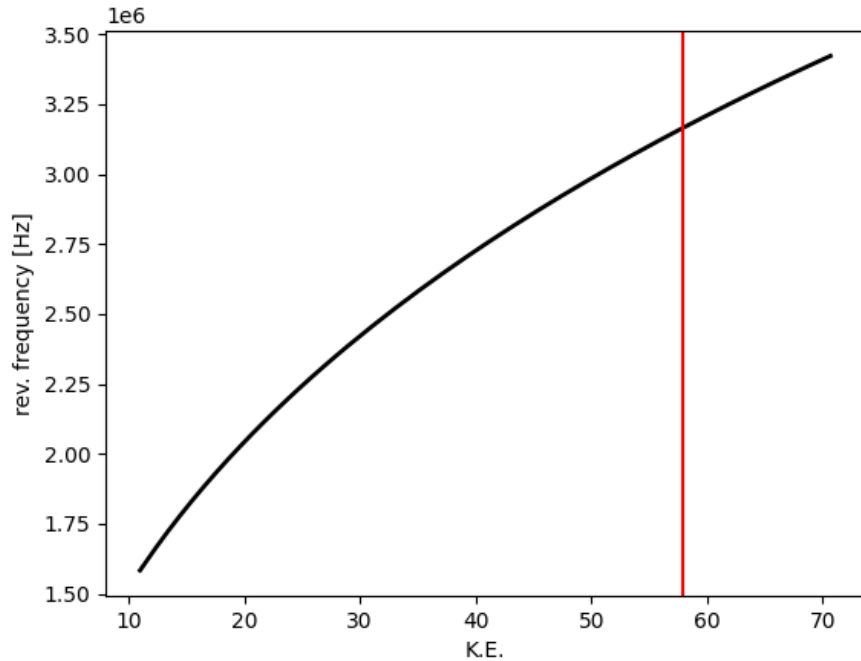


# Beam stacking study: Ramping $\phi_s$

D. Kelliher, 4/11/2022

# Stacking energy choice



From Uesugi-san's table:

- rev. frequency at injection (11 MeV): 1.58 MHz
- rev. frequency doubles at 57.9 MeV
- Harmonic 2 RF would create at injection would create a harmonic 1 bucket at this energy.

# Bucket area

- The bucket area (in Vs) is given by

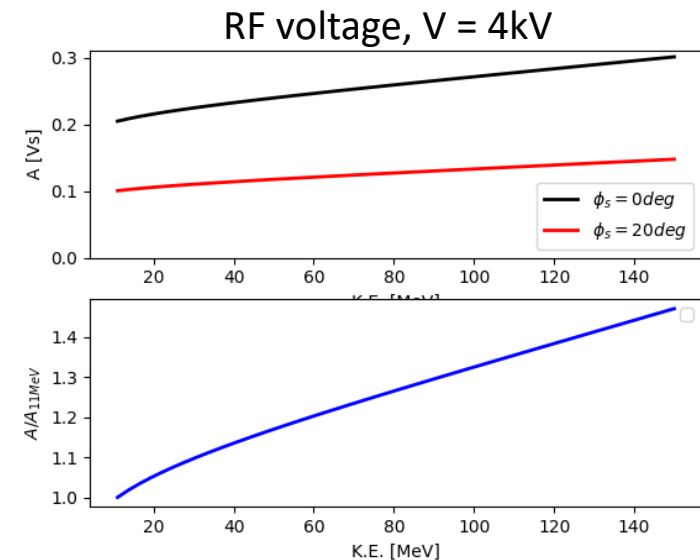
$$A = 16 \sqrt{\frac{\beta^2 E e V}{2\pi\omega_0^2 h |\eta|}} \alpha_b(\phi_s), \text{ where } \alpha_b(\phi_s) = \frac{1 - \sin\phi_s}{1 + \sin\phi_s}$$

- For  $\phi_s = 20^\circ$ ,  $\alpha_b = 0.49$ , i.e. the stationary capture bucket will have twice the area of the moving bucket if the RF voltage is the same.
- Even if the moving bucket is full, it may be possible to capture two beams (assuming the emittance is preserved during debunching and there is no gap between the two coasting beams).

