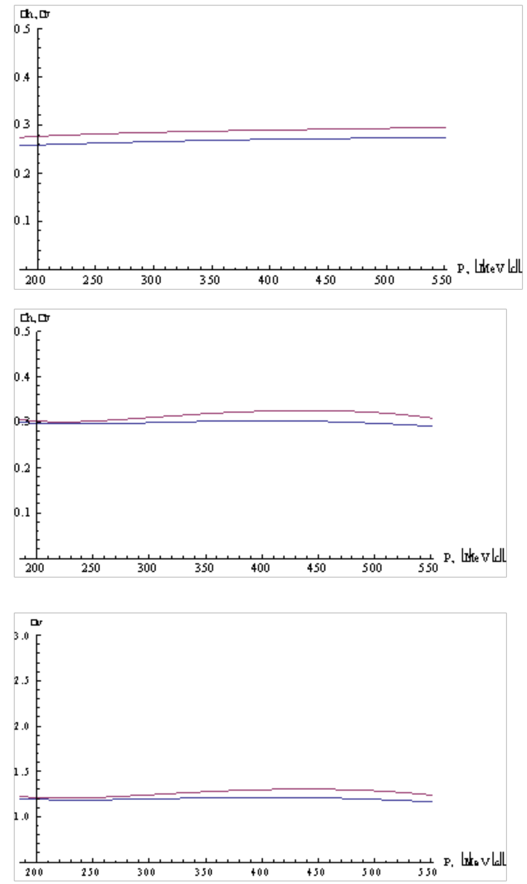
Isochronous Lattices: nonscaling nonlinear FFAGs

- First ring: 18 MeV – 150 MeV isochronous H- FFAG – this energy range was chosen to make a "standalone" therapy machine to drive "low-energy" fixed rooms and SC high-energy ring

3 m outer machine radius, 1m straights, 1.07-1.87 m injection to extraction orbits



