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FFAG School, Kyoto University Research Reactor Institute, 26, 27 Oct. 2010

Stepwise ray-tracing methods in FFAG design

Contents

1 Introduction

- We have learned from the previous lessons that the FFAG method is considered in many types of applications, as a result of the technological progress, compared to the early, 1950s, era, in domains as computional lattice design, 3-D magnet design and magnet technology, RF technology, etc.

- In this lecture I will review several of these possible applications, under the angle of beam dynamics and its specificities, and specific difficulties, in matter of FFAGs.

- That inventory of FFAG applications is not going to be exhaustive, it does not aim at that, it aims rather at pointing various questions or issues relevant to our topic today, stepwise ray-tracing methods in FFAG design, trying to show why they are a necessary step, what they bring that other methods don't, etc.

I recommend a highly educational lecture which addresses in many aspects the question of numerical calculation in FFAG design :
"O Camelot ! A Memoir of the MURA Years", by Franck Cole.

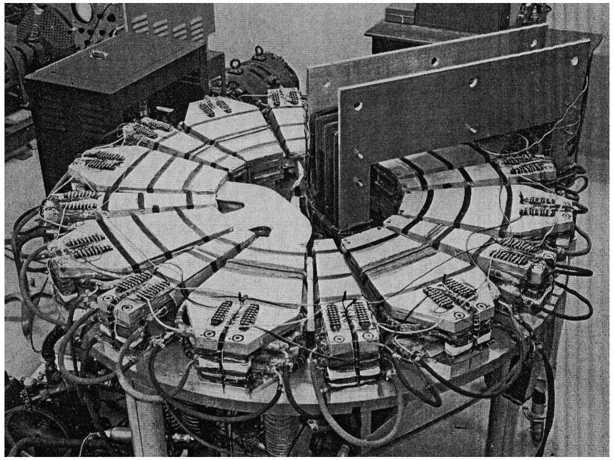
- In addition, a substantial part of the time alloted to this lecture will be devoted to tutorial activities : we will bend on numerical exercises regarding beam dynamics, computation and optimization of machine parameters, etc., using a real stepwise ray-tracing code, and producing real results !

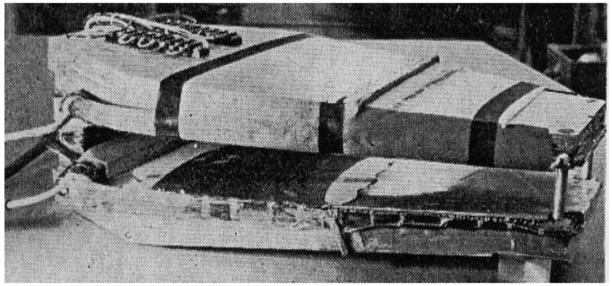
2 FFAGs. A mixed of history / theory / computational methods / introduction to the tutorial

The first model, by MURA, radial sector FFAG, Mark II

Objectives : confirm theoretical predictions ; study FFAG properties : optics, injection, test RF programs ; effects of misalignments ; effects of resonances.

First operation March 1956, U of Michigan.



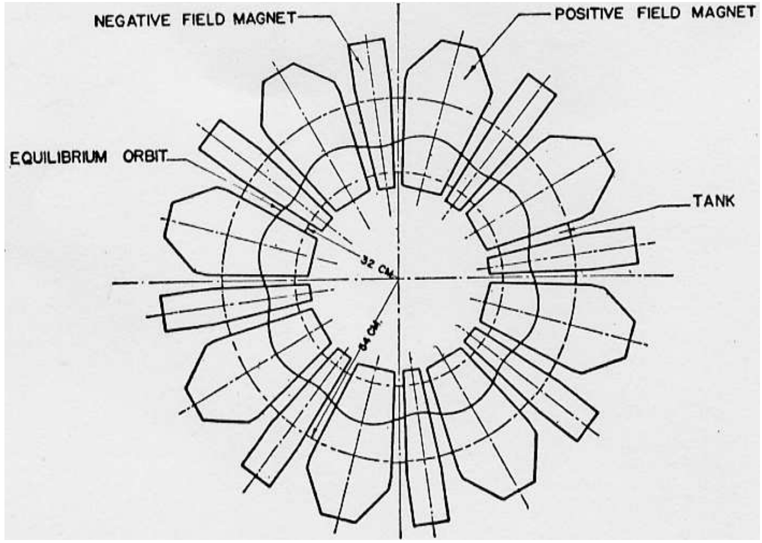


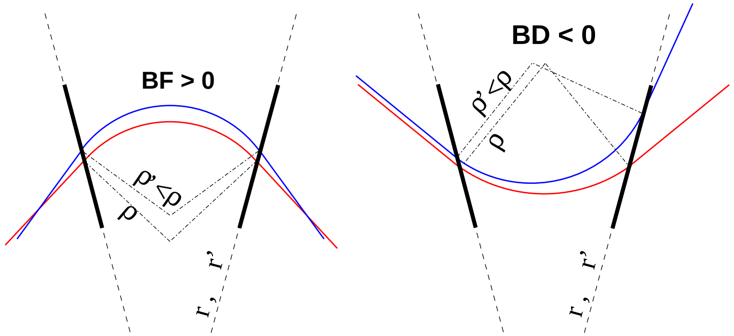
F magnet, positive field, radially focusing.

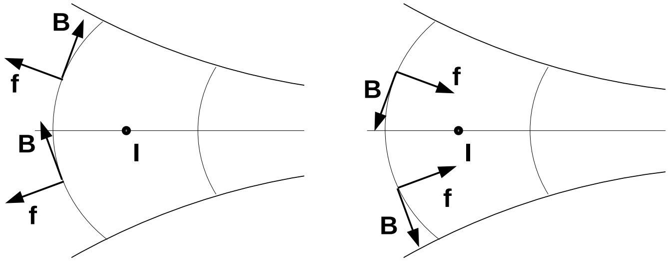
Machine parameters			criteria / comments
$E_{inj} - E_{max}$	*keV*	25 - 400	*small size, easy to build* *field not too low, ms lifetime*
orbit radius ($\mathcal{C}/2\pi$)	*m*	0.34 - 0.50	**SPIRALING ORBIT**
Optics		**STRONG FOCUSING,**	**SCALING** $\rightarrow \xi = 0$
lattice		$\frac{\mathbf{D}}{\mathbf{2}}\mathbf{F}\frac{\mathbf{D}}{\mathbf{2}}$	
number of cells		8	*16 magnets, 4.41 deg. drifts*
field index K		3.36	*gap*$\sim r$ *& coil windings*
ν_r / ν_z		2.2-3 / 1-3	*varying K, resp.* B_F/B_D *varies mostly* ν_r*, resp.* ν_z
γ_t		≈ 2	$\sqrt{1+K}$
Magnet		radial sector	$\boxed{B = B_0(r/r_0)^K\ F(\theta)}$
$\theta_F,\ \theta_D$	*deg*	25.74, 10.44	*sector angles*
$r_{F,D}/\rho$		2.85, 2.59	*at center of F, D magnets*
gap	*cm*	6 - 4	$\boxed{gap/r = Cst}$
Injection		continuous or pulsed	
Acceleration		betatron first, then RF gap	*for simplicity*
swing	*Gauss*	40 - 150	
rep. rate	*Hz*	a few 10's	
		... completed with RF acc., next	*split tank*
freq. swing	*MHz*	10 in [35, 75] MHz	*for RF stacking expts*

Radial scaling FFAG : how it works

- Magnetic field fixed in time, $\boxed{B = \pm B_0(\frac{r}{r_0})^K}$
  - from lower flux on inner orbit
  - to largest flux on outer orbit
- Transverse motion stability is insured by $\boxed{\textbf{strong, AG focusing}}$
  - as in pulsed synchrotrons, hence small beta functions
  - AG is obtained by alternance of
    - positive curvature field sectors, hence focusing, $\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} > 0$
    - negative curvature field sectors, hence defocus sing, $\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} < 0$
    - with ratio $|\int B_D\,ds| \approx \frac{2}{3} \times \int B_F\,ds$, to insure axial focusing
- The radial dependence $B = B_0(r/r_0)^K$ yields $\boxed{\text{zero chromaticity}}$ and the *scaling* property :
  - orbits are similar wrt. geometrical center
  - tunes are independent of orbit
- Corollaries
  - large *circumference factor* $\mathcal{C}/2\pi\rho$ due to alternating curvature
  - drift length is not free
  - $\alpha = 1/(1+K) \rightarrow$ larger K insures smaller $r_{max} - r_{min}$
  - transition energy $E_{tr} = E_0/\sqrt{\alpha} \approx E_0\sqrt{1+K}$ easily beyond E_{max}
- Longitudinal motion : regular synchrotron motion. In addition
  - arbitrary RF programs are possible : ω_{RF} does not track B
  - extremely high accelerating gradients are possible : B is constant







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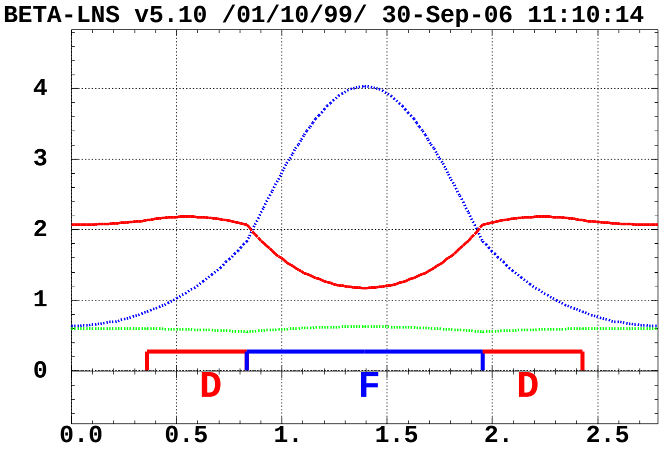
Linear optics : allows preliminary design steps based on regular “TRANSPORT” codes



First : find $\boxed{\text{a}}$ closed orbit ← from the FFAG parameters.

Then : linear approximation about that closed orbit

$$\boxed{x'' + \frac{1-n}{\rho^2}x = 0, \qquad z'' + \frac{n}{\rho^2}z = 0}$$

with $n(s) = -\frac{\rho(s)}{B(s)}\frac{dB}{dx} \approx -\frac{\rho}{B}\frac{dB}{dr}$ (scalloping is neglected)

Index $n(s)$ and K in $B = B_0(\frac{r}{r_0})^K$ relate as follows:

$\frac{dB}{dr} = K\frac{B_0}{r_0}(\frac{r}{r_0})^{K-1} = K\frac{B}{r}$ so that $\boxed{K/r = -n/\rho}$

The matrix representing a sector has the form $M = \begin{bmatrix} \cos(s\sqrt{k}) & \frac{1}{\sqrt{k}}\sin(s\sqrt{k}) \\ -\sqrt{k}\sin(s\sqrt{k}) & \cos(s\sqrt{k}) \end{bmatrix}$

with $k = (1-n)/\rho^2$ (radial motion) or $k = n/\rho^2$ (vertical motion)

The geometry provides the wedge angles, hence wedge matrices, M_{Fe1}, M_{Fe2}, M_{De1}, M_{2e2}

The product matrix representing a D-F sector yields the phase advance :

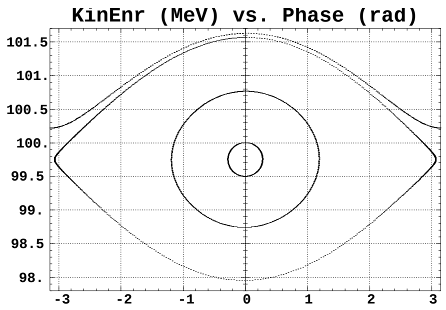
$\cos(\mu) = \frac{1}{2}Tr(M_{Fe2} \times M_F \times M_{Fe1} \times M_{De2} \times M_D \times M_{De1})/2$,...

The longitudinal motion in presence of RF satisfies, most classically

$$\Phi'' + \frac{\Omega^2}{\cos\phi_s}(\sin\phi - \sin\phi_s) = 0$$

synchrotron frequency $f_s = \Omega_s/2\pi = \frac{c}{\mathcal{L}}\left(\frac{h\eta\cos\phi_s q\hat{V}}{2\pi E_s}\right)^{1/2}$,

bucket height $\pm\frac{\Delta p}{p} = \pm\frac{1}{\beta_s}\left(\frac{2q\hat{V}}{\pi h\eta E_s}\right)^{1/2}$, etc.



Stationary bucket

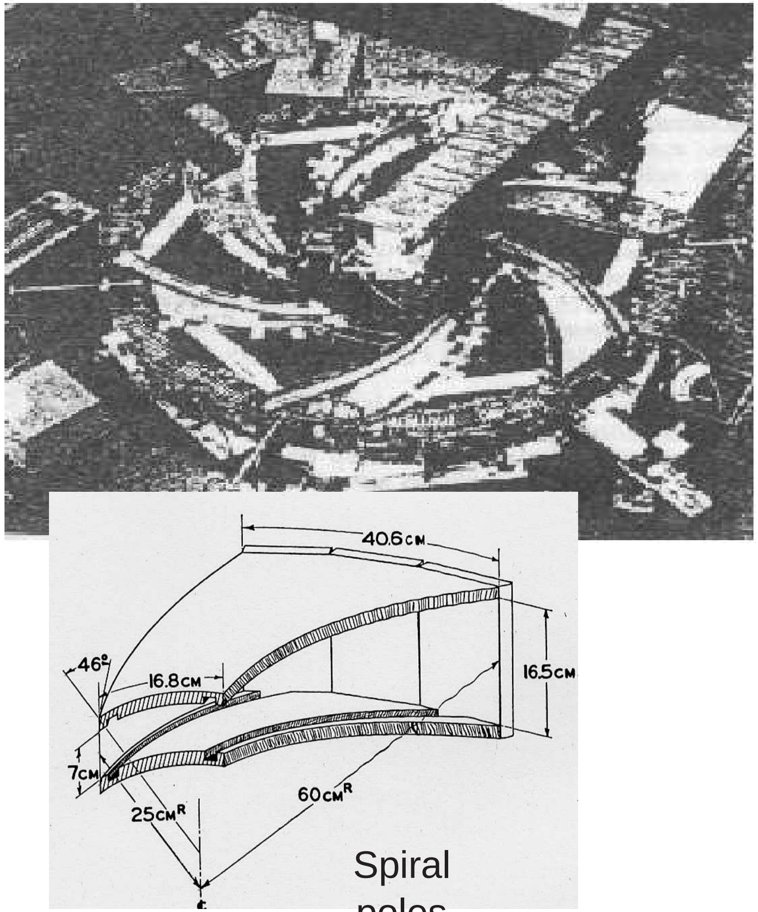
Second model, spiral sector FFAG, Mark V

The idea in the spiral FFAG was to superpose a positive field on top of the alternating sign one of the radial sector, so as to always have the good curvature sign, hence smaller accelerator.

