

# 10 YEARS OF FFAG DEVELOPMENT

Yoshiharu Mori  
Kyoto University, Research Reactor Institute

# 10 YEARS OF FFAG DEVELOPMENT

mostly in Japan

Yoshiharu Mori

Kyoto University, Research Reactor Institute

# HISTORY OF FFAG

- Backgrounds
  - Alternating Gradient(strong) focusing (by Christofolus, Courant, Snyder, Livingston)
  - Phase stability in rf acceleration (by Macmillan) :synchrotron oscillation
- Innovation of FFAG:1950's
  - Okawa(Japan), Kerst & Simon(USA), Kolomenski (USSR)
  - FODO lattice with Zero-chromaticity : betatron tunes are constant
    - Magnetic field  $\rightarrow \mathbf{B} \propto r^k f(\theta)$  : static magnetic field
  - No-isochronism : Acceleration  $\rightarrow$  Phase stability with synchrotron oscillation

FFAG'03 at KEK

T.Okawa

# FFAG'03 at KEK



T.Okawa

FFAG'03 at KEK

T.Okawa

# HISTORY OF FFAGS

- Idea

- 1950s Okawa(Japan), Kerst-Symon(USA), Kolomenskii (USSR)

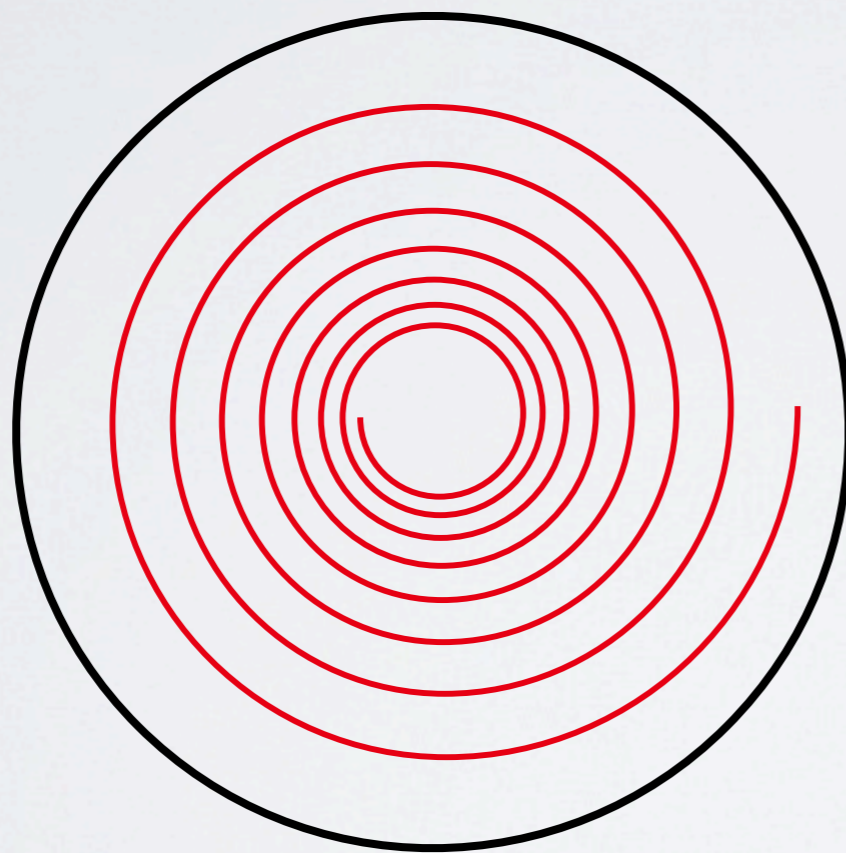
- Developments

- 1960s MURA project (USA) Electron models
- 2000 POP-FFAG (KEK, Japan) First proton FFAG
- 2004 150-MeV proton FFAGs (KEK, Kyusyu, Japan)
- 2005 R&Ds for various applications:RACAAM(Grenoble, France), PD(FNAL, USA), etc.
- 2008 Proton FFAGs for ADSR (Kyoto, Japan)
- 2008 PRISM-FFAG for muon (Osaka, Japan)
- 2009 e-FFAG(NHV, Japan)
- 2010 EMMA(Daresbury, England) First non-scaling FFAG

# FIELD INDEX

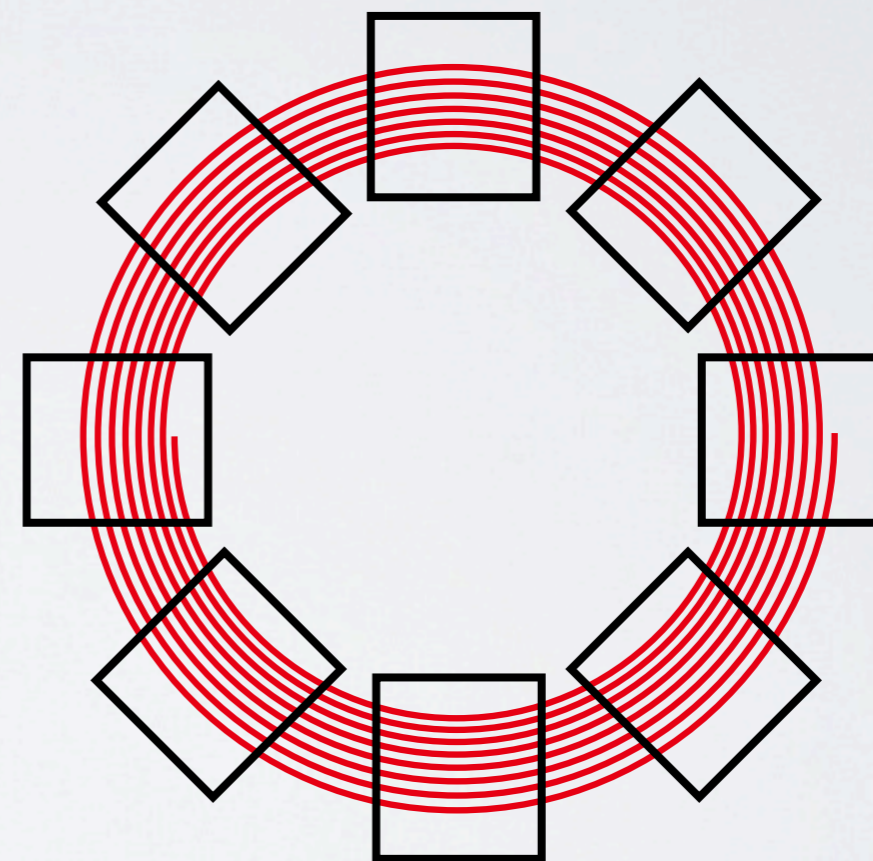
## ORBIT EXCURSION

Field Index  $\rightarrow k = -\frac{r}{B} \frac{\partial B}{\partial r}$



$$k \approx 0$$

Cyclotron



$$k \gg 1$$

FFAG

# TYPES OF FFAG OPTICS

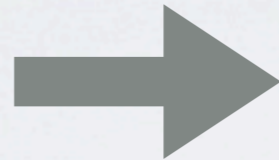
- Zero chromaticity : Scaling FFAG
  - Betatron tunes during acceleration are constant.
  - Free from resonance crossing.
  - Orbit configurations for different beam momentum(energy) are (nearly) similar.
  - Very Large momentum acceptance :  $\Delta p/p > \pm 100\%$
- Non-zero chromaticity : Non-scaling FFAG
  - Optical elements are all linear : dipole and quadrupole magnets.
  - Betatron tunes are varied during acceleration.
  - Need fast resonance crossing : very fast acceleration.
  - Large dynamic aperture

# AG FOCUSING LATTICE OF SCALING FFAG RING

$$B_z = B_0 \left( \frac{r}{r_0} \right)^k f(\theta)$$

- AG focusing : FODO lattice

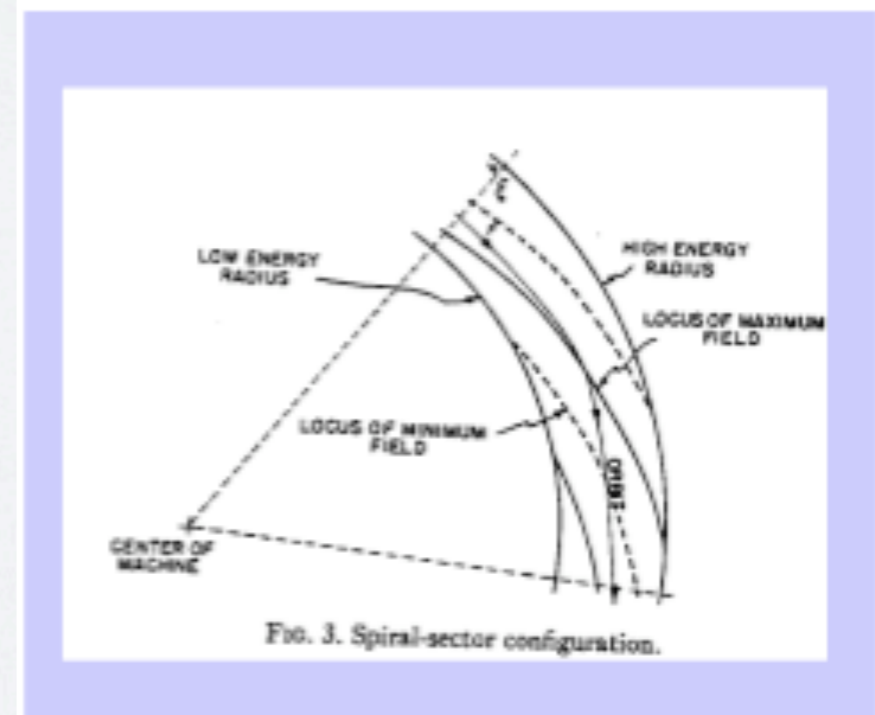
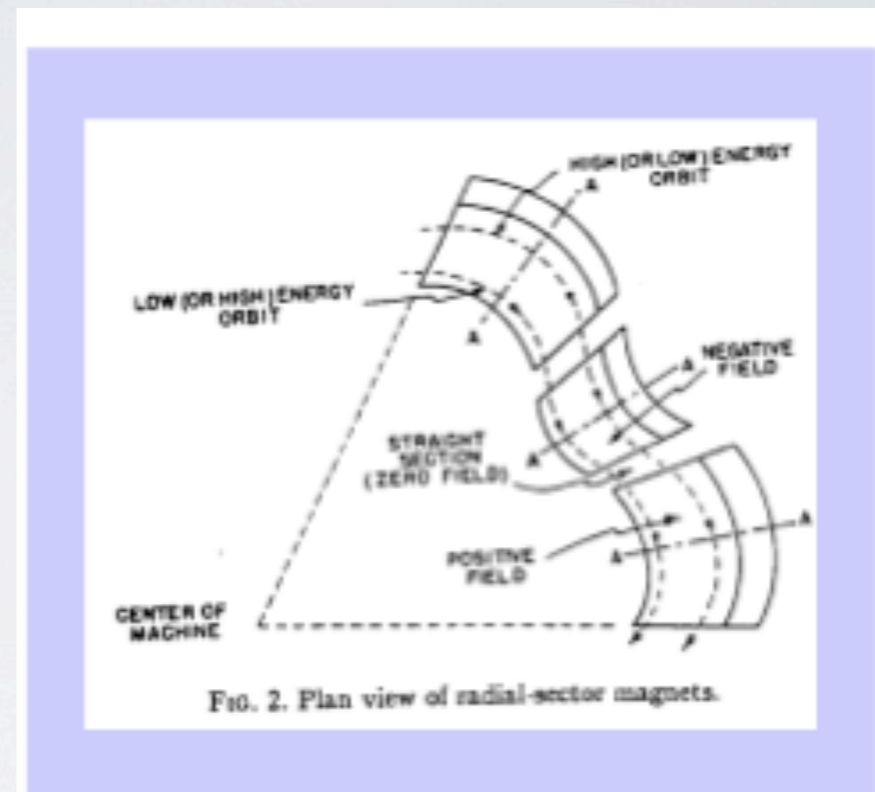
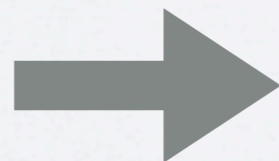
- Radial sector



- F: positive bend
- D: negative bend

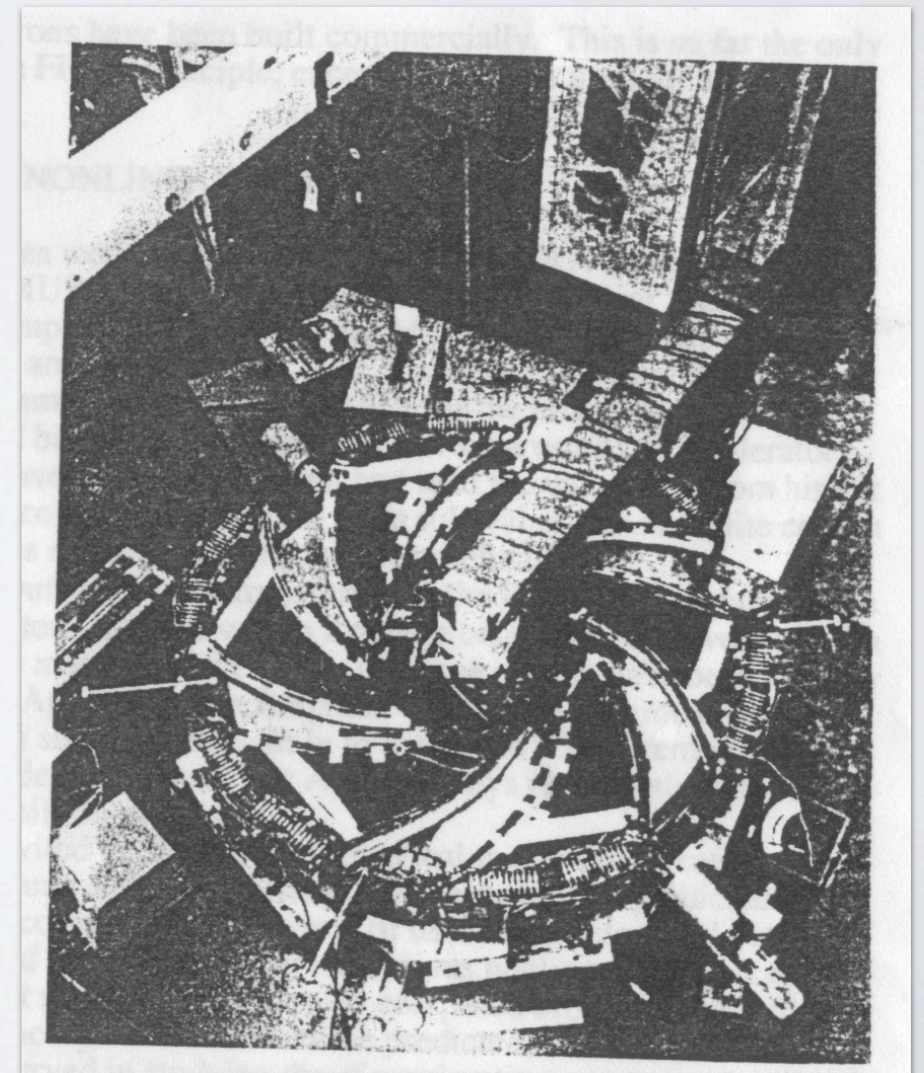
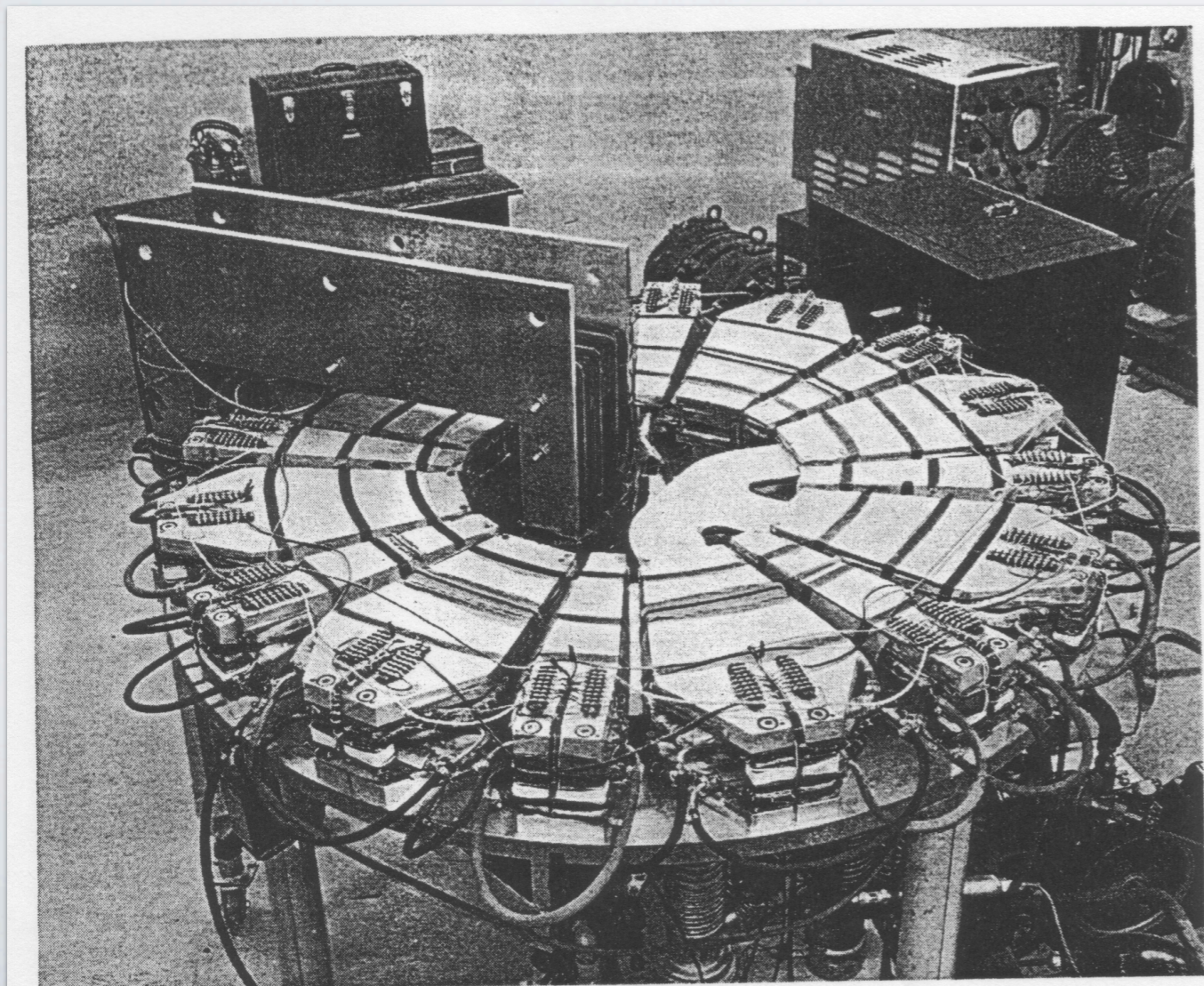
- Spiral sector

- F: positive bending
- D: edge focusing



# MURA FFAG ELECTRON MODEL

- 1960's MURA project (USA) : Cole et al.
- Acceleration      Induction (betatron)  
                            No practical RF acceleration
- No proton acceleration



# DIFFICULTIES

## HADRON(PROTON) ACCELERATION IN FFAGS

- Need a new rf accelerating cavity.
  - broad-band and high gradient
    - Particle velocity changes in wide range.
    - Rooms for the rf cavity are limited in the ring because of its compactness and high super-periodicity.
- Need a non-linear(high gradient) field magnet.
  - careful 3D design of magnetic field
    - Zero chromaticity is very needed because momentum gain per turn is relatively small compared with that of electron.

# REQUIREMENTS OF RF CAVITY

- Broad band
  - Frequency sweep of a factor.
- High gradient
  - Make it fast acceleration possible.
- Large aperture
  - Especially in horizontal to accommodate orbit excursion.
- A few MHz to have large longitudinal acceptance

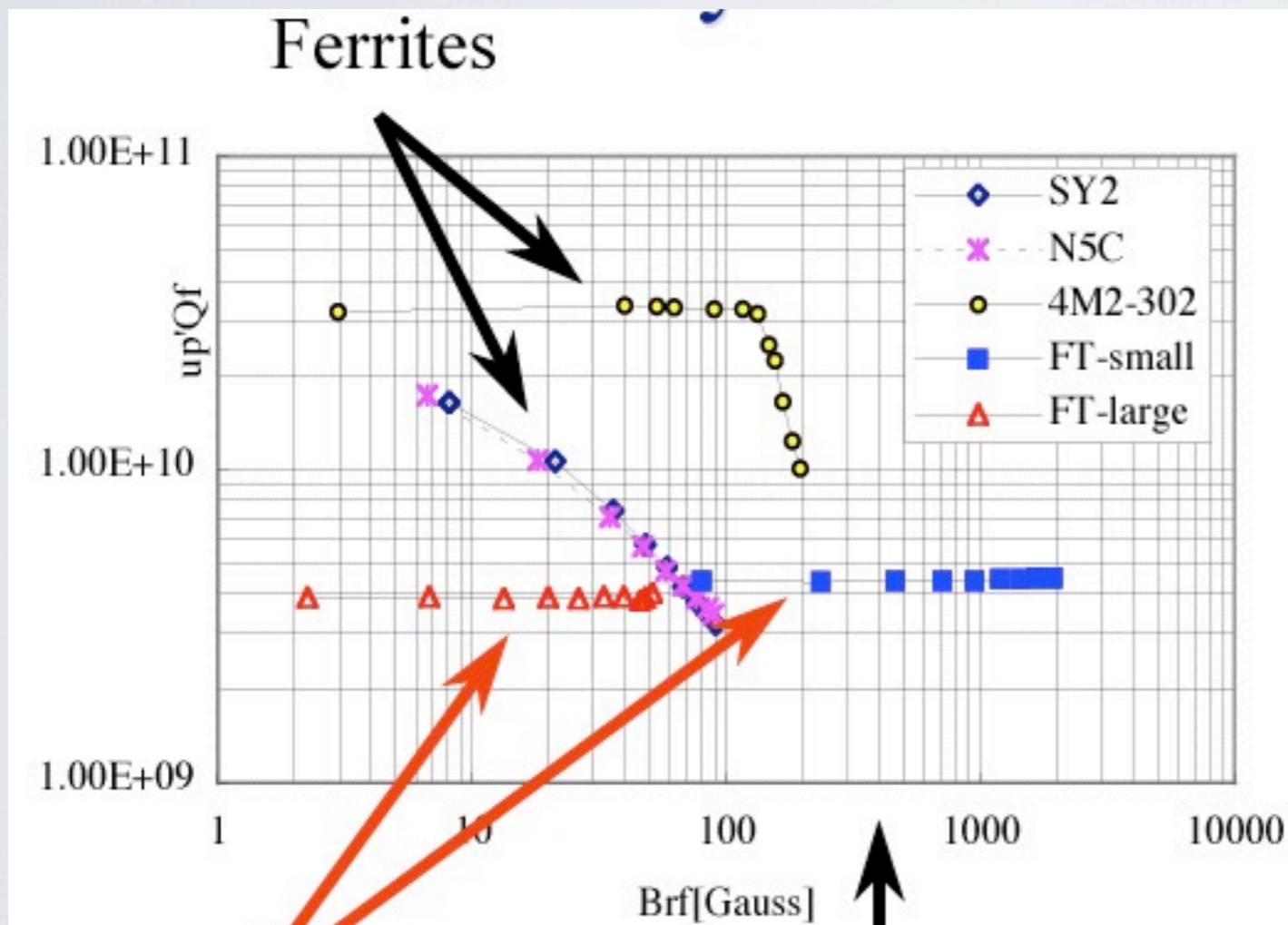
 **RF cavity with Magnetic Alloy has been developed at KEK for JPARC cavity.**

# PERSONAL HISTORY

- 1995 ~2004 Design, development and construction of J-  
PARC synchrotrons (3GeV, 50GeV)
- Colleagues: Machida, Ishi(ring design), Yoshii, Ohmori(rf),  
Tomizawa(slow extraction), Muto(magnet) .....

# MQF (SHUNT IMPEDANCE)

- A  $\mu$  QF remains constant at high RF magnetic RF (Brf) more than 2 kG.
- Ferrite has larger value at low field, but drops rapidly.
  - RF field gradient is saturated.



