Longitudinal tomography

David Kelliher, 26/7/18

Tomography

- Back projection is the technique “by which the contents of the bins of a 1D histogram are redistributed over the 2D array of cells (pixels) which comprise the reconstruction image”*.
- Typically the object is fixed and the viewing angle rotated.

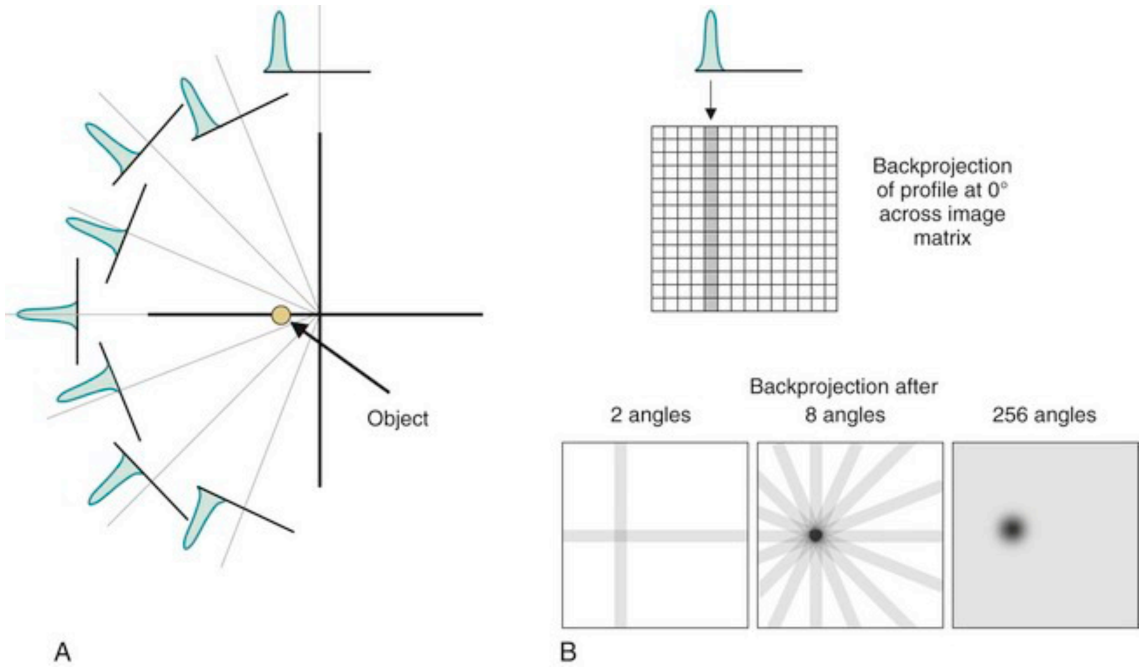


Image: https://radiologykey.com/tomographic-reconstruction-in-nuclear-medicine/

*S. Hancock, CERN PS/RF/Note 97-06

Longitudinal tomography

- In longitudinal tomography the “object” rotates whereas the projection is along a fixed line (the time axis).
- Conventional algorithms normally assume a rigid, circular motion of the 2D distribution.
- The hybrid iterative algorithm developed at CERN considers how each point in longitudinal phase space are projected into the bins of a particular profile by tracking a number of test particles. Large amplitude synchrotron motion is then taken into account.

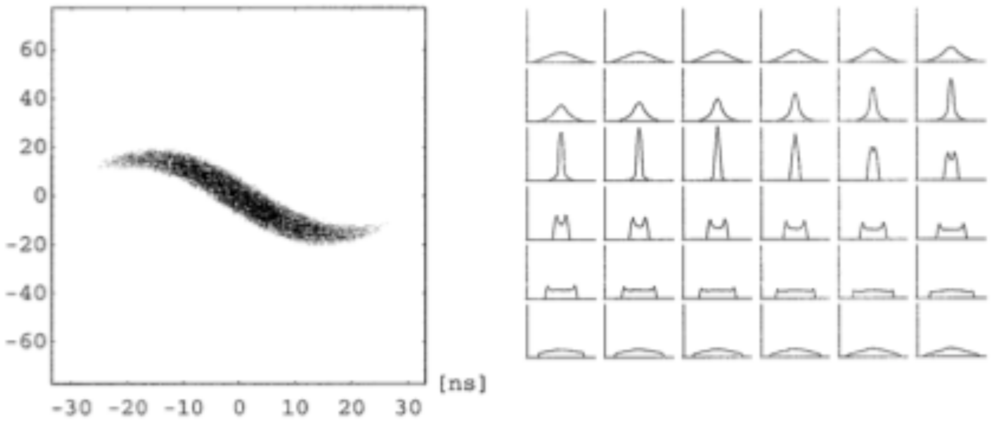


Figure 2: The initial phase space distribution and thirty-six of its projections at intervals of 5° of synchrotron phase. The latter constitute the set of “measured” data.

