KURNS meeting on 13 Oct. 2022

Longitudinal Schottky Analysis on Coasting Beam

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$\Delta P/P$ Measurement

- A Faraday Cup measures coasting beam size when the beam is accelerated by empty RF bucket.
  - The data includes a beam size information.
- Schottky Analysis gives direct measurement of $\Delta P/P$.

Beam Parameters obtained from Schottky Signal Analysis

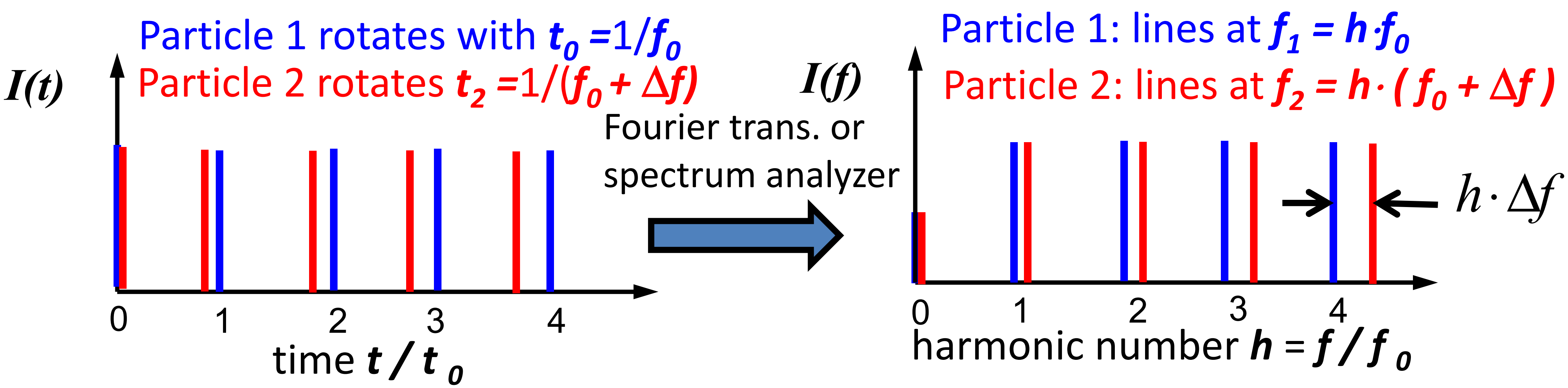
Longitudinal Schottky Spectrum delivers:

➢ Mean revolution frequency f_o, incoherent spread in revolution frequency $\Delta f\ /\ f_0$

$\Rightarrow$ in accelerator physics: mean momentum p_o, momentum spread $\Delta p\ /\ p_0$

*IBIC2017, Tutorial on Beam Measurements using Schottky Signal Analysis:
https://accelconf.web.cern.ch/ibic2017/talks/mo2ab1_talk.pdf

Schottky noise analysis is based on the power spectrum
for consecutive passage of the **same** finite number of particles



Particle 1 of charge ***e*** rotates with $t_1 = 1/f_0$:

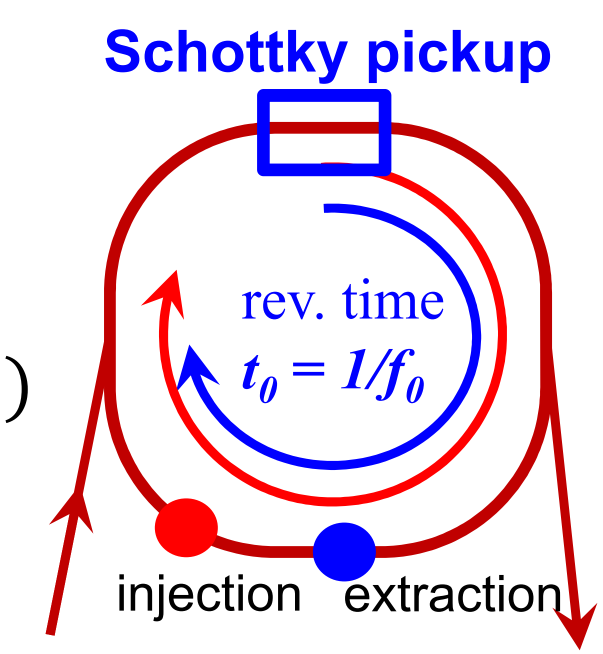
Current at pickup $I_1(t) = ef_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - ht_0)$

$$\Rightarrow I_1(f) = ef_0 + 2ef_0 \cdot \sum_{h=1}^{\infty} \delta(f - hf_0)$$

Particle 2 of charge ***e*** rotating with $t_2 = 1/(f_0 + \Delta f)$:

Current at pick-up $I_2(t) = ef_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - ht_2)$

$$\Rightarrow I_2(f) = ef_0 + 2ef_0 \cdot \sum_{h=1}^{\infty} \delta(f - h \cdot [\, f_0 + \Delta f \,])$$



Important result for 1st step:

- The **entire** information is available around all harmonics
- The distance in frequency domain scales with $h \cdot \Delta f$

Averaging over many particles for a coasting beam:

Assuming ***N* randomly** distributed particles characterized by phase $\theta_1, \theta_2, \theta_3 \dots \theta_N$
with **same** revolution time $t_0 = 1/f_0 \Leftrightarrow$ same revolution frequency f_0

The total beam current is: $I(t) = ef_0 \sum_{n=1}^{N} \cos\theta_n + 2ef_0 \sum_{n=1}^{N} \sum_{h=1}^{\infty} \cos(2\pi f_0 ht + h\theta_n)$