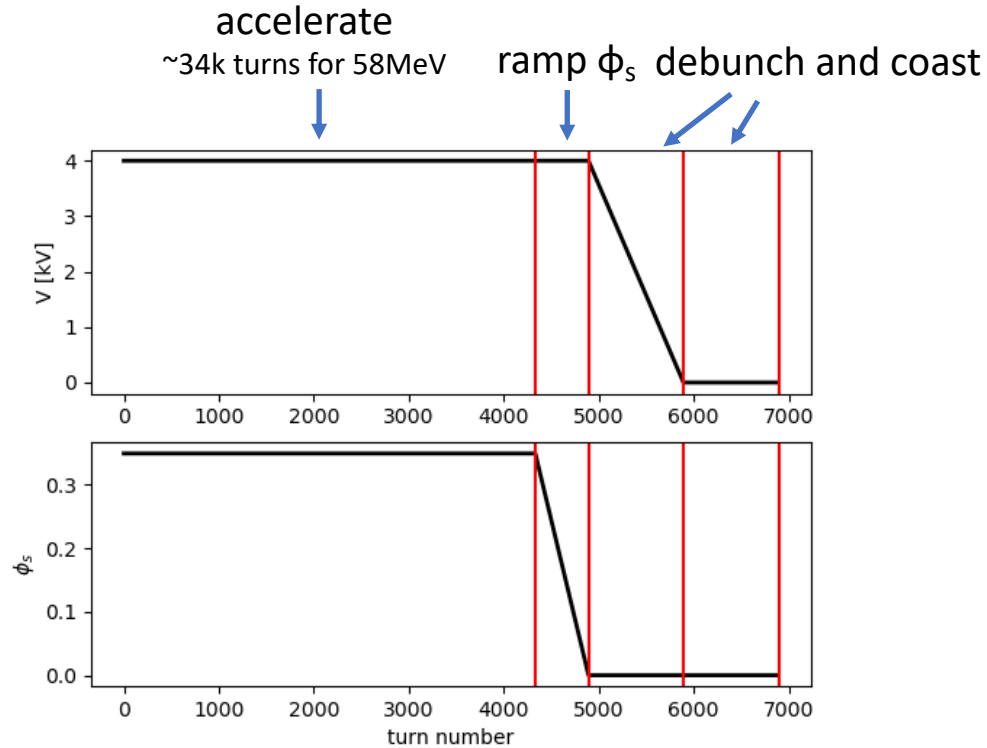
- Note, in a scaling FFA the bucket scales with momentum as

$$A \propto r \sqrt{E\eta} \propto p^{\frac{1}{k+1}} \sqrt{E \left( \frac{1}{k+1} - \frac{1}{\gamma^2} \right)}$$

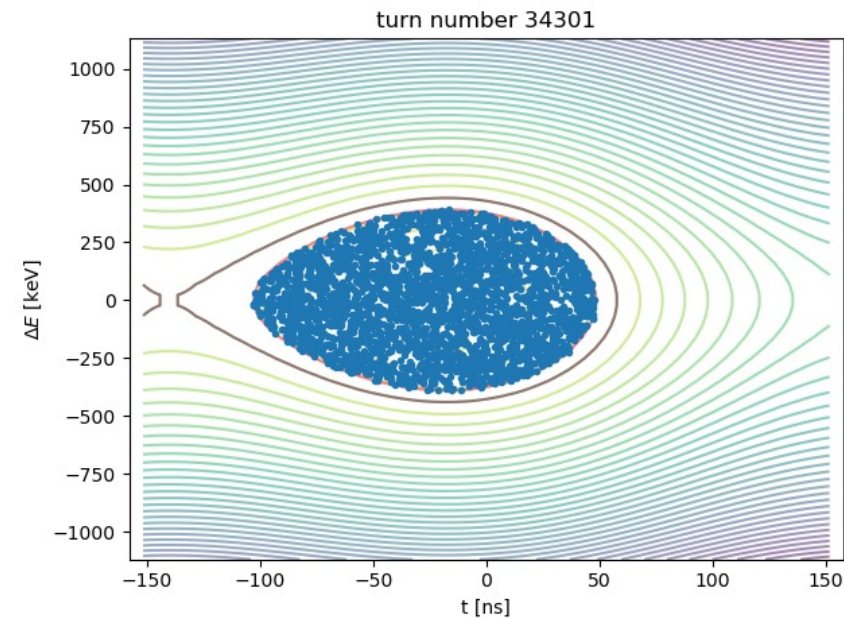
- Keeping all else constant, the bucket area at 150 MeV is 1.47 times the bucket area at 11 MeV (assuming field index  $k = 7.45$ ).