By doing so, vertical focusing is altered, hence additional edge focusing using spiral shape.

Study objectives : confirm theoretical predictions - first extensive use of computers to determine magnetic field and machine parameters ; long-term orbit stability ; RF acceleration methods.

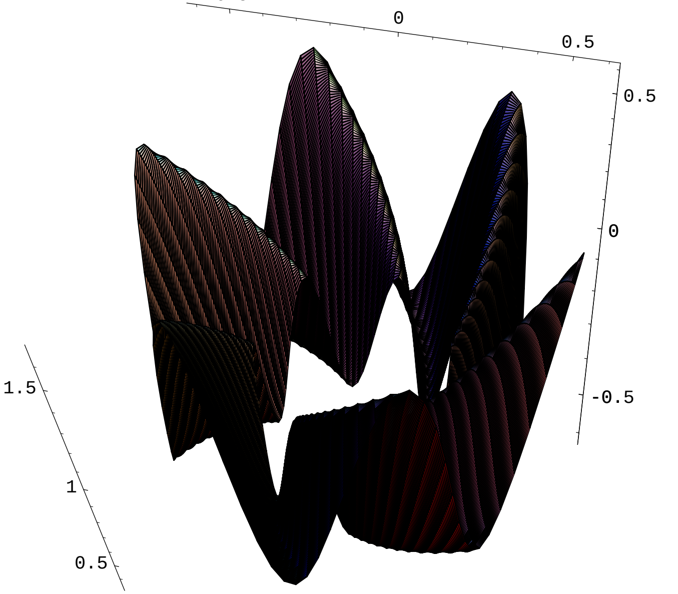
First operation Aug. 1957 at the MURA Lab., Madison.



Spiral poles

		Machine parameters	criteria / comments
$E_{inj} - E_{max}$	*keV*	35 - 180	*reasonable size magnets*
orbit radius	*m*	0.34 - 0.52	**spiraling orbit**
E_{tr} / r_{tr}	*keV / m*	155 / 0.49	*RF exprmnts* *at* $\gamma_{tr} = (1+K)^{1/2}$
Optics	**SCALING** $\rightarrow \xi = 0$		
lattice		N spiral sectors	
number of sectors		6	
field index K		0.7	*coil windings, tunable 0.2-1.16*
flutter F_{eff}		1.1	*tuning coils / 0.57 - 1.60*
ν_r / ν_z		1.4 / 1.2	*tunable*
β_r / β_z	*m*	0.45-1.3 / 0.6-1.4	*min-max*
Magnet		spiral sector	$\boxed{B=B_0(\frac{r}{r_0})^K F(\frac{1}{w}\ln\frac{r}{r_0}-N\theta)}$
$\alpha = Arctg(Nw)$	*deg*	46	*spiral angle*
$r_{min} - r_{max}$	*m*	0.25 - 0.61	
gap	*cm*	16.5 - 7	$\boxed{g/r = Cte}$
Injection		cont. or pulsed	*e-gun + e-inflector*
Acceleration		betatron and RF gap	*extensive RF prog. tests*

Spiral scaling FFAG



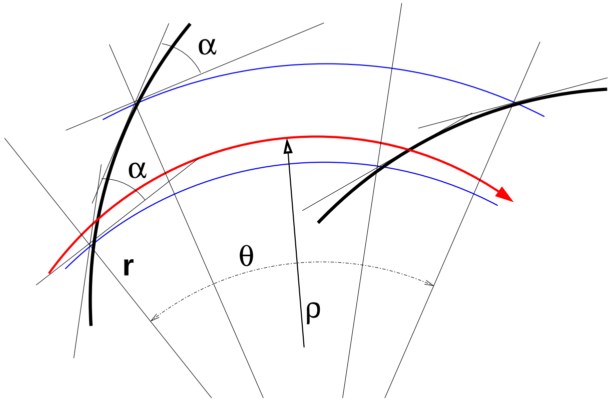
It is not AG !

Field form :

$$B(r,\theta)|_{z=0} = B_0 \left(\frac{r}{r_0}\right)^K \mathcal{F}\left(\ln\frac{r}{r_0}/w - N\theta\right)$$

$\mathcal{F}$ is the axial modulation of the field (“flutter”).

For instance, sine axial modulation : $\boxed{\mathcal{F} = 1 + f\sin(\ln\frac{r}{r_0}/w - N\theta)}$, simulates fringe fields, found often used in the litterature, makes equations easier to handle.



The logarithmic spiral edge insures constant angle between spiral sector edges and closed orbits.

Expansion of the equations of motion around the scalloped orbit in the linear approximation yields the tunes

$$\nu_r \approx \sqrt{1+K}, \quad \nu_z \approx \sqrt{-K + (f/Nw)^2/2}$$

Matrix model using hard-edge approximation and fringe field correction are efficient for reproducing first order properties.

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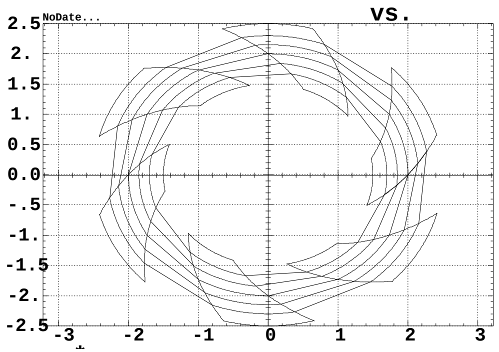
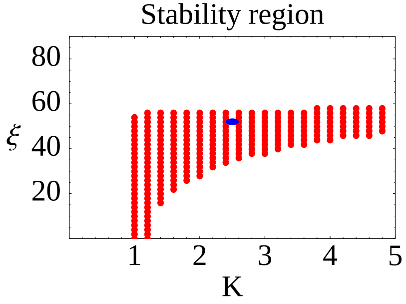
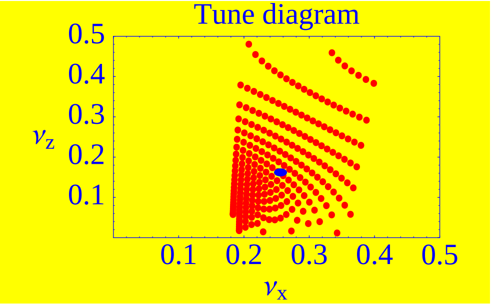
Linear optics using regular “TRANSPORT” methods

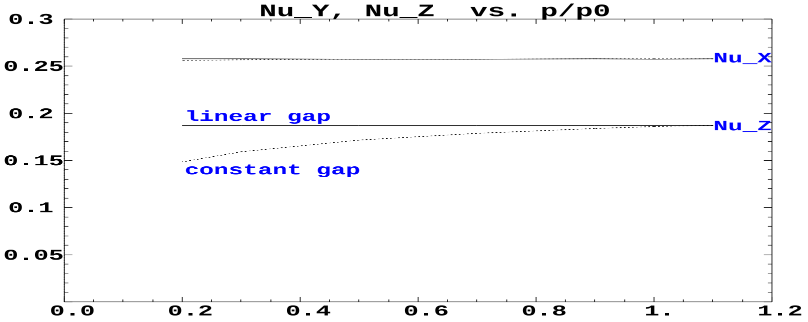
This is generally the first approach to the design problem.



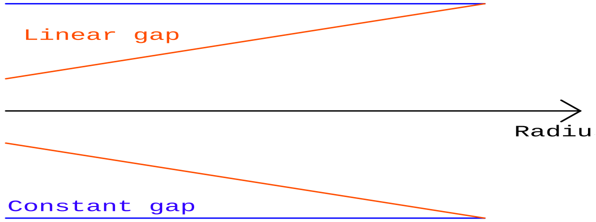


Proton, 8 cells,
3 MeV → 100 MeV
Working point :
K=2.5, ξ=52 deg
$\nu_x \,/\, \nu_z = 0.25 \,/\, 0.19$

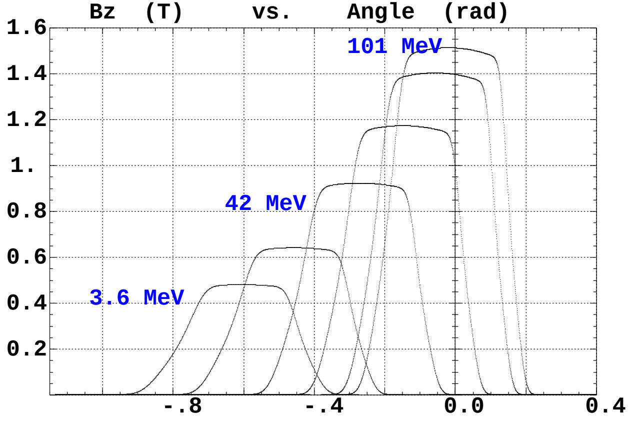




2 types of gap: constant / linear



“TRANSPORT” versus ray-tracing



(MeV) Energy	linear gap	
	Mathematica / BeamOptics	
	ν_X	ν_Z
3.55	0.263381	0.187253
7.96	0.263381	0.187253
21.0	0.263381	0.187253
42.6	0.263381	0.187253
69.4	0.263381	0.187253
85.	0.263381	0.187253
	Ray-tracing	
	ν_X	ν_Z
3.55	0.257628	0.187178
7.96	0.257625	0.187190
21.0	0.257616	0.187203
42.6	0.257616	0.187211
69.4	0.257621	0.187217
85.	0.257619	0.187220

Edge effect and Fringing Field Corrections.

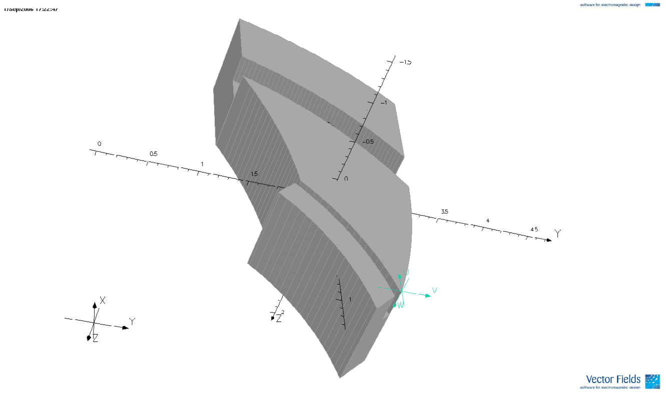
Focusing due to wedge ϵ : $\frac{1}{f}=\frac{-\tan(\epsilon)}{\rho}$.

Correction for field extent : $\epsilon \rightarrow \epsilon - \frac{gI_1(1+\sin(\alpha)^2)}{\rho\cos(\alpha)}$, with

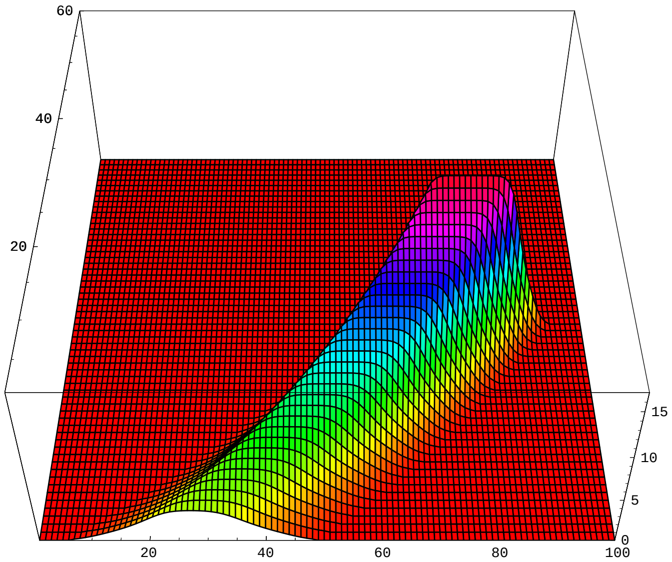
$$I_1 = \int_{-\infty}^{+\infty}\frac{B_Z(s)(B_0-B_Z(s))}{gB_0^2}ds, \qquad \alpha = \epsilon - 1.2\frac{K_1 g}{\rho}$$

g = gap, B_0 = reference field, ρ = curvature radius.

Using field maps : much closer to real world, not a simple question, however

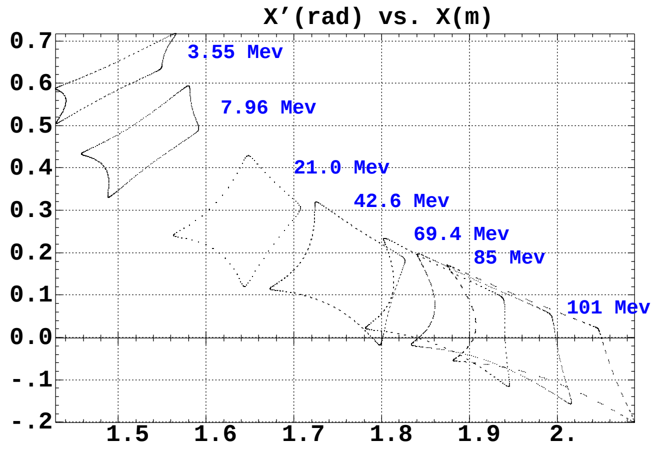


Spiral pole. TOSCA code.