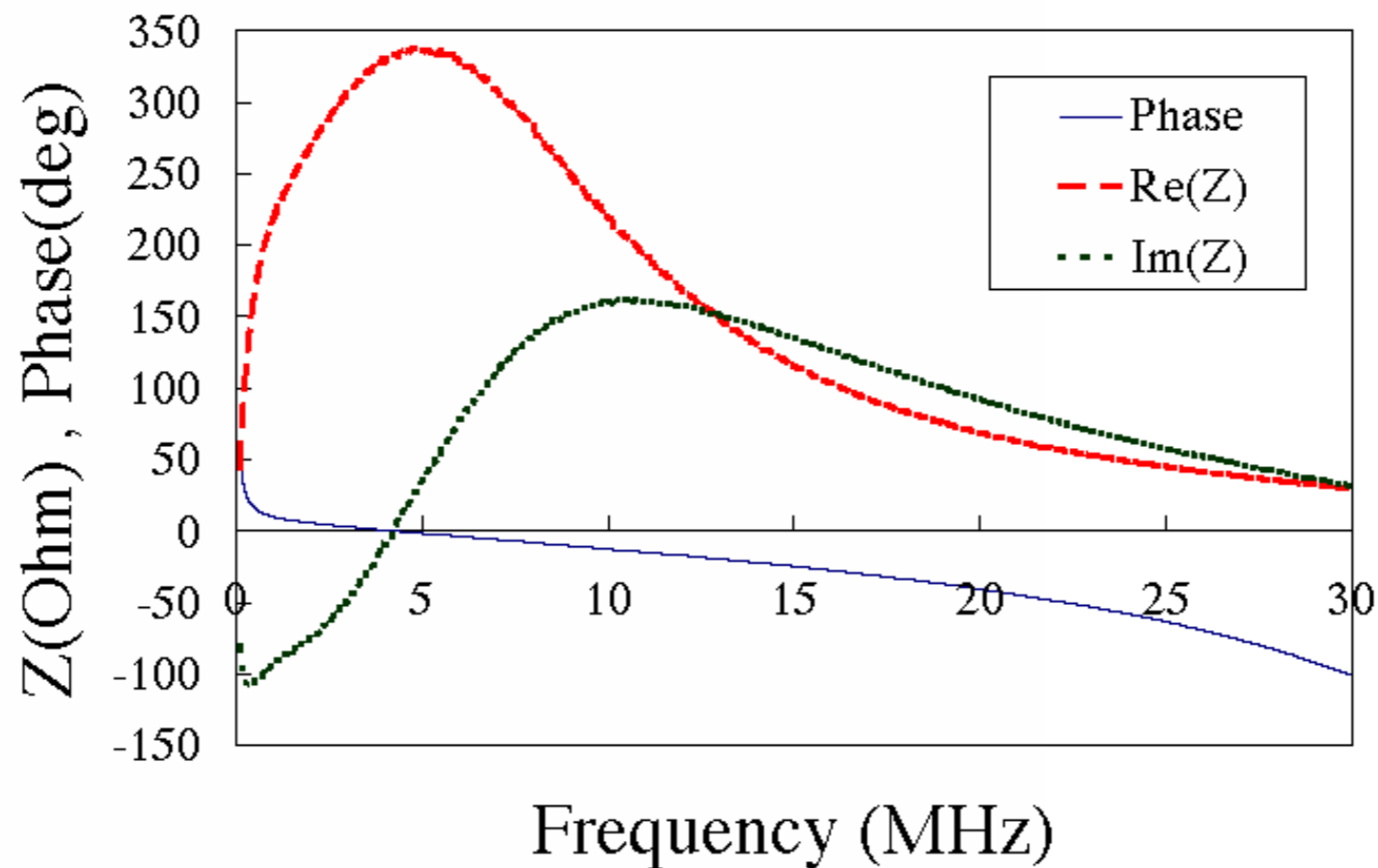
High permeability  
 ~2000 at 5 MHz  
 High curie temperature  
 ~570 deg.  
 Thin tape  
 ~18 mm  
 Q is small  
 ~0.6

Magnetic Alloys

$$B = V / \omega S = 25\text{kV} / 2\pi \times 5\text{MHz} \times 5\text{cm} \times 40\text{cm} = 400\text{Gauss}$$

# BROAD BAND

- $Q \sim 1$  : no-need to tune the rf frequency
- Rapid frequency swing is possible  $\rightarrow$  Rapid acceleration



# DEVELOPMENT OF MA CAVITY

EPAC'98 (THOB03B)

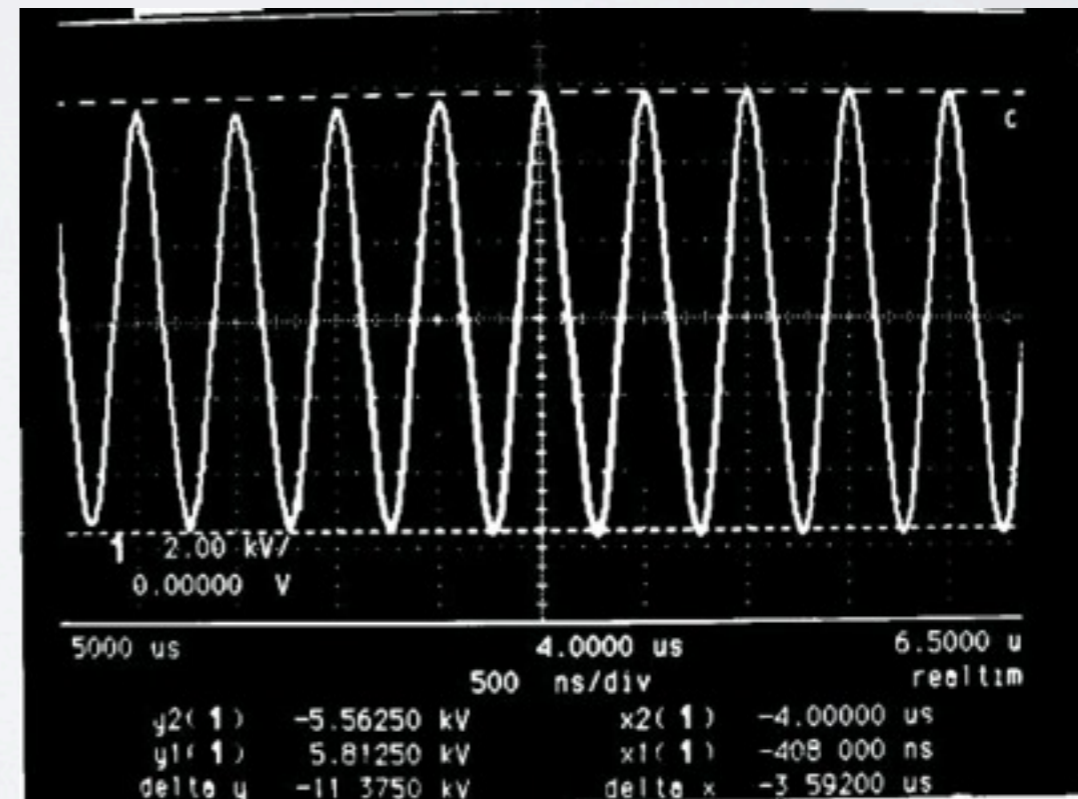
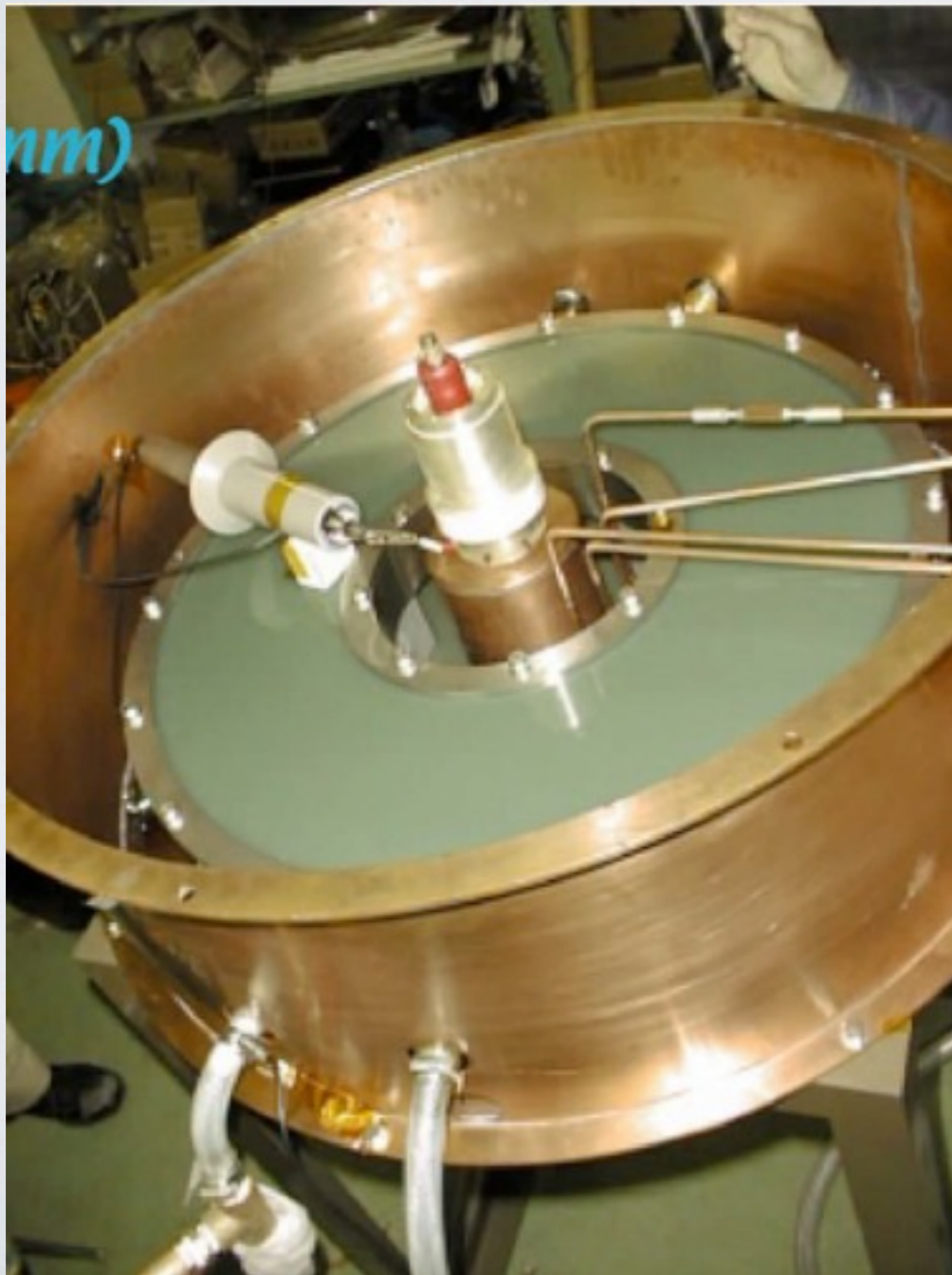
## TEST CAVITY

\*Single core (O.D=580mm,I.D=250mm, t=25mm)

\*Direct water cooling

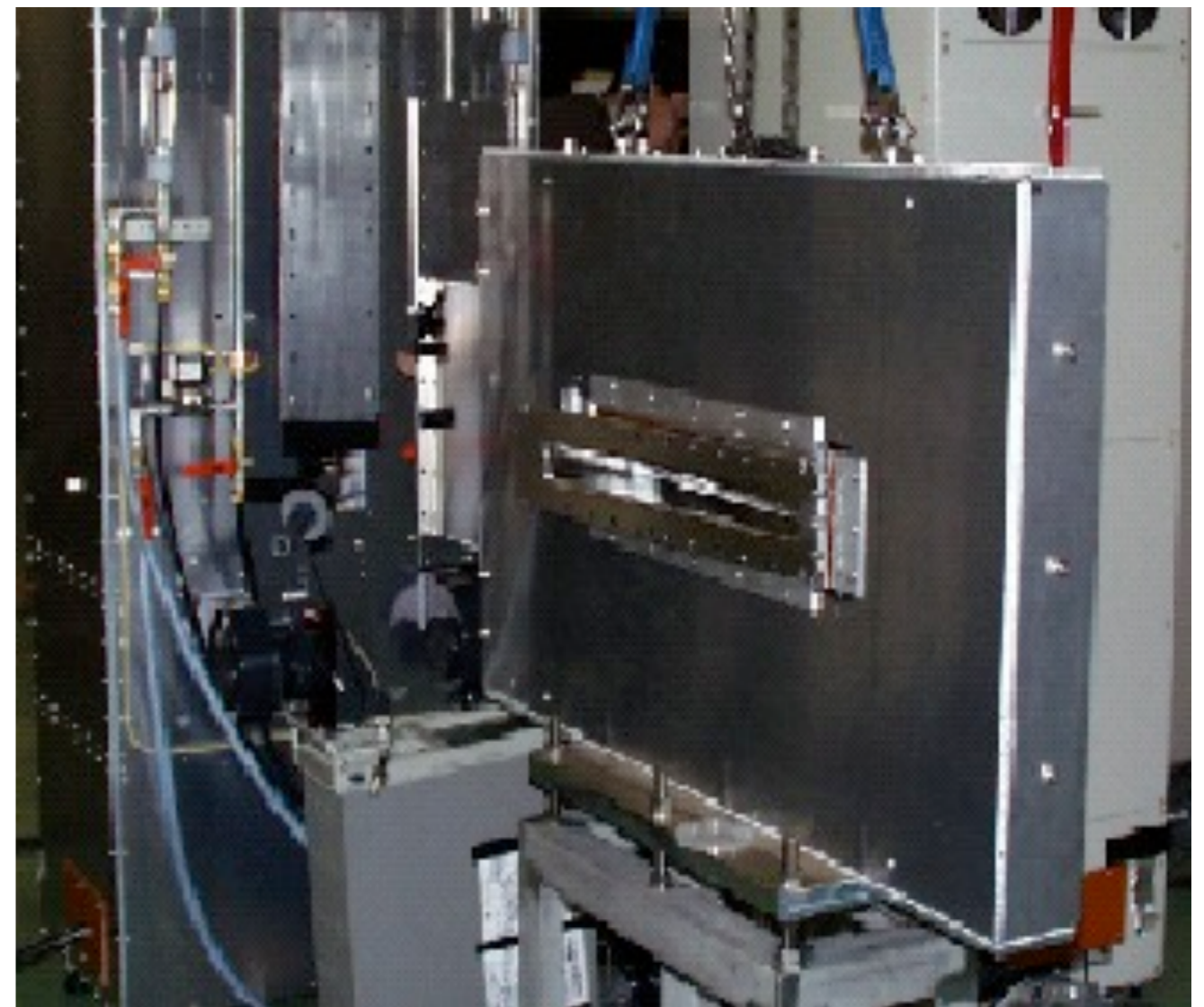
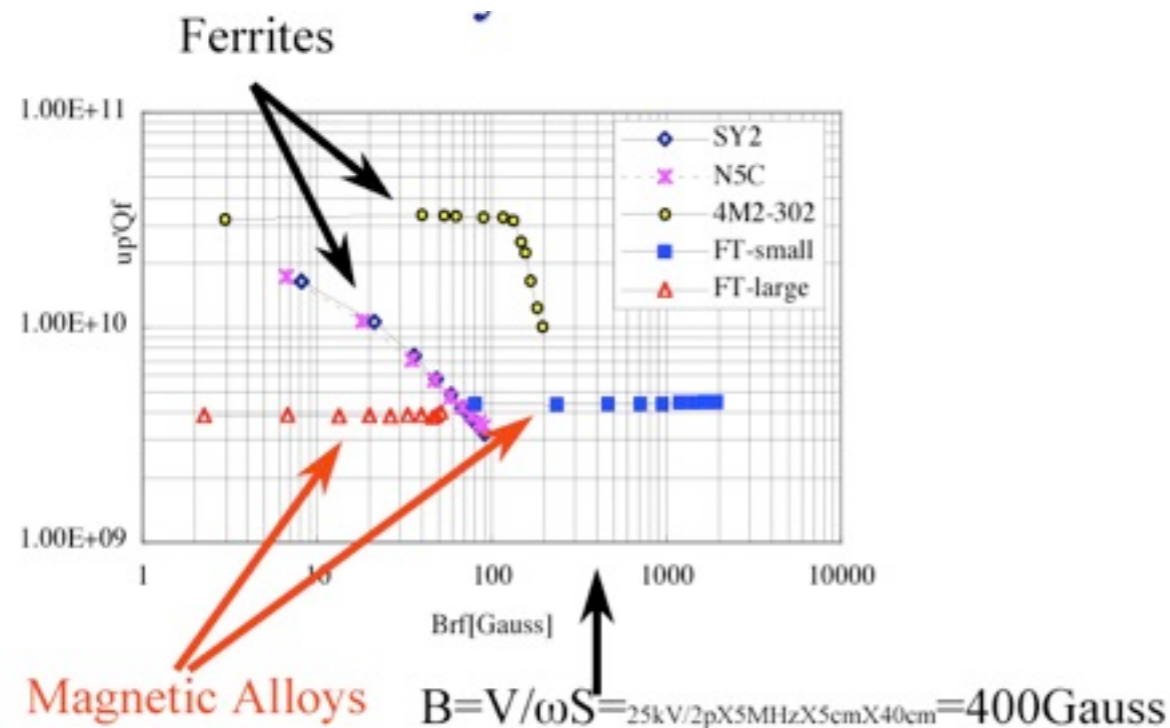
\*RF power :30kW max.(B-class)

- Direct water cooled test cavity.
- Achieved
  - 100 kV/m for CW mode
  - 220 kV/m for burst mode



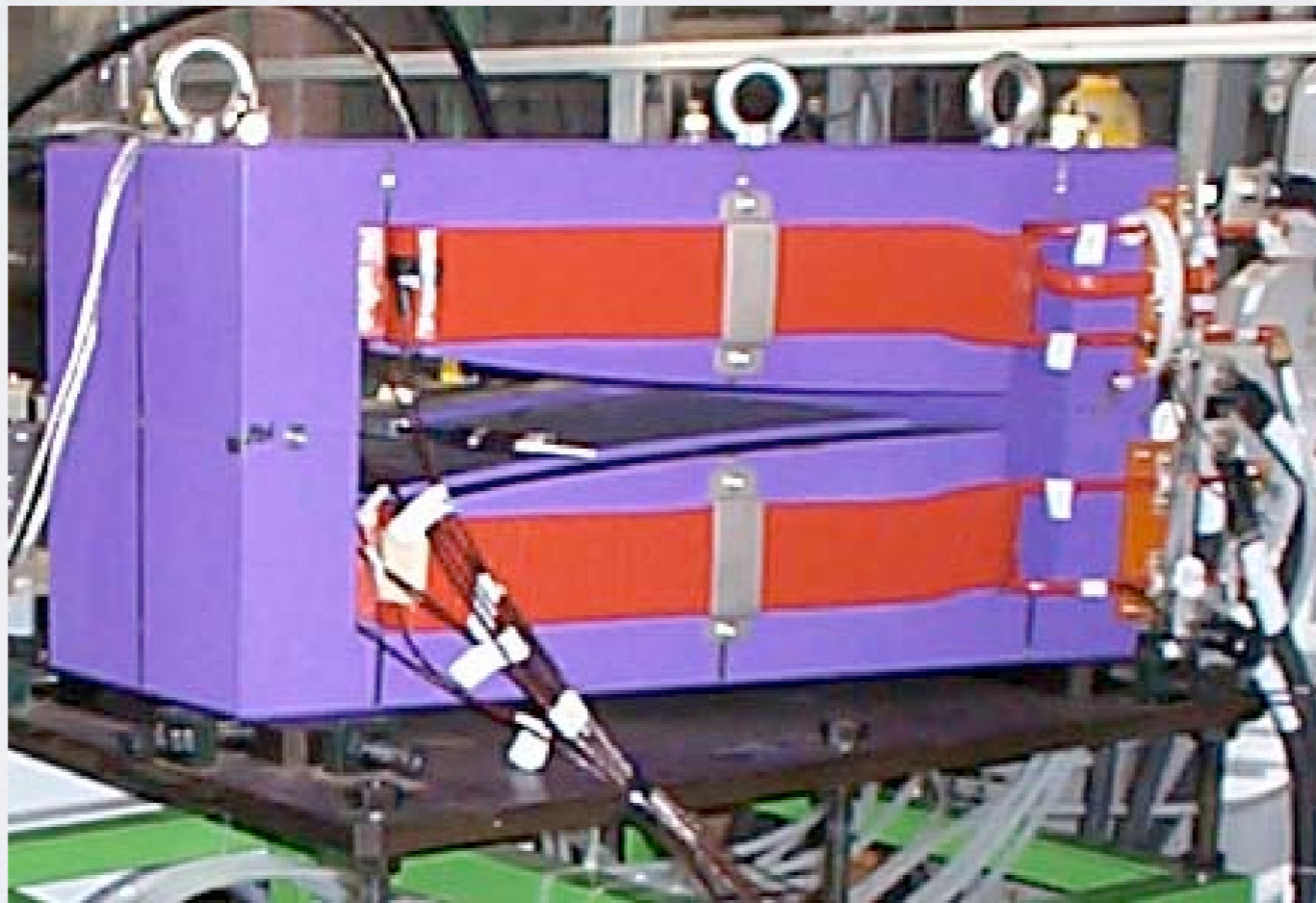
# Variable RF frequency

- Broad-band RF cavity : MA(magnetic alloy) cavity
  - Fast acceleration requires fast frequency(phase) change.
    - *Low Q ( $Q \sim 1$ ) is essential!*
  - Adequate both for scaling and non-scaling FFAGs.

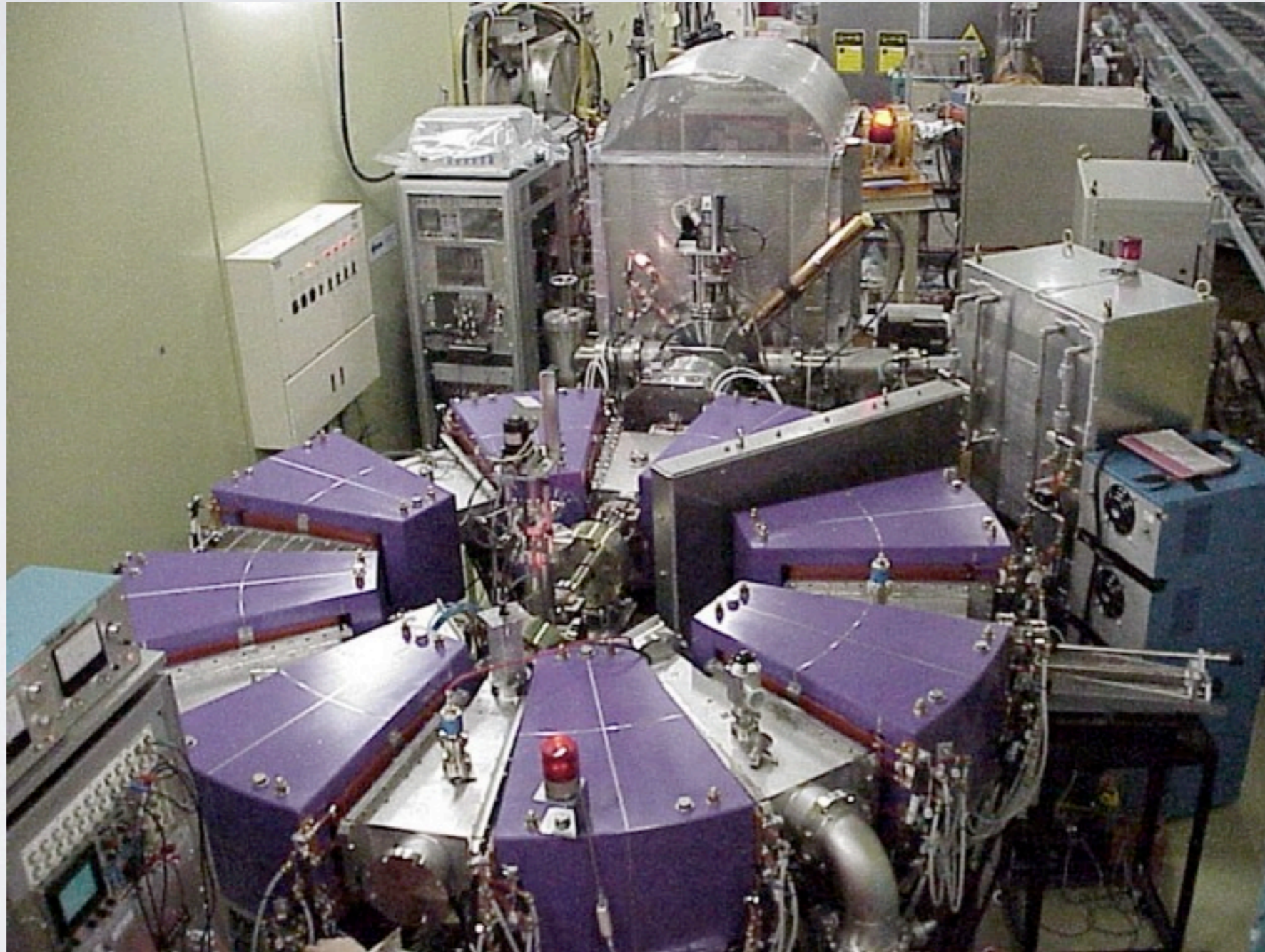


# MAGNET: TAPERED GAP

- Gap(r) is proportional to  $1/B(r)$ 
  - Easiest
  - Fringe field has wrong sign.
    - $g/r$  should be constant to have similar fringe field effects

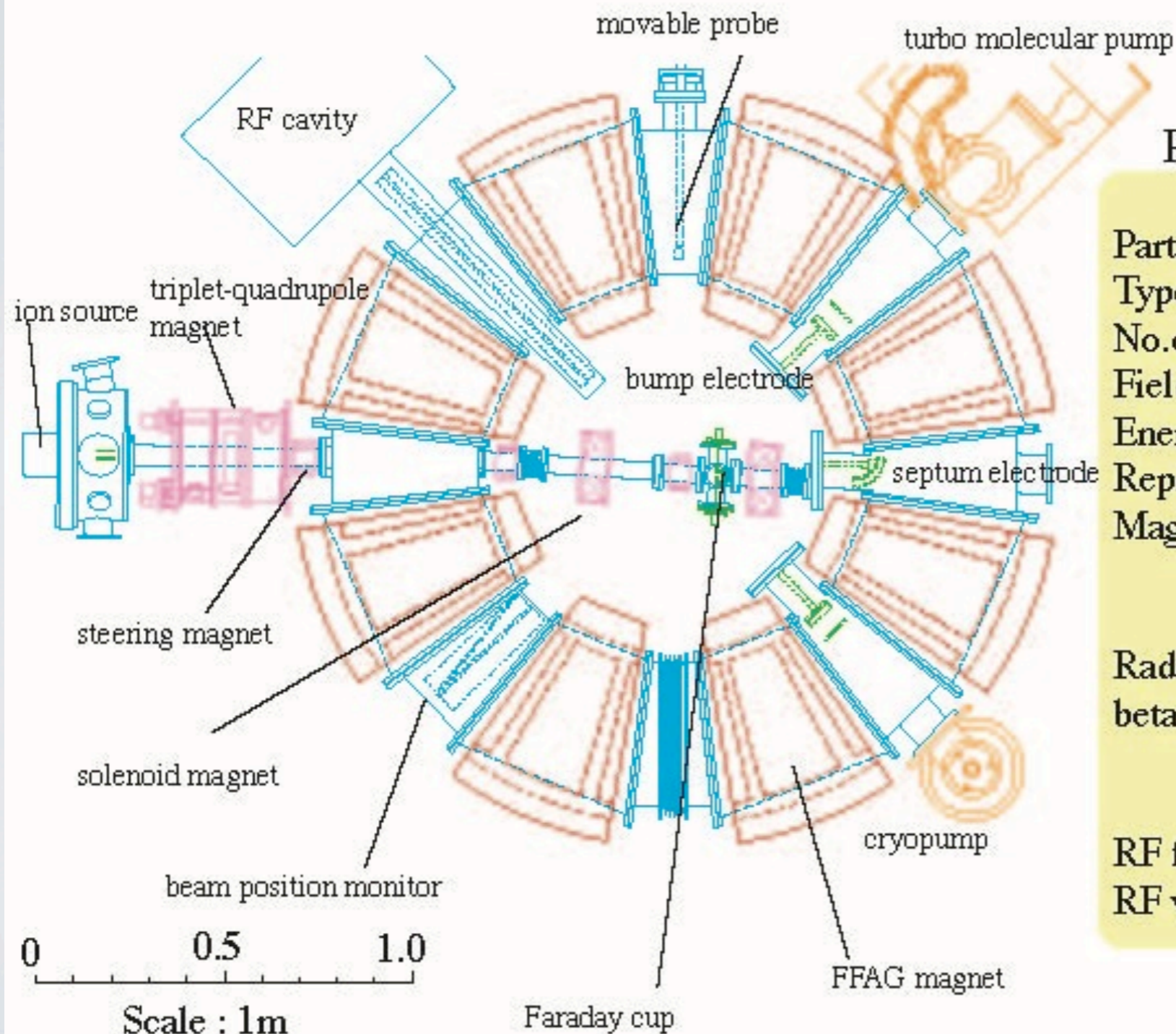


# WORLD FIRST PROTON FFAG ACCELERATOR



- PoP(proof-of-principle) FFAG :KEK 2000

# LAYOUT OF POP-FFAG



PoP-FFAG parameter table

Particle	proton
Type of magnet	radial sector type
No. of sector	8
Field index	$k=2.5$
Energy	50keV $\Rightarrow$ 500keV
Repetition rate	1kHz
Magnetic field	
Focus-mag.	0.14 - 0.32 T
Defocus-mag.	0.04 - 0.13 T
Radial of closed orbit	0.81 - 1.14m
betatron tune	
horizontal	2.17 - 2.22
vertical	1.24 - 1.26
RF frequency	0.61 - 1.38MHz
RF voltage	1.3 - 3.0kVp

# INTERNATIONAL WORKSHOP ON FFAG ACCELERATOR

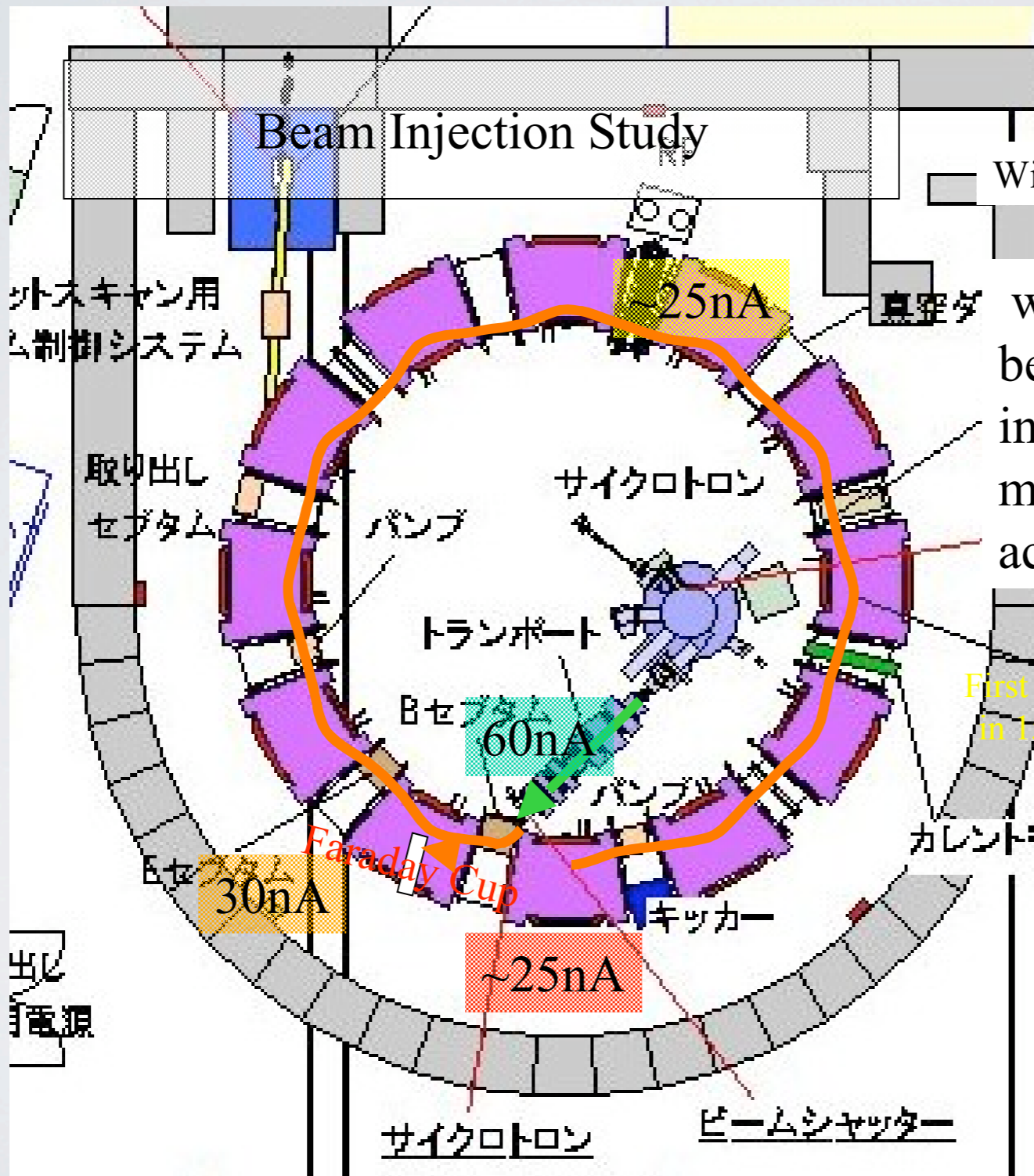
- (history)

- 1st FFAG99 (Dec. 1999) KEK PoP-FFAG first beam!
- 2nd FFAG workshop (July 2000) CERN
- 3rd FFAG00 (Oct. 2000) KEK
- 4th FFAG02 (Feb. 2002) KEK
- 5th FFAG workshop (Sept. 2002) LBL
- 6th FFAG03 (July 2003) KEK
- 7th FFAG workshop (Sept. 2003) BNL
- 8th FFAG workshop (Mar. 2004) TRIUMF
- 9th FFAG04 (Oct. 2004) KEK 150MeV proton FFAG first beam!