# RF program – constant volts during ramp



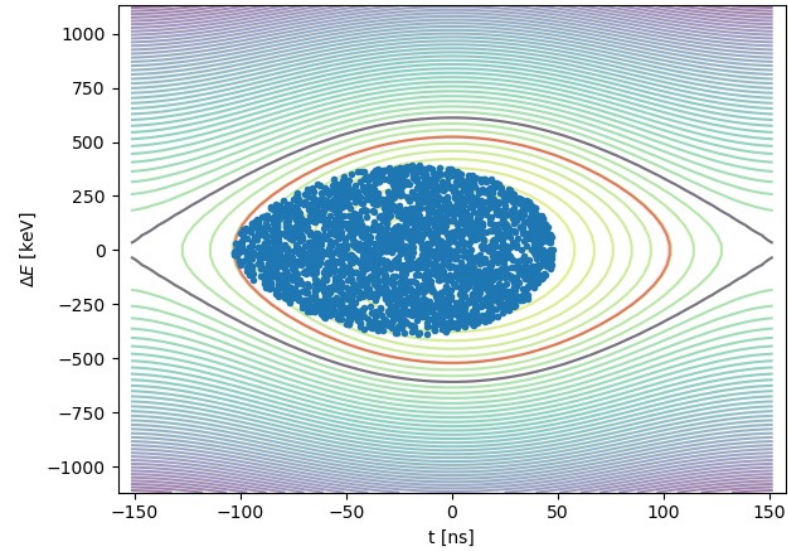
- Maintain constant voltage during  $\phi_s$  ramp
- Synchrotron period before & after  $\phi_s$  ramp is 495 & 481 turns, respectively at 58 MeV.
- Bucket area before & after ramp is 0.12 Vs and 0.246 Vs.



- Distribution just before  $\phi_s$  ramp
- Red contour defines effective 100% emittance
- Contour area is 0.089 Vs

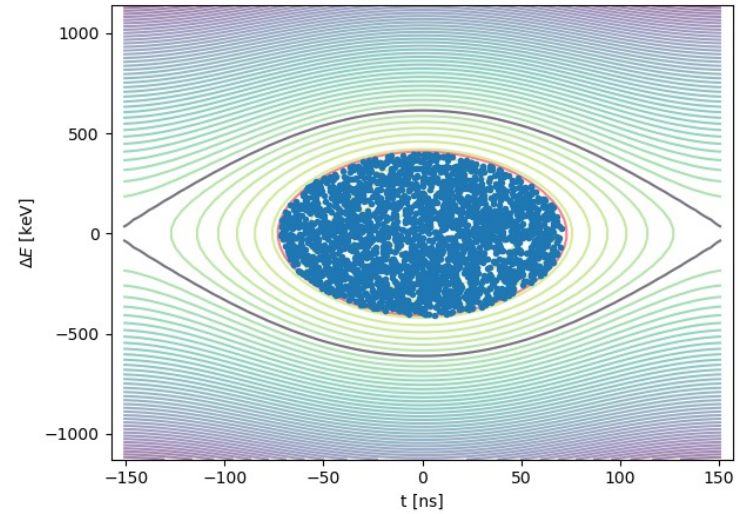
# Ramp turns

turn number 34302



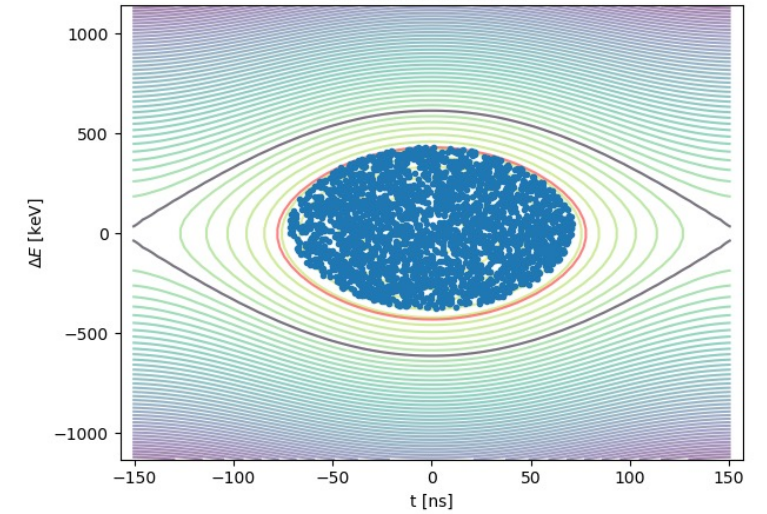
ramp turns = 1  
contour area = 0.158 Vs

