High Energy Muon Colliders

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High Energy Colliders*

- The physics:
    - Hadron collider energies are limited by their size, and technical constraints (bending magnetic fields) – it becomes impractical to obtain the required luminosities, which must rise as the energy squared. Even worse, the gluon-gluon background radiation makes it increasingly difficult to sort out the complicated decay schemes envisaged for the SUSY particles.
    - Lepton colliders in general, have the advantage that the interaction energy is given by twice the machine energy, and, because they undergo simple, single-particle interactions, - in a hadron collider, the effective energy is much lower than that of the proton.
    - The lepton collider on the other hand offers clean production of charged pairs with a cross section comparable to $\sigma_{QCD} = 100/s$ fb where s is the energy squared in TeV^2 .

* μ^+- μ^- Collider: a Feasibility Study, submitted to the Snowmass96 workshop proceedings.

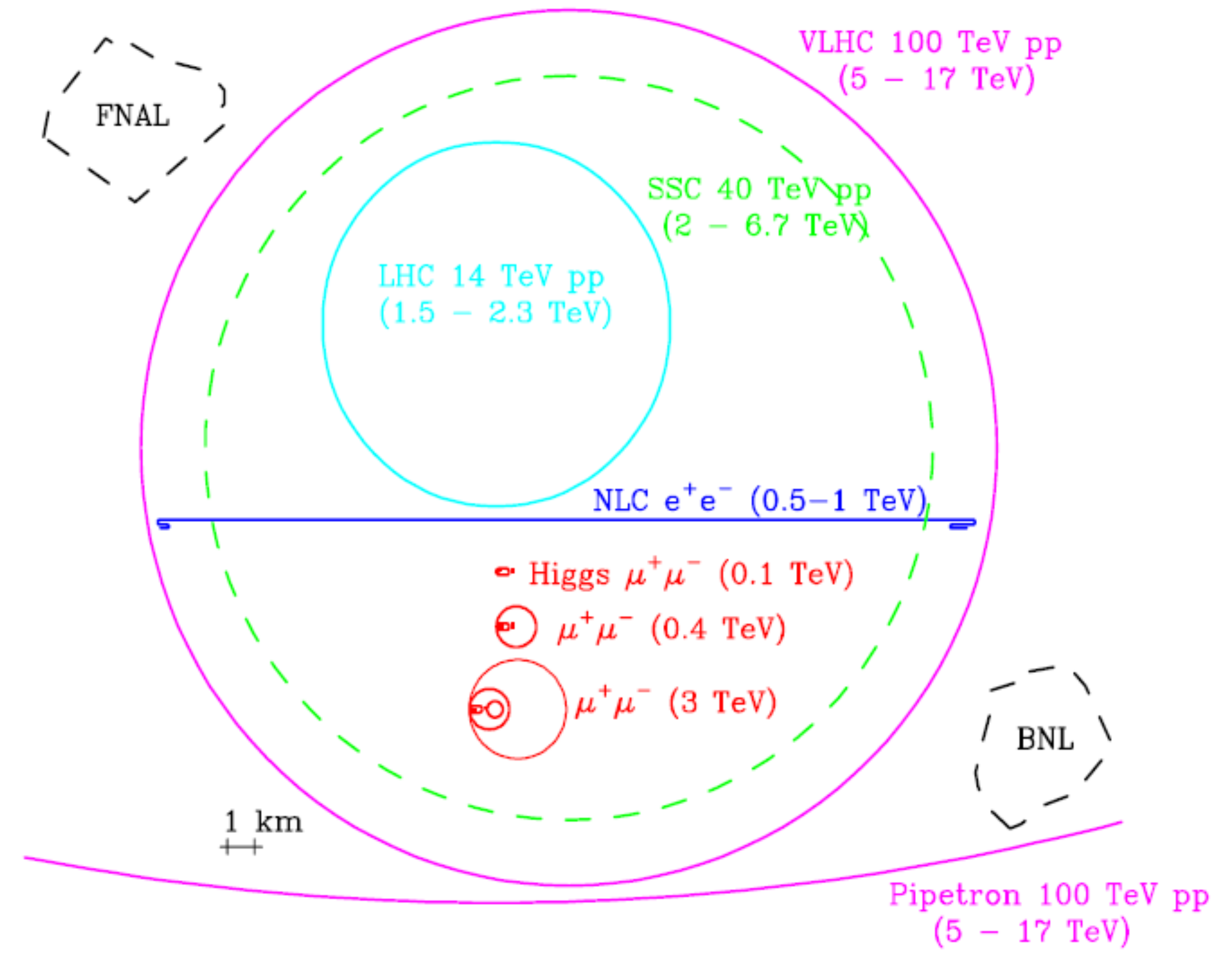
The rationale for muons

- Extension of e^+e^- colliders to multi-TeV energies is severely performance-constrained by beamstrahlung, and cost-constrained because two full energy linacs are required to avoid excessive synchrotron radiation.
- Muons ($m_\mu/m_e = 207$) have negligible beamstrahlung, can reach much higher energy than e+e− colliders, and can be accelerated /stored in rings with a much smaller radius than a hadron collider of comparable energy.
- The reduced beamstrahlung and initial state radiation leads to better energy definition of the initial state.
- When coupling is proportional to the mass (the s-channel Higgs production), muons have an advantage of $(207)^2$ over electrons.