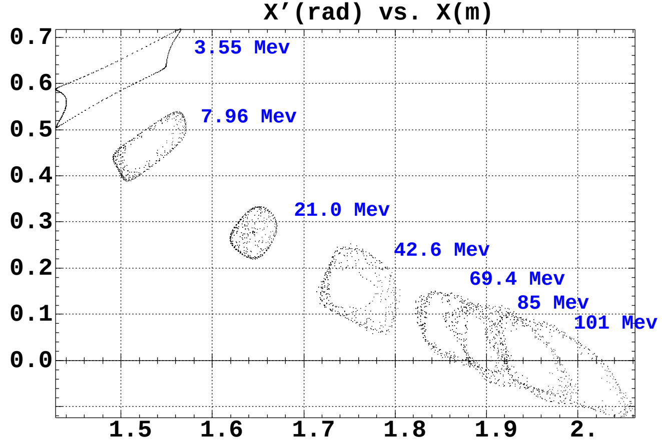


Mid plane field map, from TOSCA.
You can get very similar profile from appropriate “theoretical” model of field
(e.g. “FFAG-SPI” procedure in zgoubi)

Allows large amplitude, DA tracking :



Evolution in phase space at horizontal stability limit, z=0



Evolution in phase space at horizontal stability limit, z=ϵ

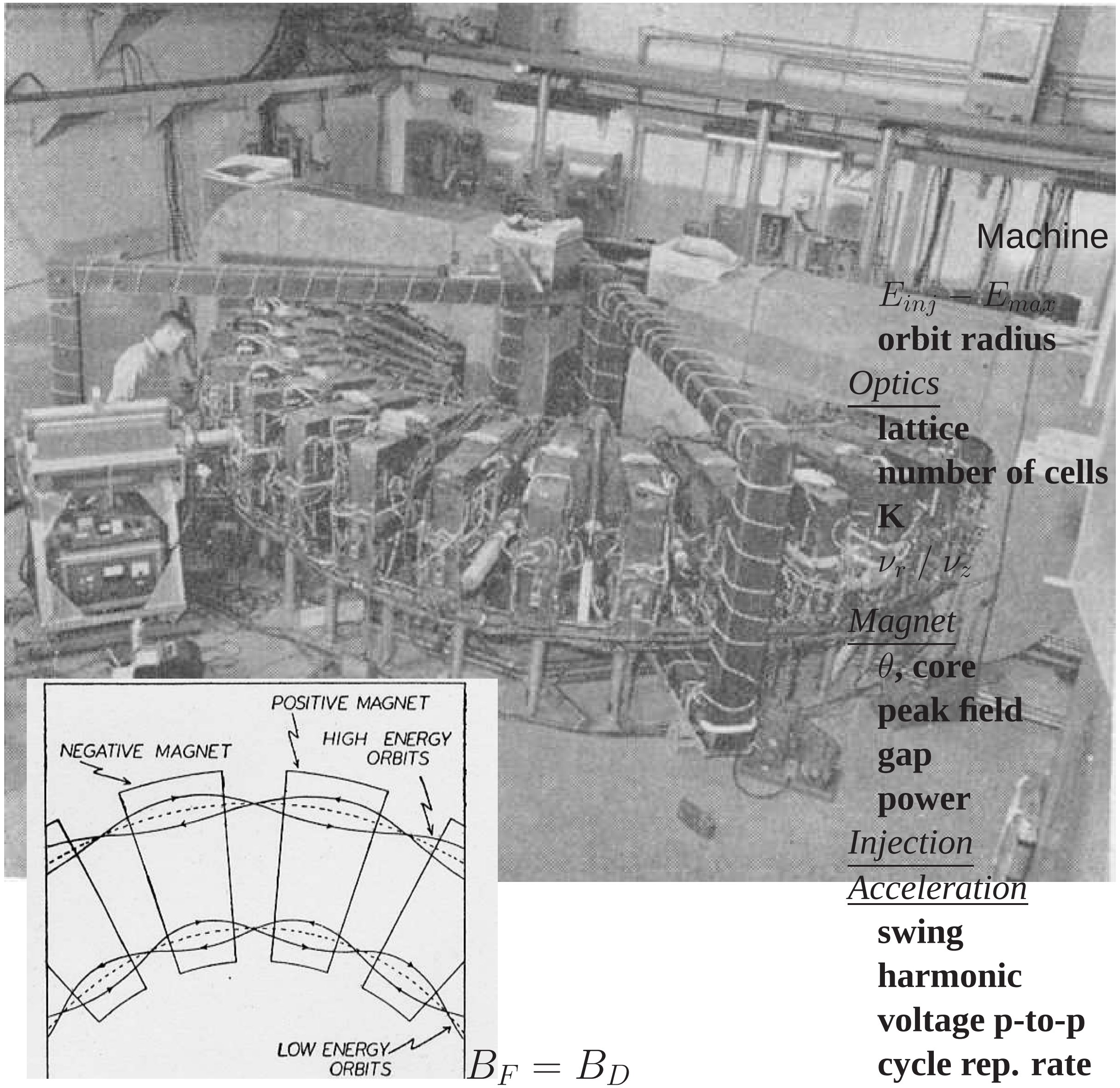
Accurate tracking method desired, in field maps as well !

Second radial sector, 50 MeV, 2-way

Preliminary studies early 1957. The spiral sector e-model was not yet completed - this determined the choice of radial sector : easier to design, better understood (?).

Study objectives : 1/ RF stacking, 2/ high circulating I, 3/ 2-way storage.

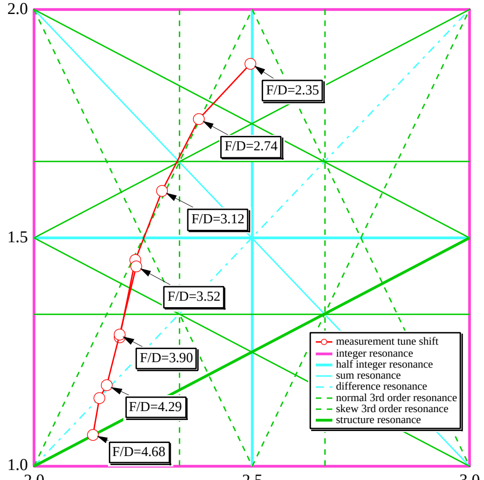
First operation Dec. 1959, 2-beam mode, 27 MeV, **could not get higher : magnets too bad** ;
Disassembled in 1960, magnets corrected ; second start Aug. 61, single beam, reached the design 50 MeV.



$B_F = B_D$

[Typical] data

Machine parameters			criteria / comments
$E_{inj} - E_{max}$	*MeV*	**0.1 - 50**	*reasonable size & beam life-time*
orbit radius	*m*	**1.20 - 2.00**	
Optics			
lattice		**FODO**	$B \approx B_0(r/r_0)^K \cos(16\,\theta)$
number of cells		**16**	*32 magnets, 3.15 deg. drifts*
K		**9.25**	
ν_r / ν_z		**4.42 / 2.75**	
Magnet		**radial sector**	
θ, **core**	*deg*	**6.3**	
peak field	*T*	**0.52**	r_{max}
gap	*cm*	**8.6**	
power	*kW*	**100**	
Injection			*e-gun + e-inflector*
Acceleration			
swing	*MHz*	**20 - 23**	
harmonic		**1**	
voltage p-to-p	*kV*	**1.3 - 3**	
cycle rep. rate	*Hz*	**60**	



R&D at KEK in recent years

First proton FFAGs, KEK

- **POP - Proof of principle. First accelerated beam 2000.**

[Typical] data

$E_{inj} - E_{max}$	keV	50 - 500	
orbit radius	m	0.8 - 1.14	
Optics			
lattice		DFD	
number of cells		8	
K		2.5	$(B = B_0(r/r_0)^K\, F(\theta))$
$\beta_r,\ \beta_z$ max.	m	0.7	
$\nu_r\ /\ \nu_z$		2.2 / 1.25	*tunable via B_F/B_D ratio*
Magnet	**high field, non-linear gradient**		*sector triplet*
$\theta_D\ /\ \theta_F$, core	deg	2.8 / 14	
$B_D\ /\ B_F$	T	0.04-0.13 / 0.14-0.32	$r_{inj} \rightarrow r_{max}$
gap	cm	30-9	$gap = g_0(r_0/r)^K$
Injection		multi- or single-turn	*electrostatic inflector* *+ 2 bumpers*
Extraction	**massless septum exprmnt**		
Acceleration			55 kW amp.
MA alloy RF core		**high $\vec{E}$, low Q, broad band, RF**	**; Ex. : 2-beam accel.**
swing	MHz	0.6 - 1.4	
harmonic		1	
voltage p-to-p	kV	1.3 - 3	
rep. rate cycle time	**ms**	**1**	*fast acceleration*
rep. rate	**kHz**	**1**	*high average current*
equiv. $\dot{B}$	T/s	180	

- **150 MeV protype FFAG : medical beams, ADS-reactor, NuFact muon accelerators - machine now at Kyushu Univ. (Y. Yonemura, this wrkshp).**

Start up 2003. Full acceleration cycle, 9-100 MeV mode, spring 2005.

[Typical] data

$E_{inj} - E_{max}$	MeV	12 - 150	
orbit radius	m	4.47 - 5.20	
Optics			
lattice		DFD/12 cells	*9.5 deg. drift*
K		7.6	$(B = B_0(r/r_0)^K\, F(\theta))$
β_r / β_z max.	m	2.5 / 4.5	
ν_r / ν_z		3.7 / 1.3	*tunable via* B_F/B_D *ratio*
$\alpha,\ \gamma_{tr}$		0.13, 2.95	$1/(1+K), \quad (1+K)^{1/2}$
Magnet	**Return yoke free magnet**		
θ_D / θ_F	deg	3.43 / 10.24	
B_D / B_F	T	0.2-0.78 / 0.5-1.63	$r_{inj} \to r_{max}$
gap	cm	23.2 - 4.2	*at* r_{inj} - r_{max} $(gap = g_0\left(\frac{r_0}{r}\right)^K)$
Injection		multi-turn	$\{$ *B-septum + E-septum* *+ 2 bumpers*
Extraction		single-turn	**fast kicker (1kG, 150 ns)**
Acceleration	*Amorphous MA*		
swing	MHz	1.5 - 4.5	
harmonic		1	
voltage p-to-p	kV	2	
ϕ_s	deg	20	
ν_s		0.01 - 0.0026	
rep. rate	**Hz**	**250**	*high average current*
equiv. $\dot{B}$	**T/s**	**300**	*fast acceleration*

“return yoke free” magnet

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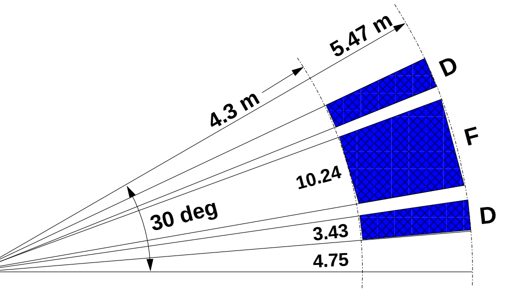
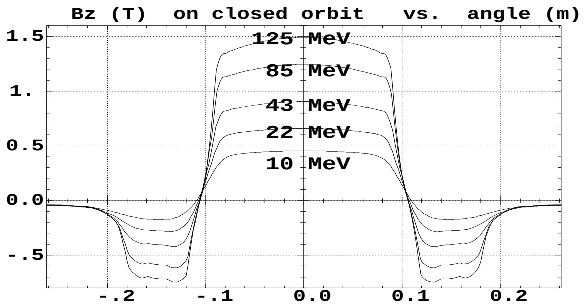
The design of such large acceptance, non-linear machine *has to* resort to tracking

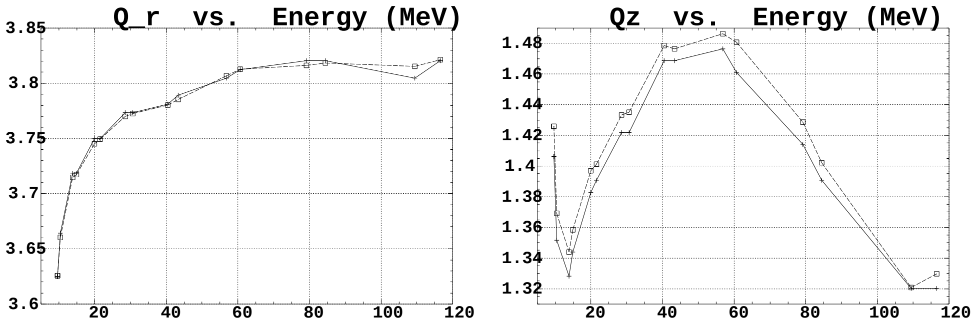
Regarding tunes (field inhomogeneities), amplitude detuning, motion stability limits (DA), 6-D transmission, etc.,



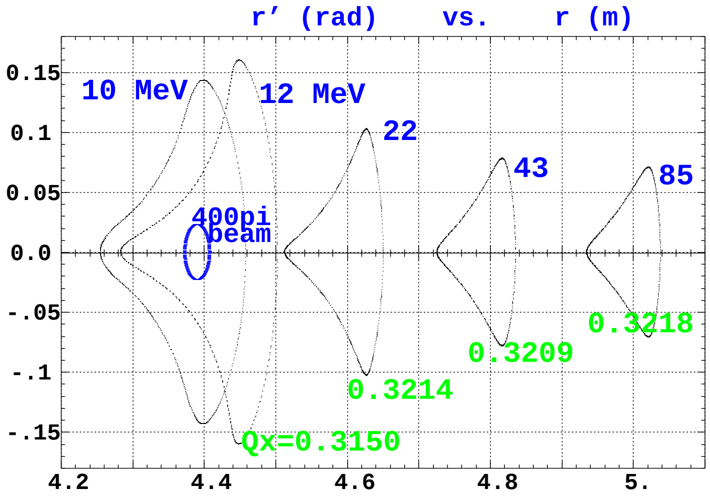
A scheme of the triplet dipole.