turn number 34881



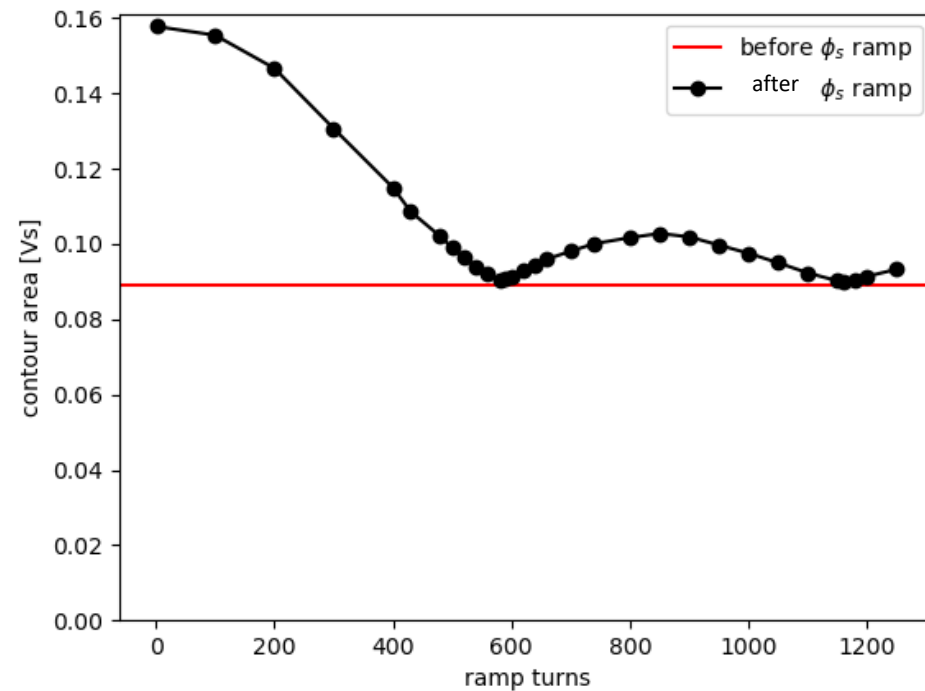
ramp turns = 580  
contour area = 0.091 Vs

turn number 35151



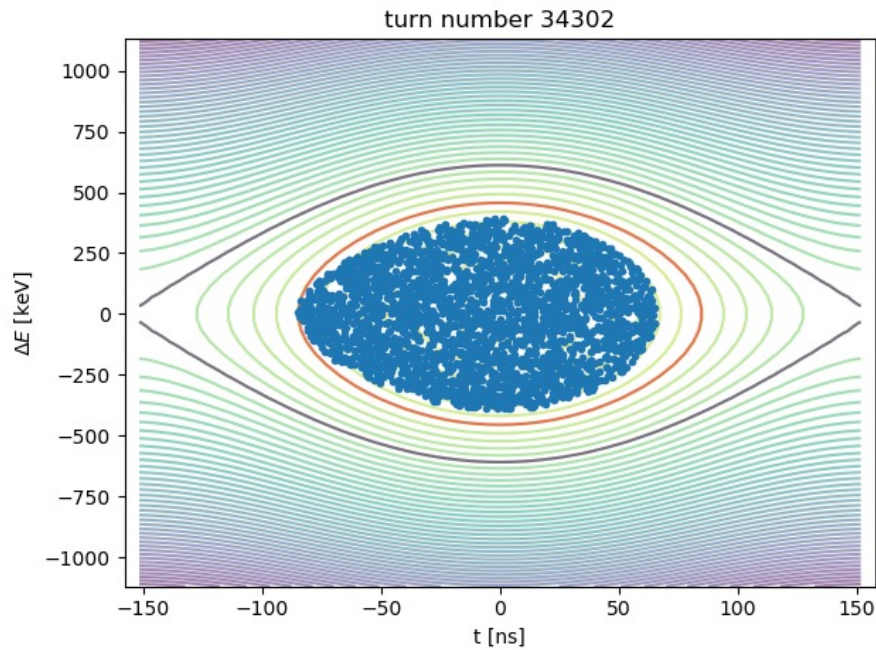
ramp turns = 850  
contour area = 0.103 Vs