Physical layout of 18-150 MeV 4 sector isochronous NC ring

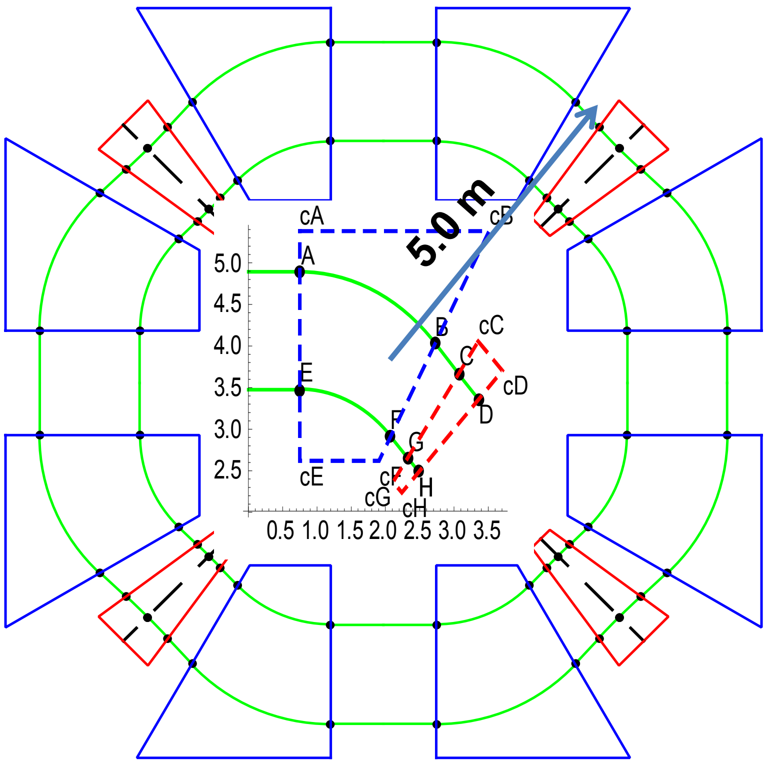


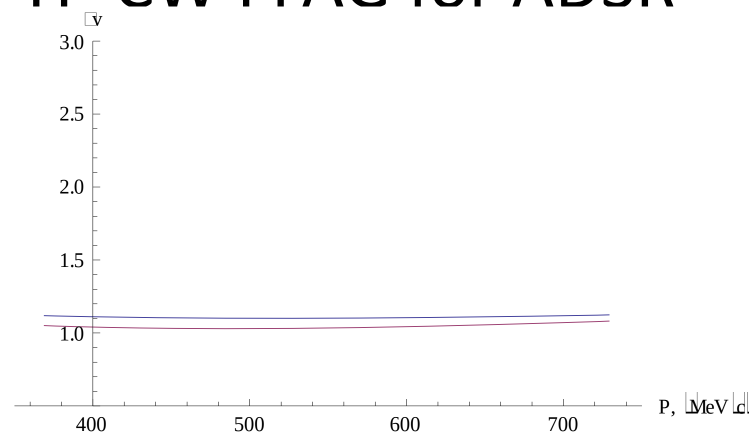
Tune per cell with just quad + sext field profile (top) and then adding octupole (middle) and ring tune (bottom)

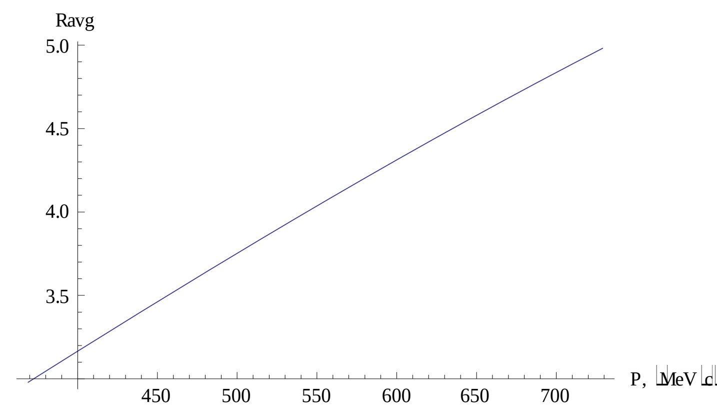
Mathematica® initial parameters: field profile up to sextupole only

A 30-250 MeV H- CW FFAG for ADSR

1.5 m straight





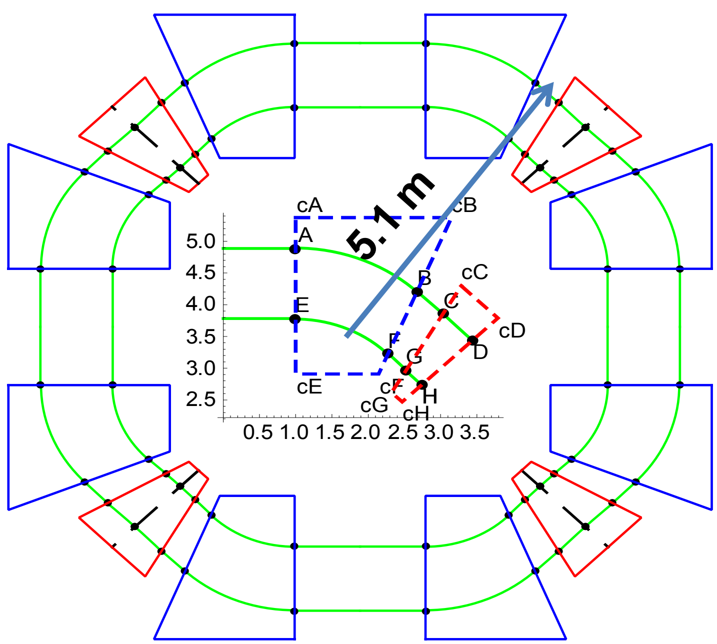


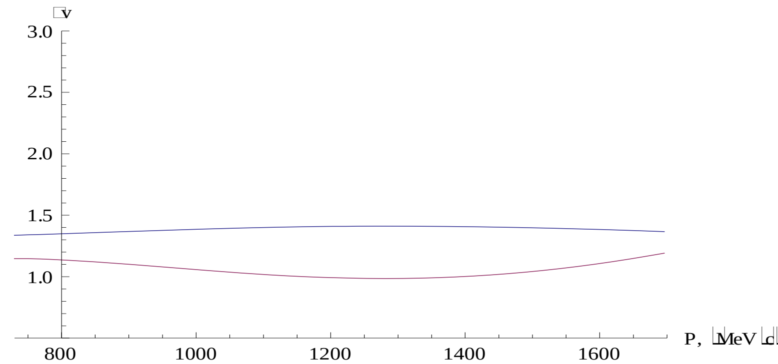
tune splitting horz/vert will drive horiz above and vertical below integer, but
radial dependence will remain well reproduced, B<9T @extractiion