**Almost twice per year! One the most active fields in accelerator physics and technology.**

# FIRST BEAM !

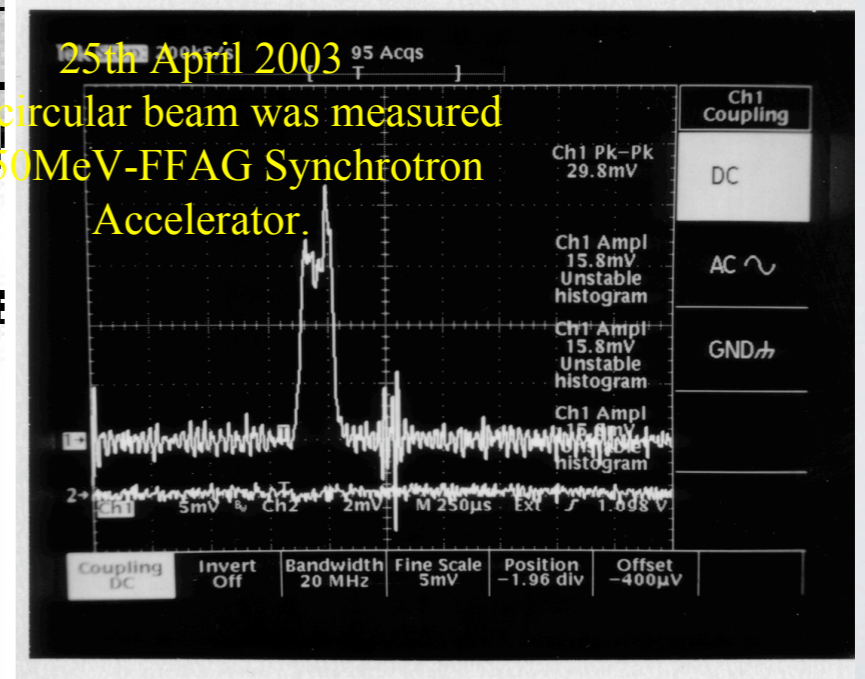
APR. 2003



With Magnetic and Electric Septum

we are studying the beam orbit in detail, and installing the bump magnets for the beam acceleration

25th April 2003  
First circular beam was measured in 150MeV-FFAG Synchrotron Accelerator.



# ALIGNMENT

JAN. 2003



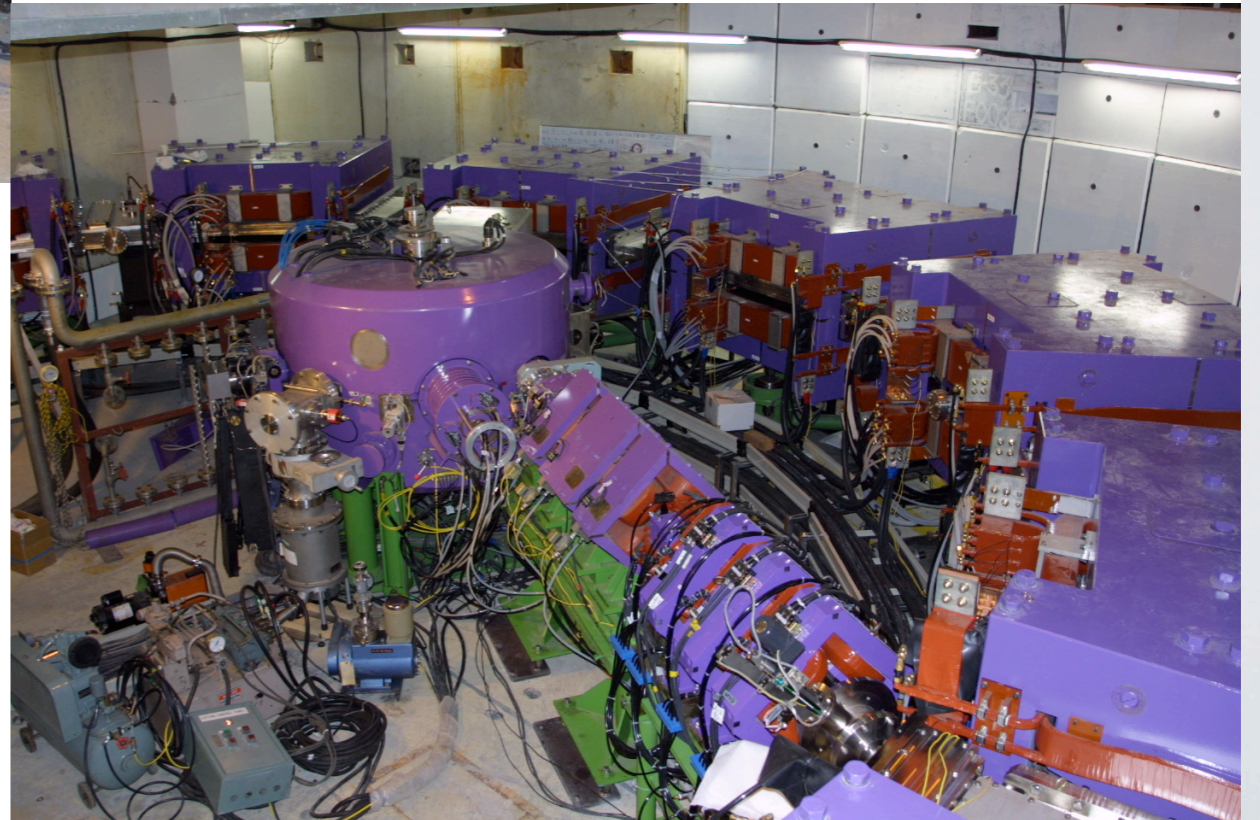
We aligned the magnets within the precision of 0.2 mm.

## Cyclotron(Injector) and Beam transport



### Cyclotron

- 10 MeV proton beam
- 250Hz pulse operation
- Max. extraction current  $0.5 \mu A$

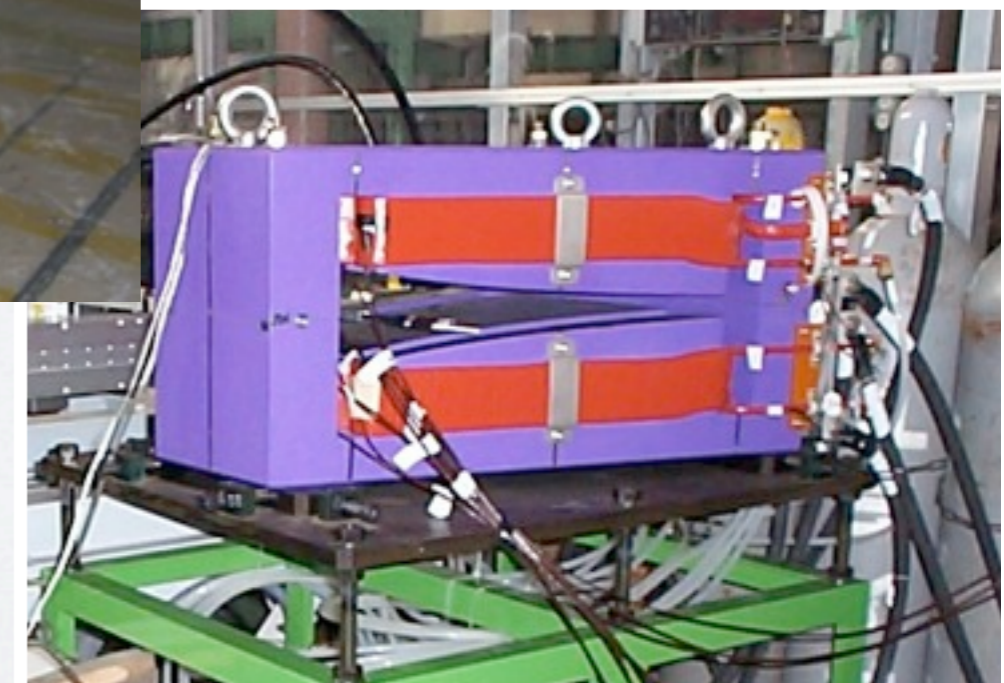


### Transport

2 sets of steering+triplet Q  
mag.

# 150 MeV FFAG - Return Yoke Free Magnet

150 MeV FFAG magnet, the view from the center of the ring.



# CAVITY ASSEMBLY



Number of cores	2~4
Outer size	1.7m x 1m
Inner size	1m x 0.23m
RF frequency	1.5 - 4.6 MHz
RF voltage	9 kV
RF output	55 kW
Power density	1 W/cm <sup>3</sup>
Cooling water	70 L/min



# COMMISSIONING 150MEV FFAG ACCELERATOR

**Shinji Machida, Yoshiharu Mori, Joe Nakano, Yasuo  
Sato, Akira Takagi, Takeidhiro Yokoi, Masahiro  
Yoshimoto, Yoshimasa Yuasa**

KEK, Ibaraki, Japan

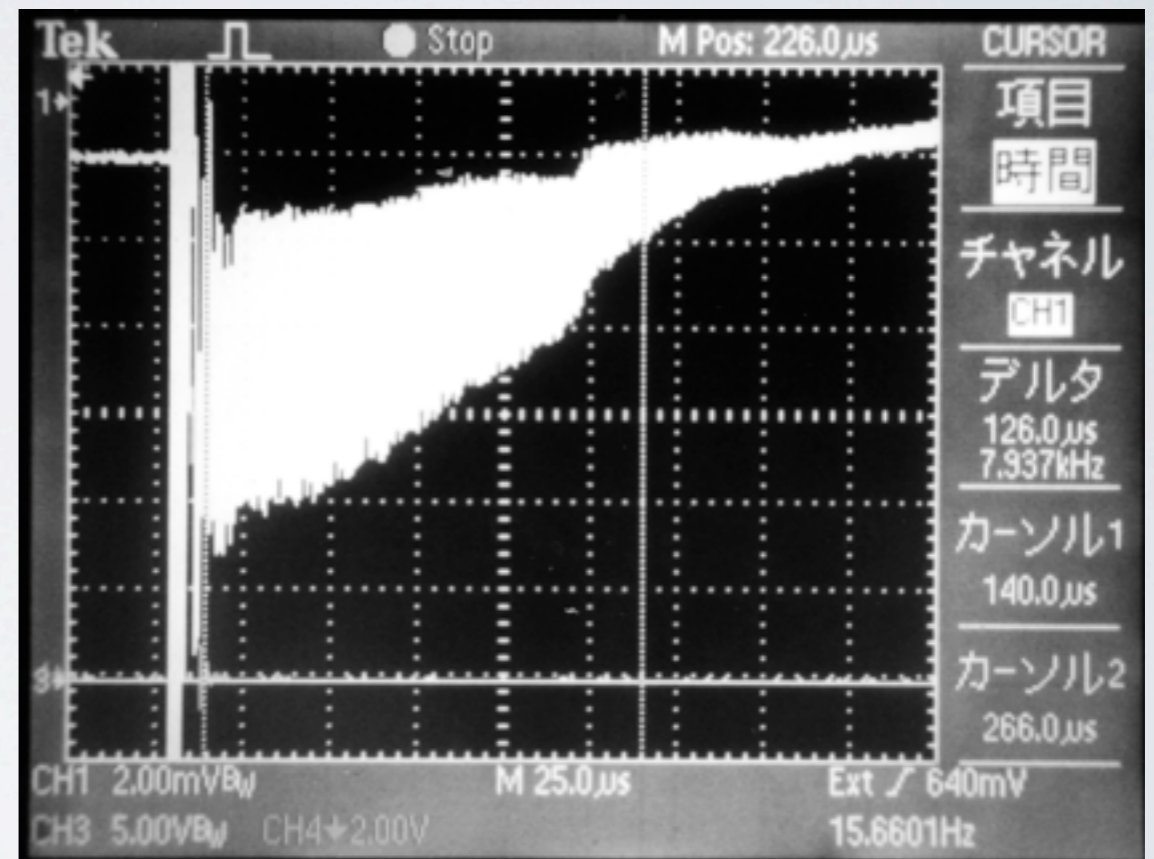
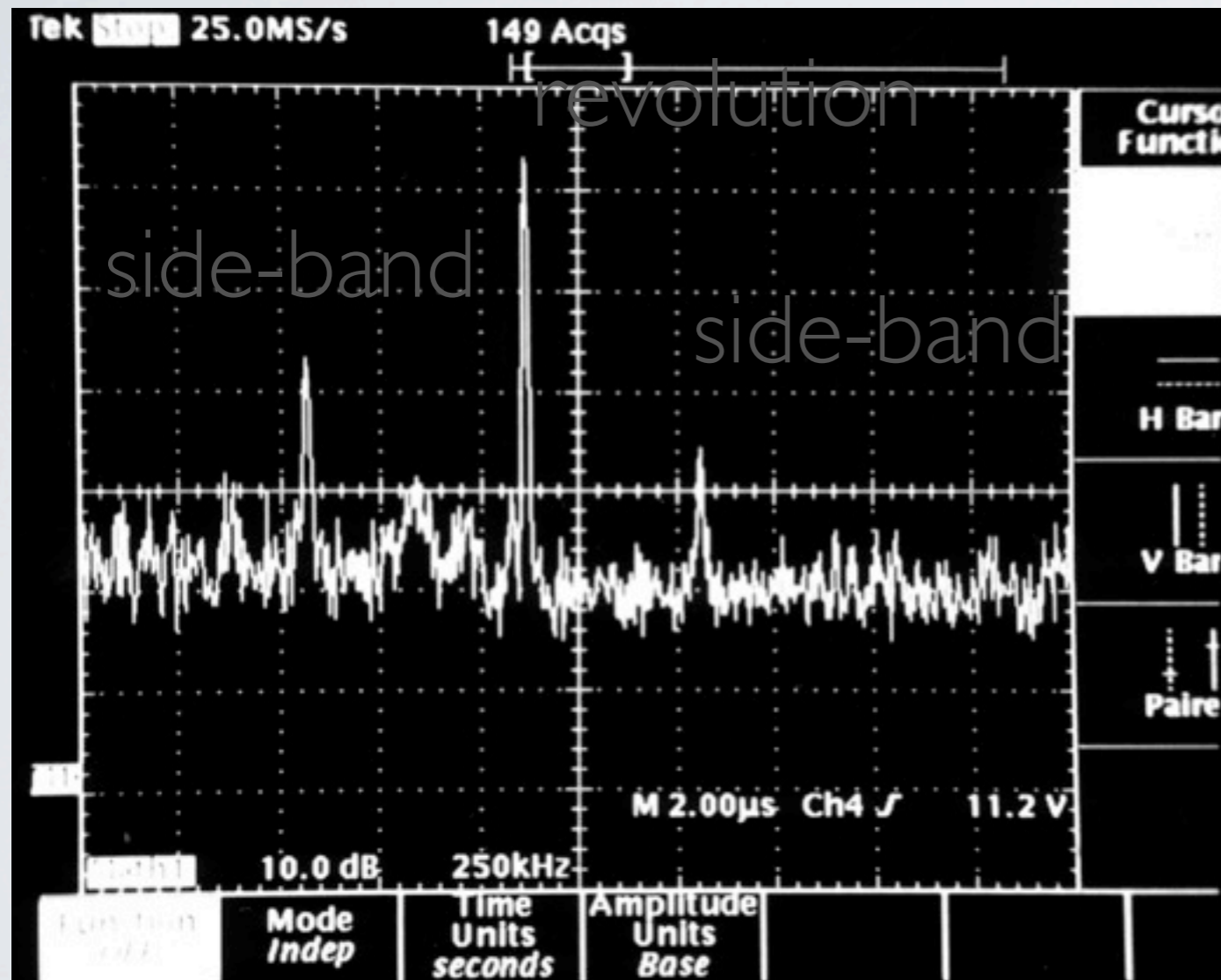
**Yujiro Yonemura**

Kyushu-univ., Fukuoka, Japan

**Masamitsu Aiba**

Univ. of Tokyo, Tokyo, Japan

# BETATRON TUNE & $\Delta P/P$ AT INJECTION

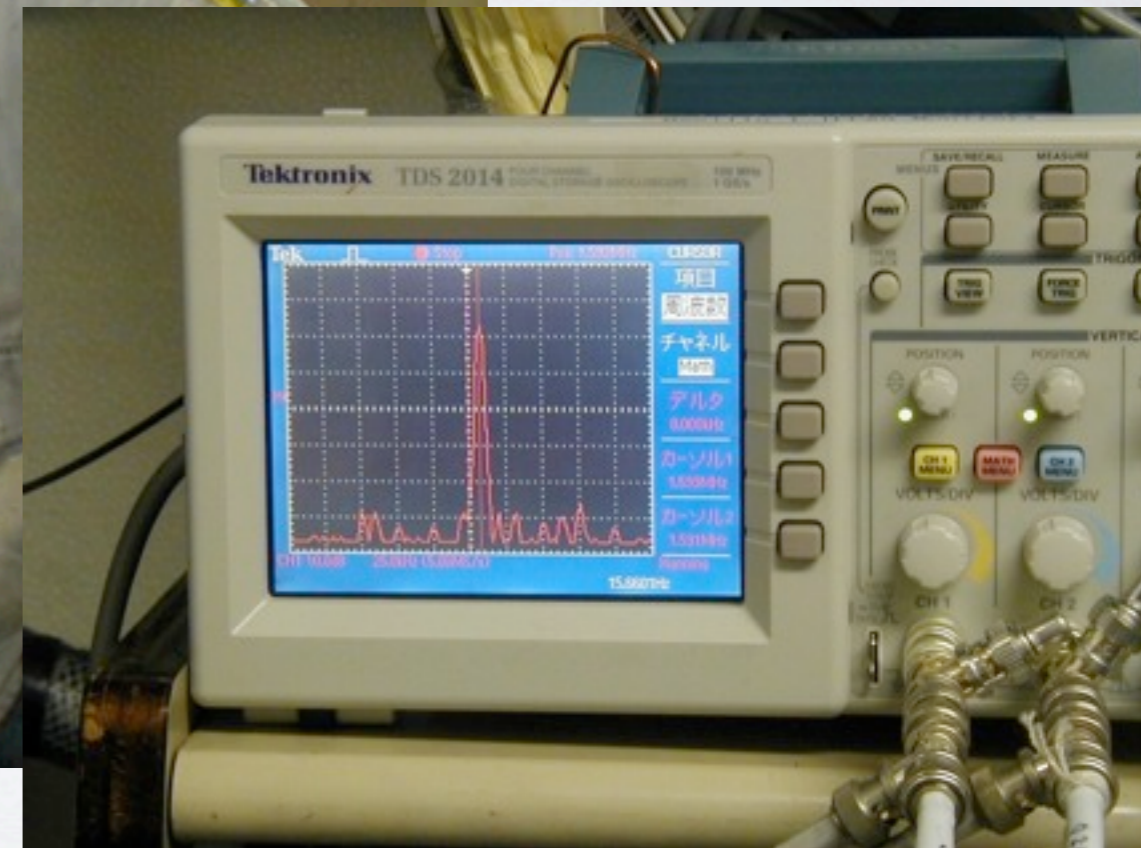
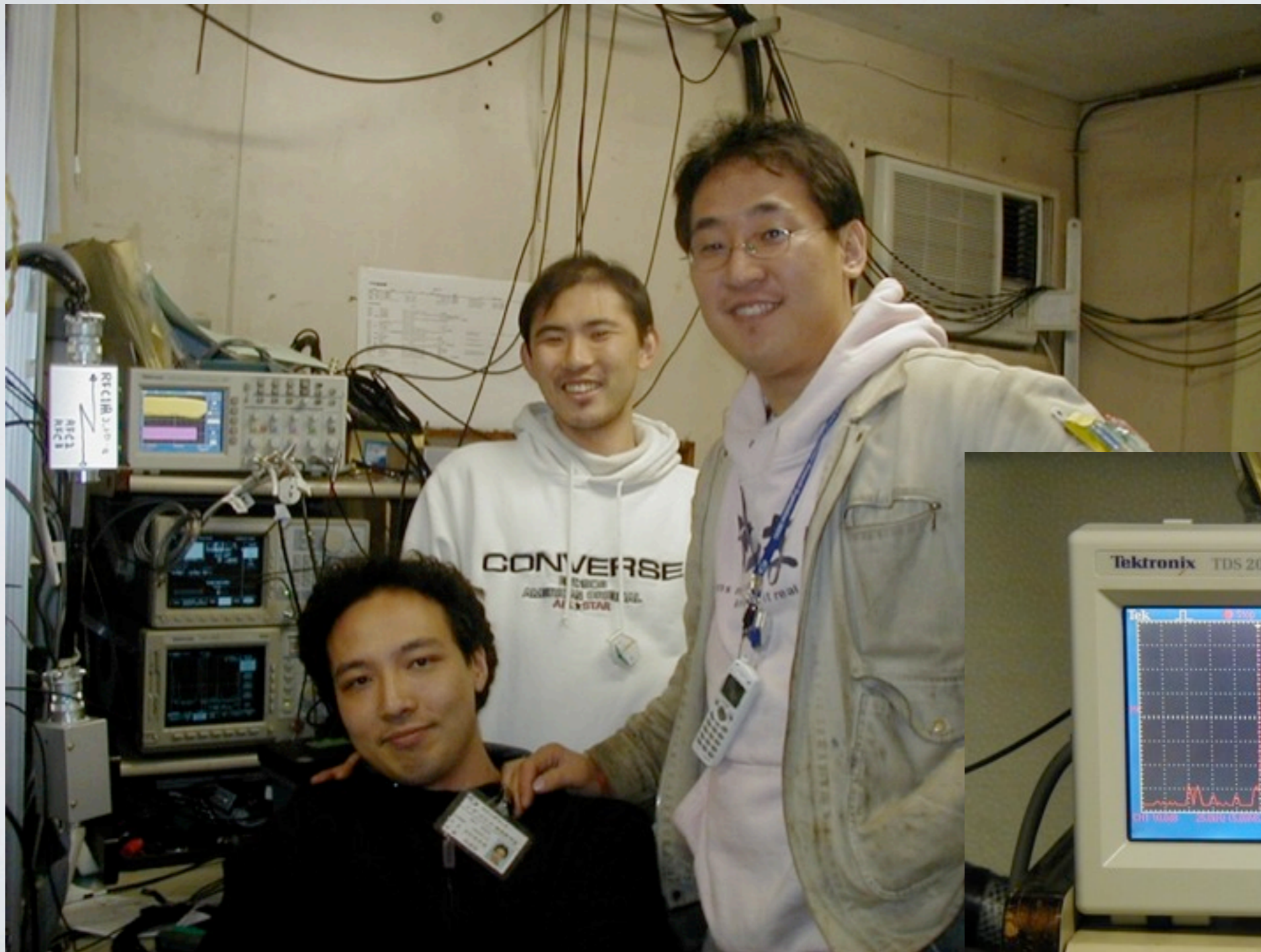