The technical challenges with muons

- Muon decay with a lifetime of 2:2 x10^{-6} s.
- Muons must be accelerated rapidly to outrace decay: at 2 TeV the muon lifetime is 0:044 s (sufficient for ~1000 storage-ring collisions).
- Muon decay products heat the superconducting magnets of the collider ring and create significant backgrounds in the detector.
- Muons are created through pion decay into a diffuse phase space, and must be captured and cooled; (conventional stochastic or synchrotron cooling is too slow for muon decay
- Muon beams will remain larger than electron beams in an e +e− collider; smaller, more severe focusing is required for collisions
- New technologies are required.

Collider comparisons

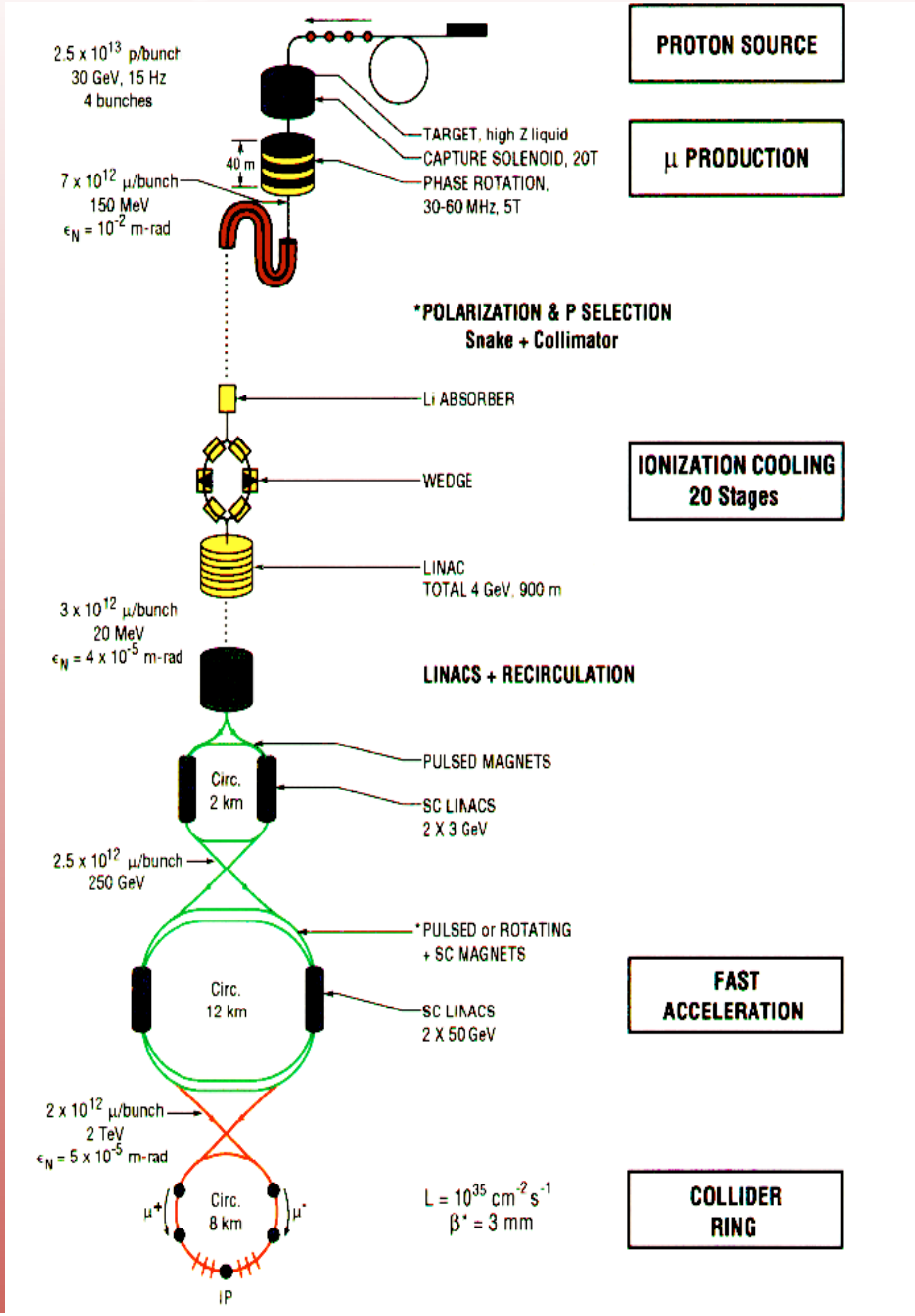


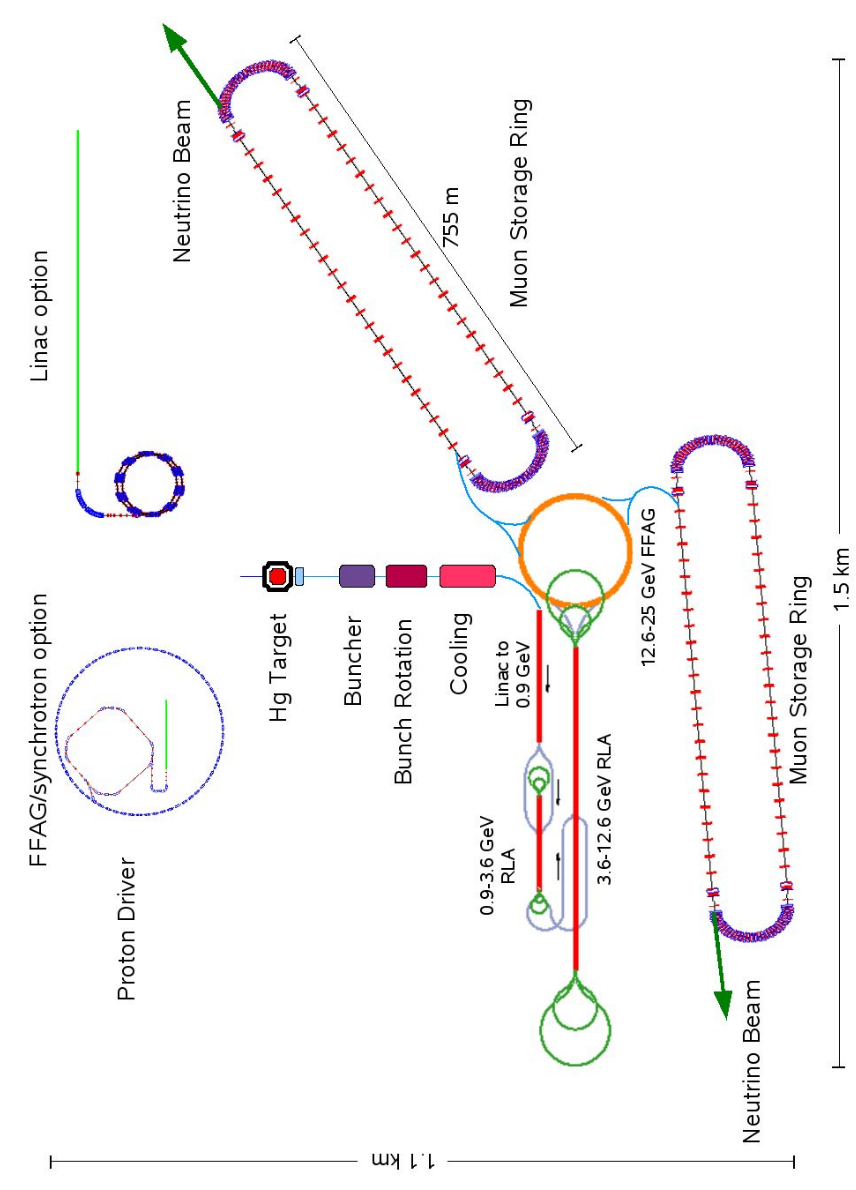
Comparative sizes of various proposed high energy colliders and FNAL and BNL sites. The energies in parentheses give for lepton colliders their CoM energies and for hadron colliders the approximate range of CoM energies attainable for hard parton-parton collisions.

Neutrino Factories

- A Neutrino Factory provides both ν_μ *and* ν_e beams of equal intensity from a stored $\mu-$ beam, and their charge-conjugates for a stored $\mu+$ beam.
- Beams from a Neutrino Factory are intense compared to other neutrino sources, have smaller divergence and therefore permit the study of non-oscillation physics at near detectors, and the measurement of structure functions and associated parameters in non-oscillation physics to unprecedented accuracy.
- NFs also permit long-baseline experiments that can determine oscillation parameters to unprecedented accuracy.
- DIFFERENT PHYSICS, DIFFERENT MACHINE!!

Early Collider and recent NF layouts





Muon Colliders and Neutrino Factories

The Collider

- At least 1000x smaller emittances required, small momentum spread
- Single circulating bunch of μ+ and also μ- in counter-rotating directions
- Collision at a single point: Interaction Point (IP) in a specially designed insertion (Interaction Region, IR).
- Maximize luminosity, minimum beam size, maximum divergence in IR

Neutrino Factory

- In contrast: large emittances, large dp/p in circulating beam
- Large beam, small divergences in long neutrino production (muon decay) straight
- Bunch train; no collisions, μ+/μ- generally in the ring independently

Comments about RLAs vs FFAGs

- FFAGs: Large emittances, longitudinal phase space distortion
  - Does NOT apply to the collider due to much smaller emittances, multiple FFAGs can be the upstream accelerator system.
  - Recent tune-stable designs may eliminate distortion for the NF.
- RLAs capable of only 4 turns due to complexity of switchyard vs. 10-15 turns in a FFAG
  - Transverse phase space blow-up issues (over full +/-10-20% momentum spread). This has not been properly tracked.