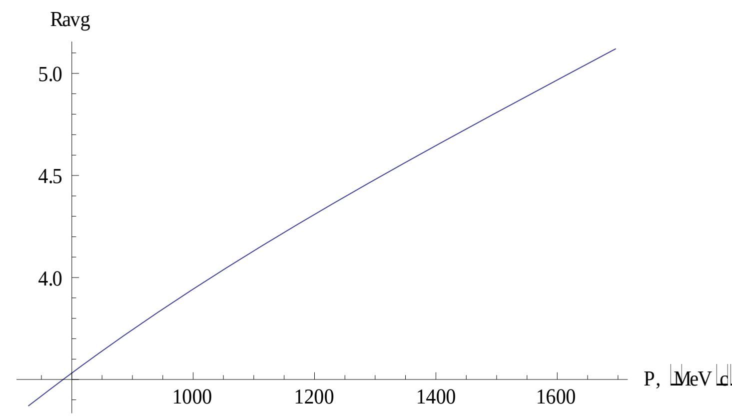
A 250 -1000 MeV Proton CW FFAG for ADSR

Mathematica® initial parameters: field profile up to octupole

2.0 m straight,
1.7 m aperture



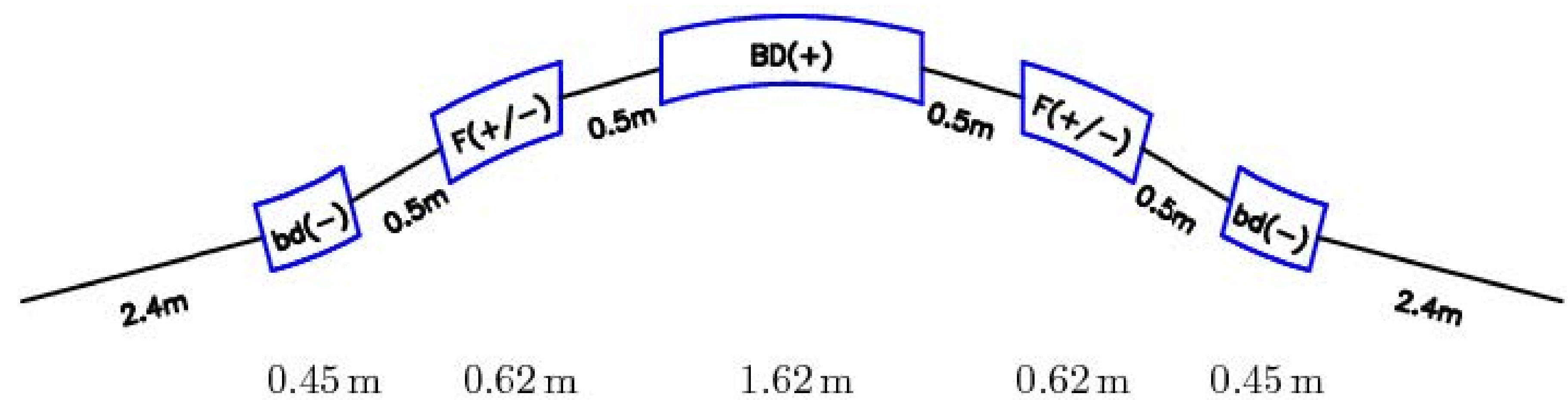




Ring tune and circumference B< 2.4 T

NON-LINEAR NON-SCALING LATTICES

G.H. Rees has designed several FFAGs using novel 5-magnet "pumplet" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius – providing great flexibility – even allowing well-matched insertions!