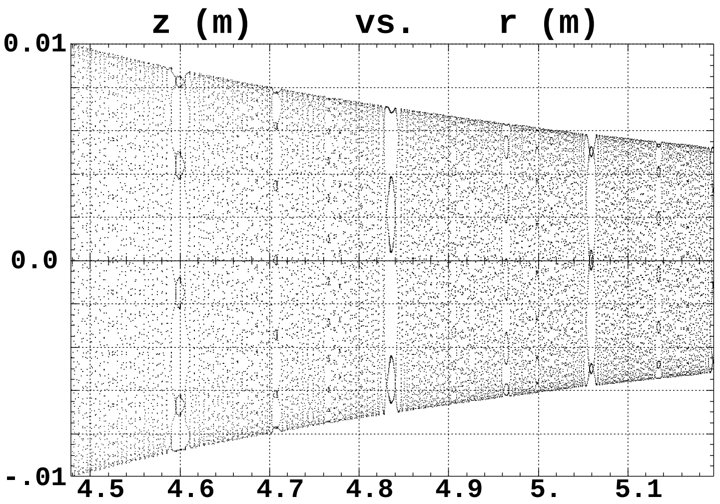
Fied on closed orbits.



- Tunes Qr(r), Qz(r) -



Motion stability limits (“DA”) about 1πcm.



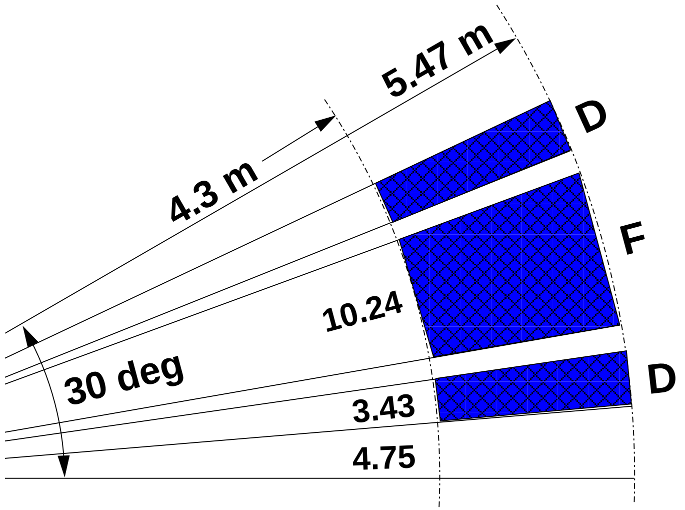
6-D acceleration, 12 to 150 MeV - adiabatic damping of vertical motion. The motion spans $\Delta R \approx 0.5$ m !.

An example of a numerical simulation method

- Main goals : simulate $B_{zi}(r,\theta) = B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)$ **AND** allow for possible overlapping of fringe fields.

  Main apps : scaling and isochronous FFAGs.

- Simulation of a scaling FFAG triplet :

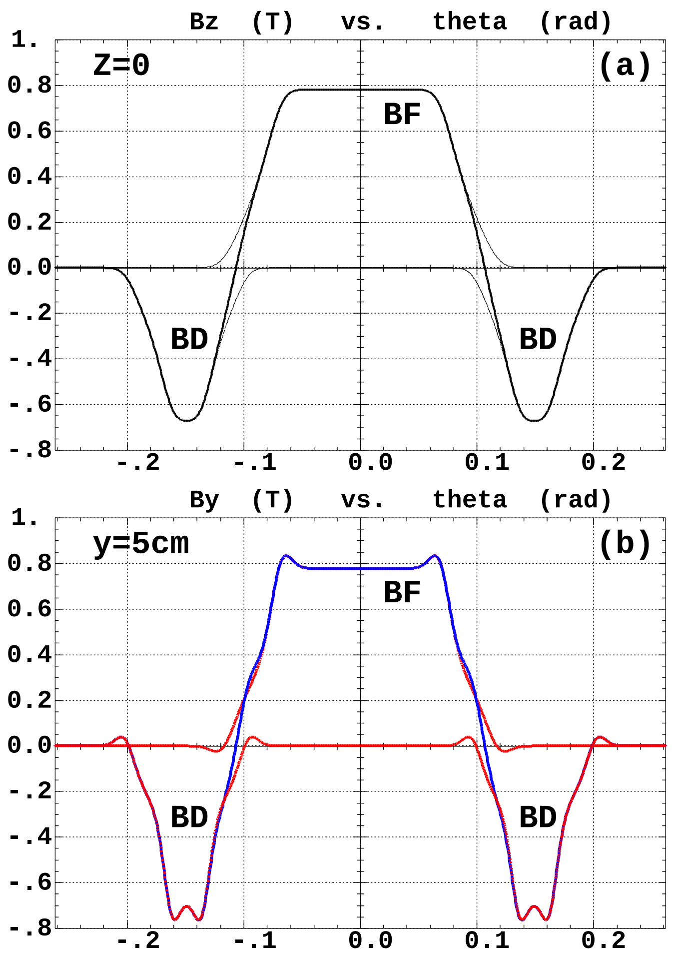


DFD triplet.

The geometrical model is based on the superposition of the independent contributions of the N dipoles :

$$\boxed{B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)}$$

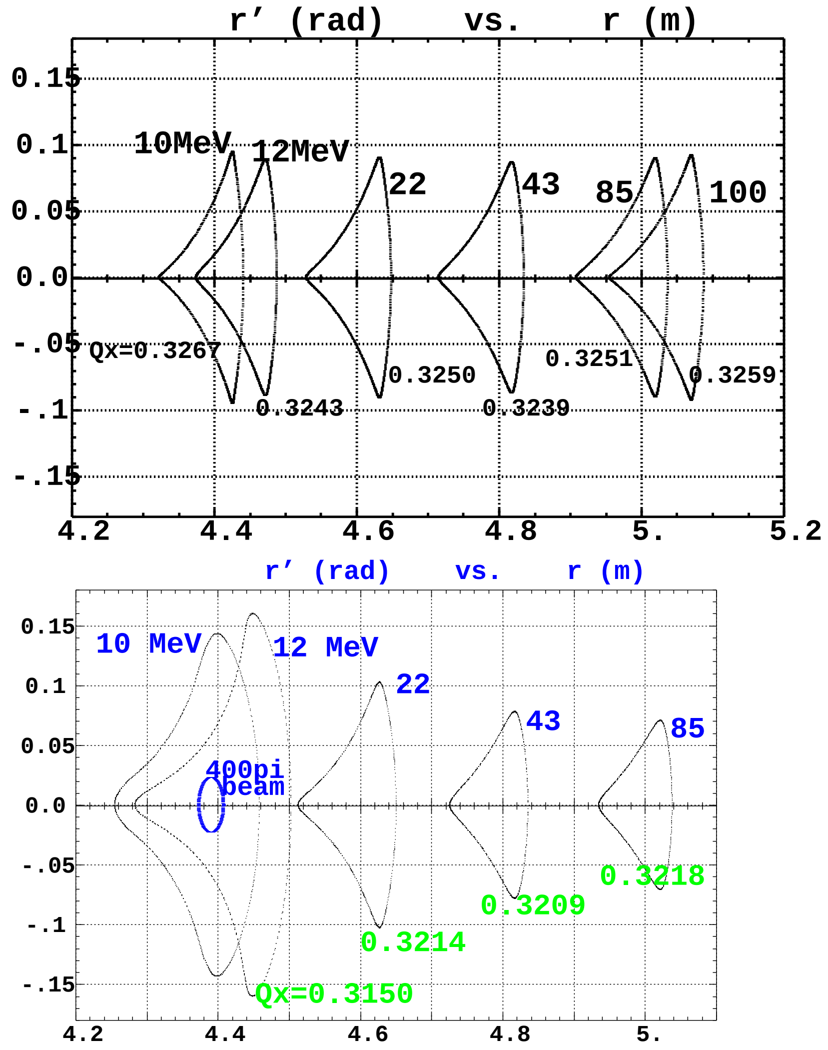
at all (r,θ) in the mid-plane. Field off mid-plane is obtained by Taylor expansion.



Field experienced for $r_0 = 4.87$ m in the DFD dipole triplet.

A superposition of $N = 3$ independent contributions, at all (r,θ,z).

Pushing further : comparison between the previous method and 3-D field maps



150MeV FFAG : horizontal phase space, the limits of stable motion, for 5 energies.

For comparison :

tracking with geometrical model (top),

or using TOSCA map (bottom).

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“FIT” techniques are convenient for optimizing machine and magnet parameters

An indispensable tool for
- preliminary adjustments (tunes, etc.) prior to 6-D simulations
- considered very useful for further assessment and optimisation of higher order behavior, DA, transmission, ...

FIT CONSTRAINTS :

Trajectory coordinates (e.g., final coordinates)

Several other types of quanttities that are deduced from trajectories, e.g. :
- first and higher order transport coefficients
- beam matrix coefficients (waist, divergence)
- particle fluxes through ellipses (→ transmission efficiency)

In the case of periodic structures :
- closed orbits
- tunes, chromaticites, anharmonicities

FIT VARIABLES :   any data !

```
  'OBJET'   * c.o., constant Gap *
 226.8235847      68MeV/c muon
2
2  1
499.377    0.    0.  0. 0.  1.2     'b'
1 1 1 1 1 1 1 1 1
 'FFAG'
 0
 3   45.     500.                        NMAG, Sector angle, R0
 18.17   0.   -0.717  5.                   mag 1 : ACNT, dum, B0, K
 6.3  0.                                EFB 1 : lambda, gap const/varbl
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 1.23 0.    1.E6  -1.E6  1.E6  1.E6
 6.3  0.                                EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-1.23  0.    1.E6  -1.E6  1.E6  1.E6
 0. -1                                   EFB 3 : inhibited by iop=0
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.    0.     0.     0. 0.
 22.5   0.   3.2    5.                   mag 2 : ACNT, B0, K,dums
 6.3  0.                                EFB 1
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 3.    0.    1.E6  -1.E6  1.E6  1.E6
 6.3  0.                                EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-3    0.    1.E6  -1.E6  1.E6  1.E6
 0. -1                                   EFB 3
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.    0.     0.     0. 0.
26.83  0.  -0.717  5.                    mag 3 : ACNT, dum, B0, K
 6.3  0.                                EFB 1
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 1.23 0.    1.E6  -1.E6  1.E6  1.E6
 6.3  0.                                EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-1.23  0.    1.E6  -1.E6  1.E6  1.E6
 0. -1                                   EFB 3
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.    0.     0.     0. 0.
   0     2  125.             KIRD anal/num, resol(mesh=step/resol)
.5                           integration step size
 2   0.  0. 0. 0.
```

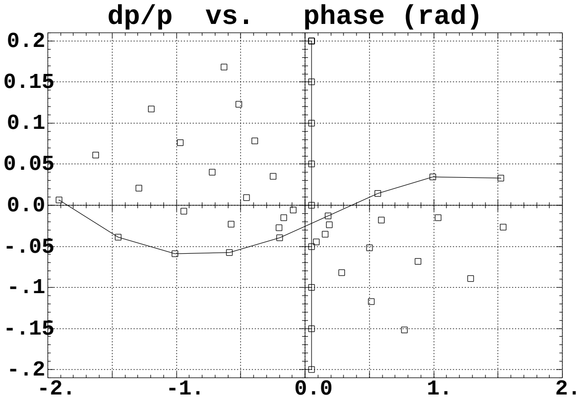
Beams for muon physics

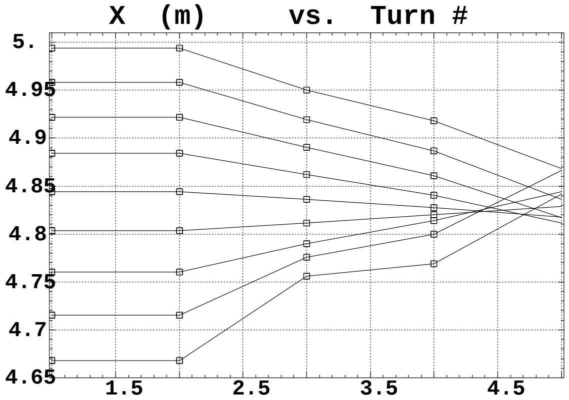
(A. Sato, this workshop)

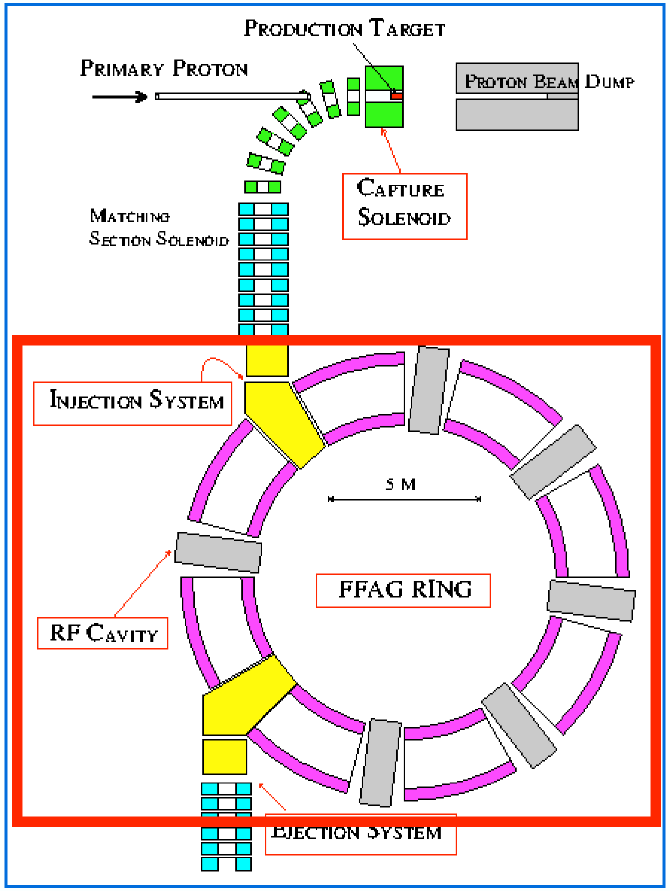
FFAG used as phase rotator, for momentum compression