$$\Delta Q_h = 0.61, \Delta Q_v = 0.34$$

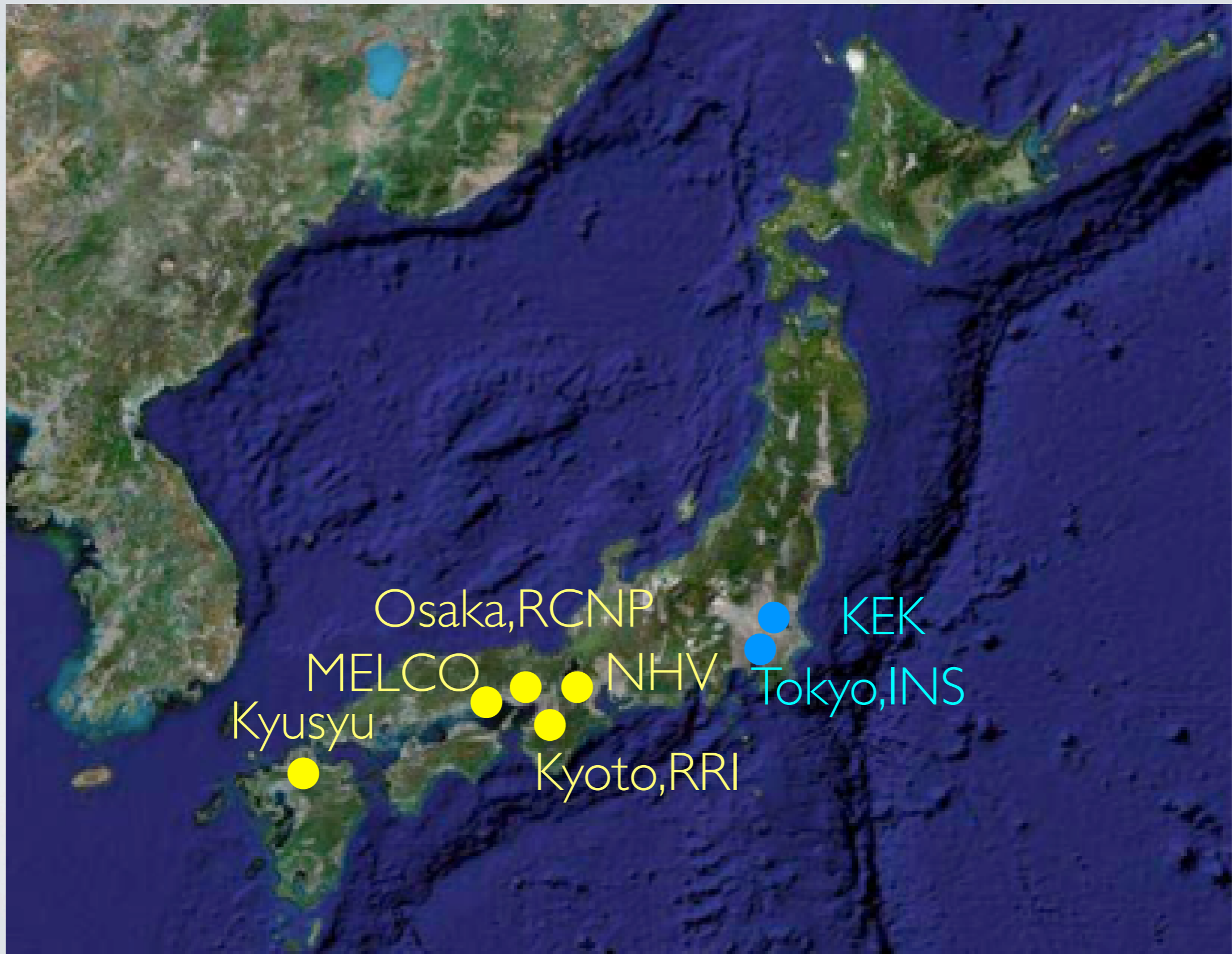
$$dp/p = 3 \times 10^{-4}$$

# CONGRATULATION!

Oct. 2003



# DEVELOPMENTS OF FFAG IN JAPAN



# KYUSYU UNIVERSITY

## Construction of new accelerator center

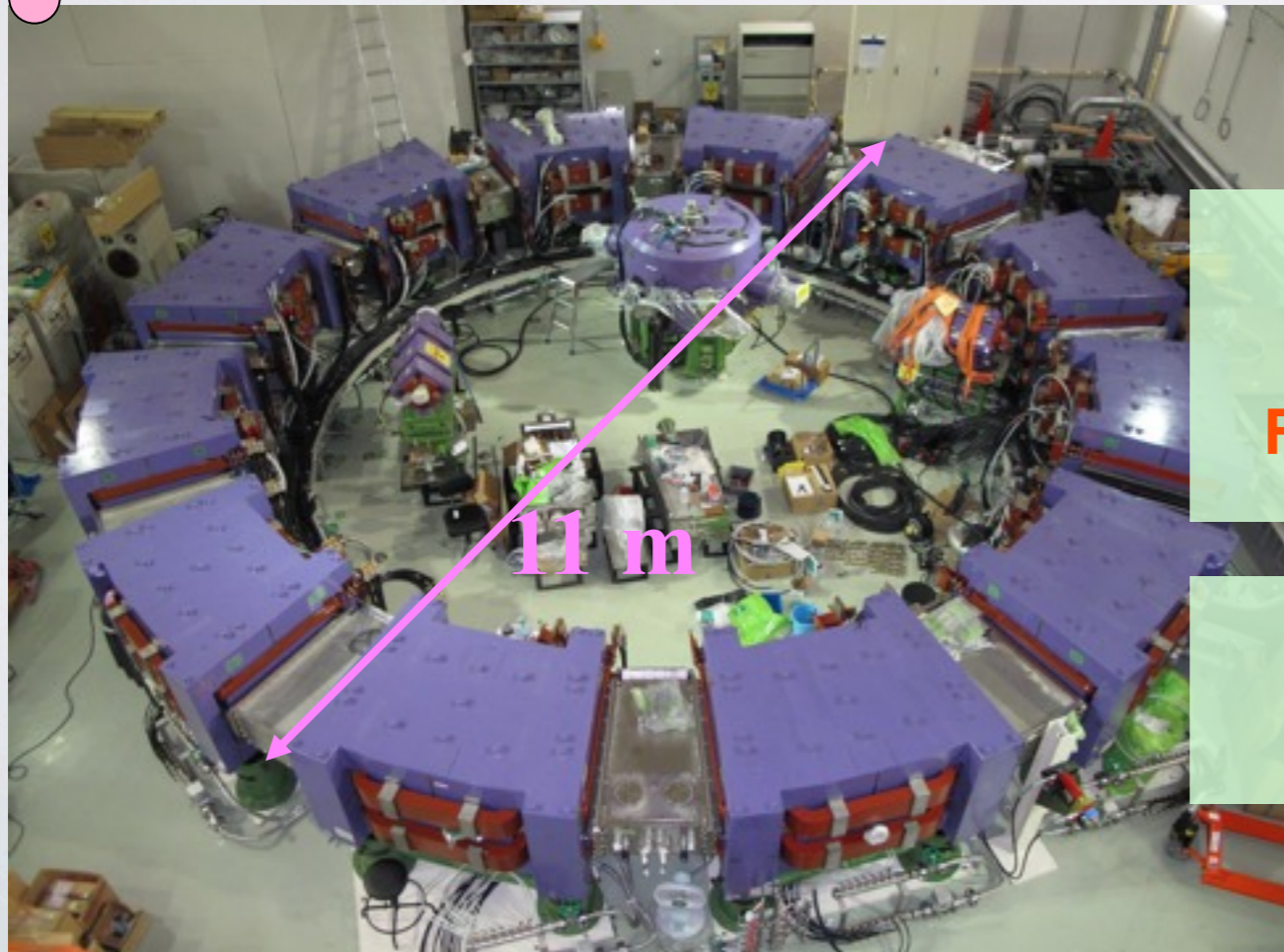
Main accelerator : FFAG Synchrotron

The test machine that Mori's group developed is under re-installation.



Developed at KEK

Moved to Kyushu



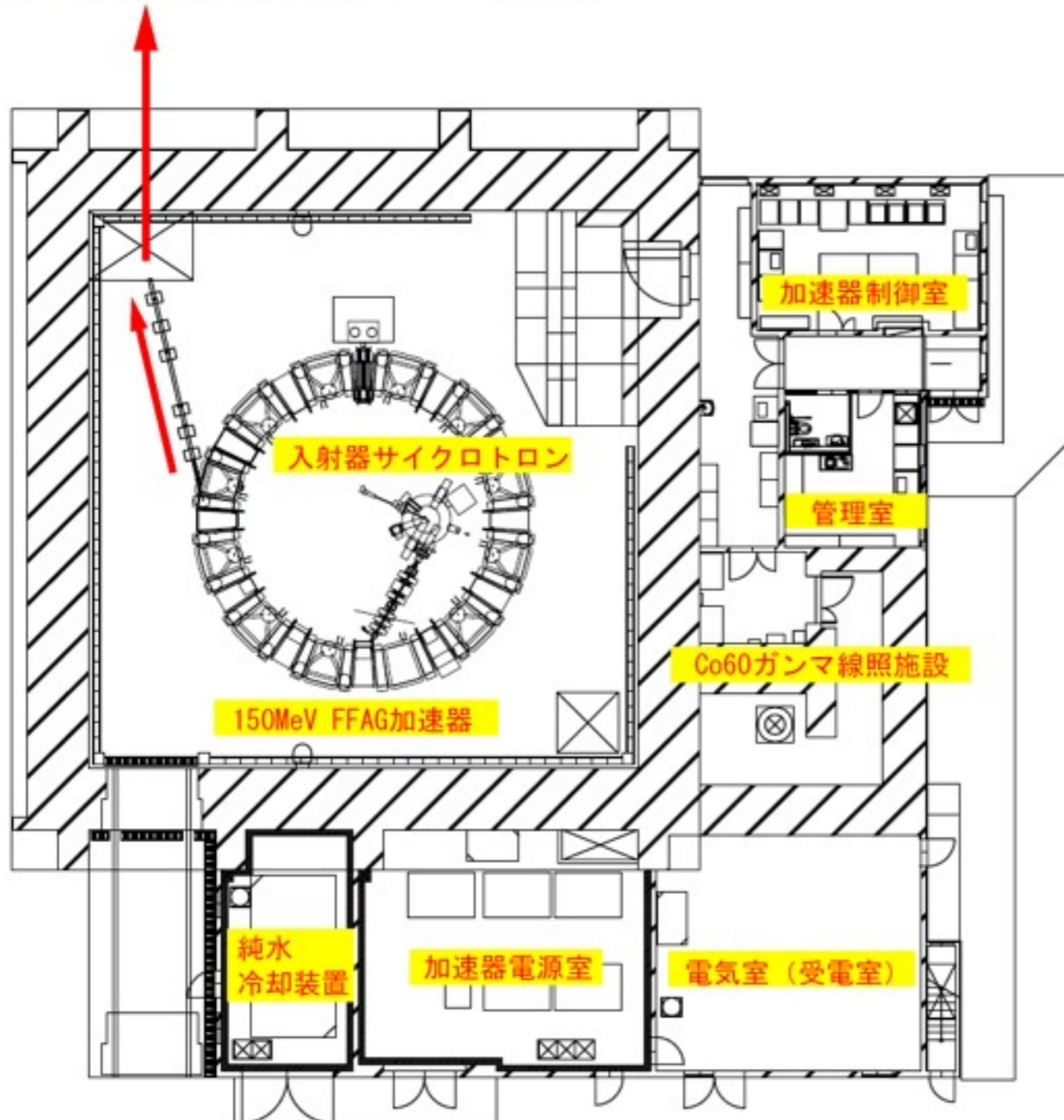
11 m

Newly constructed machine still under development  
**Further development at Kyushu**

A machine with various possibilities  
**Challenges for new usage**

# Design values of the FFAG Synchrotron

第二期以降建設される装置へビームを供給



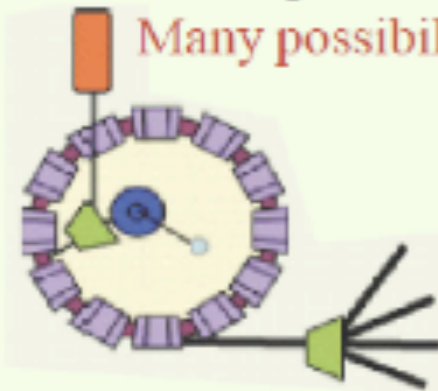
magnet	Radial sector type (DFD-triplet)
Cell	12
K-value	7.62
Beam energy	12 $\Rightarrow$ 150 MeV ( 10 $\Rightarrow$ 125 MeV)
Radius	4.47 $\Rightarrow$ 5.20 m
Betatron tune	H: 3.69~3.80 V: 1.14~1.30
Max. field	F-field: 1.63 T
(along orbit)	D-field: 0.78 T
Circ. freq.	1.55~4.56 MHz
Repetition	100 Hz

# Various field studied with FFAG

## FFAG development

## Accelerator science

Under-developed machine just born.



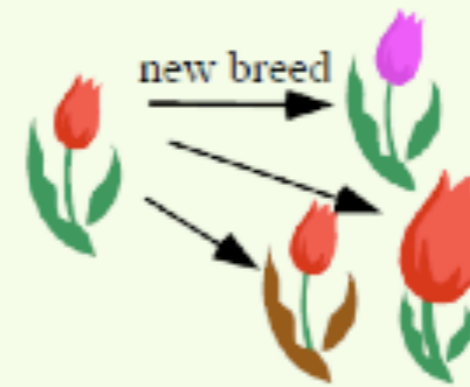
Many possibilities no other machines have.

- Flexible beam time-structure
- Large acceptance, multi-beam
- possibility as a beam delayer
- Acceleration of various beams
- High intensity by fast repetition

## DNA processing, breed improvement

## Life science

## Environment science



DNA damage  
by heavy ion

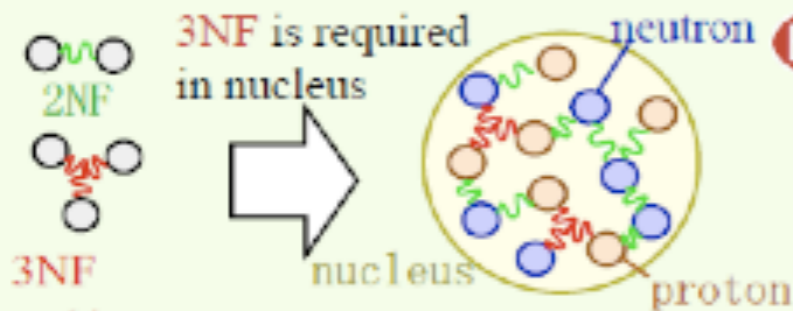


Restoration  
of DNA  
survival by  
apoptosis

## 3-body force, nuclear data

## Hadron science

## Energy science



3NF is required  
in nucleus

3NF  
working to  
3 nucleons  
simultaneously

1936: Yukawa predicted 2NF  
1994: Sagara found exp. evidence of 3NF

## Space simulation

## Life science

## Environment science

## Fronteer

## Industry use

Cosmic ray (protons)



Space-use  
devices

Space  
medicine

## biological irradiation effect energy-transfer process

## Energy science

## Life science

## Material science

## Medical use (basic)



30% of incident energy are converted to other radiation

- High quality radiation treatment
- Radiation damage process of semiconductor devices

## Monochro./white neutron field

## Hadron science

## Material science

## Environment science

## Industry use

Reactor material  
Calibration of neutron detectors  
Basic data for acc. driven reactor  
nuclear waste

? ? ?

? ? ? science

New field

# $^{36}\text{Cl}$ ( $T_{1/2} = 3 \times 10^5 \text{ y}$ ) AMS

Powerful tool for earth, environmental, biological ocean sciences, astronomy, and so on

$^{36}\text{S}$ : 744 MeV,  $\leq 16^+$   $\Rightarrow$  eliminate

$^{36}\text{Cl}$ : 744 MeV  $17^+$  ( $\sim 100\%$ )

possible contamination of particles with energies less than 475 MeV

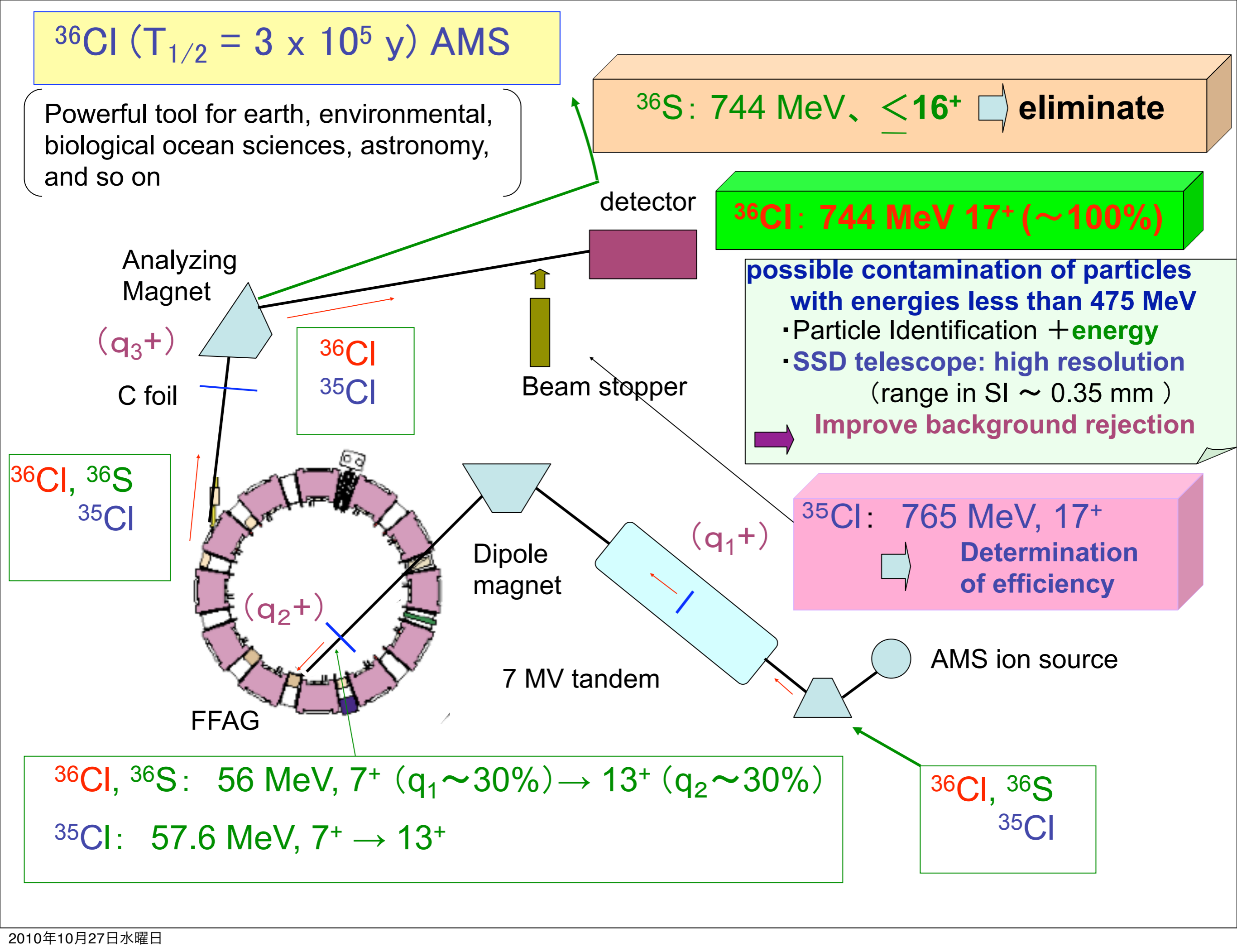
- Particle Identification + energy
- SSD telescope: high resolution (range in SI  $\sim 0.35 \text{ mm}$ )

$\Rightarrow$  Improve background rejection

$^{35}\text{Cl}$ : 765 MeV,  $17^+$   
Determination of efficiency

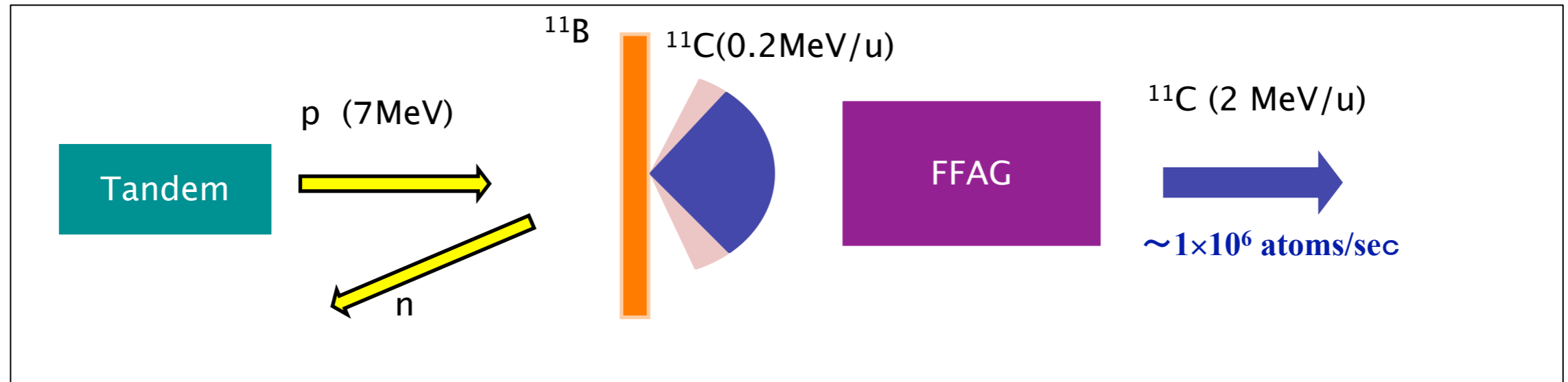
$^{36}\text{Cl}$ ,  $^{36}\text{S}$ : 56 MeV,  $7^+$  ( $q_1 \sim 30\%$ )  $\rightarrow$   $13^+$  ( $q_2 \sim 30\%$ )  
 $^{35}\text{Cl}$ : 57.6 MeV,  $7^+$   $\rightarrow$   $13^+$

$^{36}\text{Cl}$ ,  $^{36}\text{S}$   
 $^{35}\text{Cl}$



## Acceleration of unstable nuclei and isomers

### \*Acceleration of unstable nuclei



### \*Acceleration of isomers



#### Advantage:

- High quality unstable beam for all elements

#### Subjects

- Structure of high-spin isomer, Astro-nuclear data
- Diffusion process in material

#### Requirement to accelerator

- Large acceptance (longitudinal and transverse)

# OSAKA UNIVERSITY

Y.Kuno, A.Sato

- **MOTIVATION**

- Research for new physics beyond Standard Theory with  $\mu$ -e conversion rare event experiment

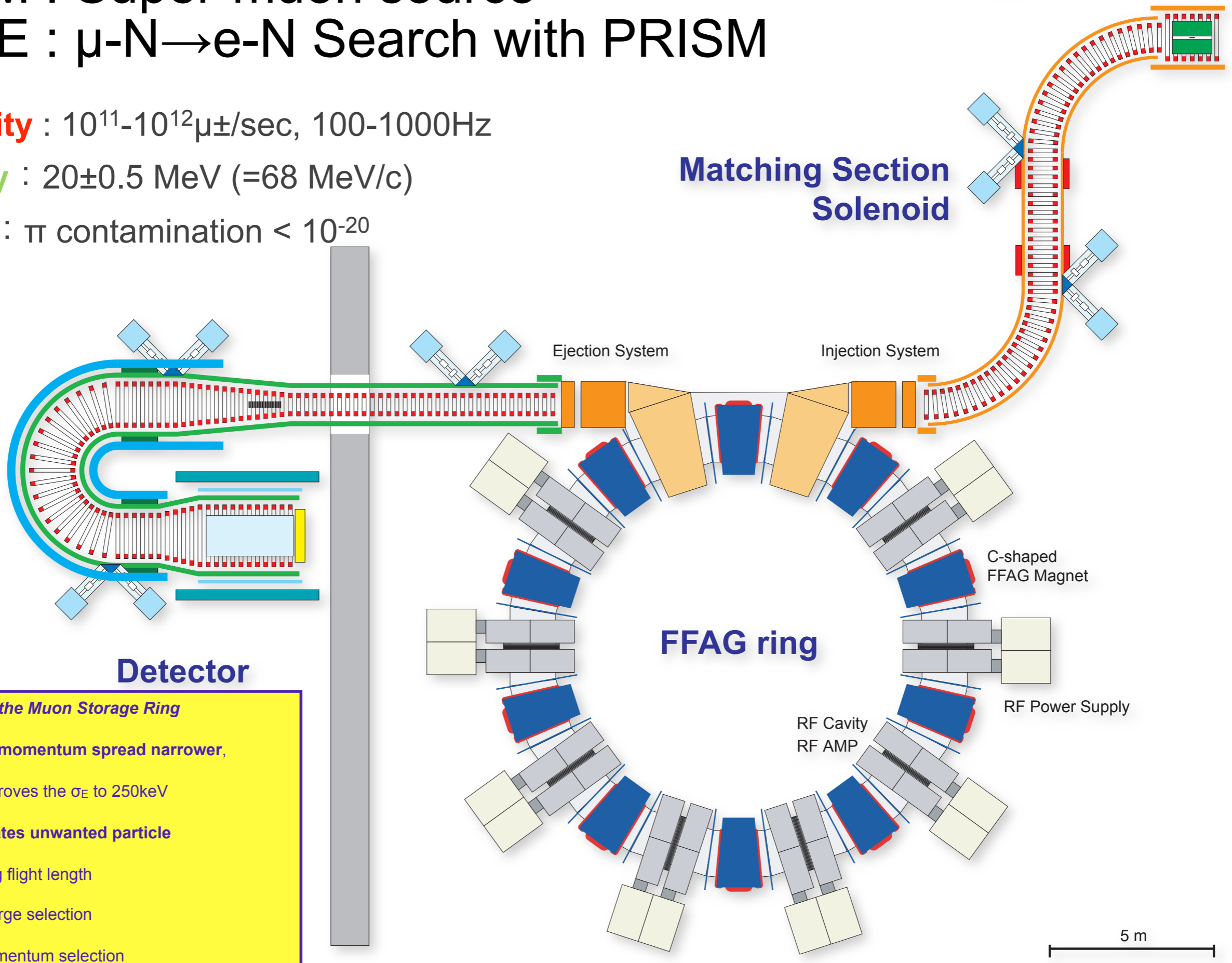
$$B(\mu^- + Ti \rightarrow e^- + Ti) < 10^{-18}$$

- To do this,
  - with a muon storage ring to reduce the energy spread and pion background.
  - with a fast-extracted pulsed proton beam.
  - need a new beamline and experimental hall.
  - Ultimate search

# PRISM : Super-muon source

## PRIME : $\mu\text{-N} \rightarrow \text{e-N}$ Search with PRISM

- **Intensity** :  $10^{11}\text{-}10^{12}\mu\pm/\text{sec}$ , 100-1000Hz
- **Energy** :  $20\pm 0.5$  MeV (=68 MeV/c)
- **Purity** :  $\pi$  contamination  $< 10^{-20}$



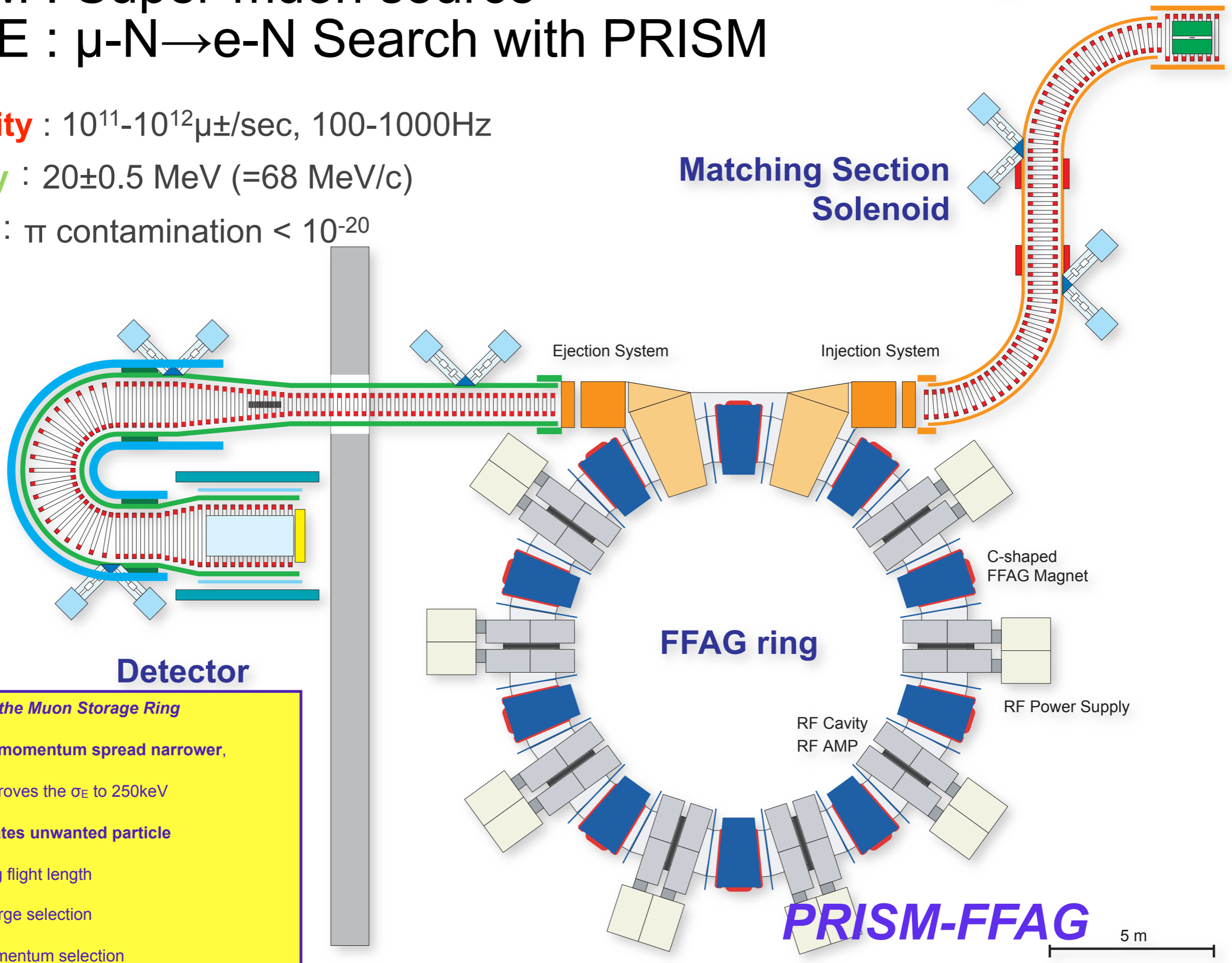
### Functions of the Muon Storage Ring

- Makes momentum spread narrower,
- improves the  $\sigma_E$  to 250keV
- Eliminates unwanted particle
- long flight length
- charge selection
- momentum selection