# Emittance vs ramp turns

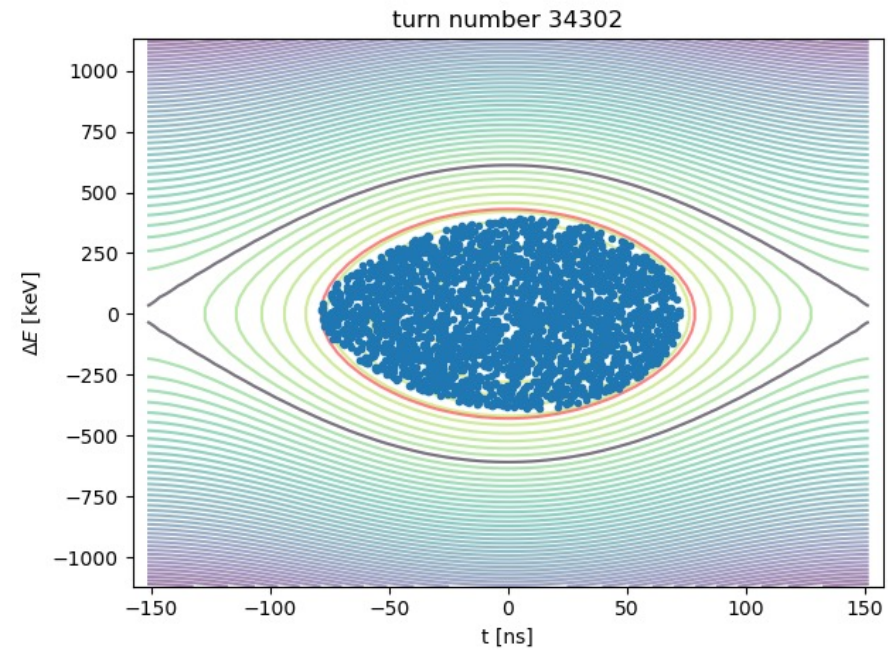


Minima at 580 turns and  $2 \cdot 580$  turns.

# Phase jump in a single turn



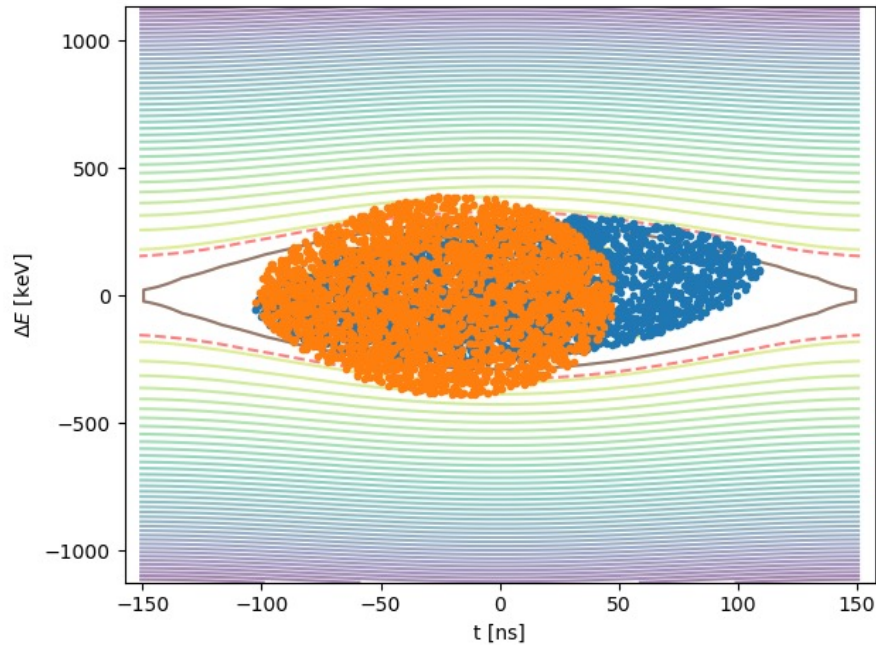
$\phi \rightarrow \phi - \phi_s$   
Contour area: 0.118 Vs