- an isochronous "IFFAG" for muons (8-20 GeV, N = 123, *C* = 1255 m, 16 turns, – as illustrated - or with insertions, N = 4 x (20 arc + 10 str.), *C* = 905 m)
- an IFFAG muon booster (3.2-8 GeV, 8 turns)
- an IFFAG electron model (11–20 MeV, N = 45, *C* = 29.3 m)
- a ν Factory proton driver (3-10 GeV, N = 66, *C* = 801 m, 50 Hz, 4 MW)
- a νF driver electron model (3.0-5.45 MeV, N = 27, *C* = 23.8 m)

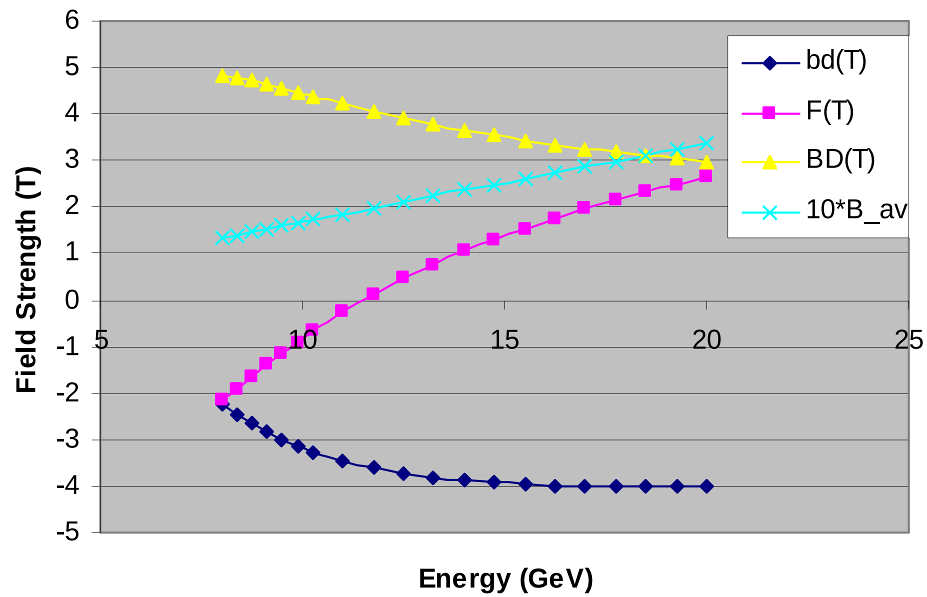
REES'S ISOCHRONOUS IFFAG

G.H. Rees's IFFAG[1] is remarkable in achieving both isochronism and vertical focusing at highly relativistic energies ($77 \leq \gamma \leq 190$) without invoking spiral magnet edge focusing

[Recall that isochronous B(r) gives $\Delta\nu_z^2 = -(r/B)(dB/dr) = -\beta^2\gamma^2$ – and that the highest energy spiral-sector isochronous cyclotron design had $\gamma \leq 15$.]

The field profiles for the bd, F and BD magnets (right) show how:

- F reverses sign at ~11 GeV
- bd focusing vanishes at high E
- BD focusing vanishes at low E
- B_{av} rises linearly with E
- The vertical defocusing associated with rising B_{av} is offset by **strong AG focusing**.