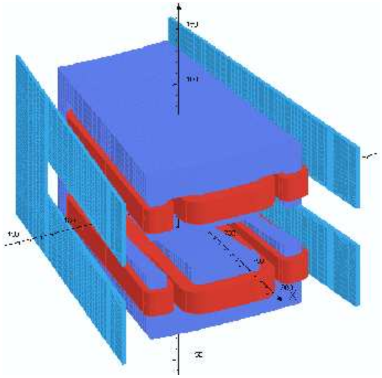
p=68MeV/c +/-20% down to +/-2% in 6 turns

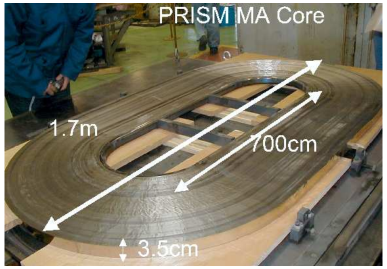
- DFD lattice 14t triplet yoke, 120 kW/triplet
- K, B_F/B_D variable $\rightarrow$ quasi-decoupled ν_x, ν_z adjustments
- H / V apertures : 1 / 0.3 m
- acceptance : 4 π cm.rad $\times$ 0.65 π cm.rad
- RF : 5-gap cavity, 33 cm gap, 150-200 kV/m, 2MV/turn









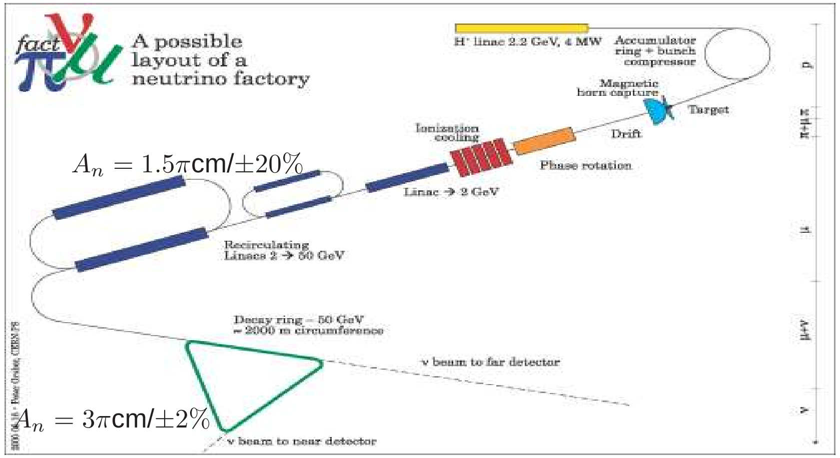


- Optics design : requested large acceptance can be achieved
- difficult task : injection & extraction

The Neutrino Factory

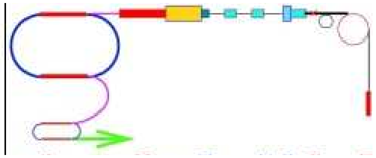
It has triggered a strong activity in the domain of FFAG design, it has lead to the linear FFAG concept.

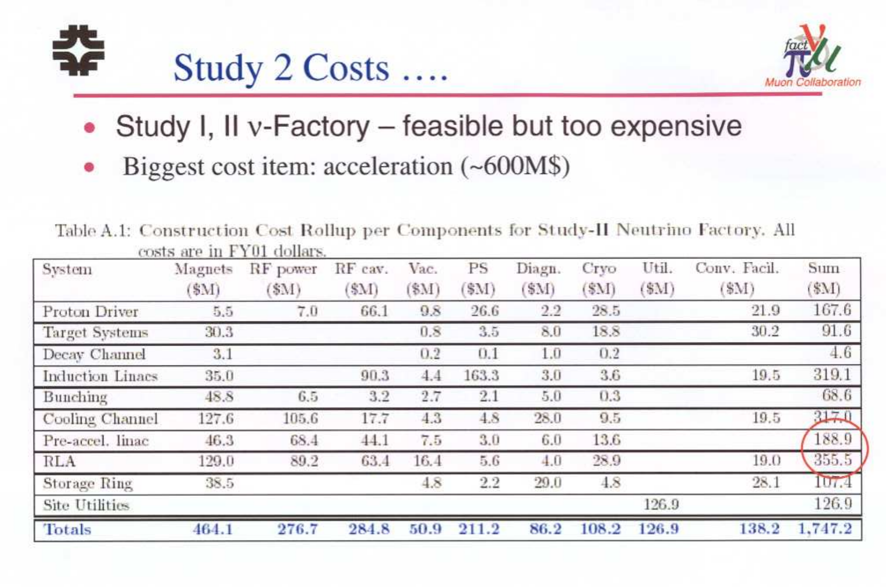
Europe NuFact



The Europe and the two US NuFact studies at first proposed to accelerate muons up to the storage energy (20 or 50 GeV) by means of one or two 4- or 5-pass RLA's. RLA's are complicated machines (spreaders, combiners), hence expensive.

US NuFact





Study 2 Costs ….

- Study I, II ν-Factory – feasible but too expensive
- Biggest cost item: acceleration (~600M$)

Table A.1: Construction Cost Rollup per Components for Study-II Neutrino Factory. All costs are in FY01 dollars.

System	Magnets ($M)	RF power ($M)	RF cav. ($M)	Vac. ($M)	PS ($M)	Diagn. ($M)	Cryo ($M)	Util. ($M)	Conv. Facil. ($M)	Sum ($M)
Proton Driver	5.5	7.0	66.1	9.8	26.6	2.2	28.5		21.9	167.6
Target Systems	30.3			0.8	3.5	8.0	18.8		30.2	91.6
Decay Channel	3.1			0.2	0.1	1.0	0.2			4.6
Induction Linacs	35.0		90.3	4.4	163.3	3.0	3.6		19.5	319.1
Bunching	48.8	6.5	3.2	2.7	2.1	5.0	0.3			68.6
Cooling Channel	127.6	105.6	17.7	4.3	4.8	28.0	9.5		19.5	317.0
Pre-accel. linac	46.3	68.4	44.1	7.5	3.0	6.0	13.6			188.9
RLA	129.0	89.2	63.4	16.4	5.6	4.0	28.9		19.0	355.5
Storage Ring	38.5			4.8	2.2	29.0	4.8		28.1	107.4
Site Utilities								126.9		126.9
Totals	**464.1**	**276.7**	**284.8**	**50.9**	**211.2**	**86.2**	**108.2**	**126.9**	**138.2**	**1,747.2**

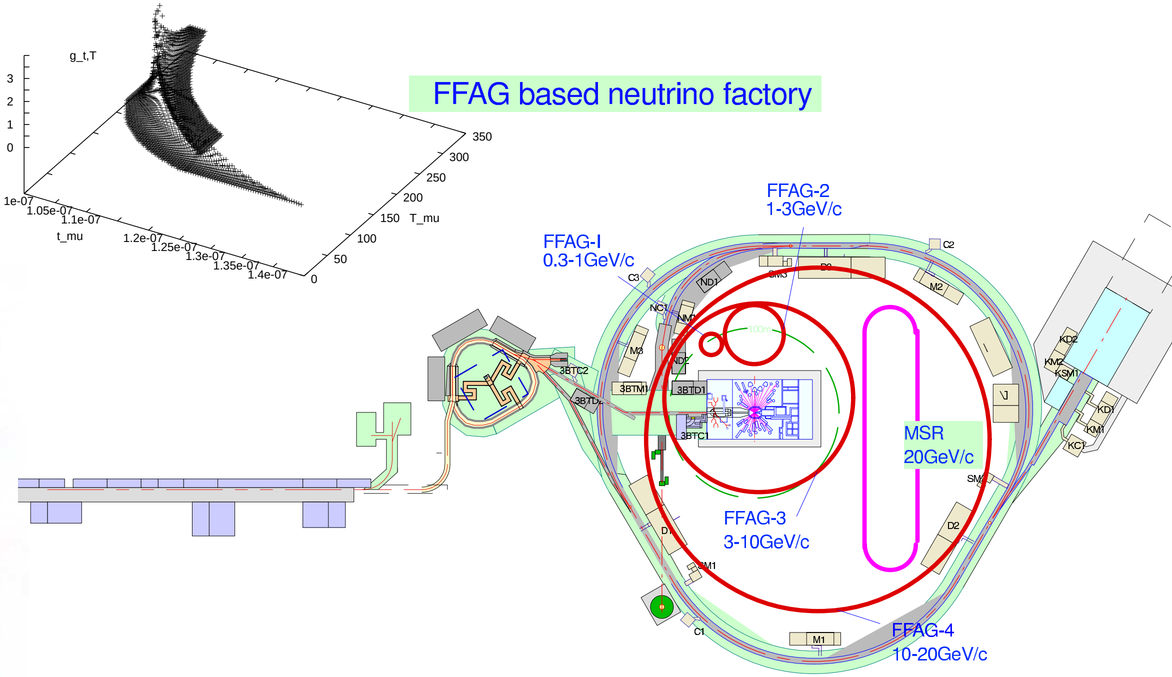
The Japan NuFact

50-GeV, $3.3\,10^{14}$ ppp with 0.3 Hz (15 μA) / 0.75 MW

Four muon FFAG's : 0.2-1 GeV, 1-3, 3-10 (SC), 10-20 (SC).

No cooling, technology simpler, compact (R≈200m)

30ns/300±50% MeV bunch



Acceleration rate is lower than RLA, requires larger distance, but, acceptance is larger both transversally (twice : DA $3\,\pi$cm norm. at $\delta p = 0$) and longitudinally (≈ 5 eV.s). Hence achieve comparable production rate : $\approx 10^{20}$ muon decays per year (1 MW p power).

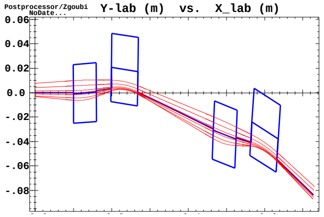
A new concept : non-scaling FFAG

NuFact activities have given rise to a new concept :

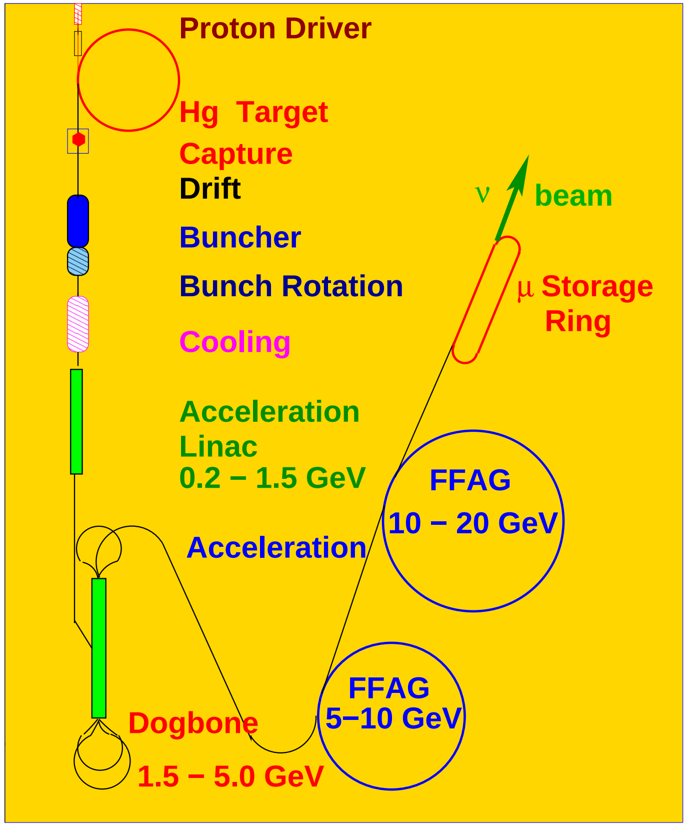
“linear, non-scaling optics”:

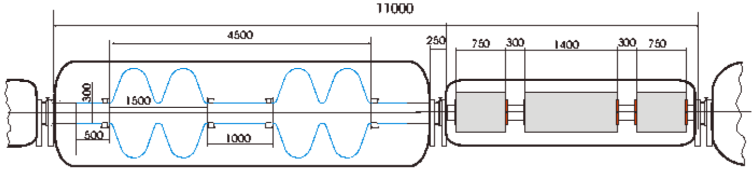
- FFAG based on linear optical elements -

- orbits no longer scale, tunes are allowed to vary with energy -



FDF / small Dx





This has a series of consequences :

- $R/\rho < 2$ - this decreases the machine size compared to classical (scaling) FFAG
- a very reasonable horizontal beam excursion (small D_x) can be designed $\rightarrow$ **magnet width is smaller compared to scaling magnets**
- small δTOF over energy span, allows **fast acceleration** thanks to use of high frequency / high gradient RF
- yields **large transverse acceptance** $\leftarrow$ fields are linear

In practice, the cell tune is allowed to decrease from just below a half-integer value at injection, to just above the lower integer (negative natural chromaticity)

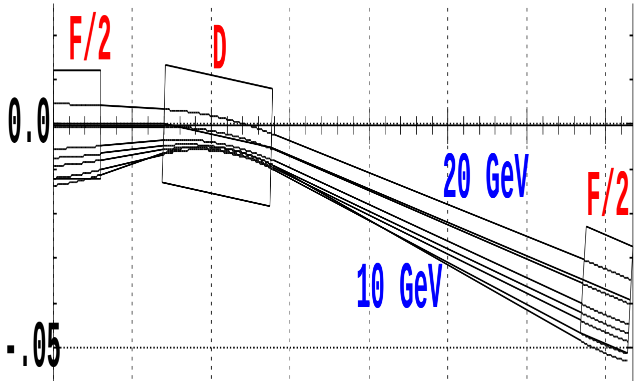
- hence tens of cells cause **resonance Xing**, tens of integer and $\frac{1}{2}$integer resonances.