# PRISM : Super-muon source

## PRIME : $\mu\text{-N} \rightarrow \text{e-N}$ Search with PRISM

- **Intensity** :  $10^{11}\text{-}10^{12}\mu\pm/\text{sec}$ , 100-1000Hz
- **Energy** :  $20\pm 0.5$  MeV (=68 MeV/c)
- **Purity** :  $\pi$  contamination  $< 10^{-20}$

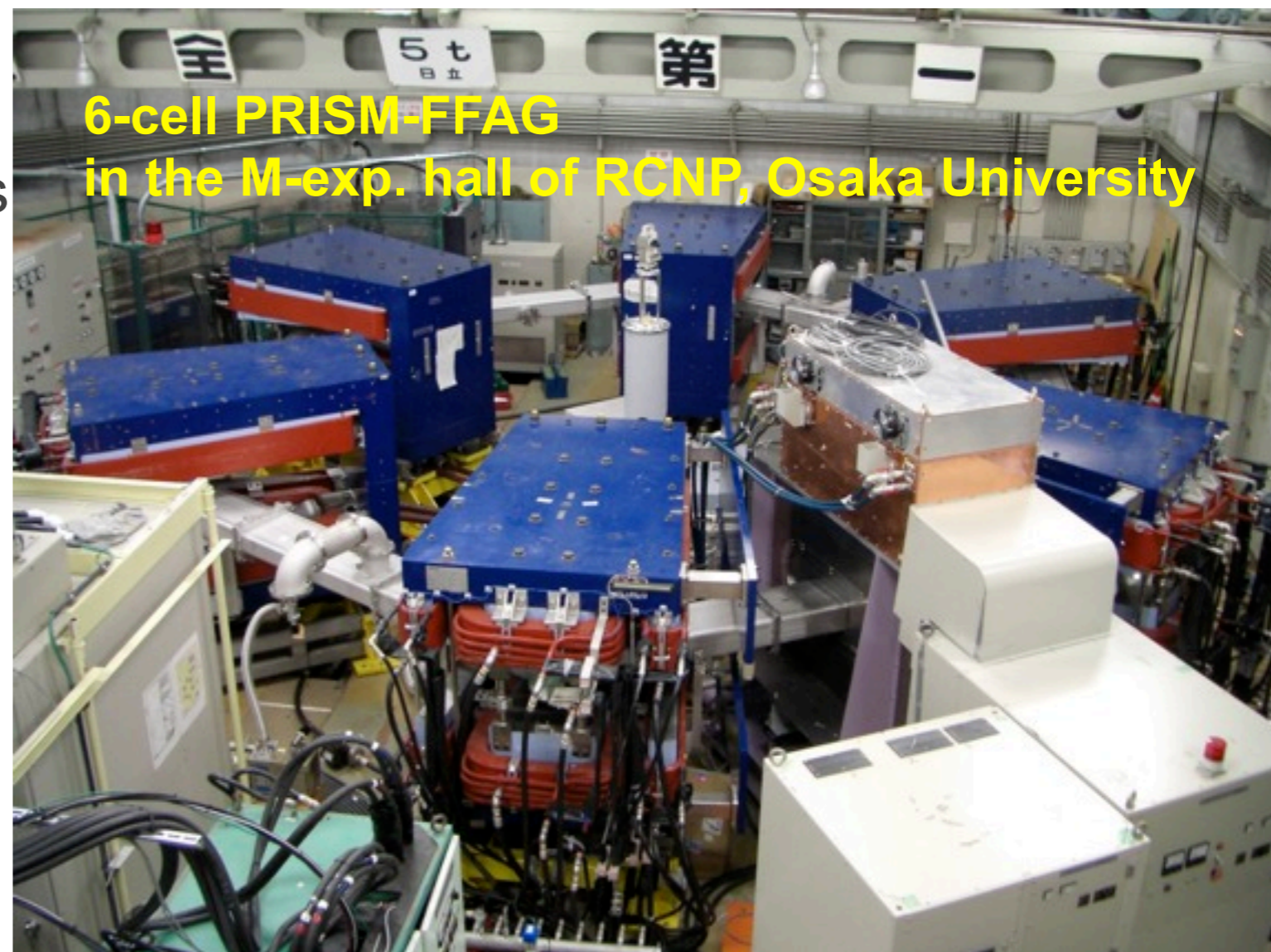


### Functions of the Muon Storage Ring

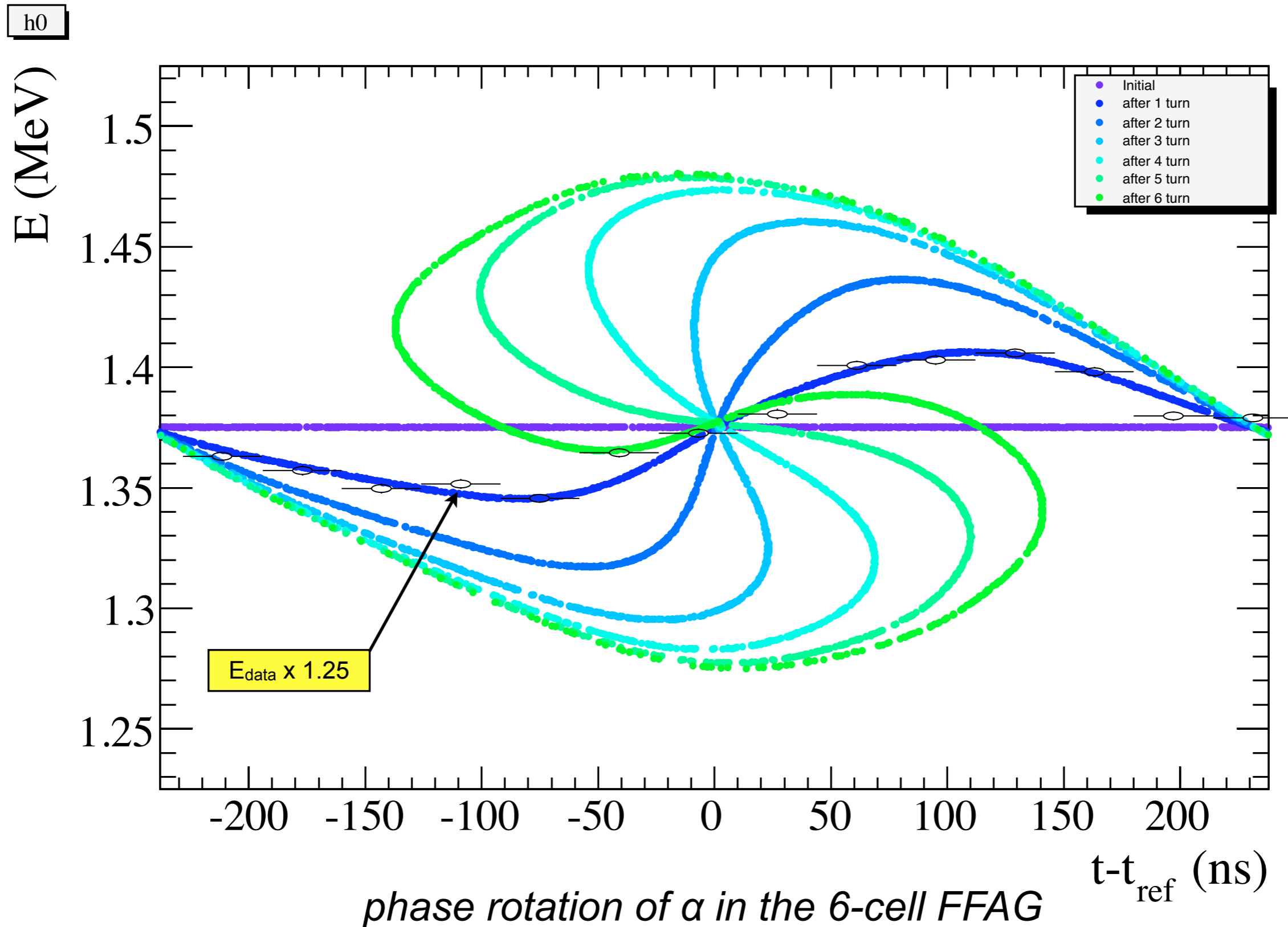
- Makes momentum spread narrower,
- improves the  $\sigma_E$  to 250keV
- Eliminates unwanted particle
- long flight length
- charge selection
- momentum selection

# Demo. of Phase Rotation with $\alpha$ -particles

- FFAG-ring
  - PRISM-FFAG Magnet x 6、 RF x 1
- Beam :  $\alpha$ -particles from radioactive isotopes
  - $^{241}\text{Am}$  5.48MeV(200MeV/c)  $\rightarrow$  degrade to 100MeV/c
  - small emittance by collimators
  - pulsing by electrostatic kickers
- Detector : Solid state detector
  - energy
  - timing



# Comparison b/w data and simulation

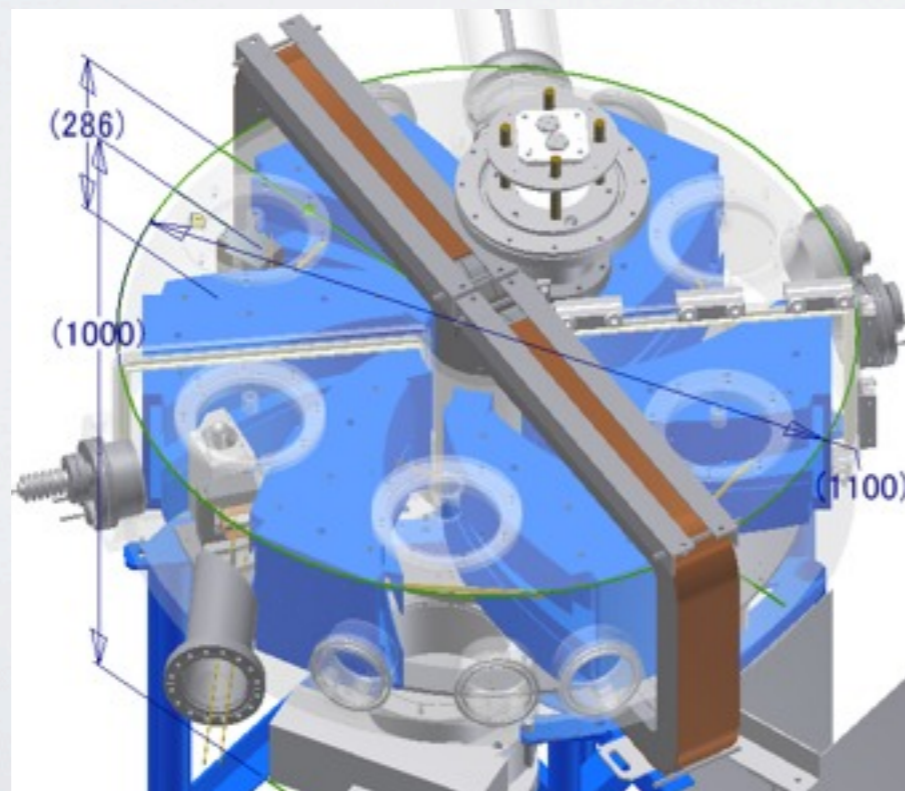
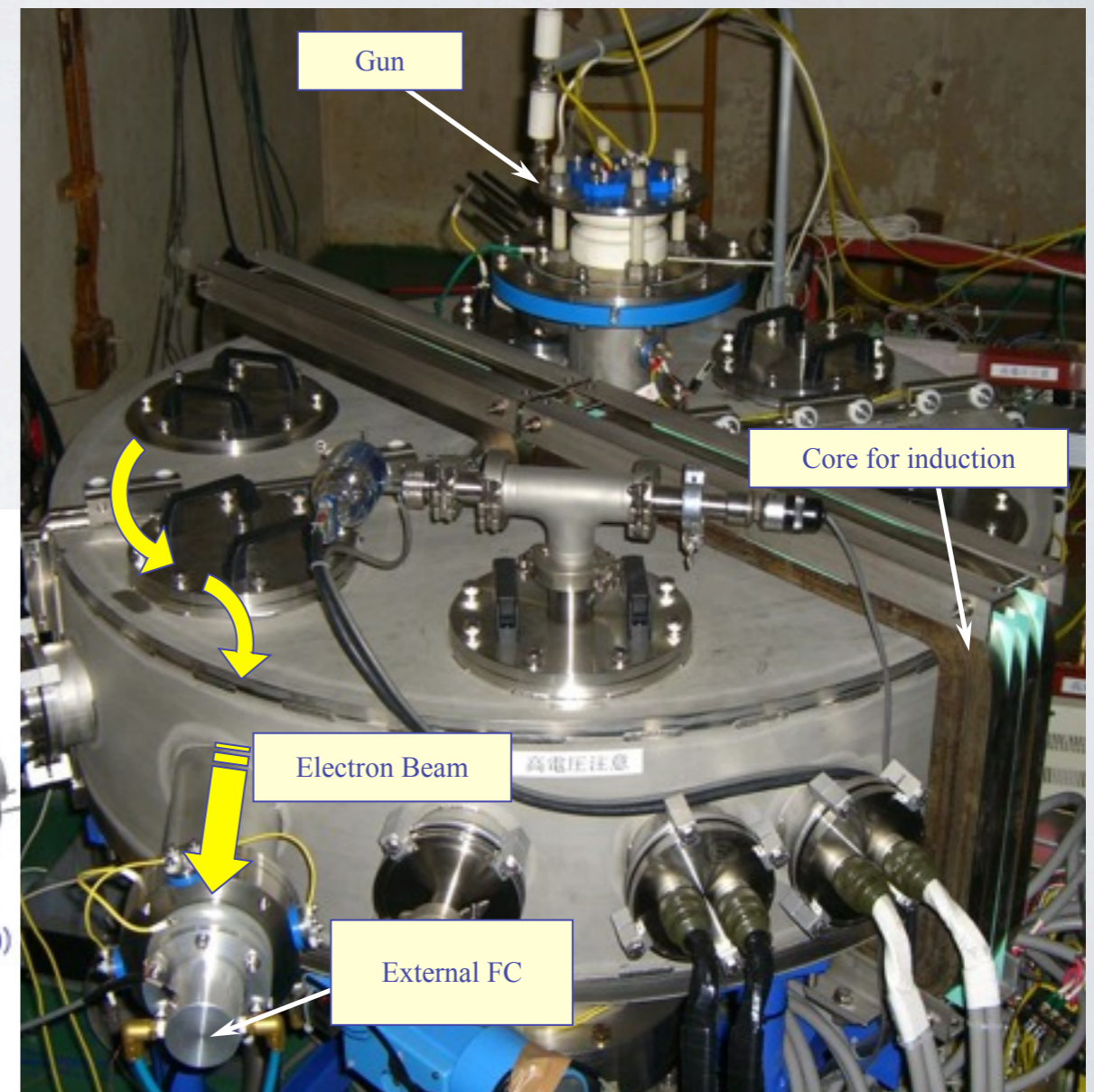


# NHV CO.

T.Baba, M.Yuasa

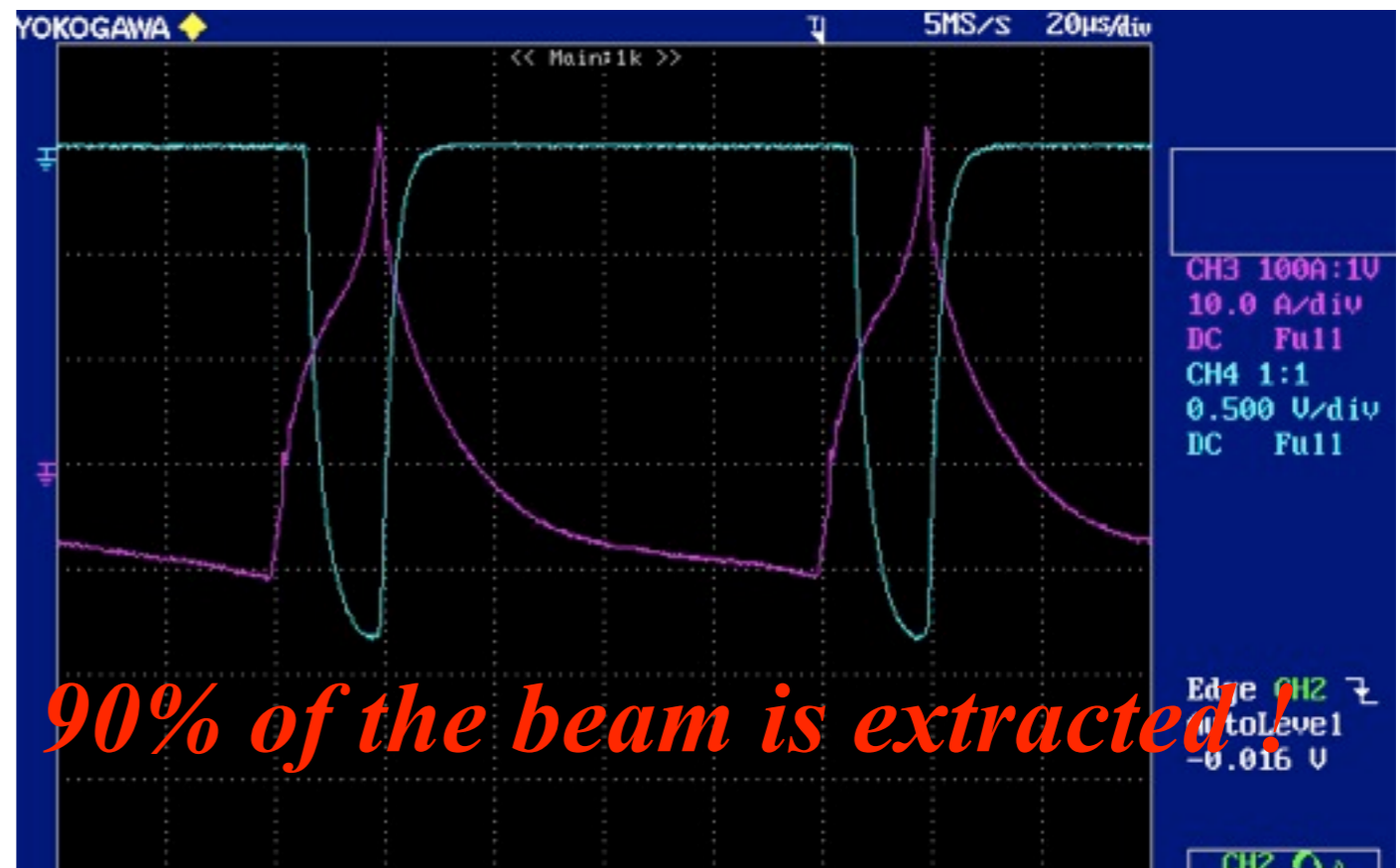
## Prototype of FFAG Electron Accelerator: sterilization etc.

Energy	Inj. / Ext.	50 / 500keV
Orbit radius	Inj. / Ext.	0.19 / 0.44m
Acceleration frequency		10kHz
Beam Current		100mA peak
Duty		20%
Outer diameter		1.1m

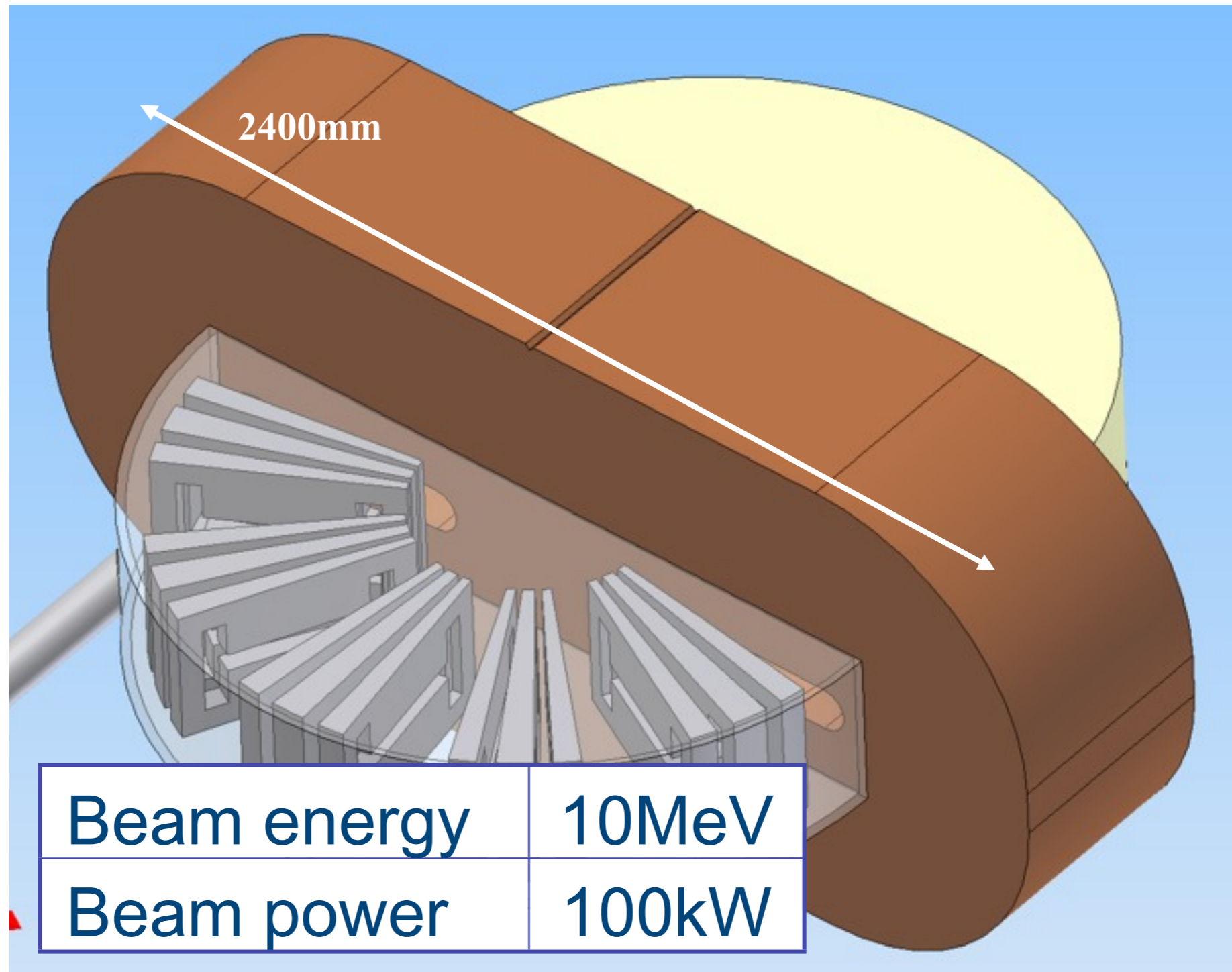


# Results of the Development

- Accelerator assembling is completed.
- Beam injection and acceleration are successful.
- 90% of the beam is extracted from FFAG ring.
- Extracted beam energy is measured as same as the specified energy.



# 10MeV Electron Accelerator



# KYOTO UNIVERSITY RESEARCH REACTOR INSTITUTE (KURRI) FFAG-ADSR PROJECT

- Purpose of the project

- Basic study of ADSR (Accelerator Driven Sub-critical Reactor) with FFAG accelerator and KUACA (Kyoto University Critical Assembly)

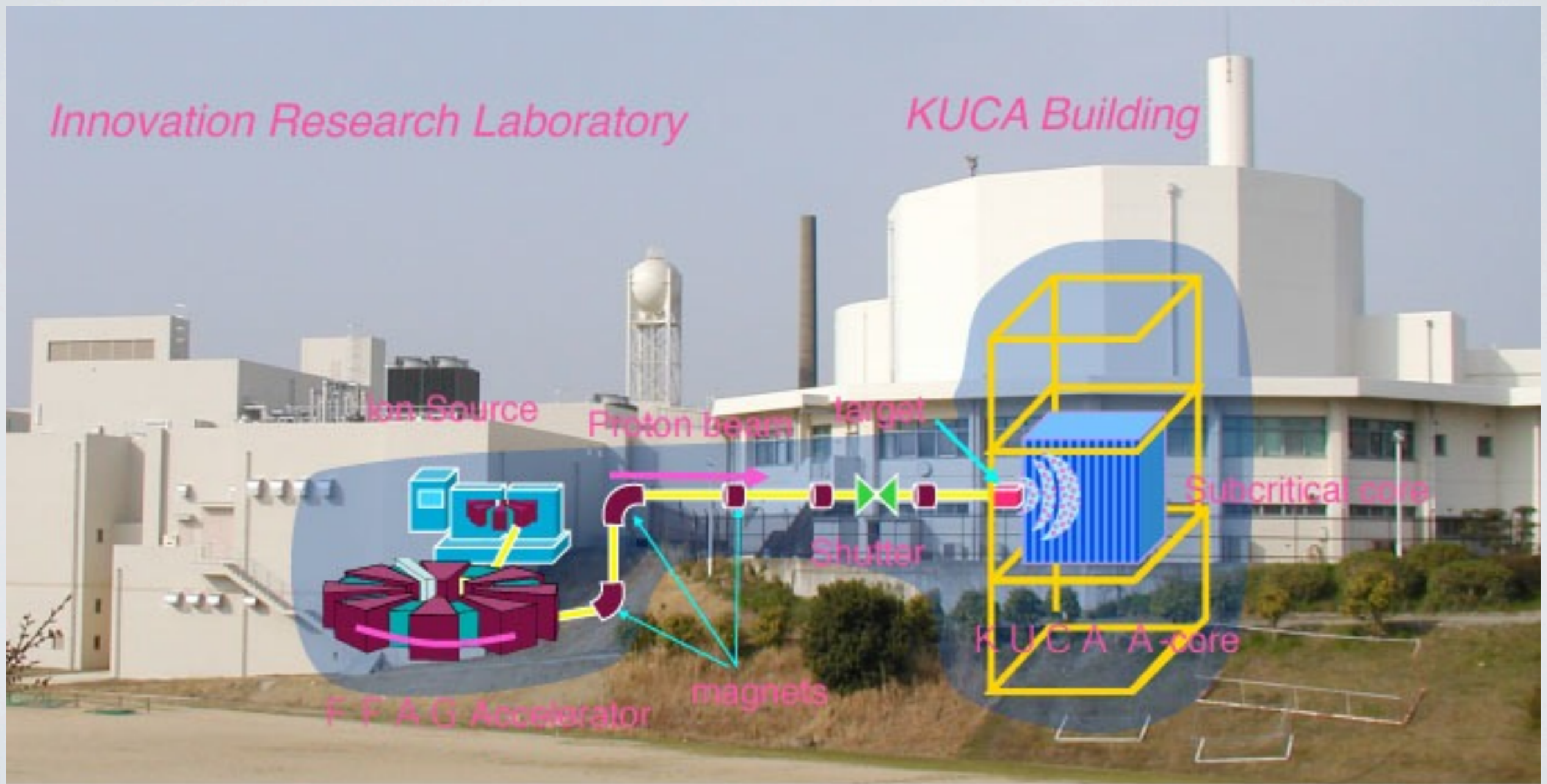
- KUACA

- Output power  $\sim 100\text{W}$
- Neutron amplification :  $\alpha = 1/(1-k_{\text{eff}})$ . If  $k_{\text{eff}}=0.99$ ,  $\alpha=100$
- Beam power should not exceed  $< 1\text{W}!!$
- Beam power is also limited by radiation safety because the beam passes only 1m away from office.
  - cf. For 100MeV proton beam,  $I < 10\text{nA}$