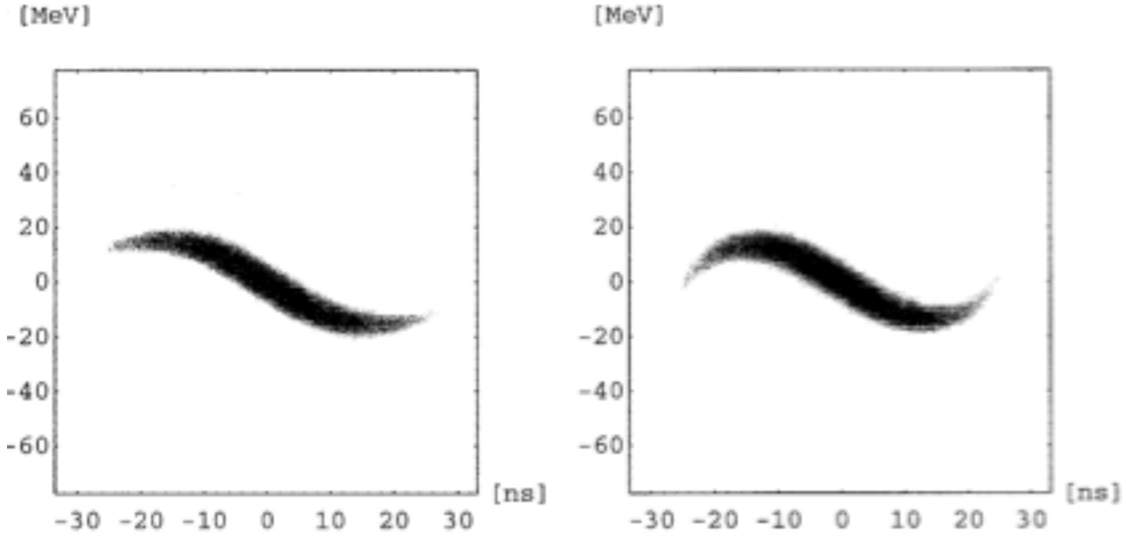


Figure 3: Reconstructed distributions assuming (i) non-linear and (ii) linear synchrotron motion.

S. Hancock, CERN PS/RF/Note 97-06

CERN tomography code
(http://tomograp.web.cern.ch/tomograp/)

- The code, based on the hybrid algorithm developed by Steve Hancock, was written by Steve and Mats Lindroos in Fortran90.
- Takes bunch monitor data as input. Each profile is separated by a fixed number of turns. The time step between data points is fixed.

Phase and energy increment at each cavity crossing

$$\Delta\phi_{i,m+1} = \Delta\phi_{i,m} - 2\pi h\left(\frac{\eta_{0,m}\Delta E_{i,m}}{\beta_{0,m}^2 E_{0,m}}\right),$$

$$\begin{aligned}\Delta E_{i,m+1} = \Delta E_{i,m} + q[&V_{\mathrm{rf},m+1}(\phi_{0,m+1} + \Delta\phi_{i,m+1}) \\ &- V_{\mathrm{rf},m+1}(\phi_{0,m+1}) \\ &+ V_{\mathrm{self},m+1}(\phi_{0,m+1} + \Delta\phi_{i,m+1})],\end{aligned}$$

Self-field voltage

$$V_{\mathrm{self},m}(\phi) = qh^2\omega_{0,m}\left[\frac{gZ_{\mathrm{vacuum}}}{2\beta_{0,m}\gamma_{0,m}^2} - \left|\frac{Z_{\mathrm{wall}}}{n}\right|\right]\frac{d\lambda_m(\phi)}{d\phi}.$$

Discrepancy parameter

$$d' = \sqrt{\frac{1}{M'}\sum_{i=1}^{M'}(e_i - r_i)^2},$$

Testing the code

Top level python script

Set machine and tracking parameters (e.g. number of turns)

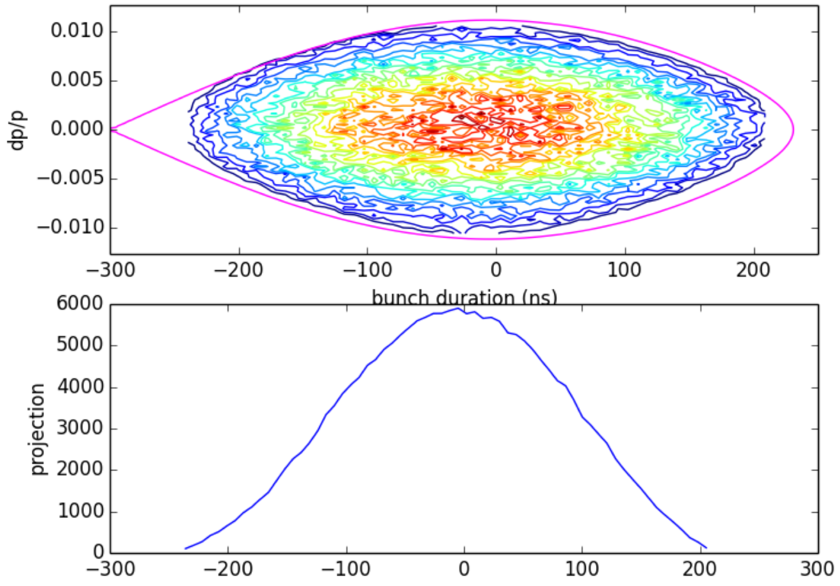
Longitudinal tracking code (PyHEADTAIL)