Muon Collider luminosity: General

- The collider ring allows for (about) 1000 collisions per bunch, rather than the single collision that is possible in a linear collider geometry. If the transverse beam size at the collision = that in an electron-positron linear collider, there would be a full increase in luminosity of order 1000. ***This is not the case.***

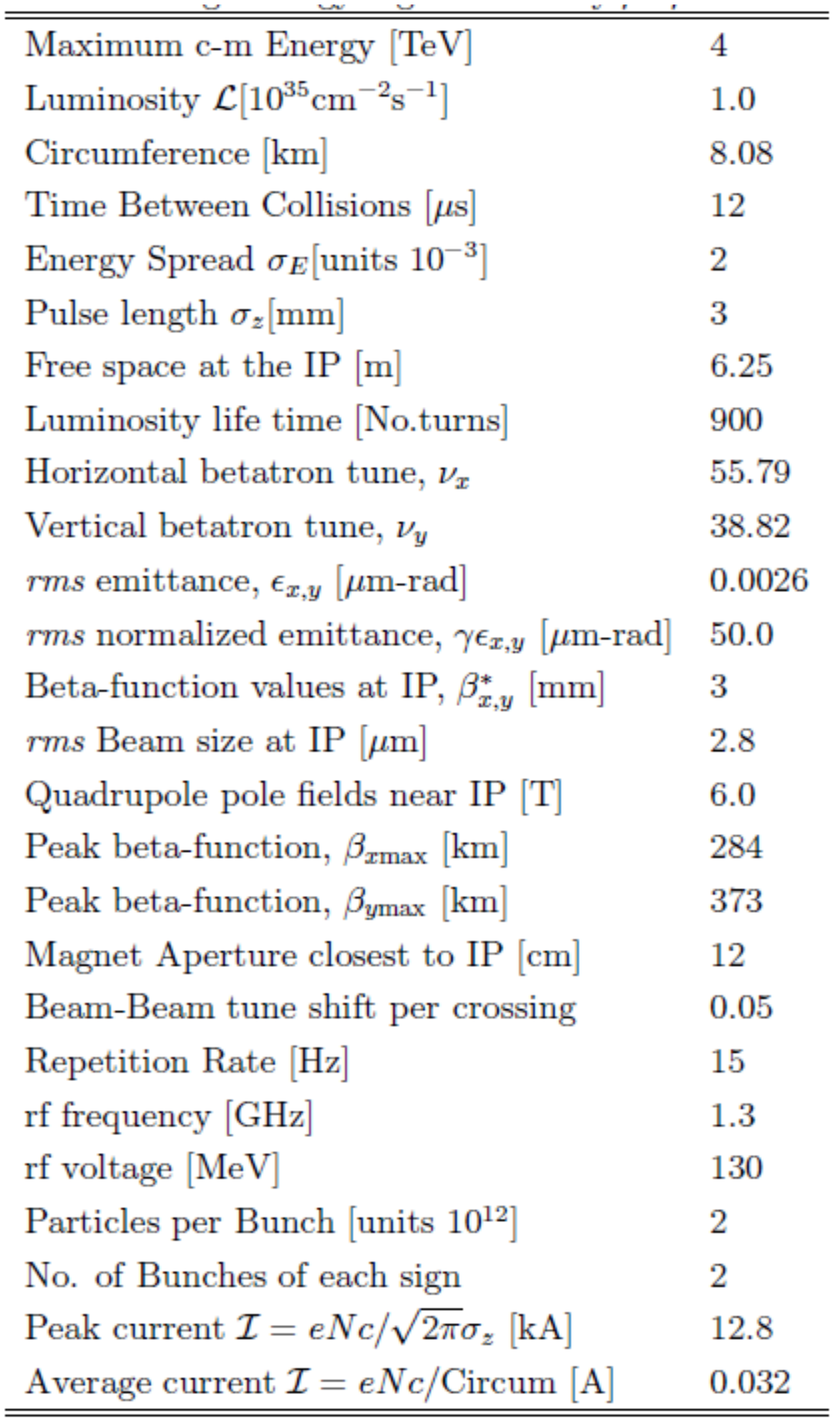
- The muon bunch is cooled as much as possible, but still has an emittance that is significantly larger than the extremely low emittances in an electron-positron linear collider.

- The luminosity scales as $L = f\,N^2 / ((\varepsilon_{nx}\beta_x)(\varepsilon_{ny}\beta_y))^{1/2}$, where $\varepsilon_{x(y)}$ is the beam emittance in the x(y) phase plane, $\beta^*_{x(y)}$ the corresponding beta-function at the interaction point (which is limited, by the hour-glass effect, to be no less than the bunch length), and f the collision frequency.