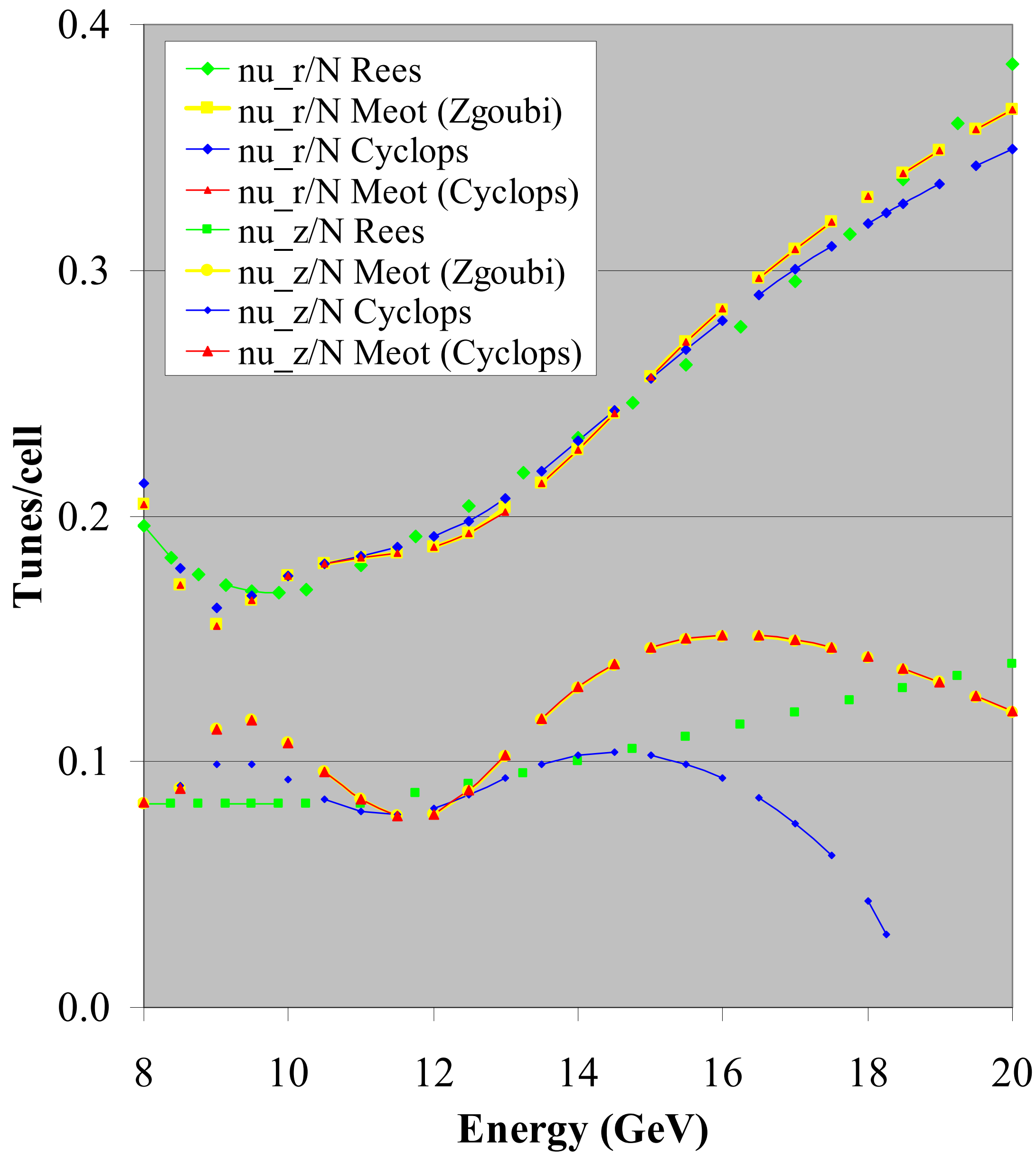


1. G.H. Rees, *FFAG'04* (2004); *FFAG'05* (2005); *ICFA-Beam Dynamics Newsletter* **43**, 74 (2007)

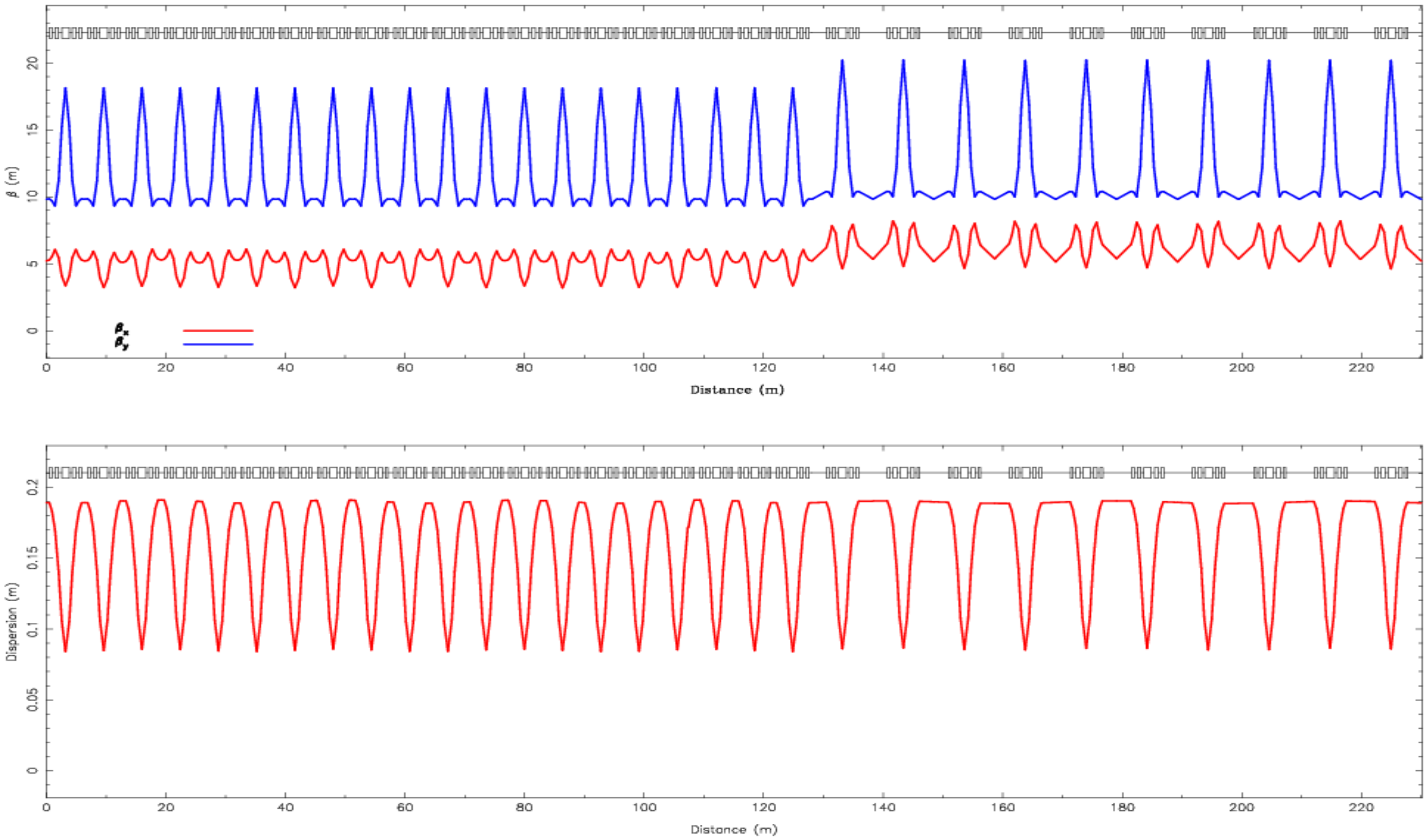
IFFAG TUNES

Tracking studies have been carried out with the ZGOUBI and CYCLOPS codes and generally confirm the tunes predicted by Rees's matrix transport code.