- yet the **crossing is fast**, this should result in not too stringent tolerance on alignments and field defects.

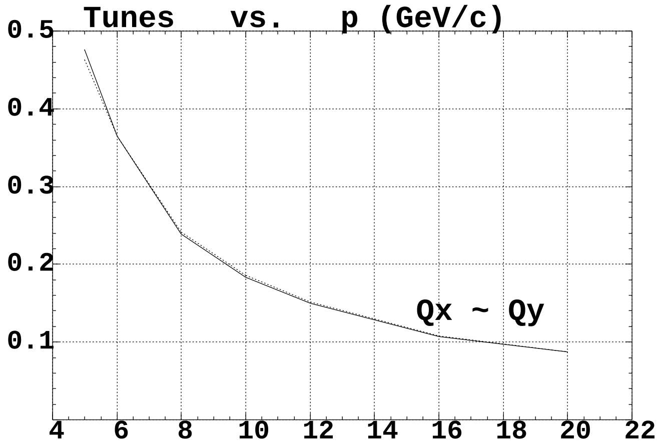
- **NEEDS BE DEMONSTRATED** $\rightarrow$ **EMMA at DL.**

- Application to a NuFact scheme : Above 5 GeV, non-scaling linear FFAG yield **lower cost/GeV than RLA.**

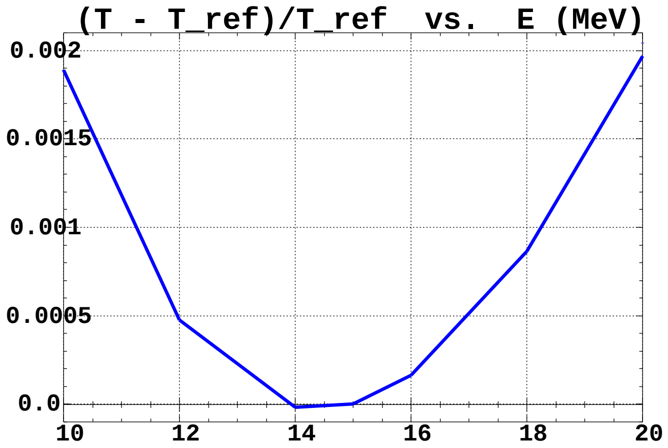
Regular “TRANSPORT” methods yield all baisc parameters of linear non-scaling FFAG



Closed orbit in FODO cell

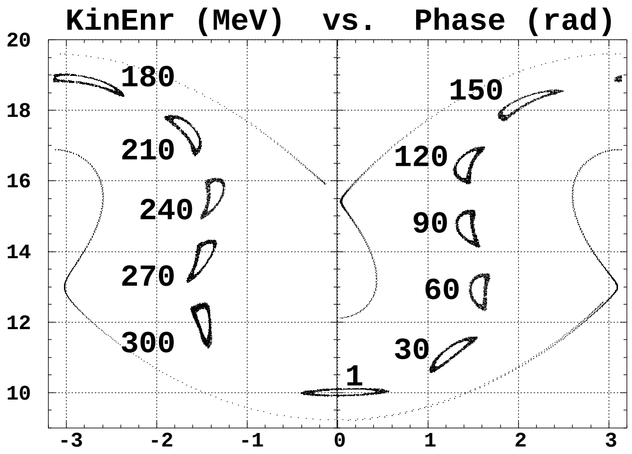


Tunes versus energy



Time of flight parabola ($\gamma \approx \gamma_{tr}$)

$$\frac{\delta TOF}{TOF} \approx \left[\eta_0 \frac{\delta p}{p}\right] + \eta_1 \left(\frac{\delta p}{p}\right)^2$$



Longitudinal motion

$$H \approx \sin^2 \pi\phi + \left[a \left(\frac{\delta p}{p}\right)^2\right] + b \left(\frac{\delta p}{p}\right)^3$$

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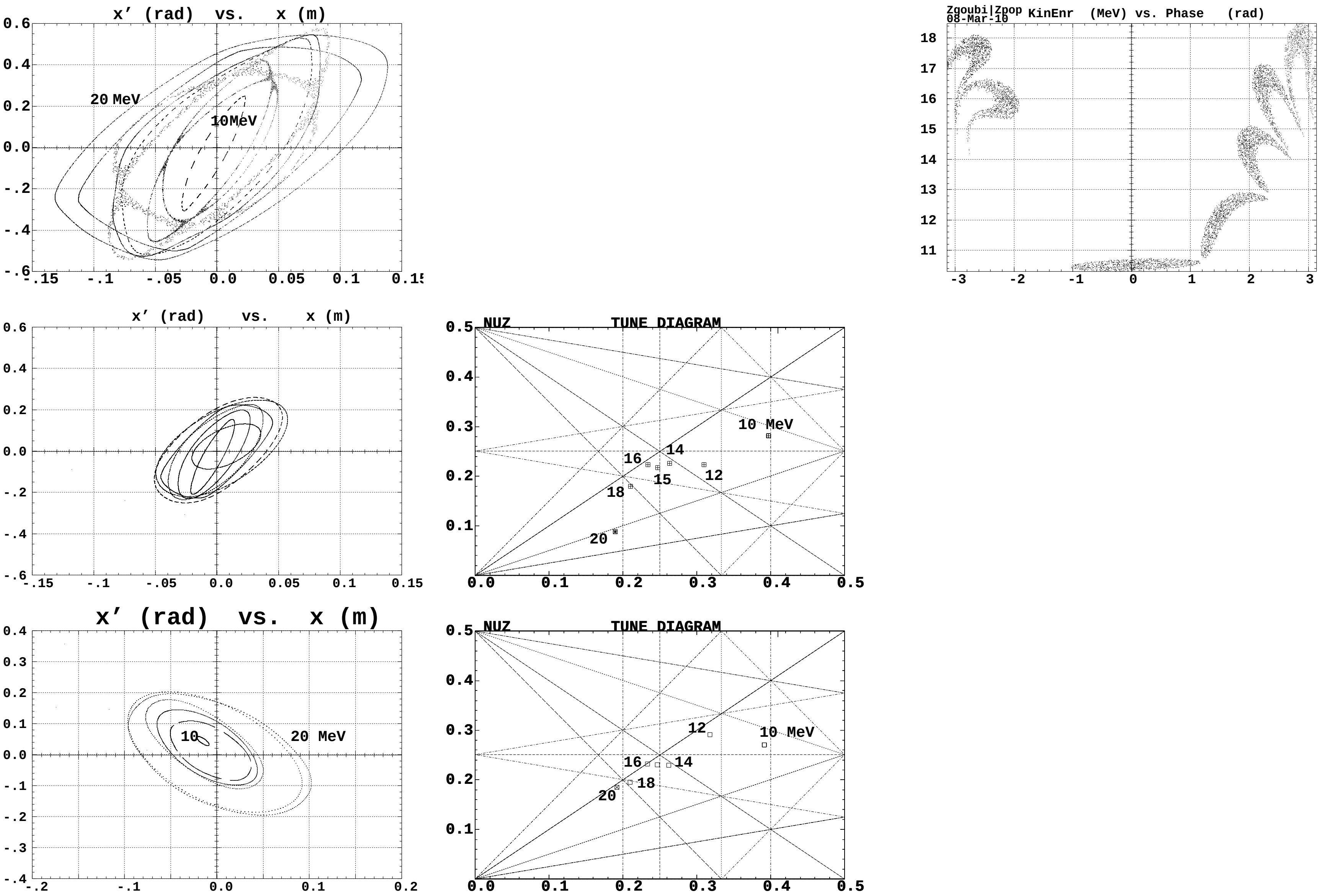
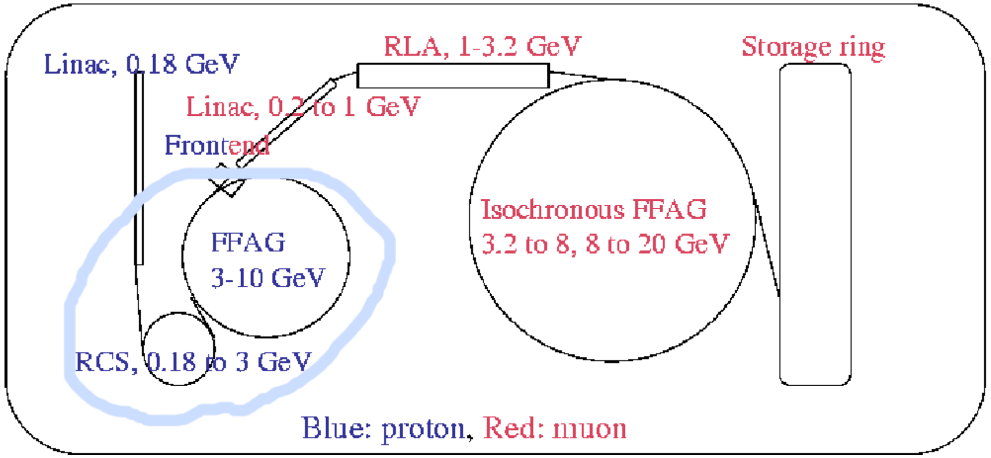
Higher order tracking yields DA, 6-D acceleration



Left : 2-D motion, 2000-cell stability limits, with $\sim 5\%$ precision in x, at various energies.
Top : pure horizontal motion, no fringe fields. Middle : including very small z motion, no fringe fields. Bottom : including very small z motion, fringe fields set.
Middle : cell tunes at stability limits (resonance lines up to 5th order are represented).
Right : longitudinal phase-space, serpentine acceleration of a large emittance bunch.

UK design of a Neutrino Factory : muon FFAGs, and proton driver FFAG

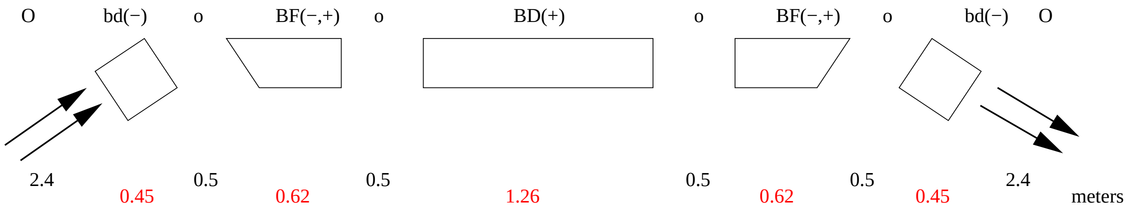


NFFAGI p-Driver		
Energy	**(GeV)**	**3→10**
Power	**MW**	**4**
Circumference	**m**	**686**
Q_x/Q_z		$19.2 \rightarrow 19.4$ / 13.7
# N/I cells per super-p		**21 / 13**
# super-periods		**2**
N-cell/I-cell length	**m**	**6.4/10.2**
RF range	**MHz**	**14-14.37 / 86.2**
RF voltage	**MV/turn**	**1**
h accel./compress.		**33 / 198**
bunch length / compressed	**ns**	**2 / 1**
# bunches		**5**
ppb		10^{13}
pulse rate	**Hz**	**50**

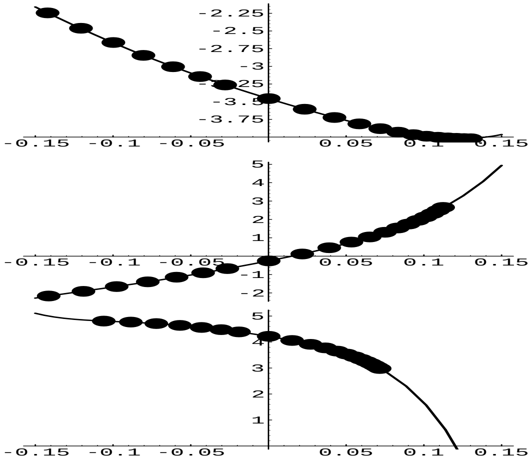
Based on non-linear lattice that yields **possibility of insertions** , with the advantages of

1. easier injection and extraction,
2. space for beam loss collimators

$$V_n(s,x,z) = (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q\, G^{(2q)}(s)(x^2+z^2)^q}{4^q q!(n+q)!}\right) \times \left(\sum_{m=0}^{n} \frac{sin(m\frac{\pi}{2})\, x^{n-m} z^m}{m!(n-m)!}\right)$$

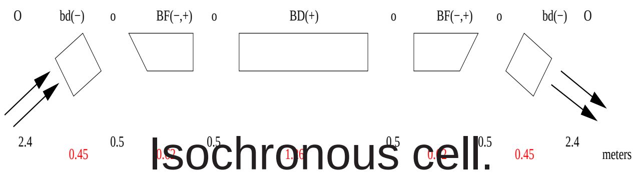


Magnetic field in bd, BF and BD.

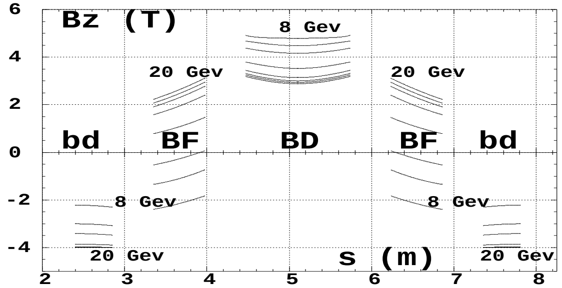


Muon lattice case : $\gamma = \gamma_{tr}$ at all energy hence $\eta \equiv 0$ (isochronism), allows on-crest acceleration

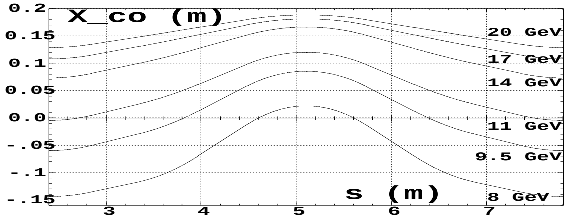
Isochronous muon FFAG lattice. Design due to G. Rees, geometry, fields, isochronism, are based exclusively on matrix methods. Ray-tracing allows more complete and more detailed insight.