2x2 TeV Collider Requirements

- Luminosity: $10^{35}\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ for 2 TeV x 2 TeV.
  - *f* = 2 x 1000 x 15 = 30,000 Hz

  (# bunches x # collisions x 15 Hz)
  - N = $2\mathrm{x}10^{12}$ muons/bunch
  - β^*=0.003 m
  - 15Hz challenging, $2\mathrm{x}10^{12}$ muons/bunch really challenging



Maximum c-m Energy [TeV]	4
Luminosity $\mathcal{L}[10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}]$	1.0
Circumference [km]	8.08
Time Between Collisions [μs]	12
Energy Spread σ_E[units 10^{-3}]	2
Pulse length σ_z[mm]	3
Free space at the IP [m]	6.25
Luminosity life time [No.turns]	900
Horizontal betatron tune, ν_x	55.79
Vertical betatron tune, ν_y	38.82
rms emittance, $\epsilon_{x,y}$ [μm-rad]	0.0026
rms normalized emittance, $\gamma\epsilon_{x,y}$ [μm-rad]	50.0
Beta-function values at IP, $\beta^*_{x,y}$ [mm]	3
rms Beam size at IP [μm]	2.8
Quadrupole pole fields near IP [T]	6.0
Peak beta-function, $\beta_{x\mathrm{max}}$ [km]	284
Peak beta-function, $\beta_{y\mathrm{max}}$ [km]	373
Magnet Aperture closest to IP [cm]	12
Beam-Beam tune shift per crossing	0.05
Repetition Rate [Hz]	15
rf frequency [GHz]	1.3
rf voltage [MeV]	130
Particles per Bunch [units 10^{12}]	2
No. of Bunches of each sign	2
Peak current $\mathcal{I} = eNc/\sqrt{2\pi}\sigma_z$ [kA]	12.8
Average current $\mathcal{I} = eNc/\mathrm{Circum}$ [A]	0.032

Achieving ultra-low 3 mm beta IP

- 6 m drift is required for the detector
- Local chromaticity correction is required, change in tune as a function of momentum:

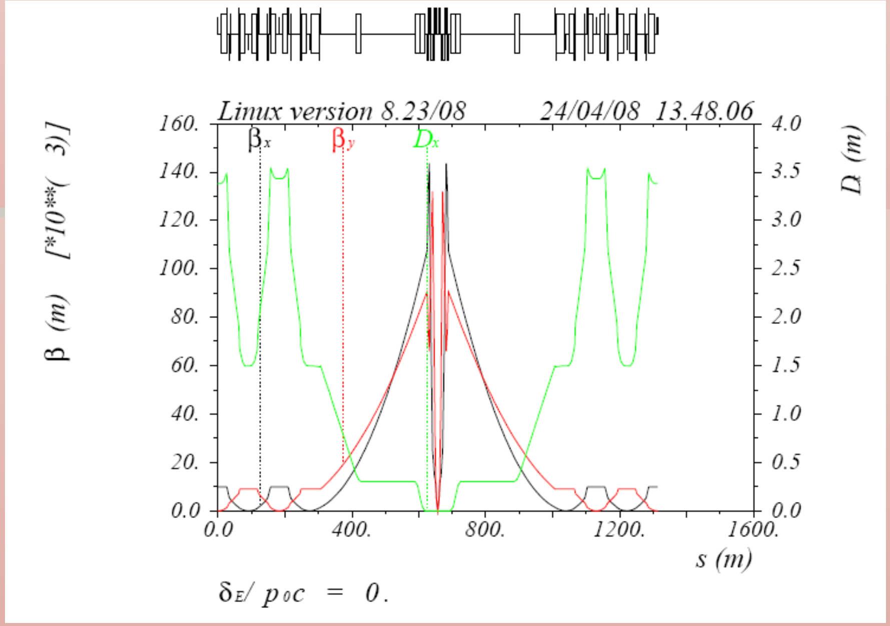
  $\xi \propto \beta kl$

where k and l are the strength

And length of a quadrupole,

respectively

β_{Max} *scales with* $1/\beta^*$

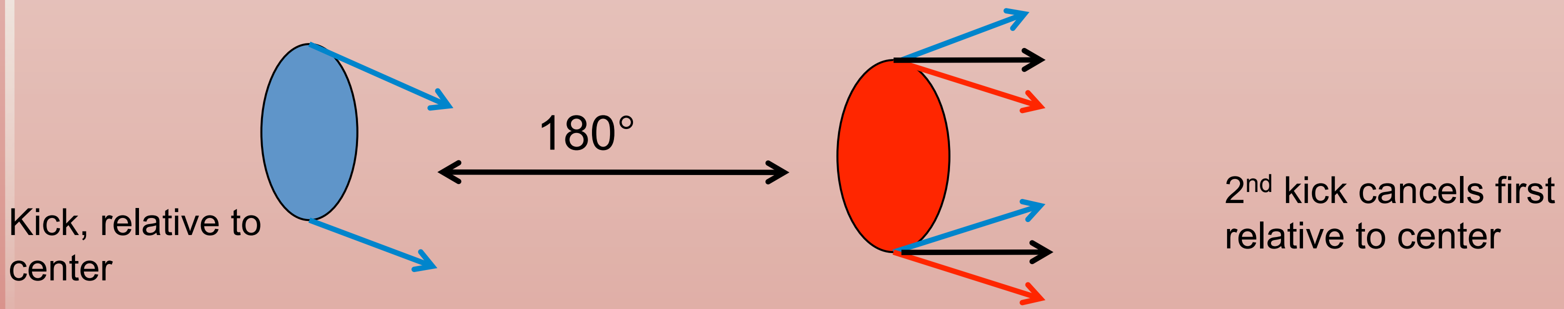


Chromaticity of Muon IRs

- β^*= 3 mm, ξ ~ -2,000 (vertical)
- β^*= 1 cm, ξ ~ -666
- For comparison the LHC IR (β^*= 0.5 m) has a chromaticity of ~60.
- No colliders are currently operating at such low β^*s.
- This high a chromaticity cannot be corrected in the arc – no off-momentum stability

Chromatic correction

- Sextupoles in –I pairs on either side of the IR quads.
- Dispersion causes off-momentum beam to travel off the reference trajectory; in a sextupole there is a quadrupole feed-down for off-axis beam:
- $B \propto x^2$; $B' \propto x$
- The "local" gradient increases / decreases with offset and therefore high / low momentum relative to the central momentum
- However, this also means that as the particle amplitude increases, the gradient and phase advance also change known as a tune-shift-with-amplitude; this is countered using sextupole in 180° pairs.

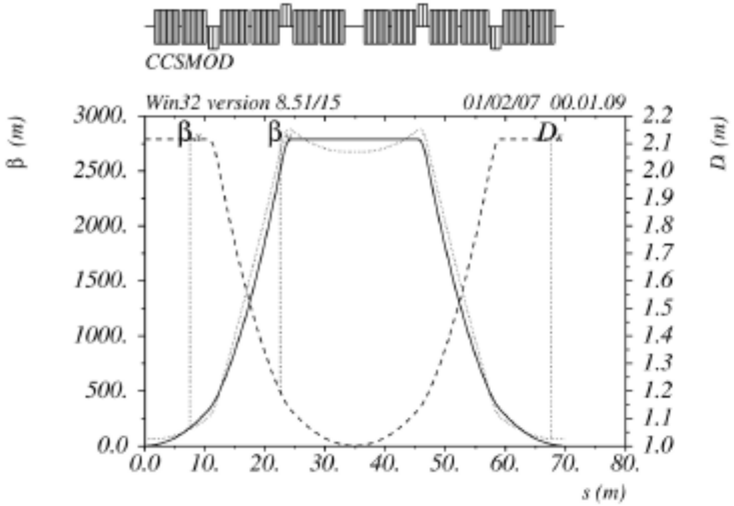


Kick, relative to center

2nd kick cancels first relative to center

Chromatic Correction section

- Large β ratio allows interleaving of horz/vert sextupoles for compactness



- Large β ratio $\Rightarrow$ clean chromatic correction, minimal cross correlation;
- The sextupoles are π phase advance apart $\Rightarrow$ no second order geometric aberrations;
- Repetitive symmetry + transfer map unity $\Rightarrow (x|\delta\delta) = 0$.

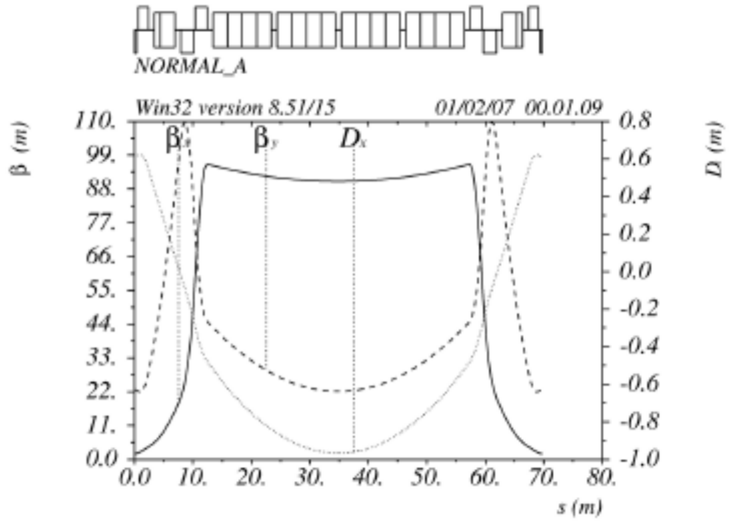
Isochronous condition for Collider

- In order to have very short 3mm bunches in the 2 TeV muon collider, the storage ring must be quasi-isochronous, which requires that the momentum compaction be very close to 0, where α is defined in terms of the offsets of the momentum *p* and equilibrium orbit circumference, C, by $$\alpha(p) = \frac{p}{C}\frac{dC}{dp}.$$
- To first order the time difference with respect to the reference particle is given by $$\frac{\Delta T}{T_0} = \eta\frac{\Delta p}{p_0} = (\alpha_0 - \frac{1}{\gamma^2})\frac{\Delta p}{p_0}$$
- where T_0 and α_0 are the revolution period and momentum compaction of the reference particle, ΔT and Δp are the time and momentum deviations, respectively, of the off-momentum
- particle relative to the synchronous particle with momentum p_0; *η* is the phase slip factor; γ is the Lorentz relativistic factor, and $\alpha_0 = \alpha(p_0)$.