Create tomography code input file

Generate bunch monitor “data”

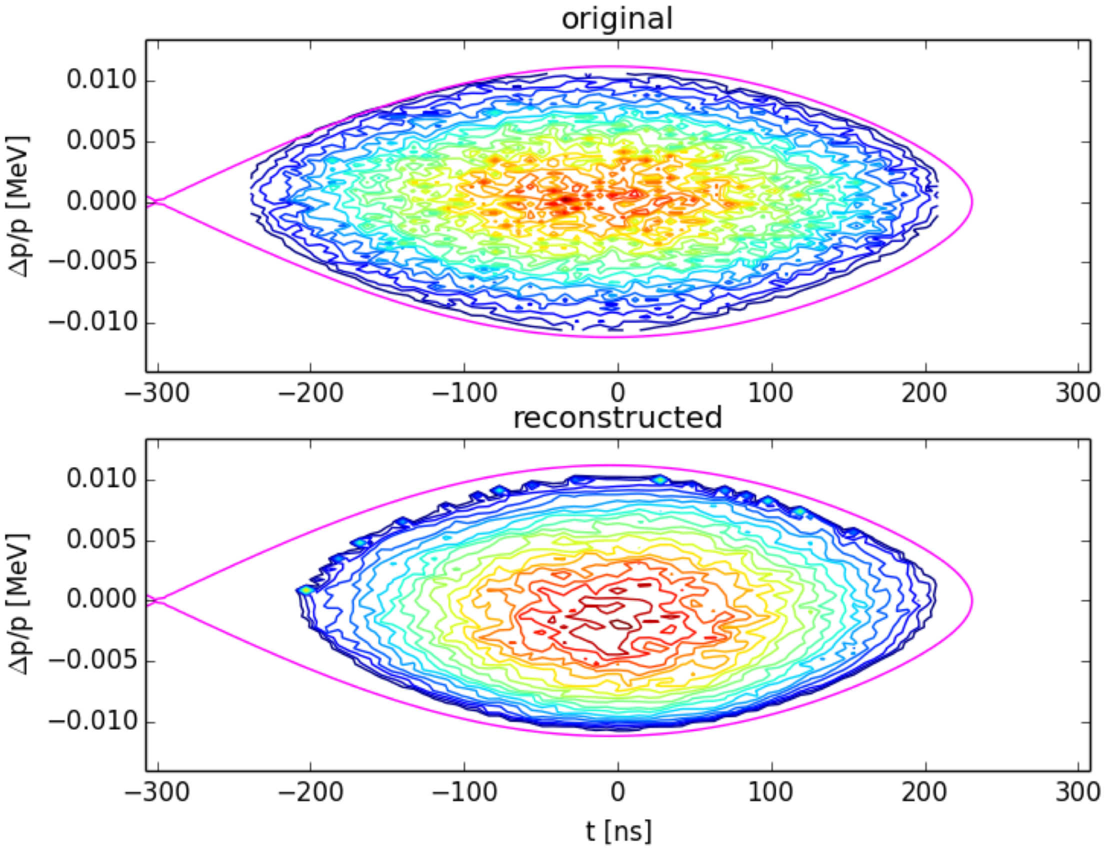
Run tomography code

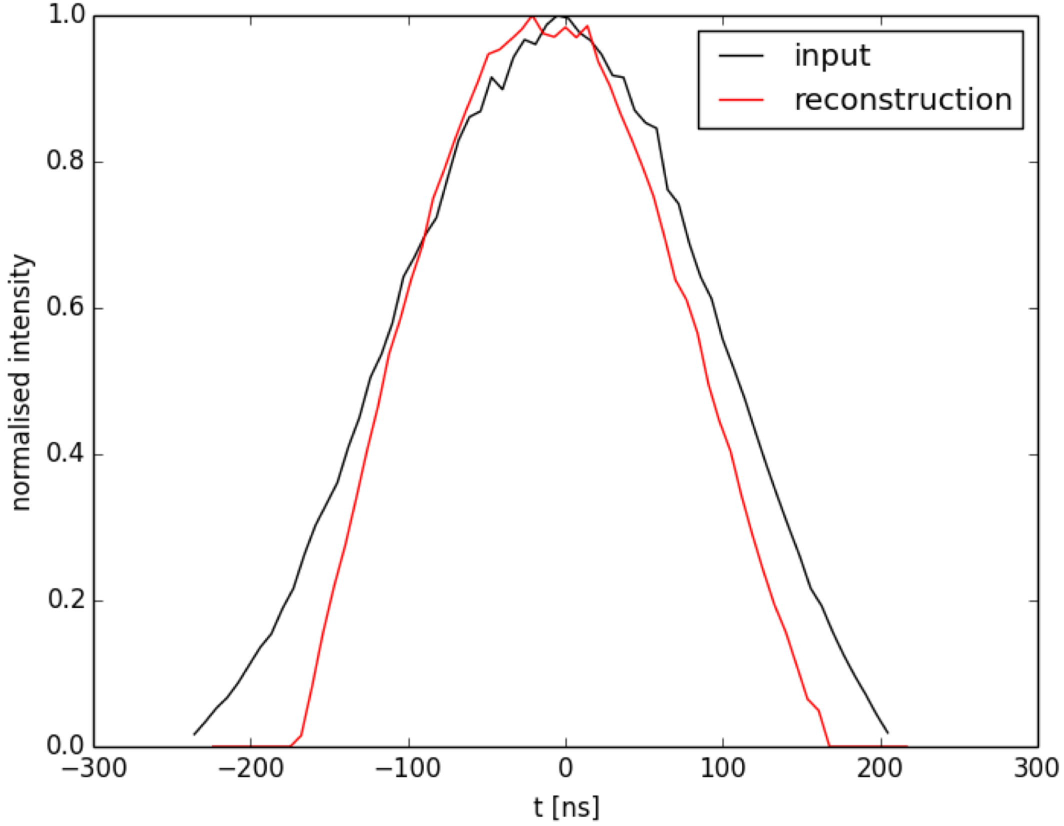
Compare reconstructed phase space with original