- FFAG Accelerator Complex

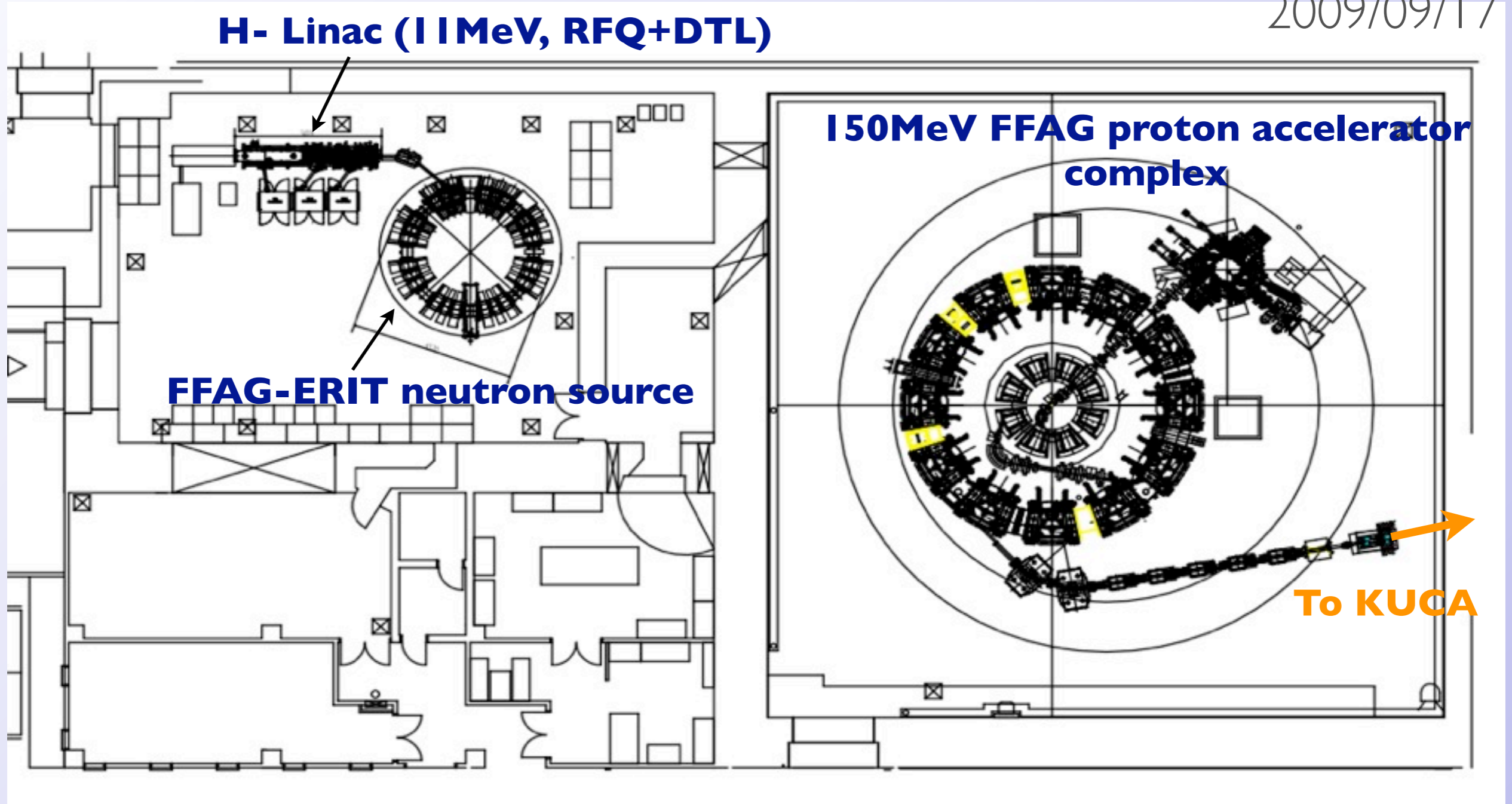
- Beam energy 100-150MeV (variable)
- Beam current 1nA

# FFAG-KUCA ADSR PROJECT AT KURRI



# Layout of FFAG Accelerators in Innovation Laboratory

2009/09/17



# FFAG accelerator complex

To KUCA

- Accelerated Particle proton
- Beam Energy (variable) 100-150 MeV
- Beam Intensity  $\sim 10^9$  ppp
- Pulse Width(duration) 30 nsec
- Repetition Rate 1-60 Hz (variable)
- Circumference 27m

Booster(11MeV)

MAIN RING(100MeV)

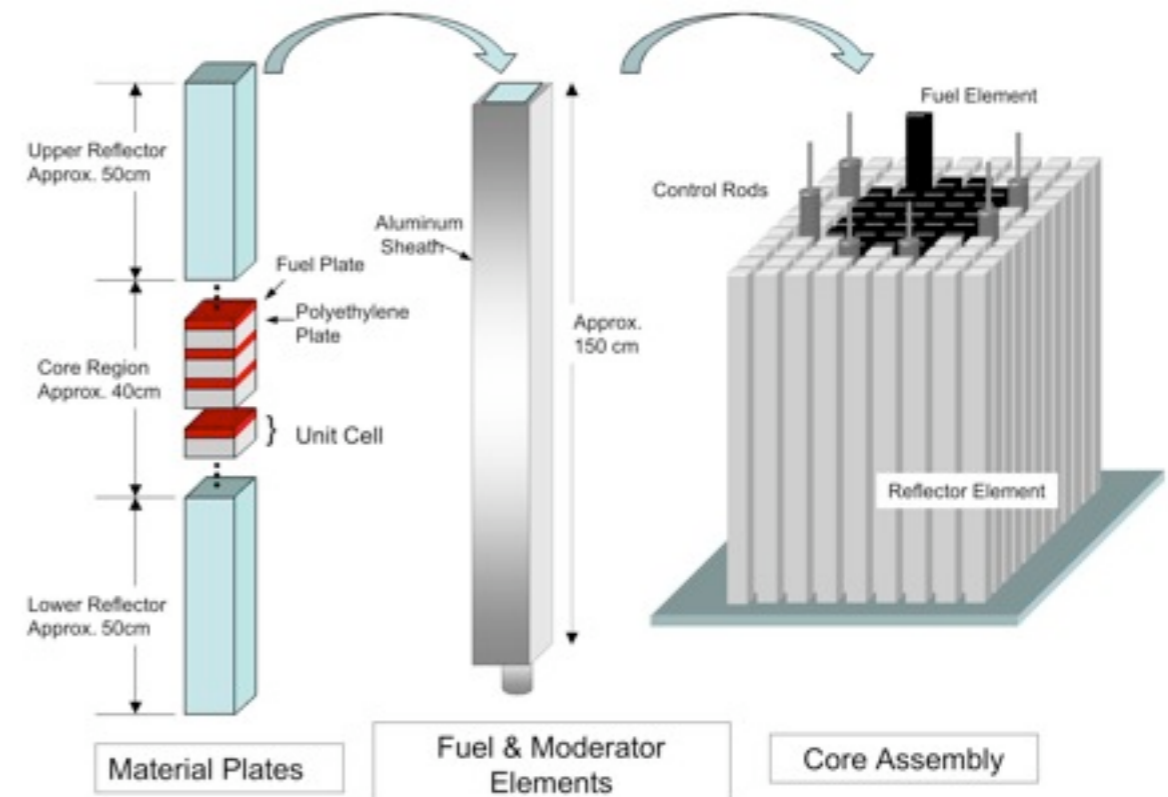
Ion beta

# KUCA-A Core - solid moderated and reflected -



## Items of ADSR experimental study

- High energy neutron spectrum
- Reactivity distribution, neutron distribution and proton profile at the reactor core
- Reactor response for abrupt changes in reactivity: beam trip, negative reactivity introduction, etc.
- Sub-criticality measurement with pulsed neutron method
- Dynamical behaviors with Feynman- $\alpha$  method



# ADSR EXPERIMENT

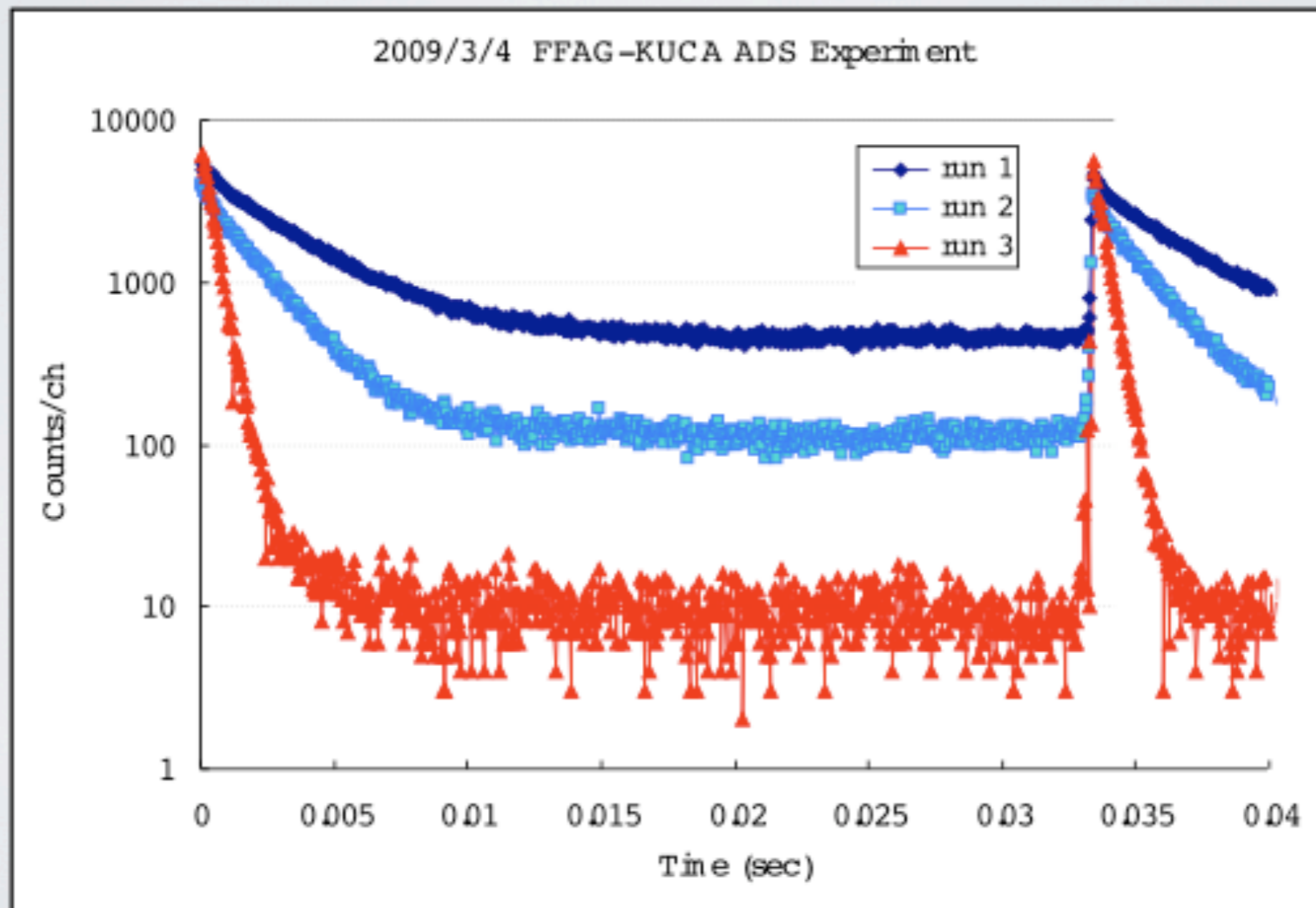
WORLD FIRST ADSR EXPERIMENT WITH SPALLATION NEUTRONS  
-THE FIRST FFAG USED FOR APPLICATION-



# FIRST DATA

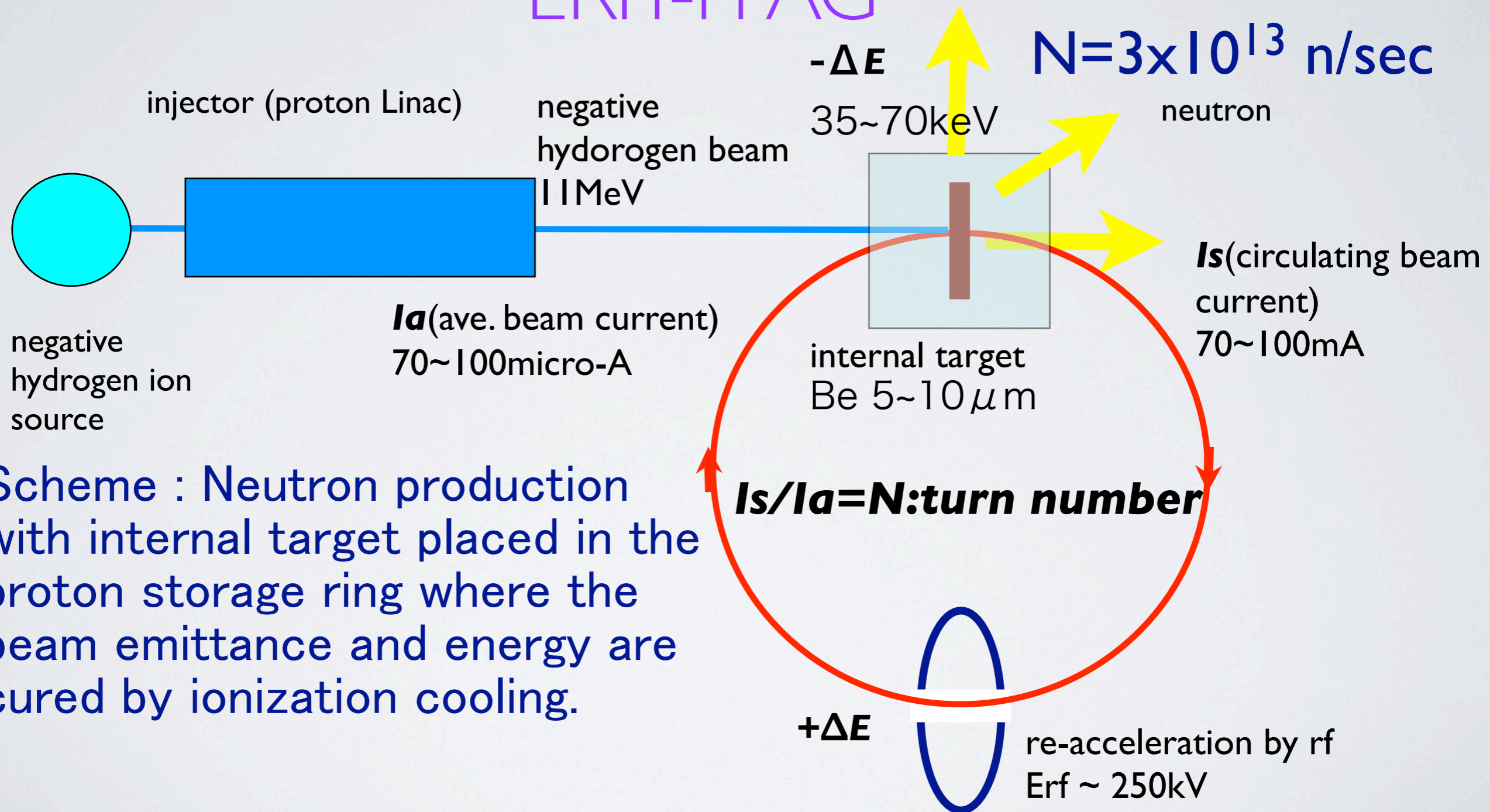
Journal of Nuclear Science and Technology, Vol.46 No.12, pp.1091-1093(2009).

- Measurement of neutron multiplication



# NEUTRON SOURCE WITH EMITTANCE RECOVERY INTERNAL TARGET

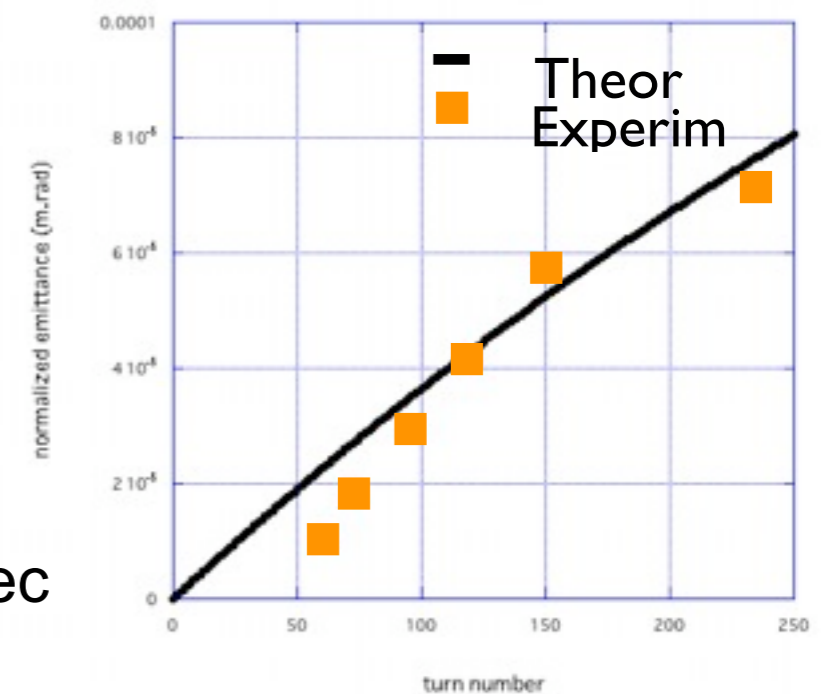
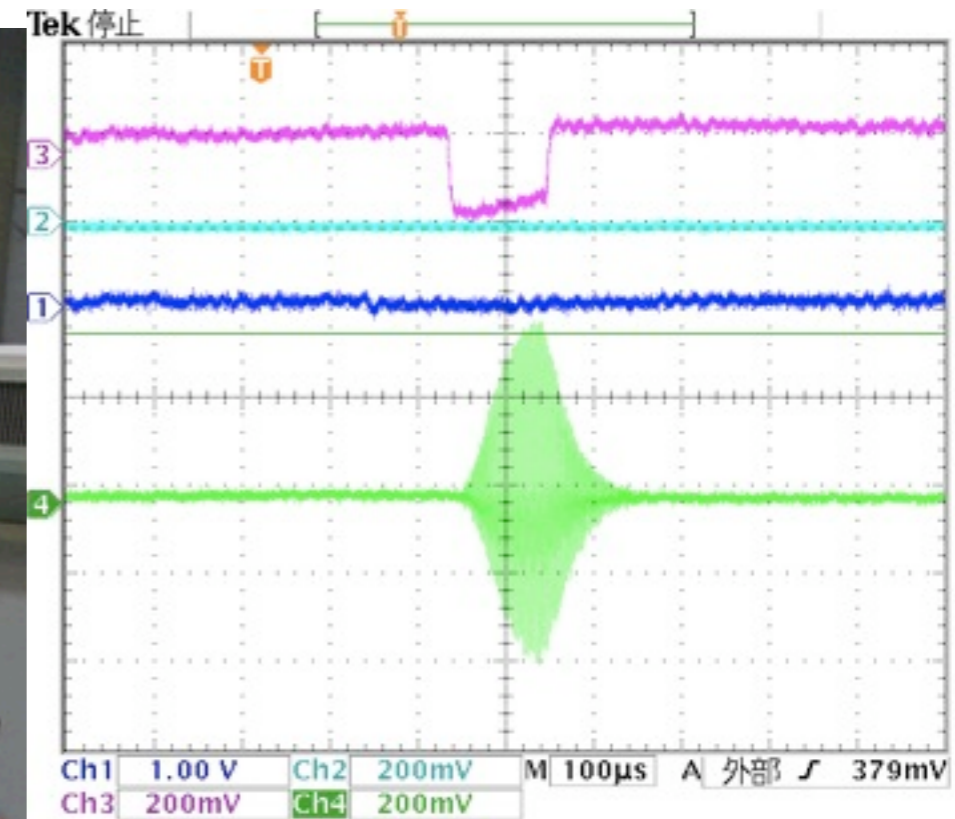
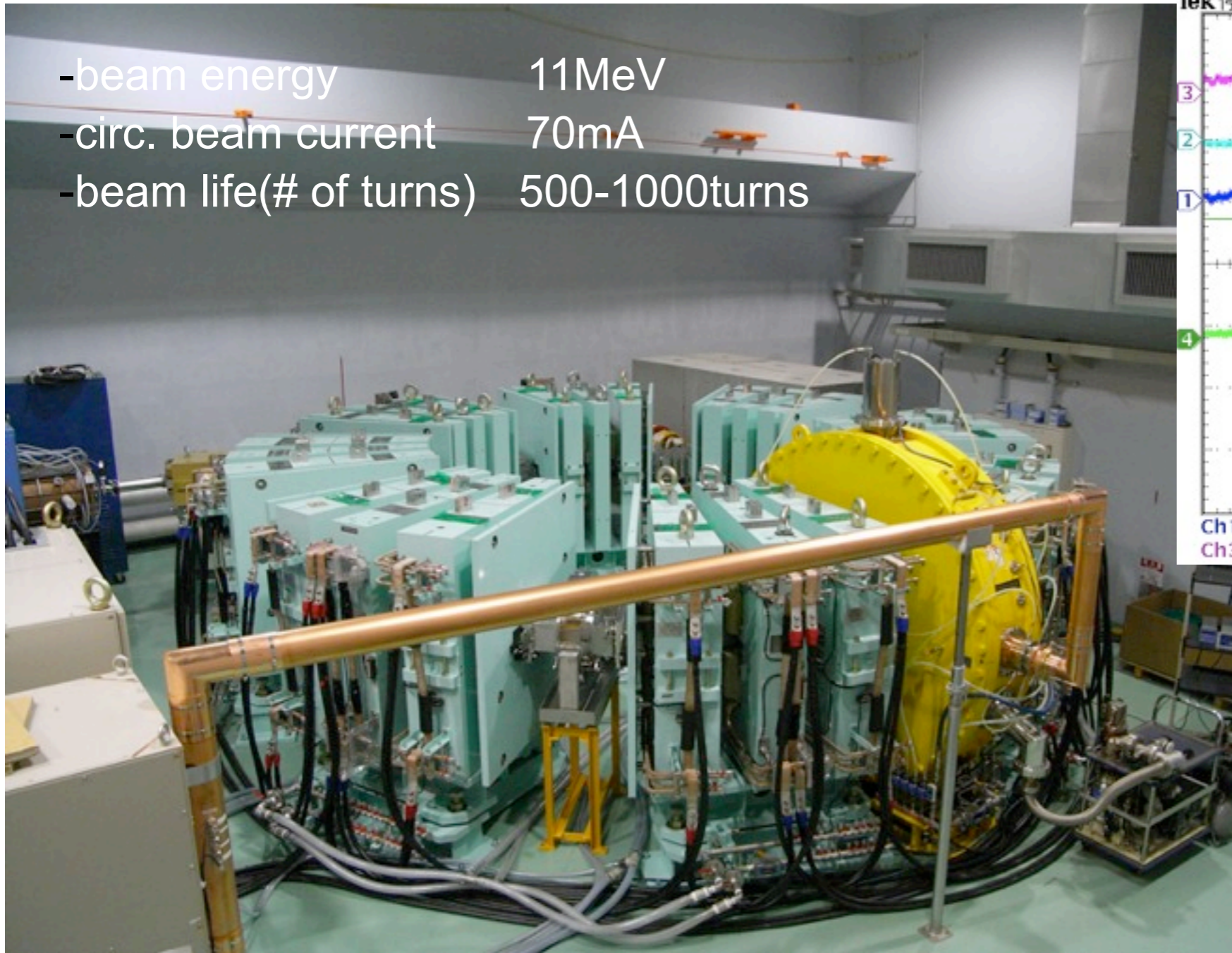
## ERIT-FFAG



Scheme : Neutron production with internal target placed in the proton storage ring where the beam emittance and energy are cured by ionization cooling.

# FFAG-ERIT RING

- beam energy 11MeV
- circ. beam current 70mA
- beam life(# of turns) 500-1000turns



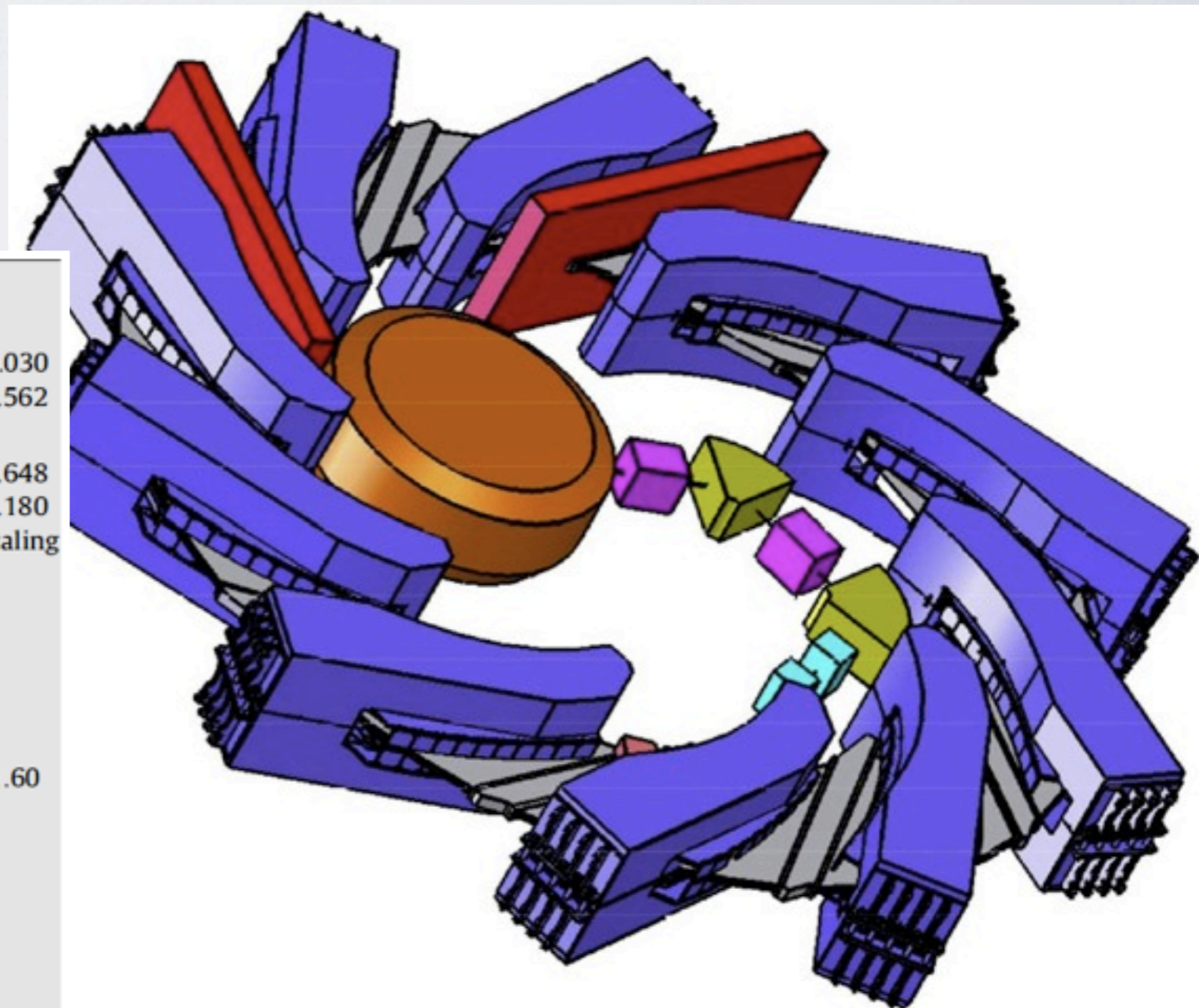
- acceptance  $A_v > 3000 \text{ mm.mrad}$ ,  
 $dp/p > \pm 5\%$ (full)
- $v_x, v_y$  1.77, 2.27