Isochronous condition continued

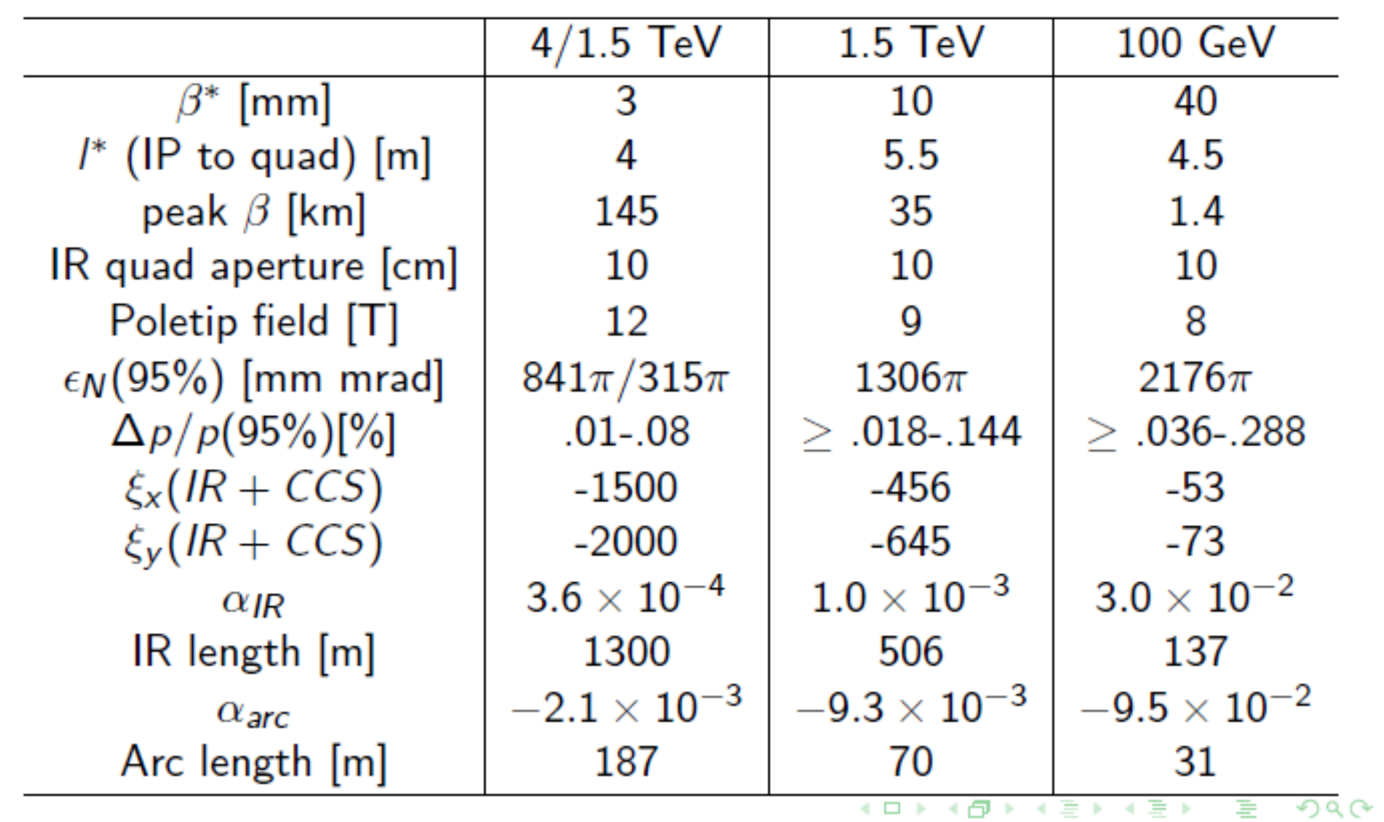
- In an isochronous ring $\eta = 0$, so to first order the arrival time is independent of the momentum; i.e., $\gamma_t = \gamma$. The collider operates at transition.
- For 2 TeV muons $\gamma \approx 2 \times 10^4$; so $\alpha \approx 2.5 \times 10^{-9}$
- To bring the first order value of α to zero requires that the $\langle D/\rho \rangle = 0$ through all of the dipoles be equal to zero, where D is the dispersion and ρ the radius of curvature.