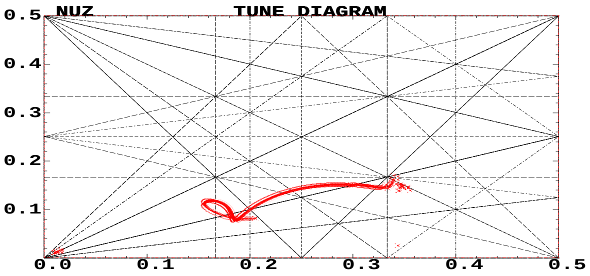
Isochronous cell.



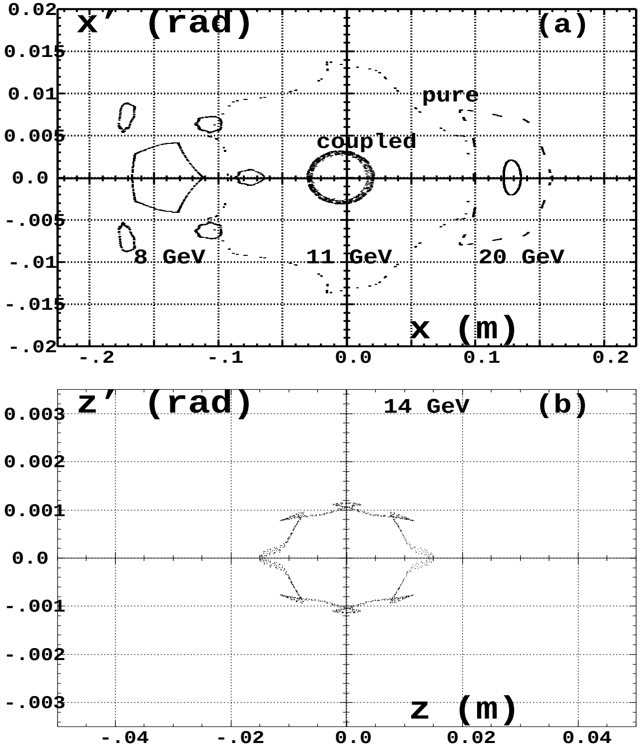
Magnetic field on closed orbits at various energies.



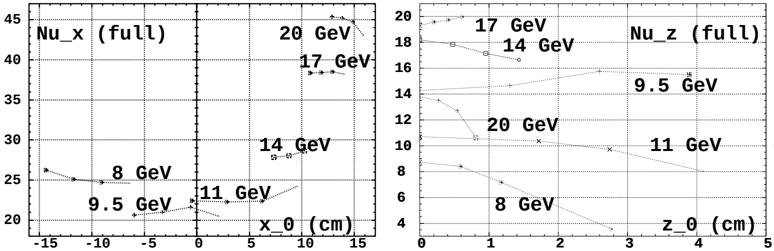
Closed orbits



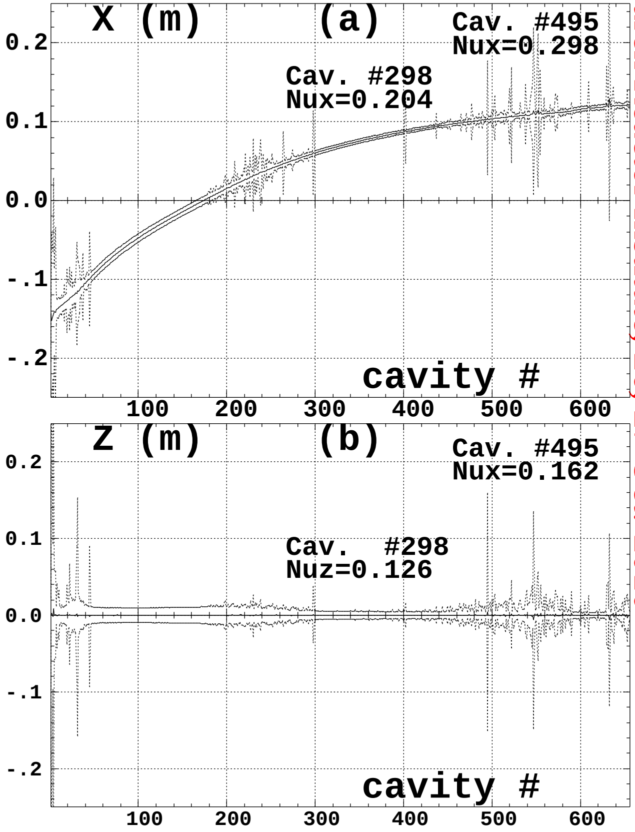
Beam trajectory in tune diagram, from 6 to 20 GeV (10 turns)



Stability limits, H and V.



Amplitude detuning and its energy dependence.



Muon rate transmission

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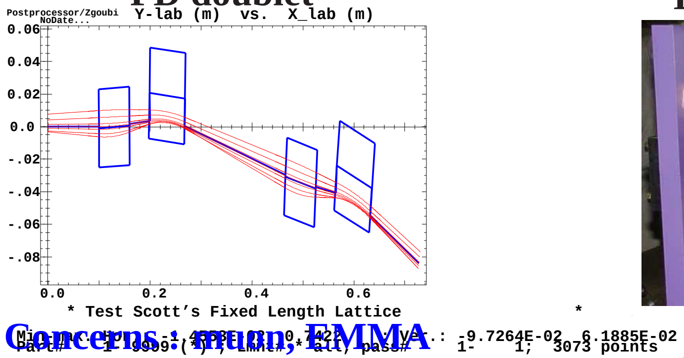
3 A brief review of FFAG lattices under R&D

Goals : design and optimisation of lattice, orbits, focusing, *and also* of magnets. DA tracking. Errors.

Linear, non-scaling

(natural $\xi_{x,z}$)

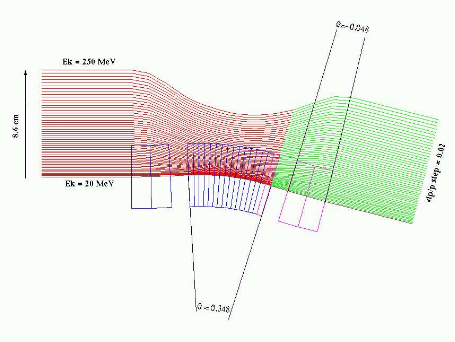
FD doublet



Concerns : muon, EMMA

Apps. : NuFact

FDF triplet



Concerns : proton, p-model

Apps. : hadrontherapy, p-driver

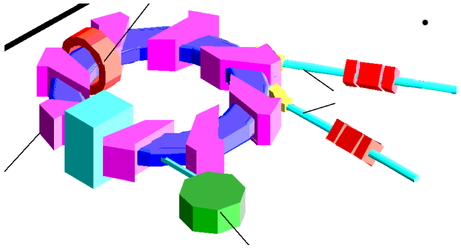
Scaling - $\frac{B}{B_0} = \left(\frac{r}{r_0}\right)^K$

(zero chromaticity, $\forall p$)

DFD triplet, doublet

SC technology

Spiral

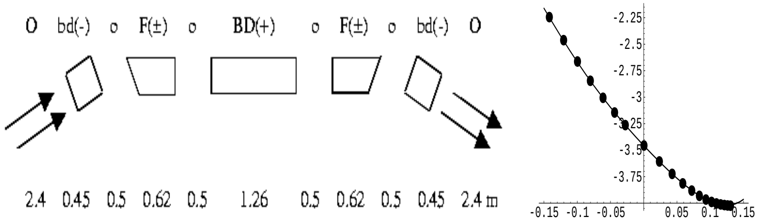


Concern : muon, e and p

Apps. : muon phys., NuFact, high power e and p, hadrontherapy, [R]Ions

Non-linear, non-scaling

Pumplet lattice

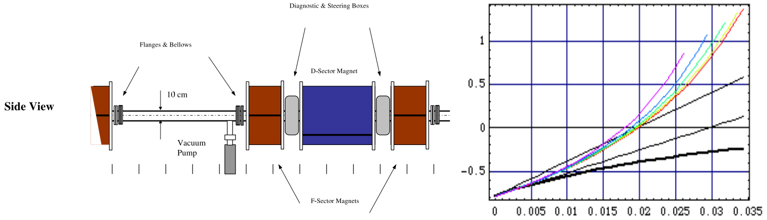


Concerns : muon and e-model (isochronous, $\xi_x > 0$, $\xi_z \to 0$) ; proton

Apps. : NuFact, p driver

Adjusted field profile ?

($\xi_{x,z} \to$ small)



Concerns : proton

Apps. : p driver, hadrontherapy

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4 DA hunting : orders of magnitude

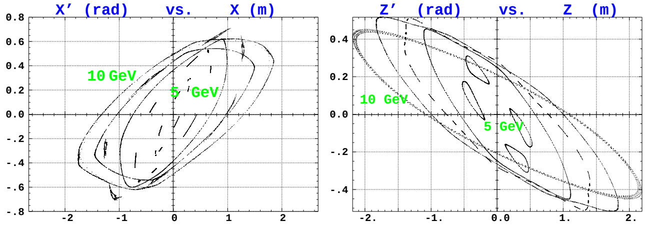
- DA’s to explore are large, sometimes *very* large - a key interest of FFAGs.

Linear, non-scaling

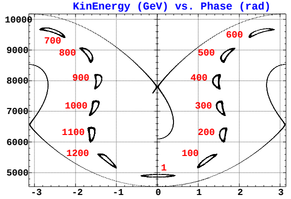
FD doublet

Muon :

$\gg 3\pi$cm norm.



$> 0.5\ \pi$eV.s



EMMA (electron) :

$\gg$200-300 πmm.mrad norm.

Proton :

10s πmm.mrad norm.

Non-linear, scaling

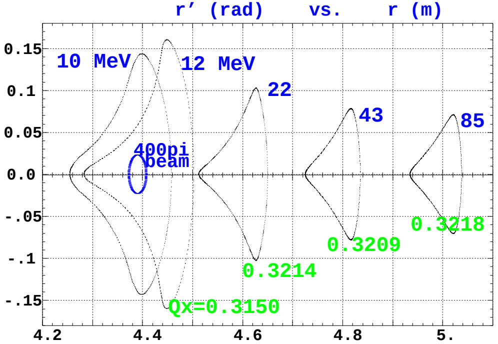
DFD triplet, doublet, spiral

$0.3 - 20$ GeV muon :

$> 3\pi$cm norm., $1.5\ \pi$eV.s

Proton :

10s πmm.mrad norm.



Non-linear, non scaling

Pumplet lattice

$8 - 20$ GeV muon

isochronous

$\approx \pi$cm norm. $-0.5\ \pi$eV.s

p-Driver :

10s πmm.mrad norm.

electron model :

100-300 πmm.mrad norm.

Adjusted field profile

p apps.,

10s πmm.mrad norm.

- A straightforward remark with these types of optics :

given $\left\{ \begin{array}{l} (i) \\ (ii) \end{array} \right.$ **the large excursions,**
strong non-linearities, be they transverse or longitudinal,
better use an accurate tracking method !

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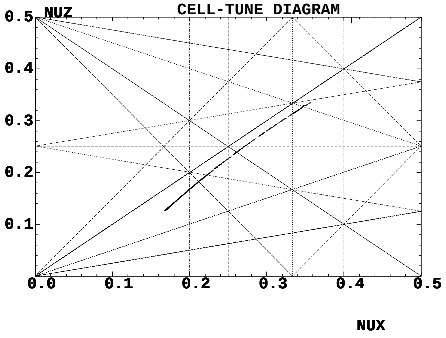
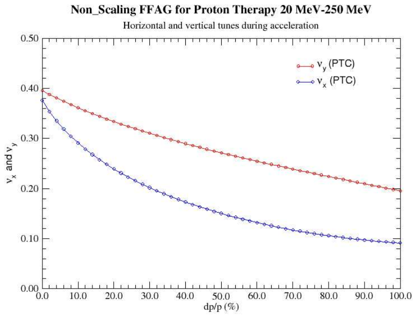
- In all cases : FFAG tracking methods need to provide means for 6-D simulation in presence of
  - fast acceleration
  - orbit change with E
  - proximity and/or crossing of resonances

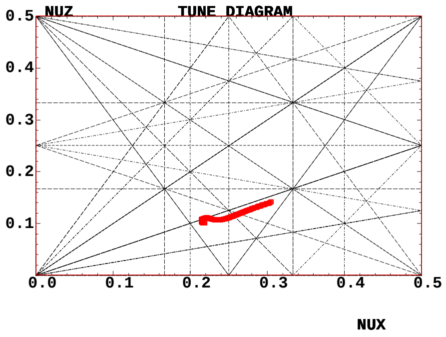
muon, EMMA,
linear, non-scaling
FD doublet



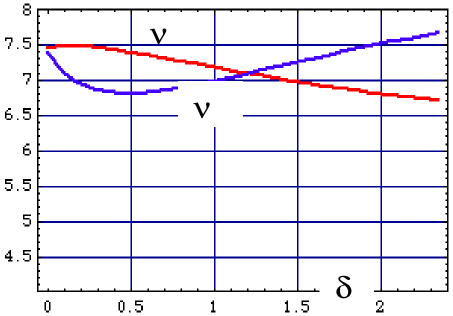
250MeV p-therapy
linear, non-scaling
FDF triplet



pumplet, e-model
non-linear
non-scaling



AFP
non-linear
non-scaling



- May also need : Fringe gield overlapping, case of e.g.,
  - scaling FFAGs, cf. PRISM
  - linear FDF triplet, cf. 250MeV p-Therapy

- And also : sooner or later the FFAG designer will reach a stage where he needs symplectic tracking using *magnetic field maps.*

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TRACKING CODES

5 Tracking codes

...known to (or to have) handle(-d) FFAG problems

code	**stepwise**	**seen in company of (sort of POP)**	**method**
COSY		**linear FFAG**	**Taylor mapping needs reference orbit**
ICOOL	**yes**	**muon, lin. & scal.**	**RK4**
MAD-PTC	**yes**	**muon, EMMA**	**kick-drift, symplectic, z-type**
J-RK4	**yes**		
S	**yes**	**linear ; scaling**	**kick-drift, sympl., s-type**
Zgoubi	**yes**	**all types of FFAGs**	**Taylor series, s-type**
MAD	**no**	**lin. FFAG / E. Keil**	**Transport methods**

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. . .