Neutron Yield  $> 10^{13} \text{ n/sec}$

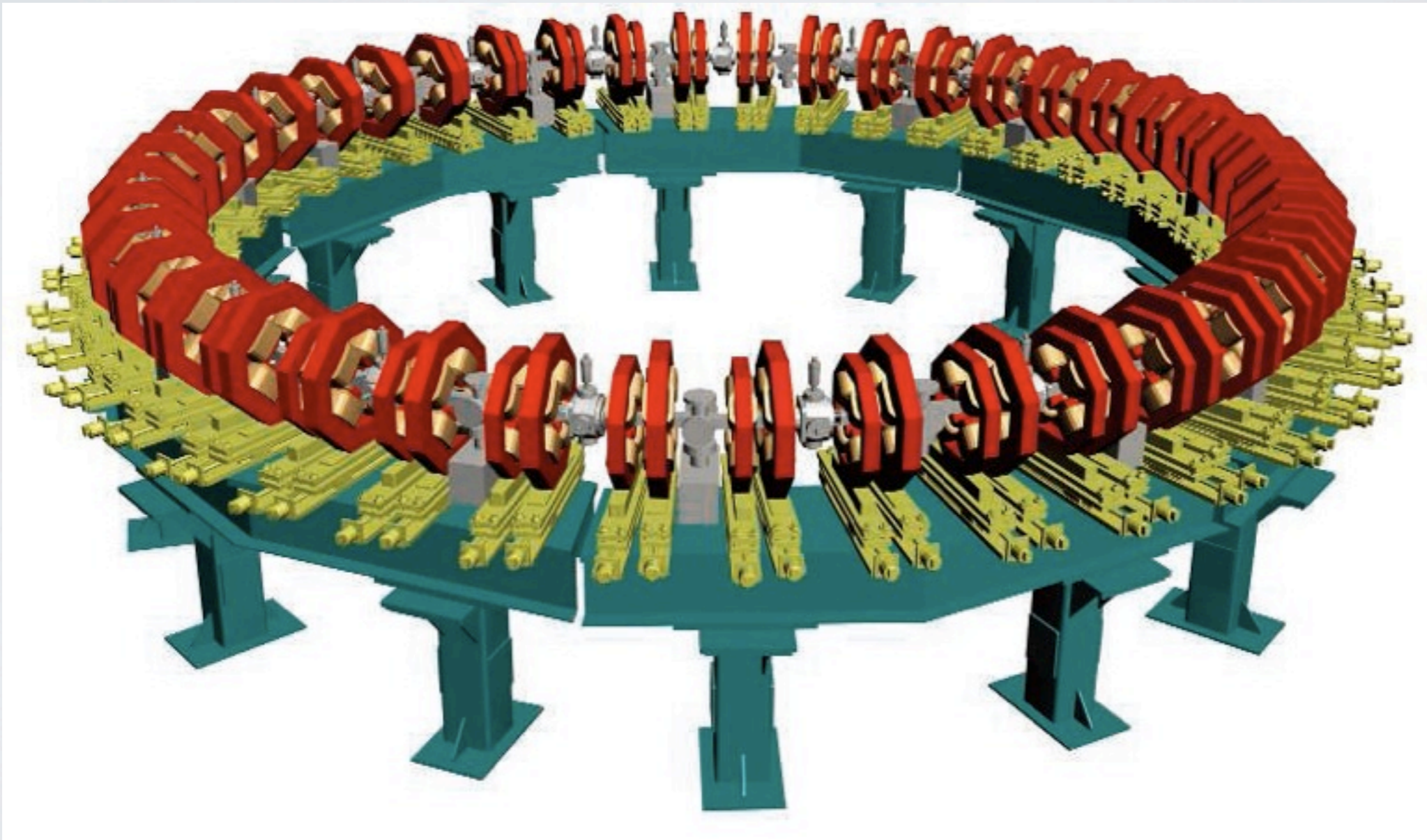
# RACCAM PROJECT GRENOBLE

- Hadron therapy

Extraction energy, variable ( $E_{xtr}$ )	MeV	70–180
Injection energy, variable ( $E_{inj}$ )	MeV	5.55–15
Extraction $B\rho$ ( $B\rho_{xtr}$ )	T m	1.231–2.030
Injection $B\rho$ ( $B\rho_{inj}$ )	T m	0.341–0.562
$B\rho_{xtr}/B\rho_{inj}$	T m	3.612
Extraction $\beta\gamma$		0.393–0.648
Injection $\beta\gamma$		0.109–0.180
Lattice type		Spiral, scaling
Number of cells ( $N$ )		10
Packing factor ( $pf$ )		0.34
Field index ( $k$ )		5.00
Spiral angle ( $\zeta$ )	deg.	53.7
Dipole sector angle ( $A$ )	deg.	12.24
$v_r$		2.76
$v_z$ for 15 → 180 MeV		1.55 → 1.60
Transition gamma ( $\gamma_{tr}$ )		2.45
Drift length on extraction orbit	m	1.42
Drift length on injection orbit	m	1.15
Radius on extraction orbit ( $r_{xtr} \equiv r_0$ )	m	3.460
Radius on injection orbit ( $r_{inj}$ )	m	2.794
Orbit excursion	m	0.667



## EMMA: Electron Model for Muon Accelerator under construction at UK



# PARAMETERS OF A 3.6 TO 12.6 GEV MUON RING

Lattice type	FDF triplet
Injection/extraction energy	3.6/12.6 GeV
RF frequency	200 MHz
Number of turns	6
RF peak voltage (per turn)	1.8 GV
Synchronous energy	8.04 GeV
Mean radius	$\sim 160.9$ m
$B_{max}$ (@ 12.6 GeV)	3.9 T
Field index $k$	1390
Total orbit excursion	14.3 cm
Harmonic number $h$	675
Number of cells	225
Long drift length	$\sim 1.5$ m
Horiz. phase adv. per cell	85.86 deg.
Vert. phase adv. per cell	33.81 deg.

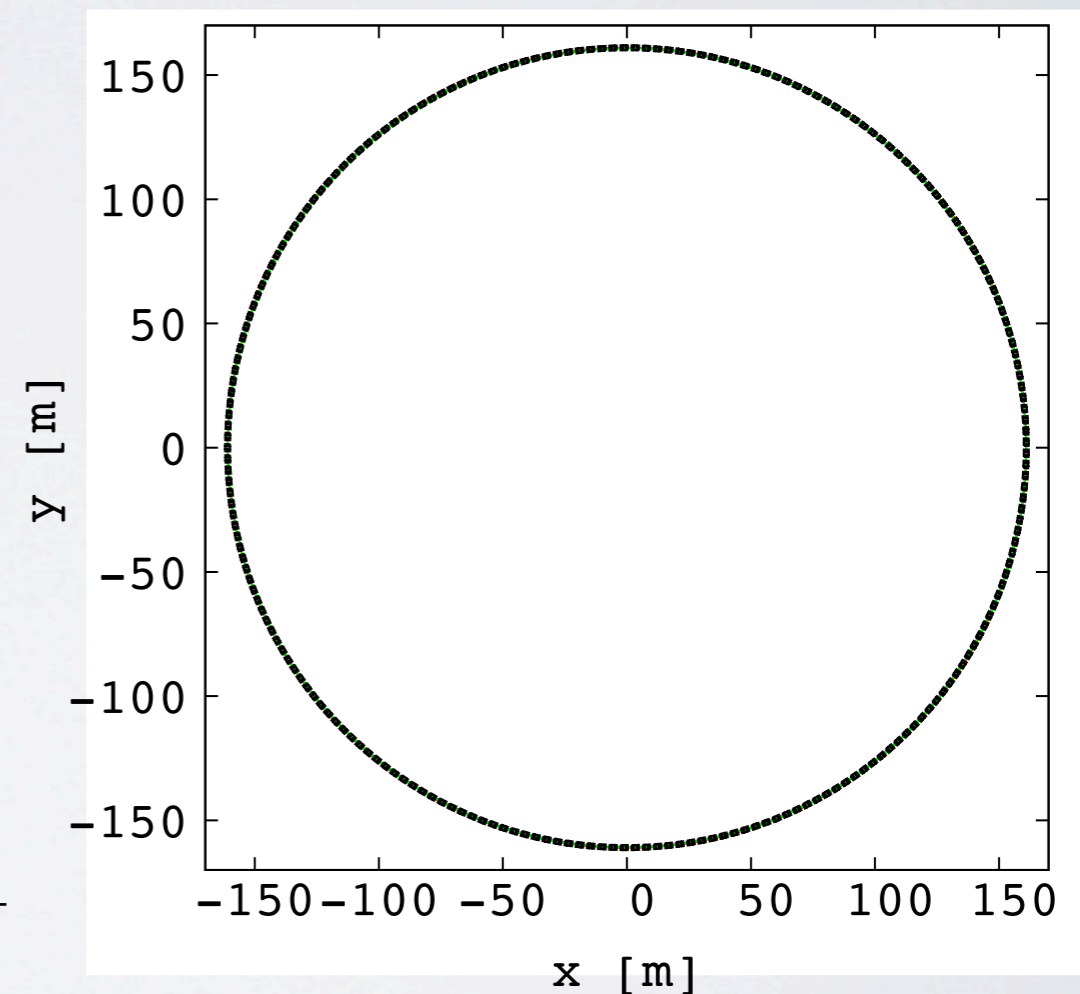
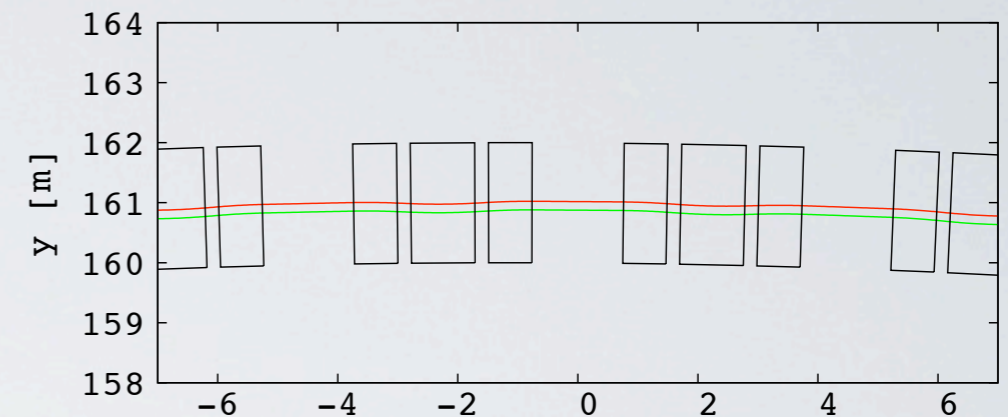


Table 1 - Example of 3.6 to 12.6 GeV muon scaling FFAG ring parameters.

Figure 3 - Ring layout.

# PARAMETERS OF A 3.6 TO 12.6 GEV MUON RING

Lattice type	FDF triplet
Injection/extraction energy	3.6/12.6 GeV
RF frequency	200 MHz
Number of turns	6
RF peak voltage (per turn)	1.8 GV
Synchronous energy	8.04 GeV
Mean radius	$\sim 160.9$ m
$B_{max}$ (@ 12.6 GeV)	3.9 T
Field index $k$	1390
Total orbit excursion	14.3 cm
Harmonic number $h$	675
Number of cells	225
Long drift length	$\sim 1.5$ m
Horiz. phase adv. per cell	85.86 deg.
Vert. phase adv. per cell	33.81 deg.

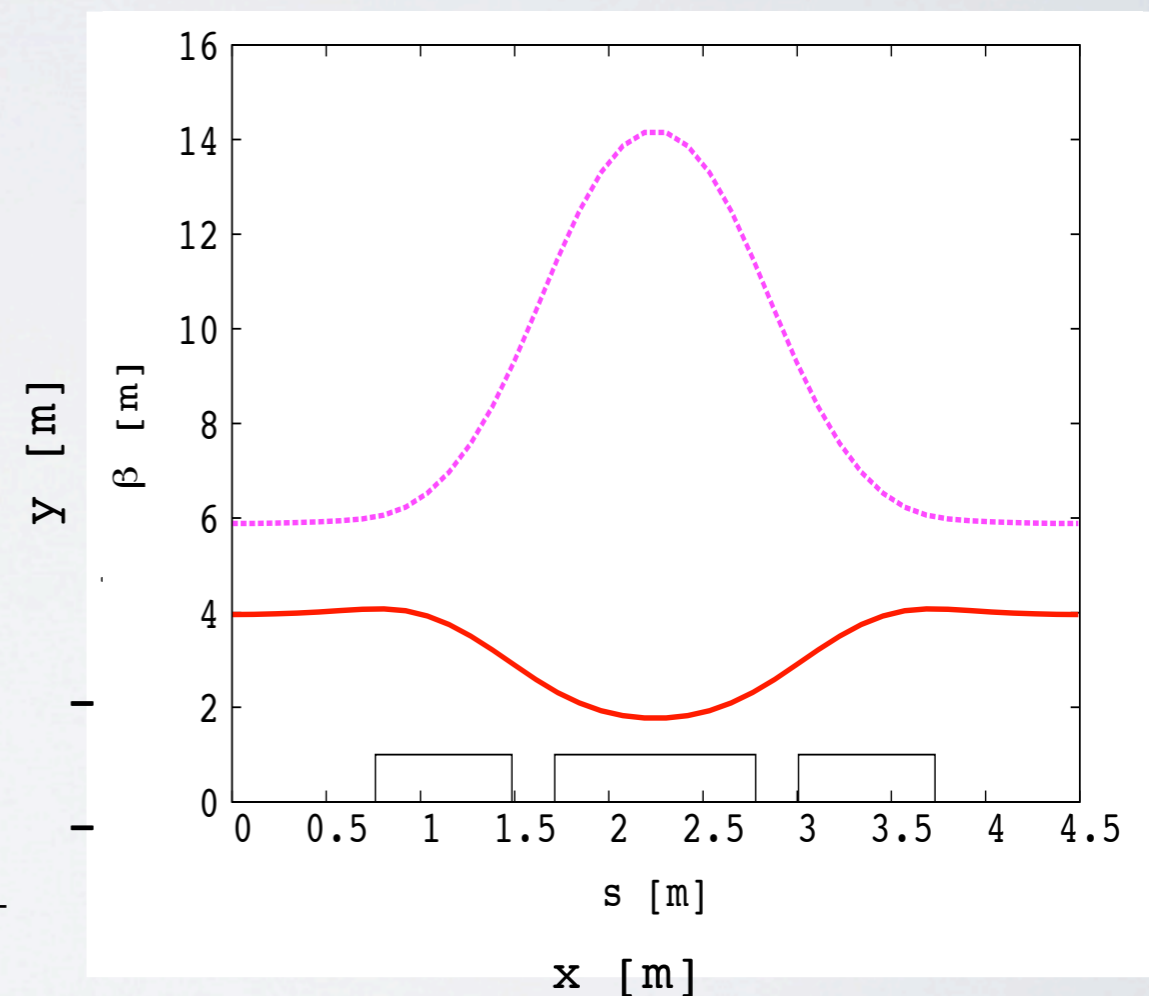
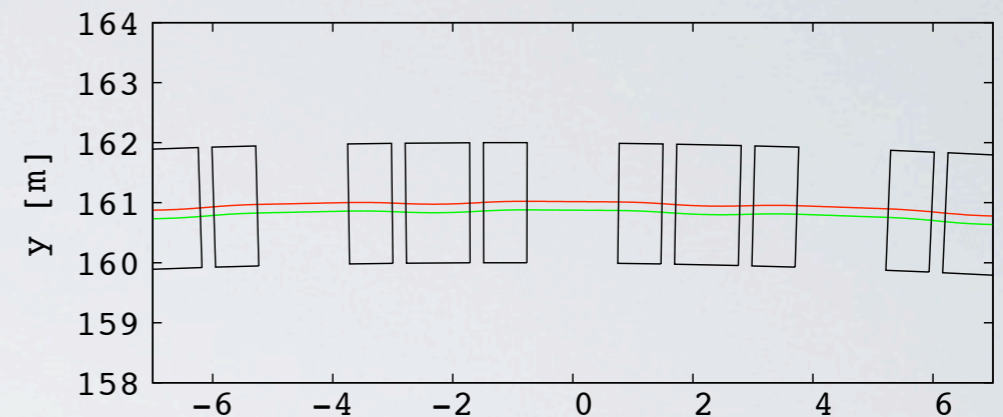


Figure 3 - Ring layout.

Table 1 - Example of 3.6 to 12.6 GeV muon scaling FFAG ring parameters.

# PARAMETERS OF A 3.6 TO 12.6 GEV MUON RING

Lattice type	FDF triplet
Injection/extraction energy	3.6/12.6 GeV
RF frequency	200 MHz
Number of turns	6
RF peak voltage (per turn)	1.8 GV
Synchronous energy	8.04 GeV
Mean radius	$\sim 160.9$ m
$B_{max}$ (@ 12.6 GeV)	3.9 T
Field index $k$	1390
Total orbit excursion	14.3 cm
Harmonic number $h$	675
Number of cells	225
Long drift length	$\sim 1.5$ m
Horiz. phase adv. per cell	85.86 deg.
Vert. phase adv. per cell	33.81 deg.

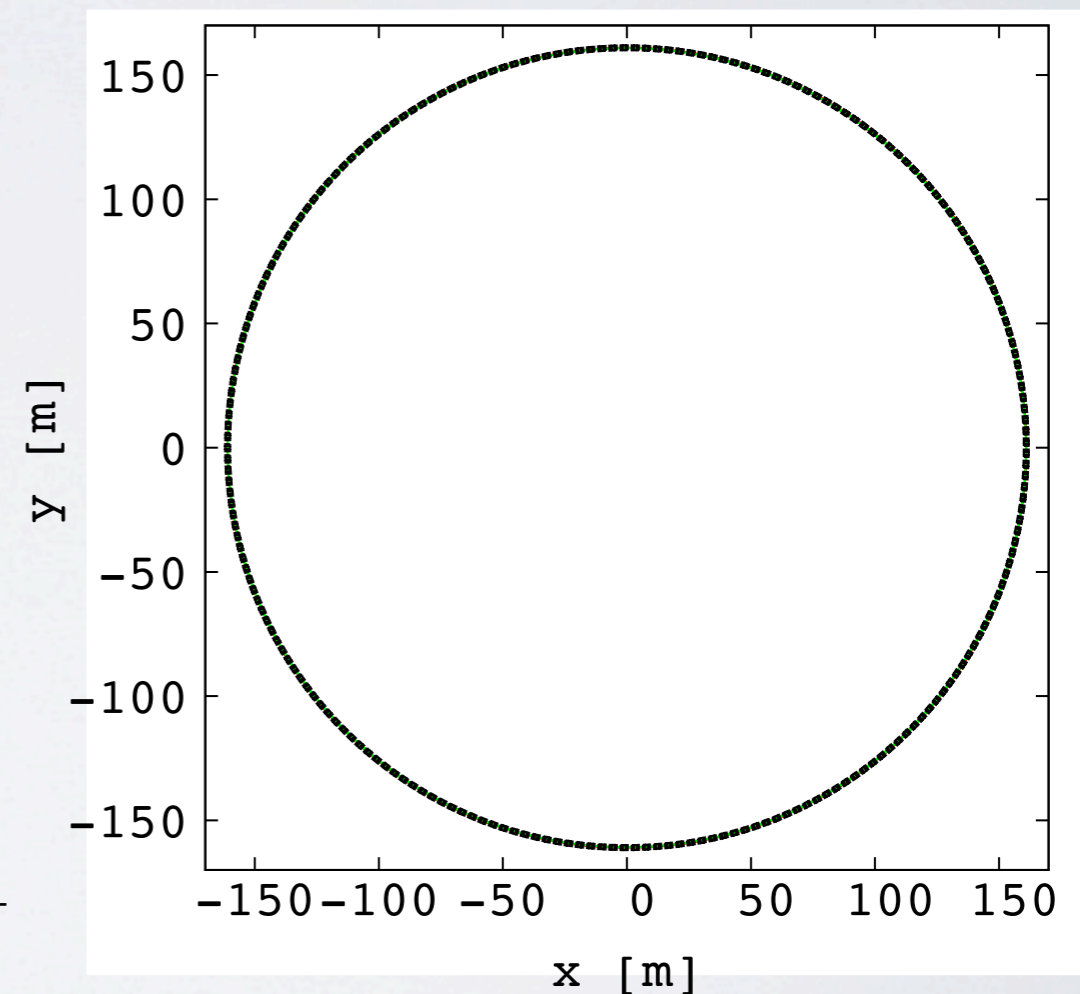
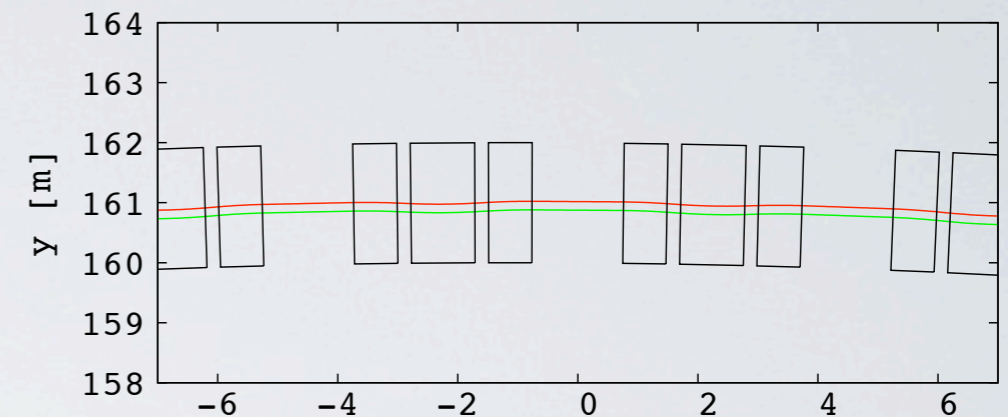
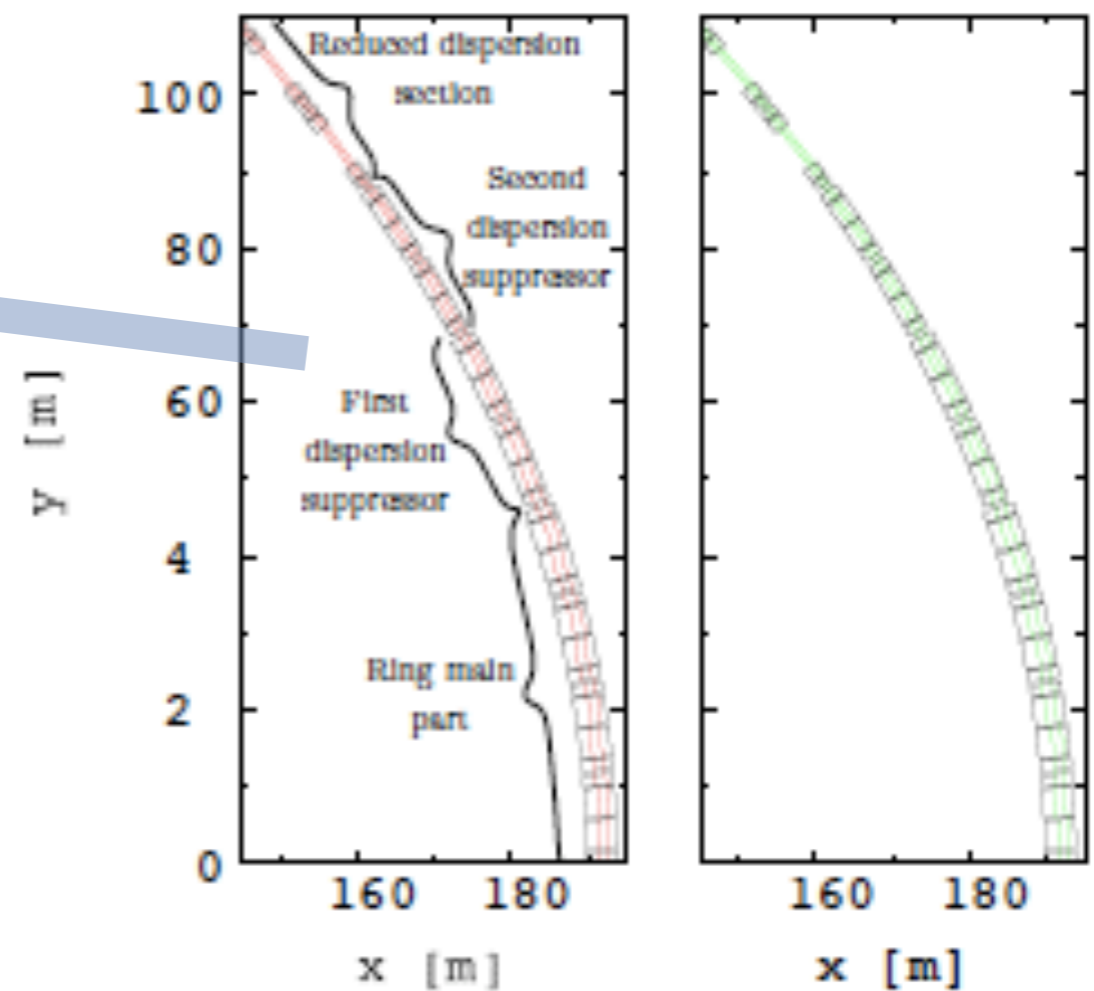
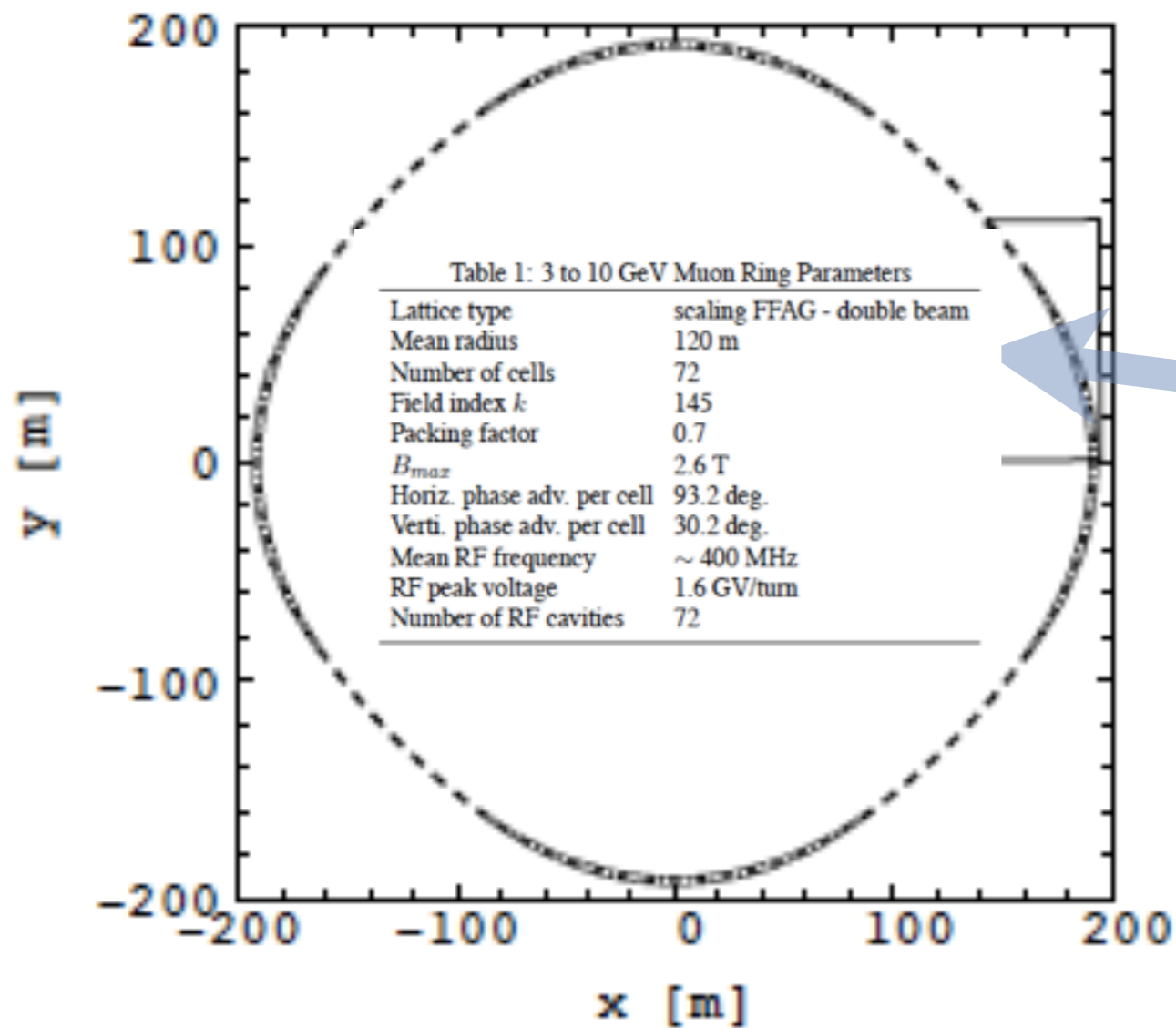


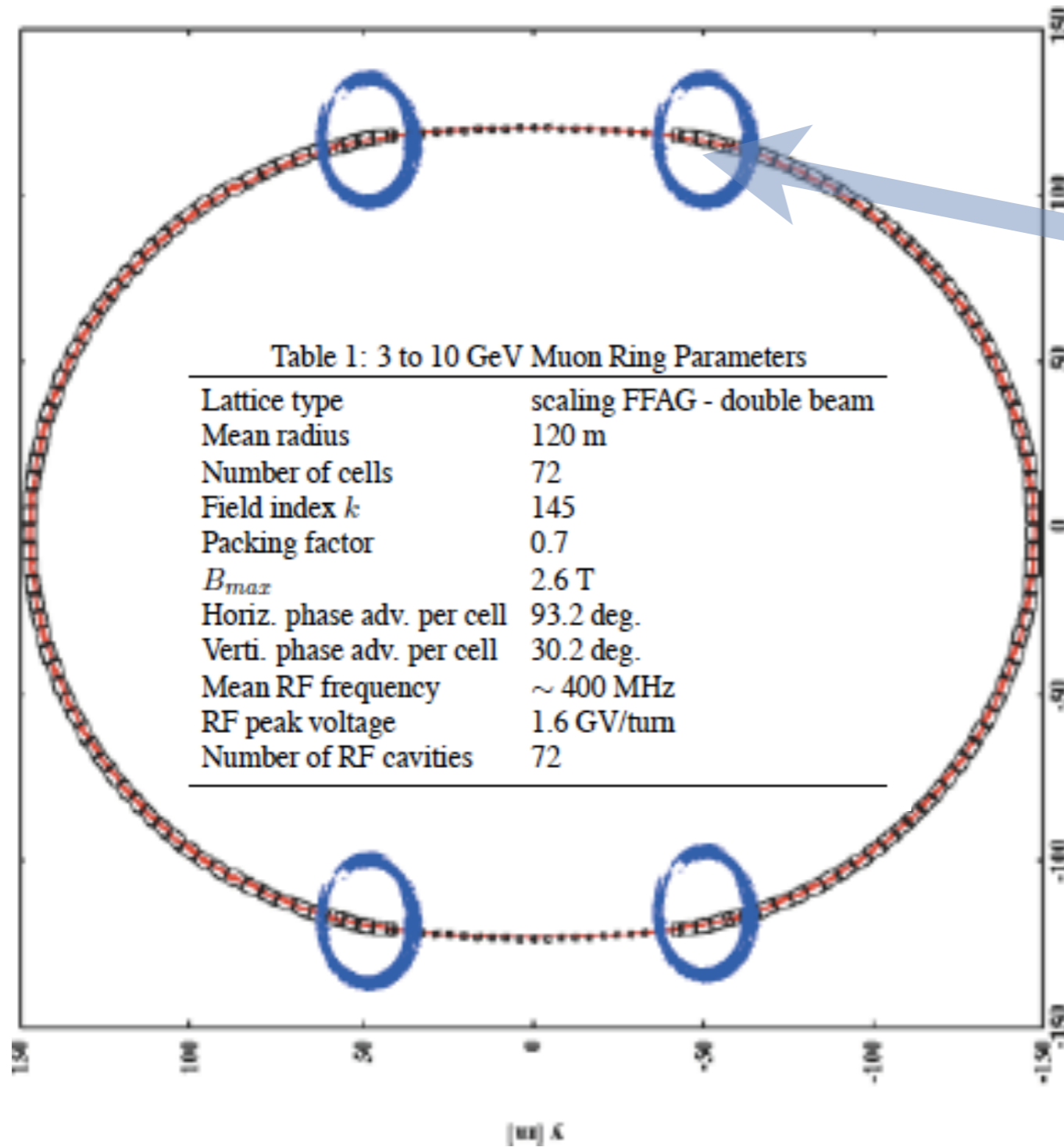
Table 1 - Example of 3.6 to 12.6 GeV muon scaling FFAG ring parameters.