PyHEADTAIL distribution

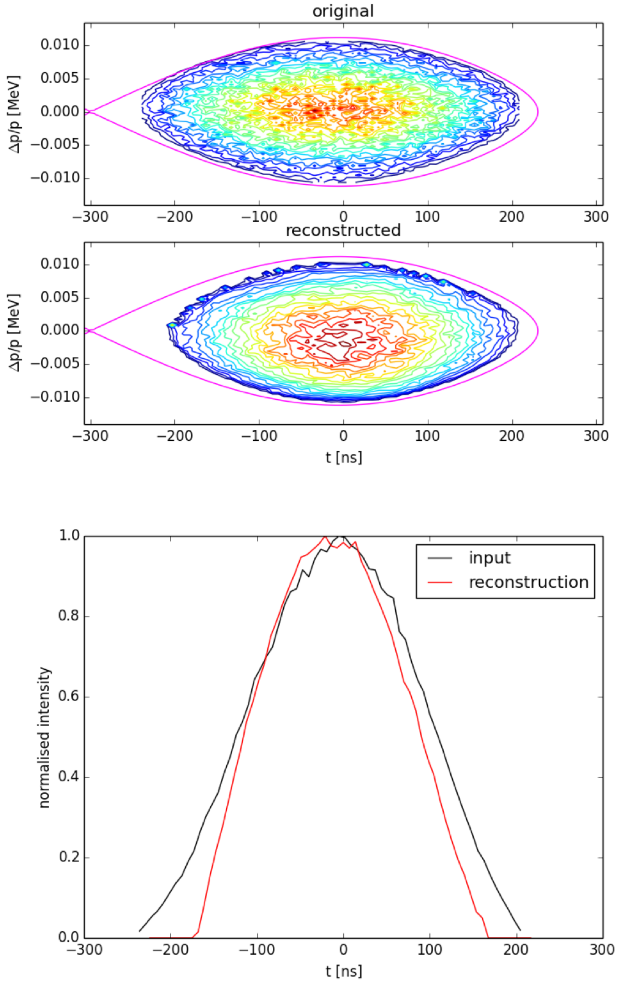
Stationary thermal distribution



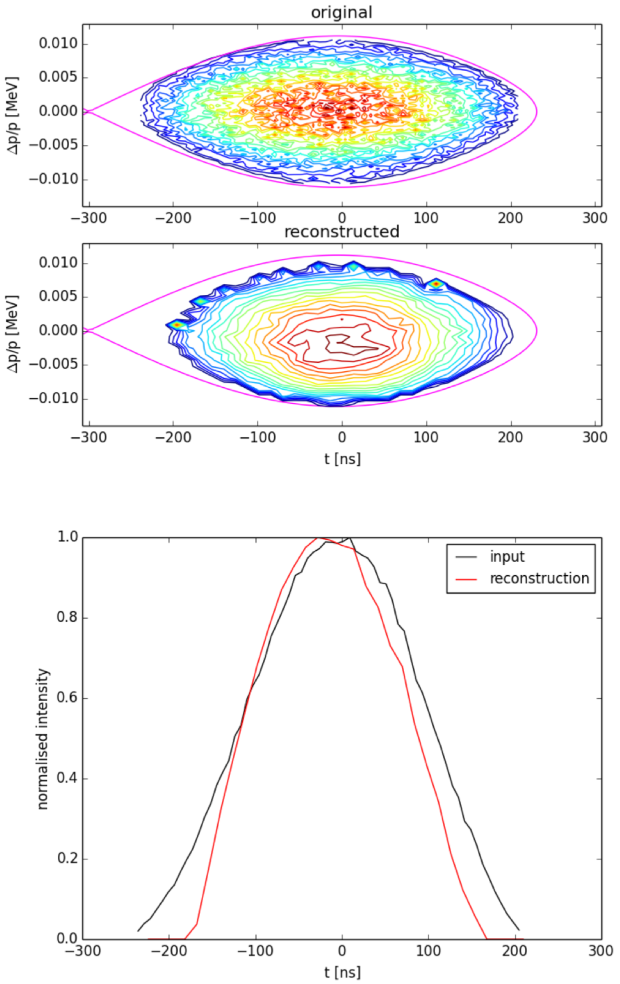


Qs: 0.005 (Ts=200), phi_s: 0.05 rad.