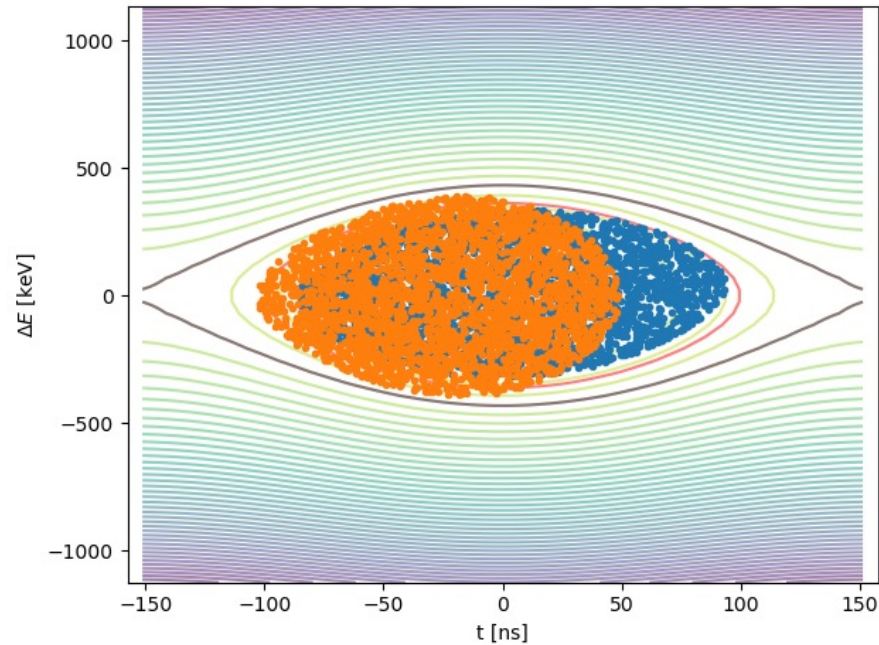
$\phi \rightarrow \phi - \langle \phi_{\text{bunch}} \rangle$   
Contour area: 0.102 Vs

Emittance increase is unavoidable because of mismatch.

# Reduce voltage during $\phi_s$ ramp



Bucket area before & after ramp is 0.12 Vs and 0.116 Vs.

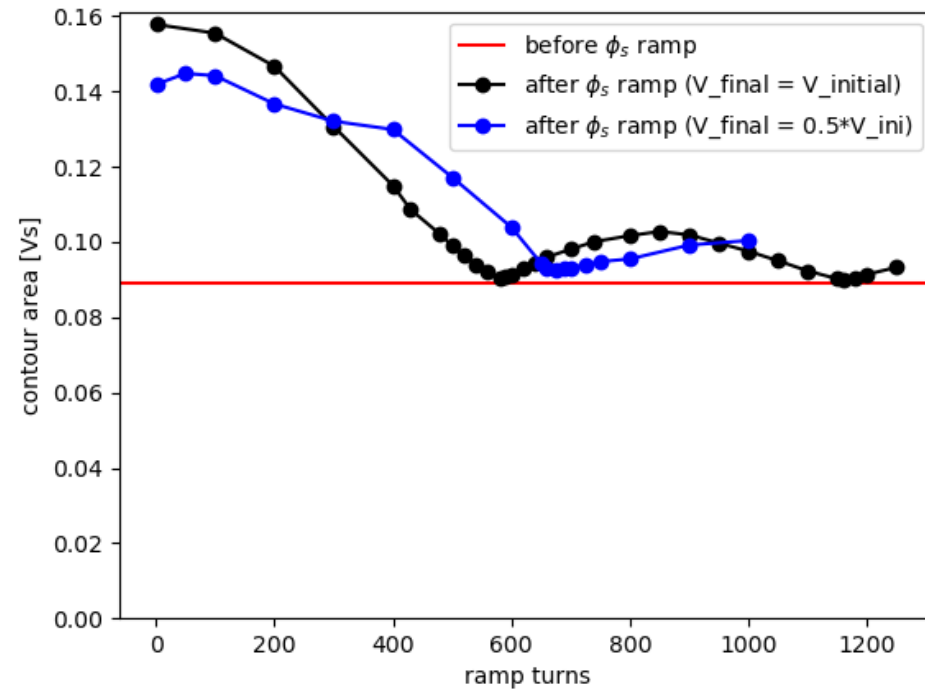


Bucket area before & after ramp is 0.12 Vs and 0.174 Vs.

Distribution before ramp in orange, after ramp in blue.  
Fix ramp turns at 580



# Emittance vs ramp turns



Minimum of blue curve at around 675 turns.