Figure 3 - Ring layout.

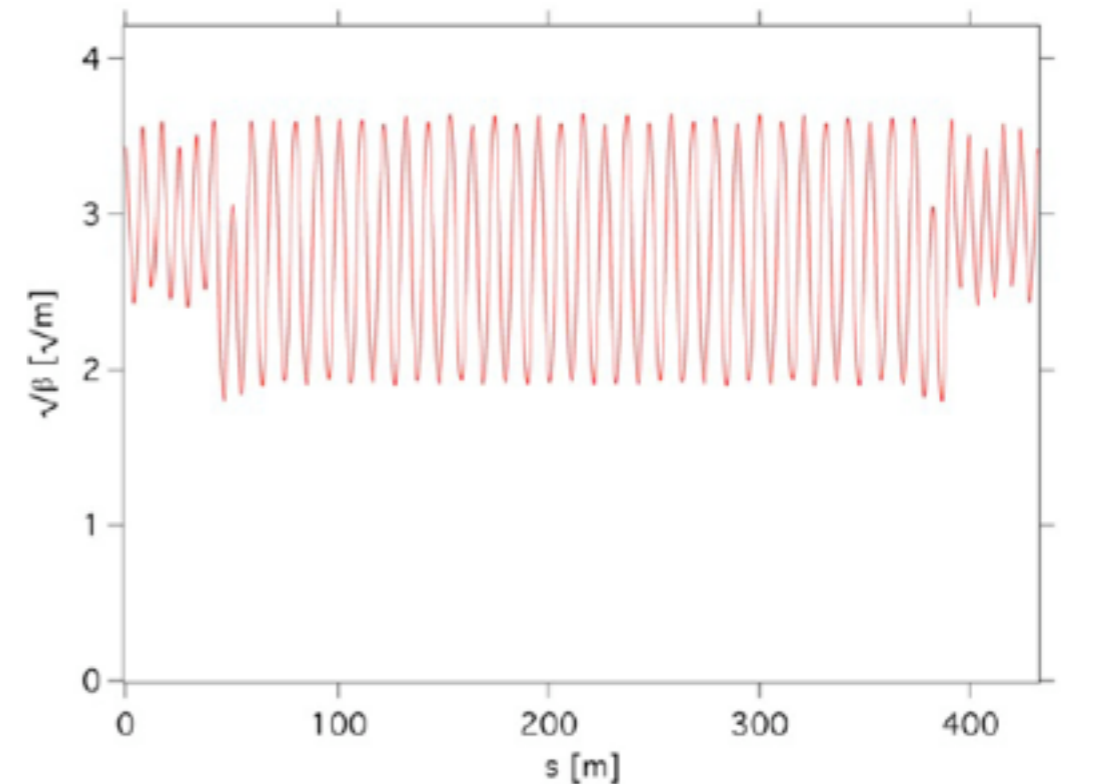
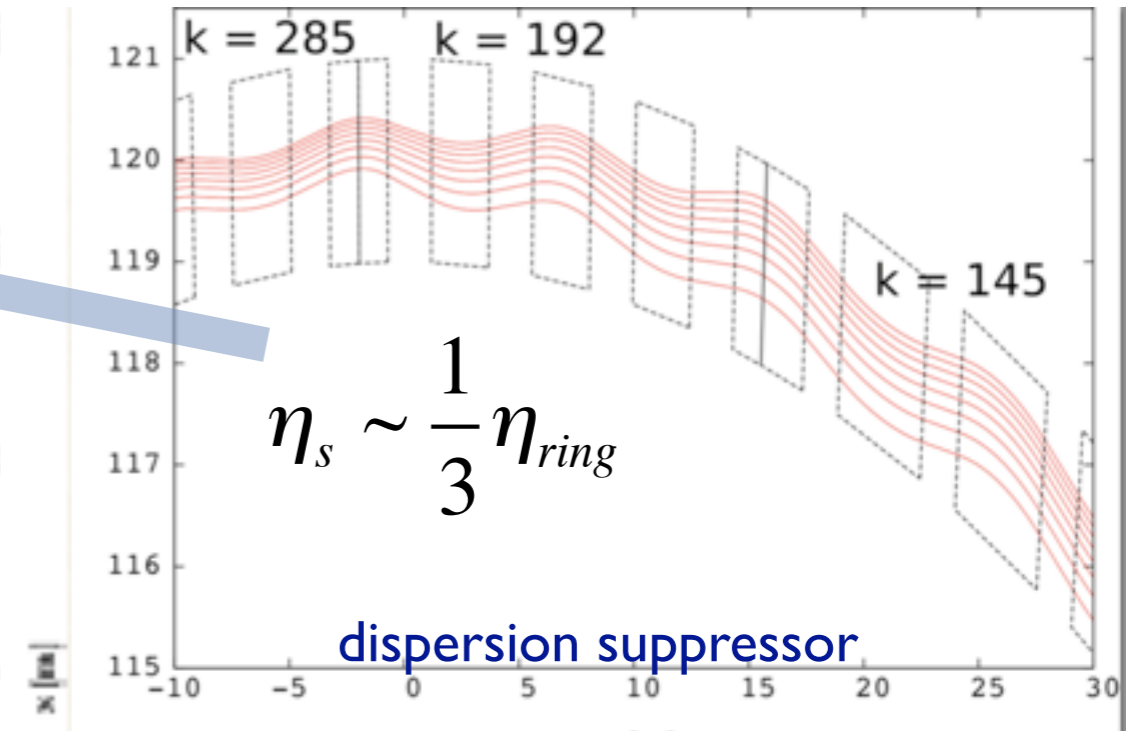
# Muon accelerator: 3.6-12.6 GeV neutrino factory



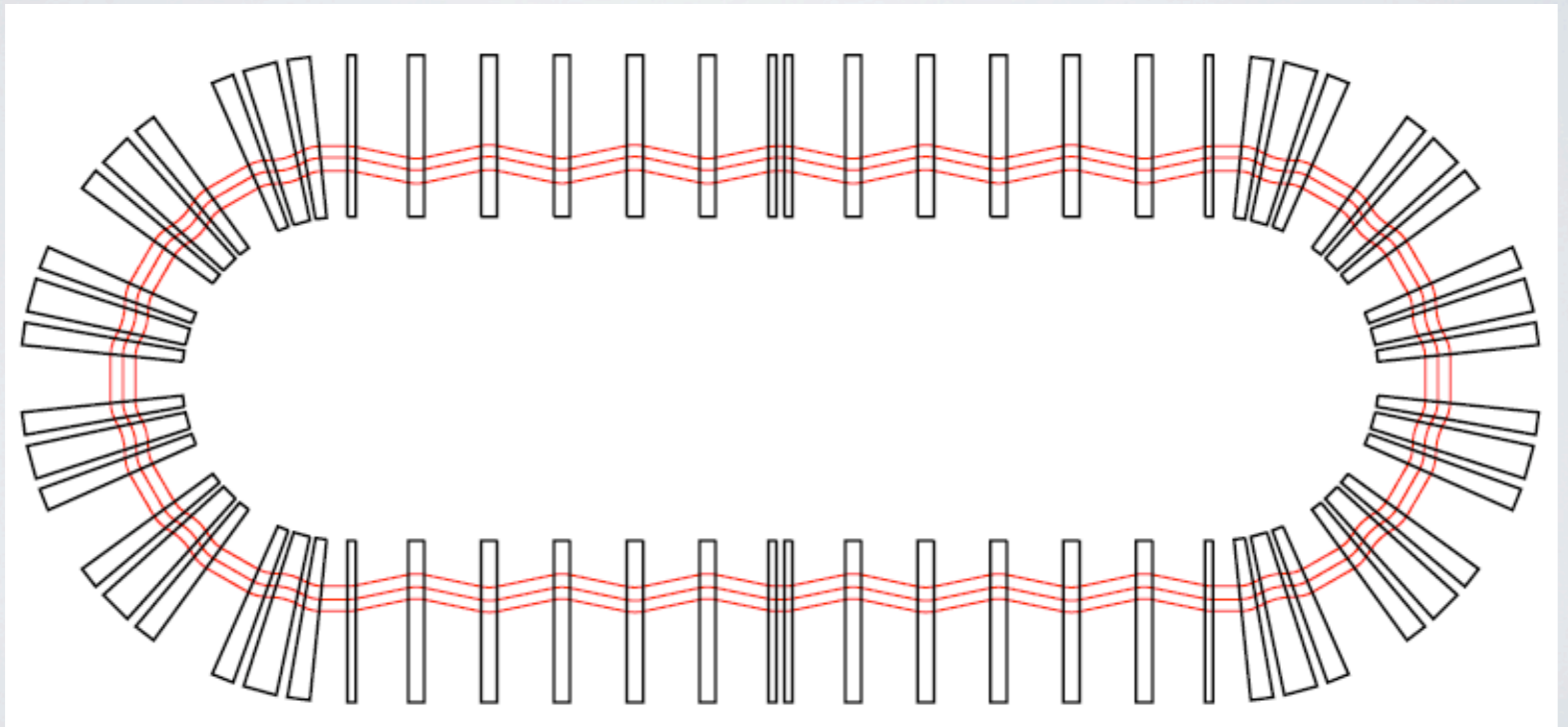
# Muon accelerator neutrino factory



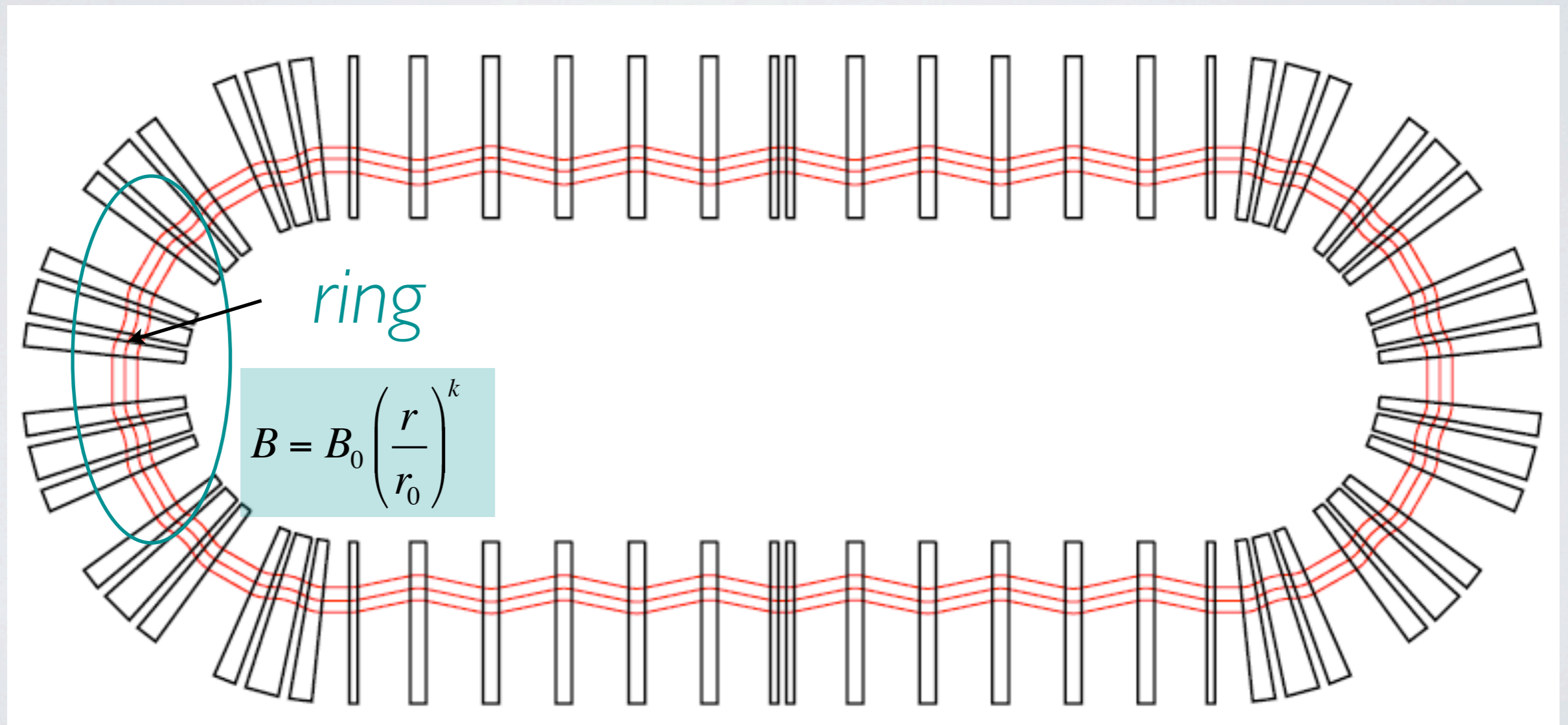
Planch, Mori



# ADVANCEMENT OF SCALING FFAG

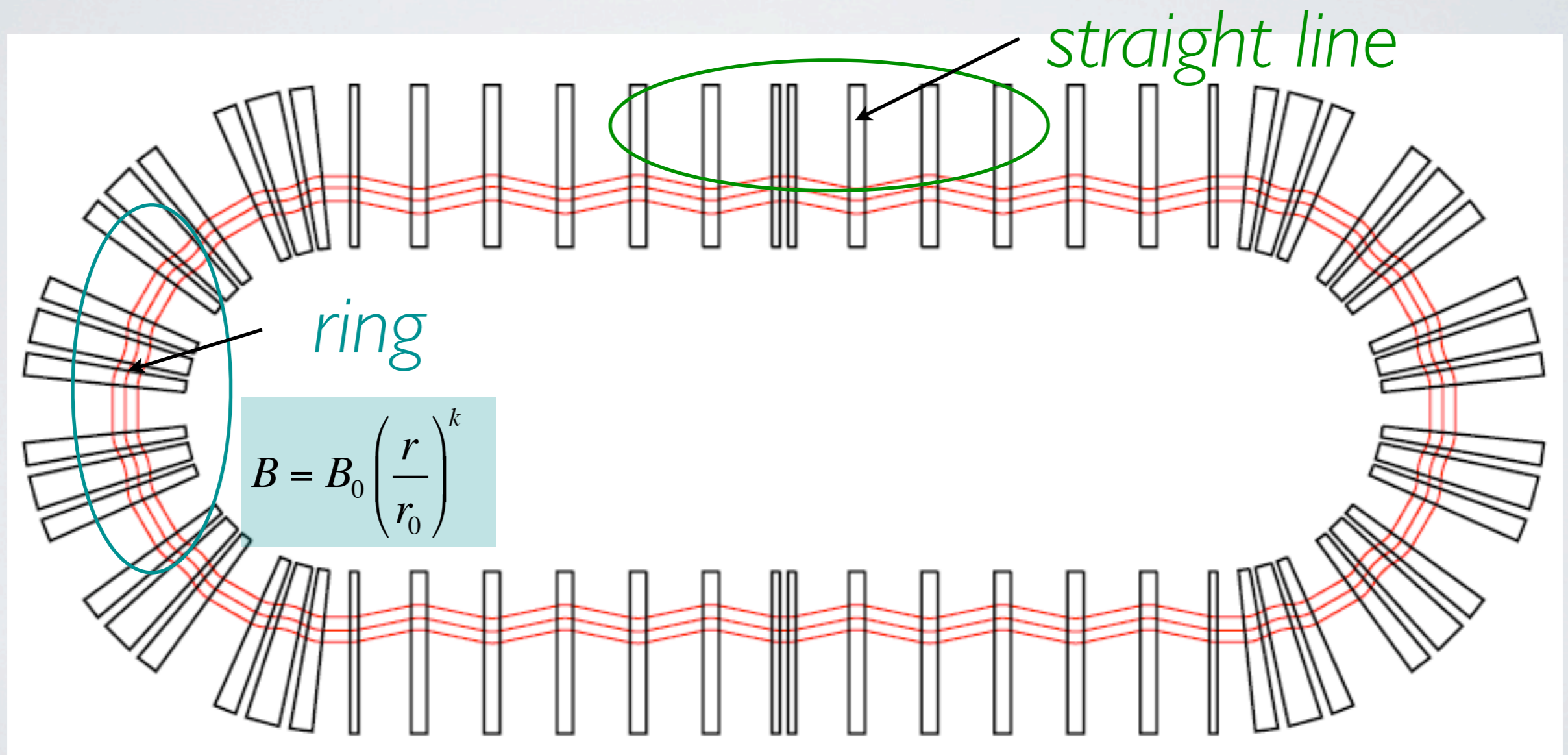


# ADVANCEMENT OF SCALING FFAG



# ADVANCEMENT OF SCALING FFAG

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$



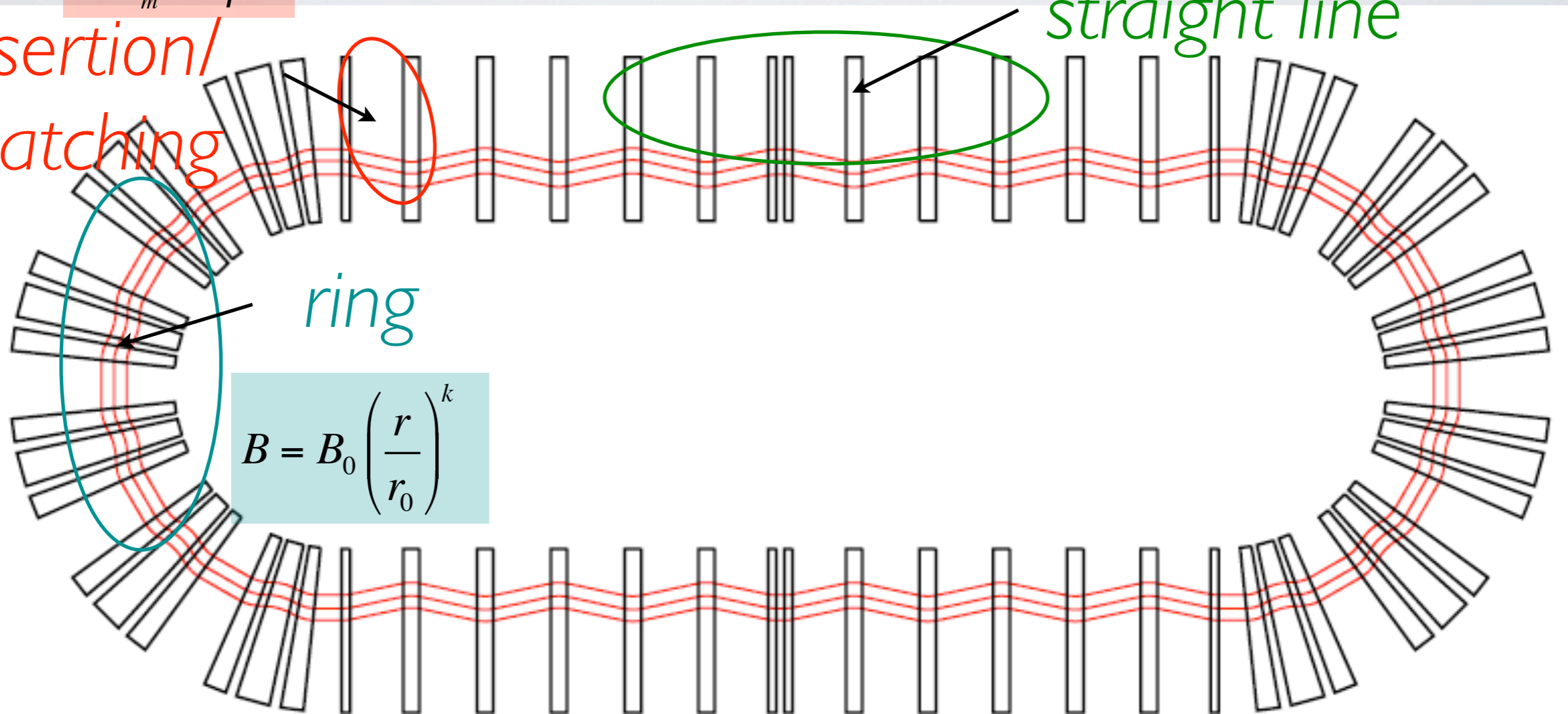
# ADVANCEMENT OF SCALING FFAG

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

insertion/  
matching

straight line



## ADVANCEMENT OF SCALING FFAG

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

insertion/  
matching

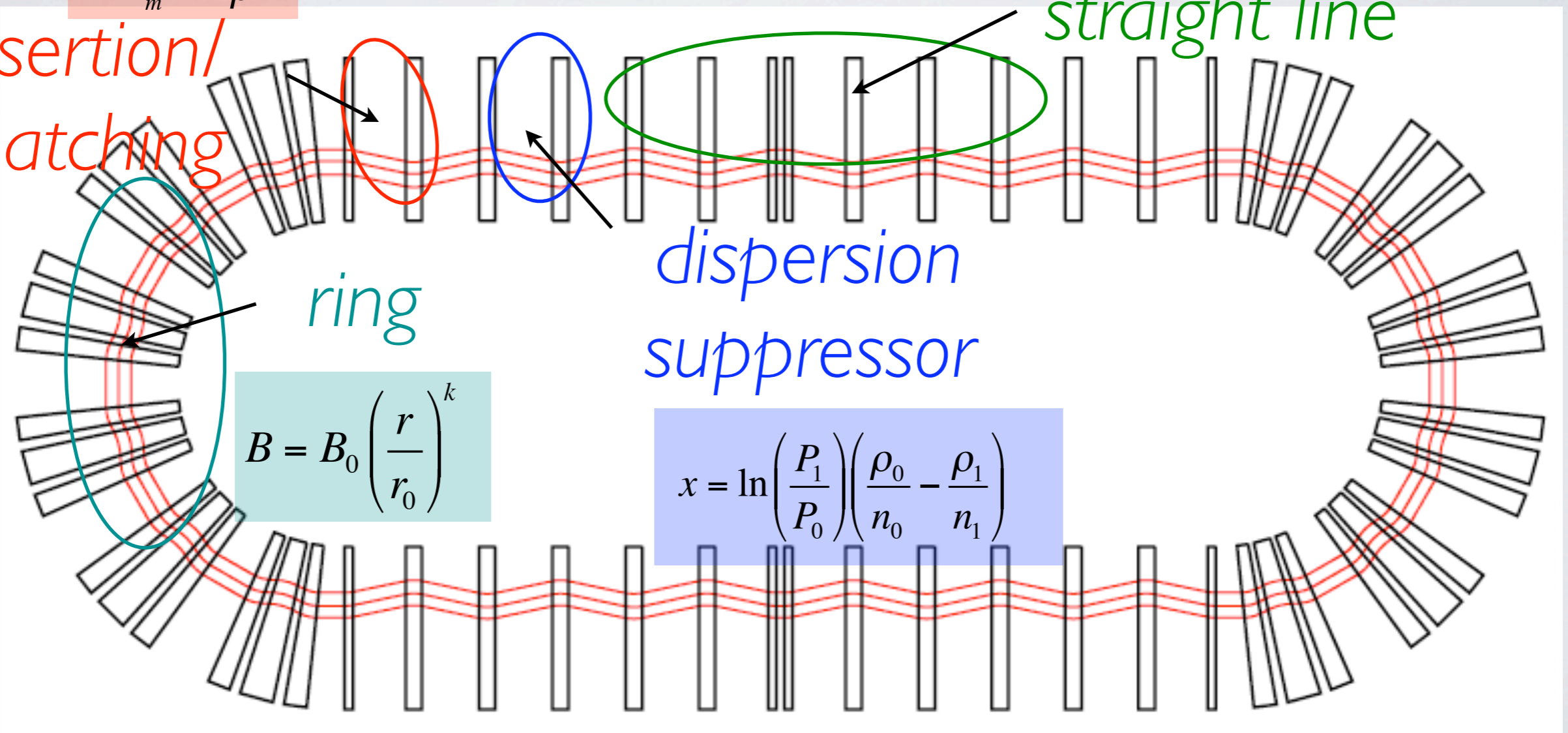
straight line

dispersion  
suppressor

ring

$$B = B_0 \left(\frac{r}{r_0}\right)^k$$

$$x = \ln\left(\frac{P_1}{P_0}\right) \left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$



# FFAG IN FUTURE

- G.W. Friedrich Hegel : Ideology must be essentially overcome.
- ヘーゲル：理想理念は必ず乗り越えられねばならない。