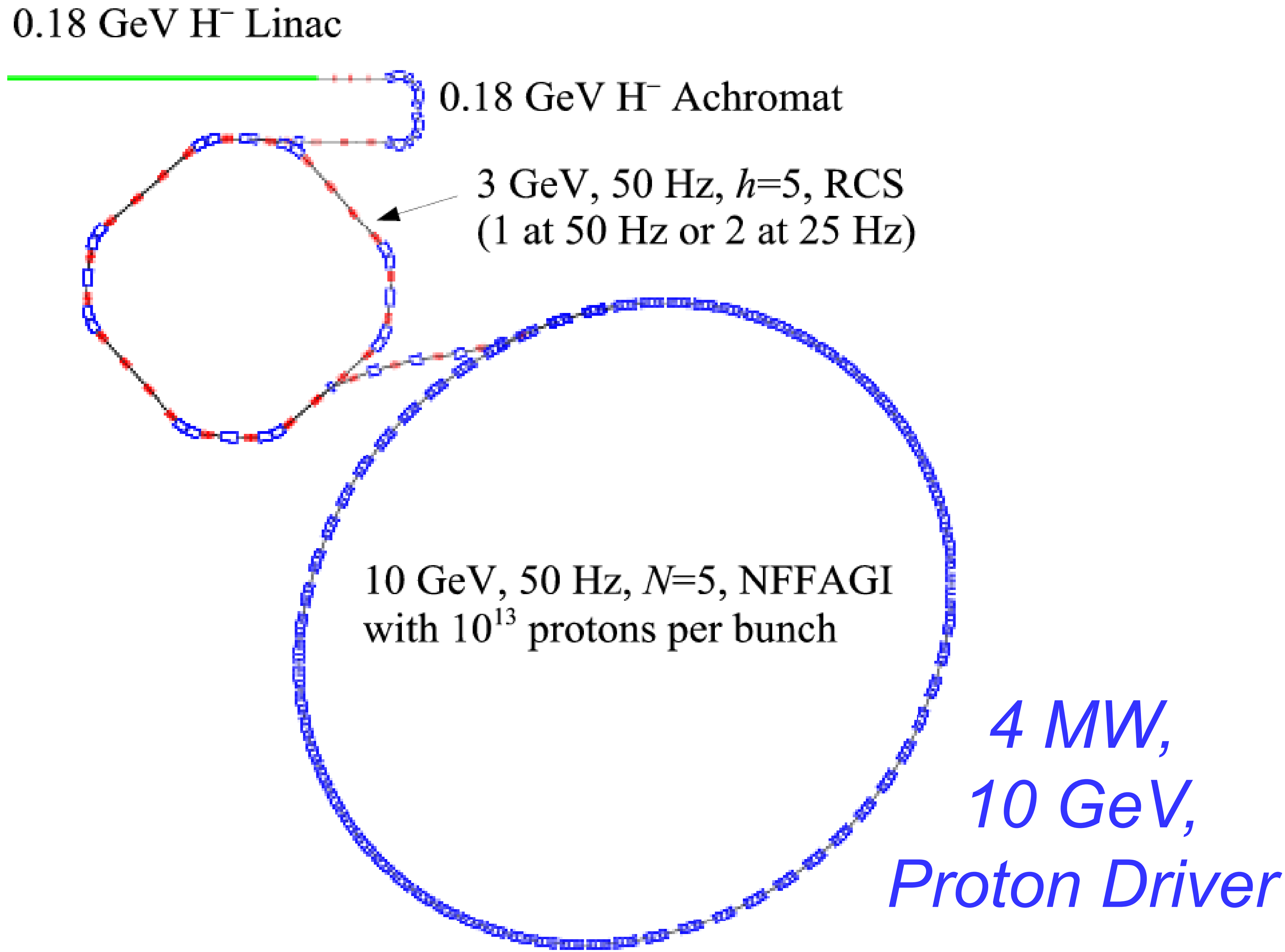
The agreement is very good for ν_r, but for ν_z there's a discrepancy. The tunes found by ZGOUBI (-■-) and CYCLOPS (-▲-) agree with each other, but oscillate around Rees's values (-■-)



Lattice Functions at 14.75 GeV



Rees (2005) has successfully incorporated long-drift insertions in an FFAG


0.18 GeV H⁻ Linac
0.18 GeV H⁻ Achromat
3 GeV, 50 Hz, h=5, RCS
(1 at 50 Hz or 2 at 25 Hz)
10 GeV, 50 Hz, N=5, NFFAGI
with 10^13 protons per bunch
4 MW,
10 GeV,
Proton Driver

FURTHER READING

- K.R. Symon, D.W. Kerst, L.W. Jones, L.J. Laslett, K.M. Terwilliger, *Fixed-Field Alternating-Gradient Particle Accelerators*, Phys. Rev. **103**, 1837-59 (1956): http://prola.aps.org/abstract/PR/v103/i6/p1837_1
- C.H Prior (ed.) *FFAG Accelerators*, ICFA Beam Dynamics Newsletter **43**, 19-133 (2007) – 14 review articles: http://www-bd.fnal.gov/icfabd/Newsletter43.pdf
- M.K. Craddock, K.R. Symon, *Cyclotrons and FFAGs*, Rev. Acc. Sci. Tech. **1**, 65-97 (2008) – a review for the general reader: http://www.worldscinet.com/rast/01/preserved-docs/0101/S1793626808000058.pdf or http://trshare.triumf.ca/~craddock/RAST-cycFFAG.pdf