Momentum compaction in the Collider arcs



- Flexible Momentum Compaction module provides negative momentum compaction values compensating for the positive momentum compaction generated by the CCS;

Basic parameter comparisons for different designs and β*



	4/1.5 TeV	1.5 TeV	100 GeV
β^* [mm]	3	10	40
l^* (IP to quad) [m]	4	5.5	4.5
peak β [km]	145	35	1.4
IR quad aperture [cm]	10	10	10
Poletip field [T]	12	9	8
ϵ_N(95%) [mm mrad]	$841\pi/315\pi$	1306π	2176π
$\Delta p/p$(95%)[%]	.01-.08	$\geq$ .018-.144	$\geq$ .036-.288
$\xi_x(IR + CCS)$	-1500	-456	-53
$\xi_y(IR + CCS)$	-2000	-645	-73
α_{IR}	3.6×10^{-4}	1.0×10^{-3}	3.0×10^{-2}
IR length [m]	1300	506	137
α_{arc}	-2.1×10^{-3}	-9.3×10^{-3}	-9.5×10^{-2}
Arc length [m]	187	70	31

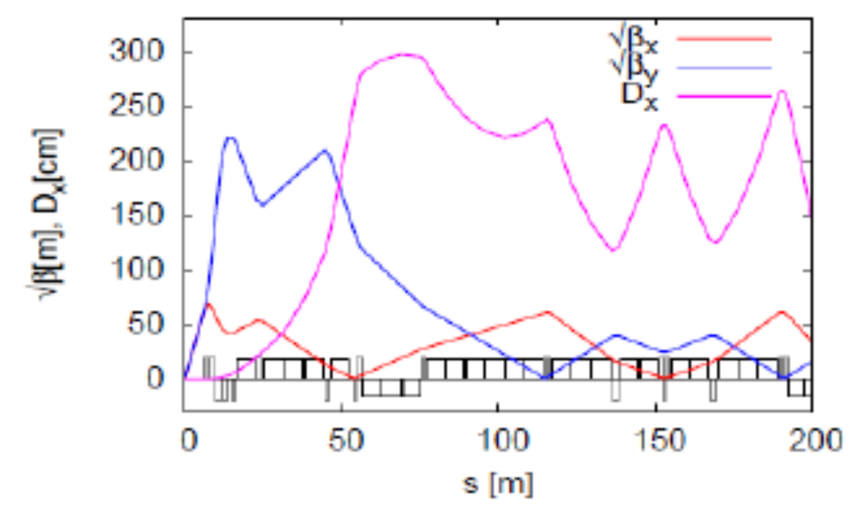
Recent Collider design work: Y. Alexahin and E. Gian-felice

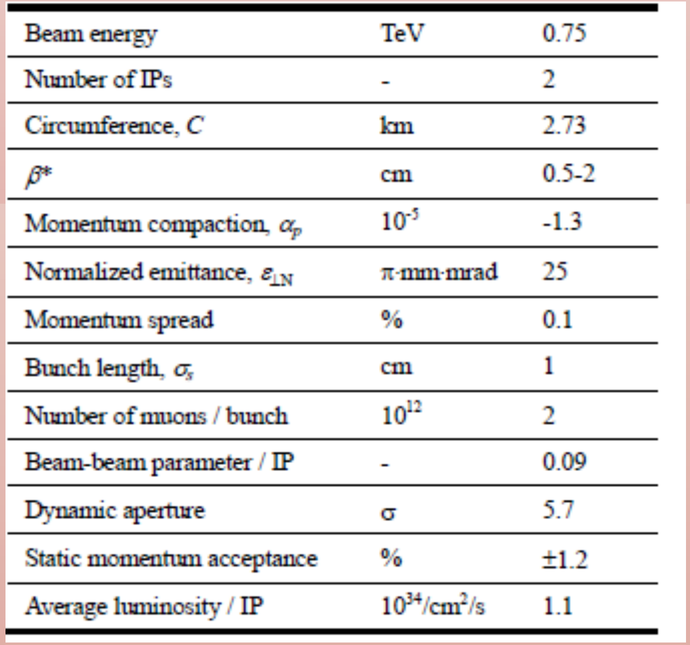
- Borrows heavily from

Oide design where $\beta_y >> \beta_x$; hence Chromaticity is large in one plane only.

The approach and correction scheme is detailed in IPAC10 papers:

- tupeb021/tupeb022
- Has achieved required DA and
- Momentum stability





Beam energy	TeV	0.75
Number of IPs	-	2
Circumference, C	km	2.73
β^*	cm	0.5-2
Momentum compaction, α_p	10^{-5}	-1.3
Normalized emittance, $\varepsilon_{\perp N}$	π·mm·mrad	25
Momentum spread	%	0.1
Bunch length, σ_s	cm	1
Number of muons / bunch	10^{12}	2
Beam-beam parameter / IP	-	0.09
Dynamic aperture	σ	5.7
Static momentum acceptance	%	±1.2
Average luminosity / IP	10^{34}/cm^2/s	1.1

Summary of current collider work

- Complex correction scheme to achieve required performance.
- No quadrupole fringe fields in simulation
    - This has always been devastating
    - No confidence that existing lattices will survive
    - Asymmetric emittances are being investigated.