Number of bins in projection: 64
Number of turns: 100 (=0.5*Ts)

Number of bins



64



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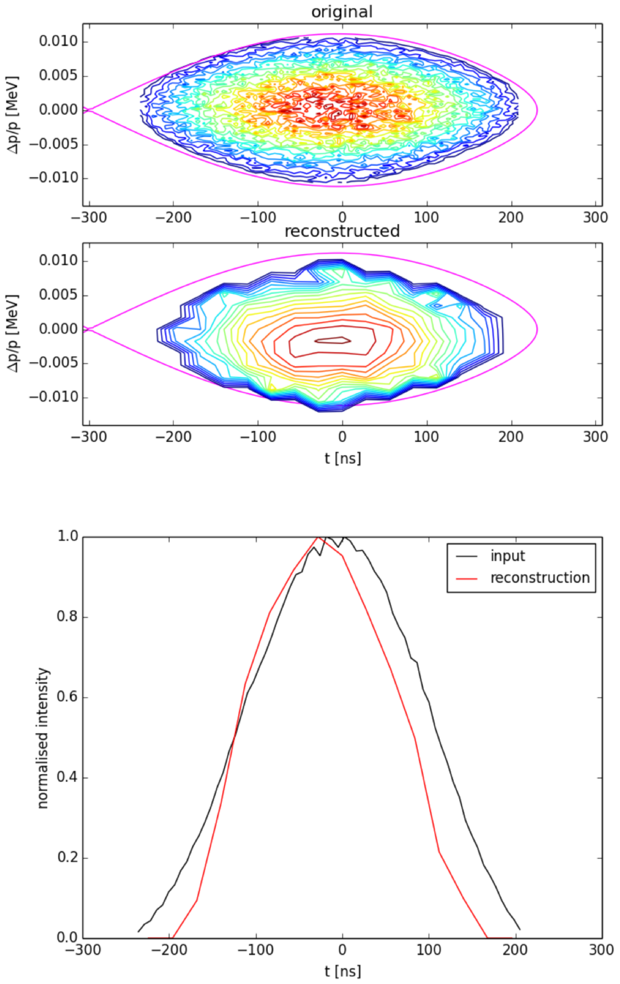