Other codes may have been tried, SYNCH, TEAPOT, etc. : see Trbojevic et als., PAC03

Codes known to (or to have) handle(-d) FFAG problems

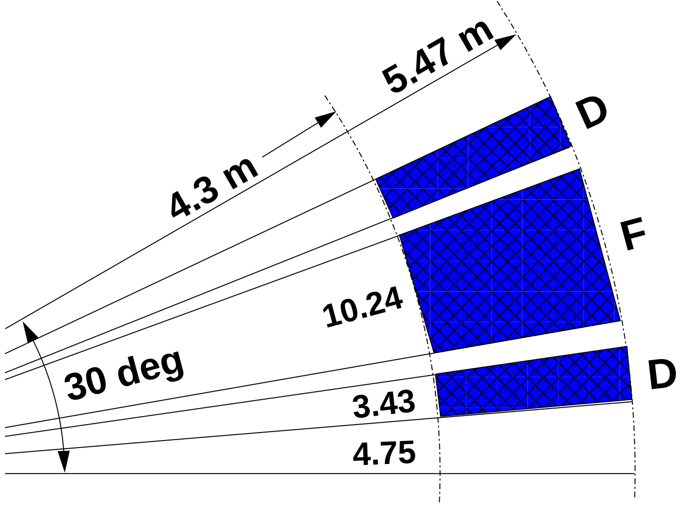
code	stepwise	seen in company of (sort of POP)	allows fringe-field overlap	
COSY		linear FFAG		
ICOOL	yes	muon, lin. & scal.	yes	*Fourier expansion of fringe fields*
MAD-PTC	yes	muon, EMMA		
J-RK4	yes			
S	yes	linear ; scaling		
Zgoubi	yes	all types of FFAGs	yes	*Linear superimosition of field sources*
MAD	no	lin. FFAG / E. Keil	no	
. . .				
. . .				
. . .				

Example - “`FFAG`” and “`DIPOLES`” procedures in Zgoubi

- **Two main goals :** $\begin{cases} (i) \text{ simulate } B_{zi}(r,\theta) = B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r) \text{ (e.g., scaling, pumplet)} \\ (ii) \text{ allow for possible overlap of fringe fields} \end{cases}$

  Main apps : scaling and isochronous FFAGs.

- **An example : simulation of a scaling FFAG triplet :**

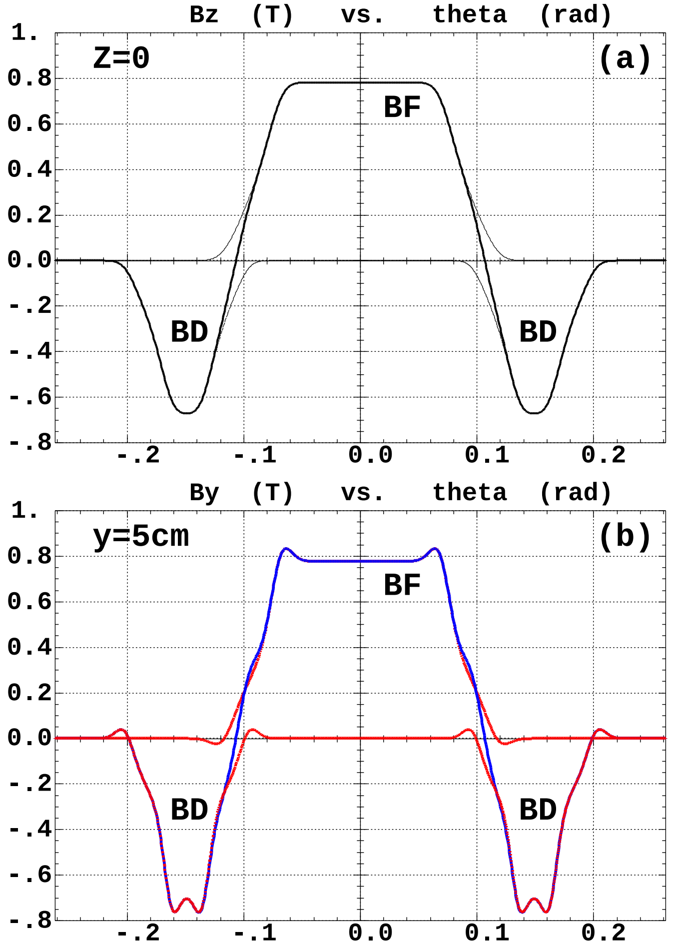


DFD triplet.

The geometrical model is based on the superposition of the independent contributions of the N dipoles :

$$\boxed{B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)}$$

at all (r,θ) in the mid-plane.
Field off mid-plane is obtained by Taylor expansion accounting for Maxwell’s eqs.



Field experienced for $r_0 = 4.87$ m in a DFD dipole triplet.

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Codes known to (or to have) handle(-d) FFAG problems

	code	stepwise	seen in company of (sort of POP)	allows FF overlap	allows field map
	COSY		linear FFAG		
	ICOOL	yes	muon ; scaling DFD	yes	
	MAD-PTC	yes	muon, EMMA		did
	J-RK4	yes	typical of J R&D		yes
	S	yes	linear ; scaling		
	Zgoubi	yes	all types of FFAGs	yes	yes
Probably needs completion	. . .				
	. . .				
	. . .				

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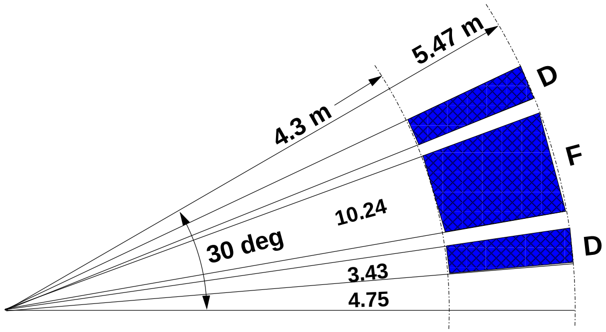
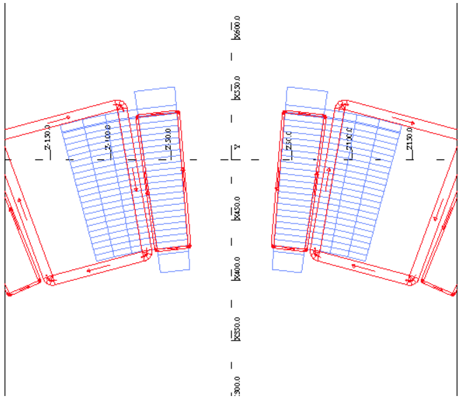
Tracking in TOSCA 3-D field maps

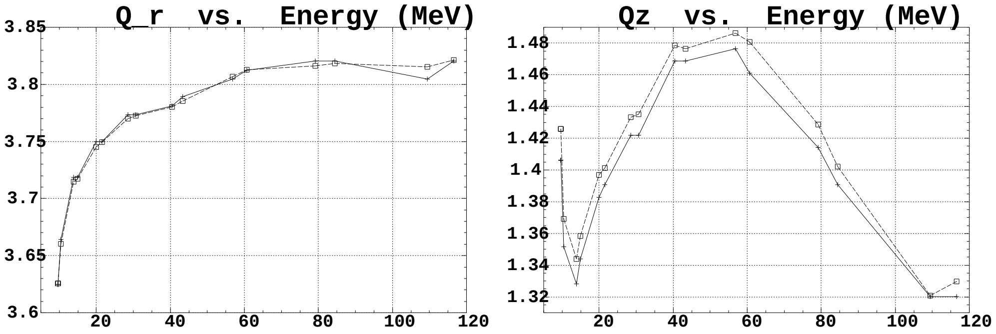
Accuracy can be very good, depends on (i) accuracy of the integration method. (ii) density of map mesh, (iii) smoothness of magnetic field sampling.



DFD sector triplet, 30 degrees sector cell.



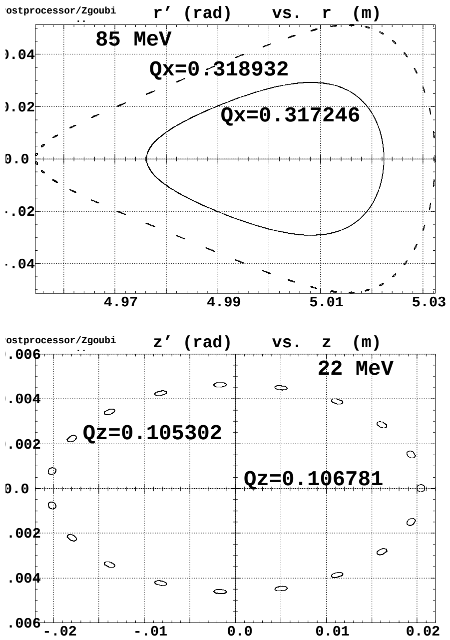
Geometry of TOSCA field map, covering half the angular extent.



Radial tune (left plot) and axial tune (right) as a function of energy, as obtained using

RK4 integration (solid lines/crosses),

or using Zgoubi (dashed line/squares).



Sample multiturn tracking using field maps, more than 3000 passes in a DFD triplet cell.

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Codes known to (or to have) handle(-d) FFAG problems

code	seen in company of (sort of POP)	allows FF overlap	allows field map	has performed 6-D tracking
COSY	linear FFAG			
ICOOL	muon ; scaling DFD	yes		yes
MAD-PTC	muon, EMMA		no	
J-RK4	typical of J R&D		yes	yes
S	linear ; scaling		yes	yes
Zgoubi	all types of FFAGs	yes	yes	all FFAG types
. . .				
. . .				
. . .				

Probably needs completion

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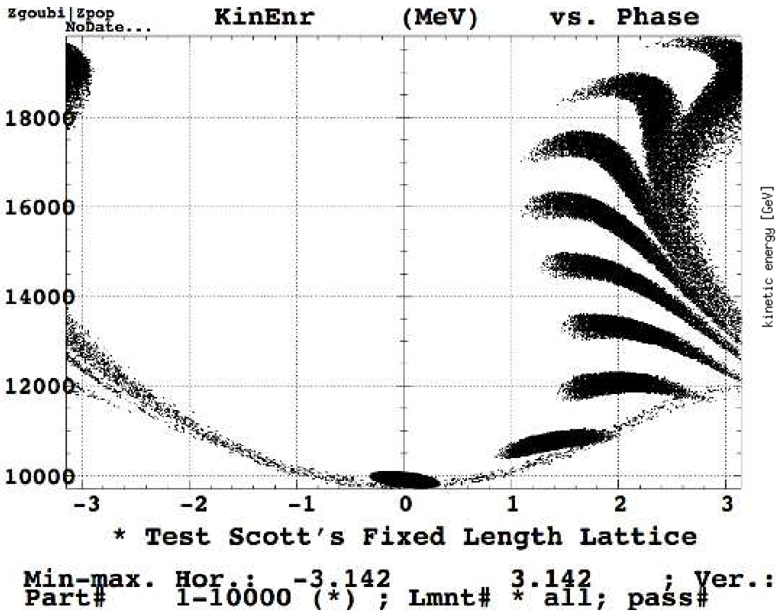
Example : 6-D acceleration in linear non-scaling muon FFAG / code comparisons

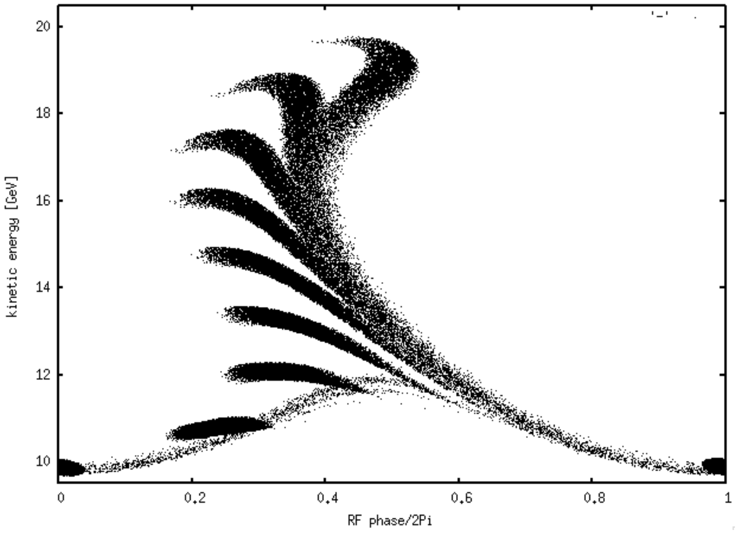
10 to 20 GeV muon ring

Particles are uniformly filled in each phase space independently.

Zgoubi



S-code



(30 p mm in transverse, 0.05 eVs in longitudinal.)

Other features of interest :

• **In-flight decay**	*yields pollution by muon-decay electrons*
• **Spin tracking**	*muons have spin. Protons too...*
• **Collimation**	*quantify beam loss, look for hot spots*
• **Field and alignement errors**	*compute tolerances*
• **Synchrotron radiation**	*e-FFAG based eRHIC, LHeC ?*
• **...**	

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6 A few concluding remarks

- **Regular "TRANSPORT" type methods allow preliminary design parameter (magnets, lattice)**
- **Resorting to ray-tracing is mandatory, in general early in the design process**
- **3-D field maps is a necessary stage : non-linear fields, field inhomogeneities, huge amplitudes, etc.**
- **Needs symplectic tracking in highly non-linear fields, involving tremendous emittances (πcm range), and possibly highly non-linear lonitudinal motion**
- **Design and tracking codes exist. Not a lot, though**
- **Carry on upgrade of codes and of optics libraries**

Thank you