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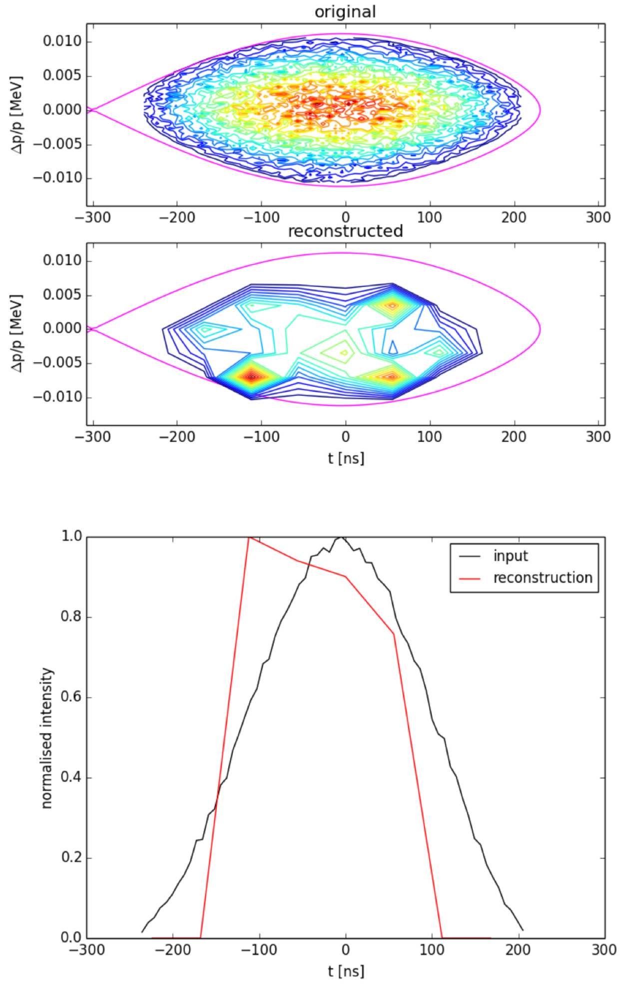
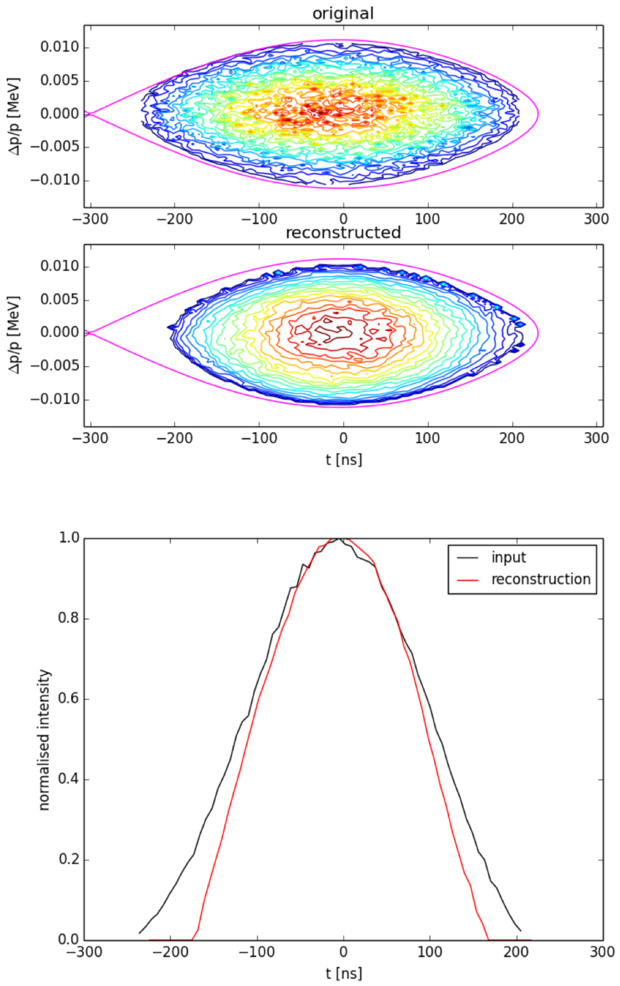
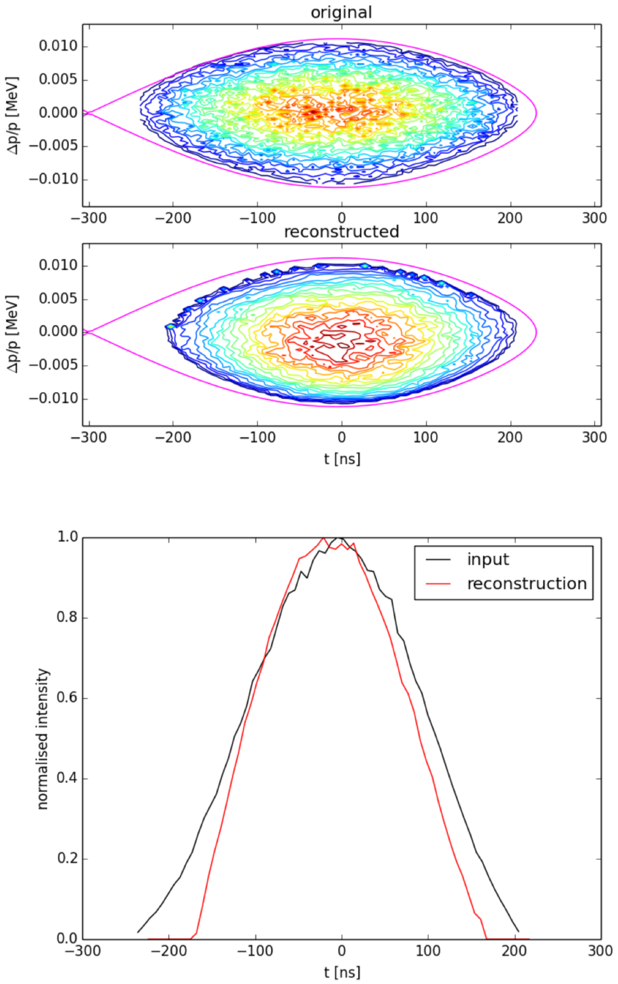


8

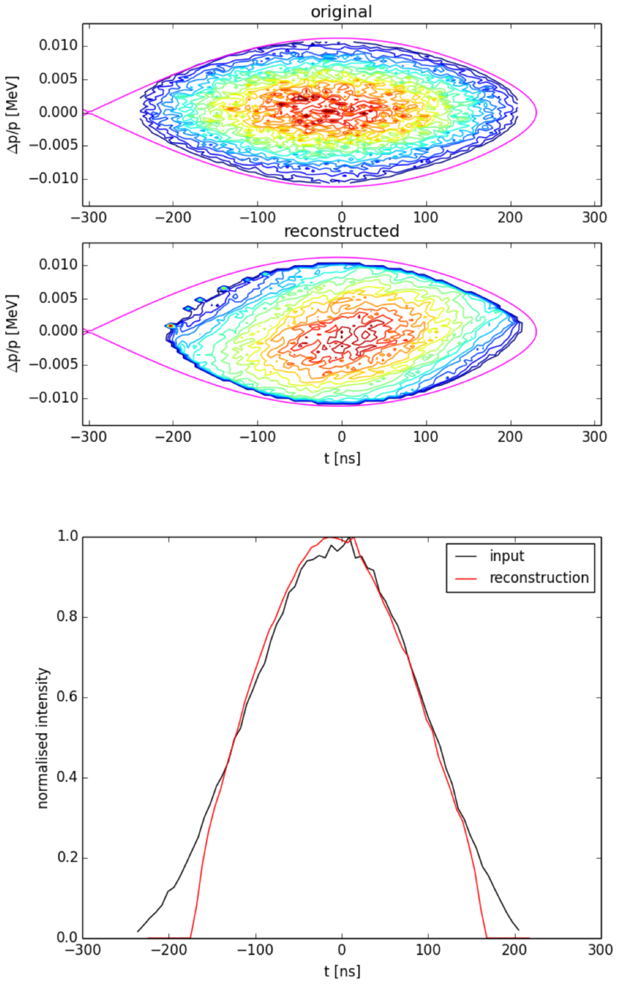
Number of turns/synchrotron period



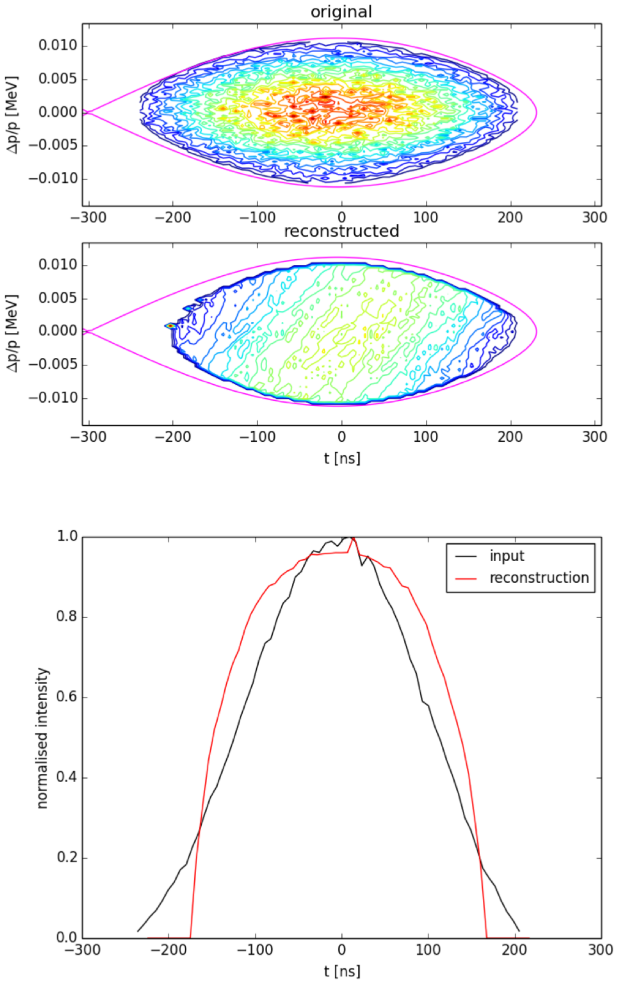
1



0.5

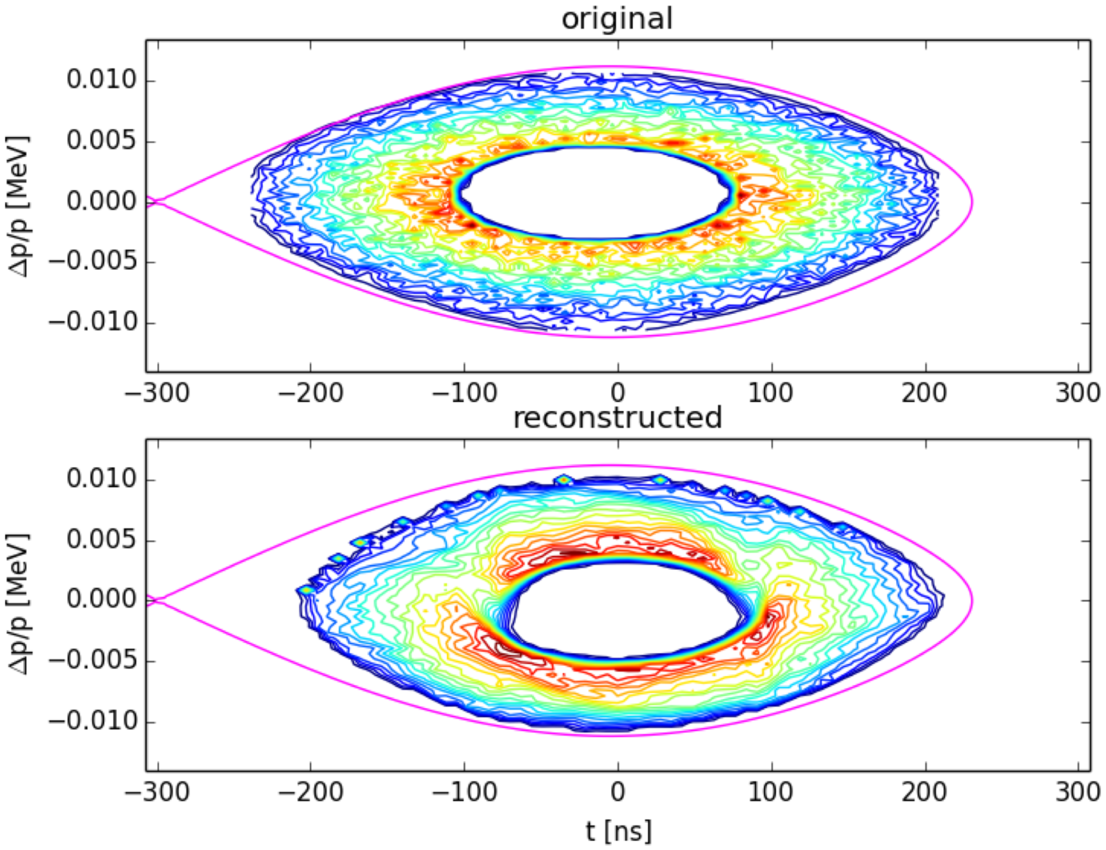


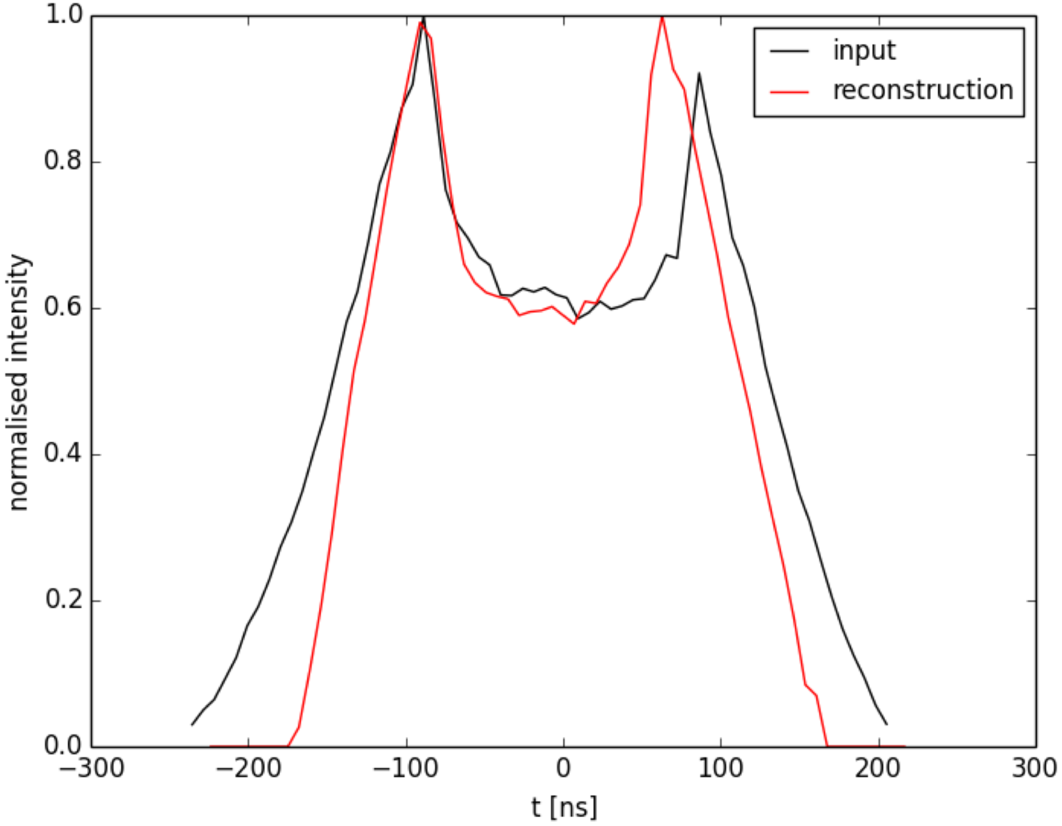
0.25



0.125

Hollow distribution





Qs: 0.005 (Ts=200), phi_s: 0.05 rad.
Number of bins in projection: 64
Number of turns: 100 (=0.5*Ts)

Tomography code issues/future work

- Resolve discrepancy between input and reconstructed profile widths (perhaps an error in translating between codes?).
- The code assumes a fixed synchronous phase.
- The time point within the data corresponding to the synchronous phase must be specified (apparently the results are fairly robust to errors in this guess).
- Tests should be extended to include non-stationary distributions and longitudinal space charge.