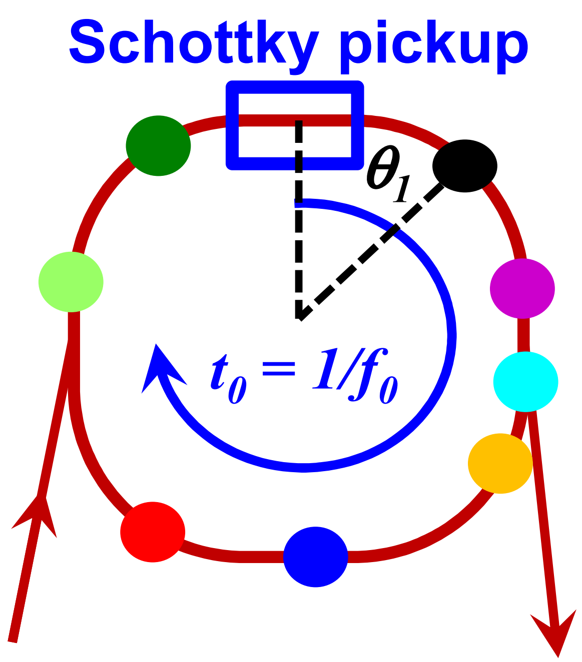
For observations much longer than one turn: average current $\langle I \rangle_h = 0$ for **each** harm. $h \neq 1$
but In a band around **each** harmonics ***h*** the *rms* current $I_{rms}(h) = \sqrt{\langle I^2 \rangle_h}$ remains:

$$\langle I^2 \rangle_h = \left(2ef_0 \sum_{n=1}^{N} \cos(h\theta_n)\right)^2 = (2ef_0)^2 \cdot (\cos h\theta_1 + \cos h\theta_2 + \dots \cos h\theta_N)^2$$

$$\equiv (2ef_0)^2 \cdot N \langle \cos^2 h\theta_i \rangle = (2ef_0)^2 \cdot N \cdot \frac{1}{2} = 2\,\mathrm{e}^2 f_0^2 \cdot N \quad \text{due to the random phases } \theta_n$$

The power at each harm. *h* is: $P_h = Z_t \langle I^2 \rangle_h = 2\, Z_t\, e^2 f_0^2 \cdot N$

measured with a pickup of transfer impedance Z_t



Important result for 2nd step:

- The **integrated** power in each band is constant and $\propto N$

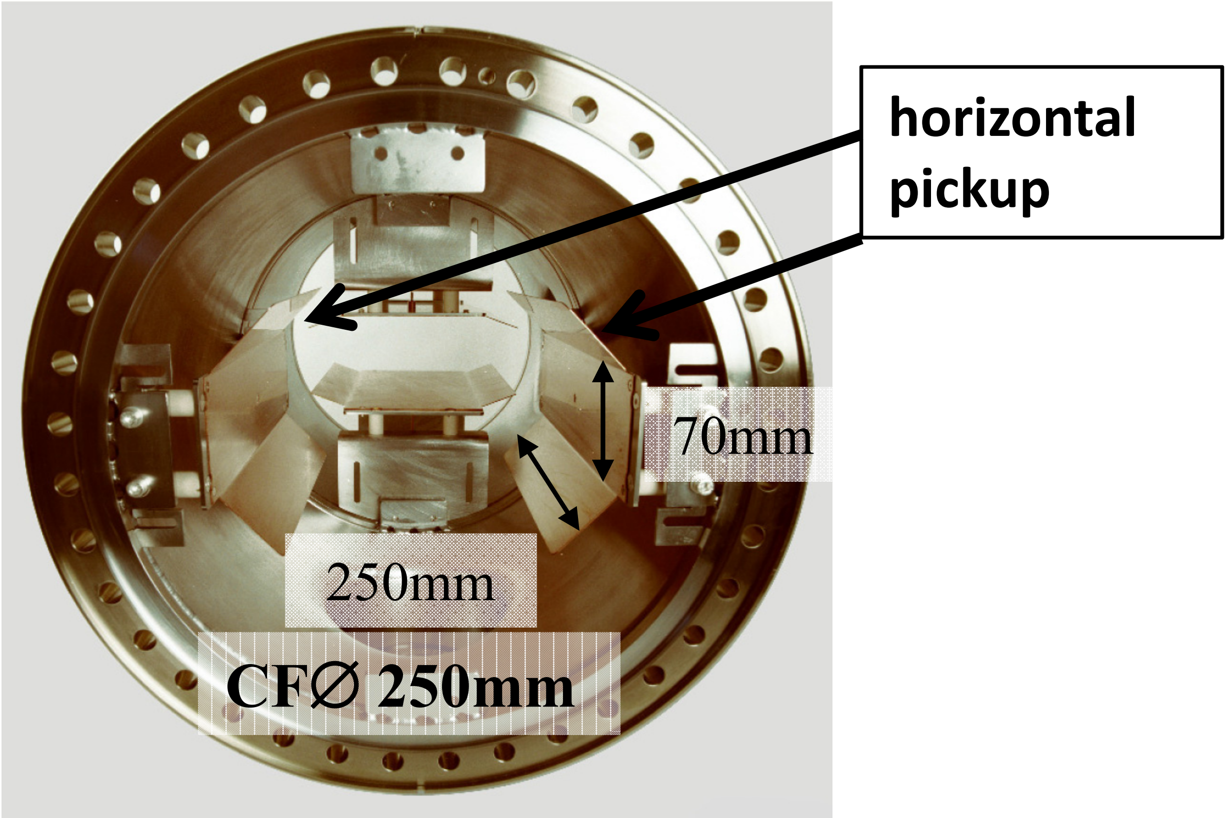
Remark: This random distribution is the connection
to shot noise as described by W. Schottky in 1918

Pickup for Schottky Signals: Capacitive Pickup

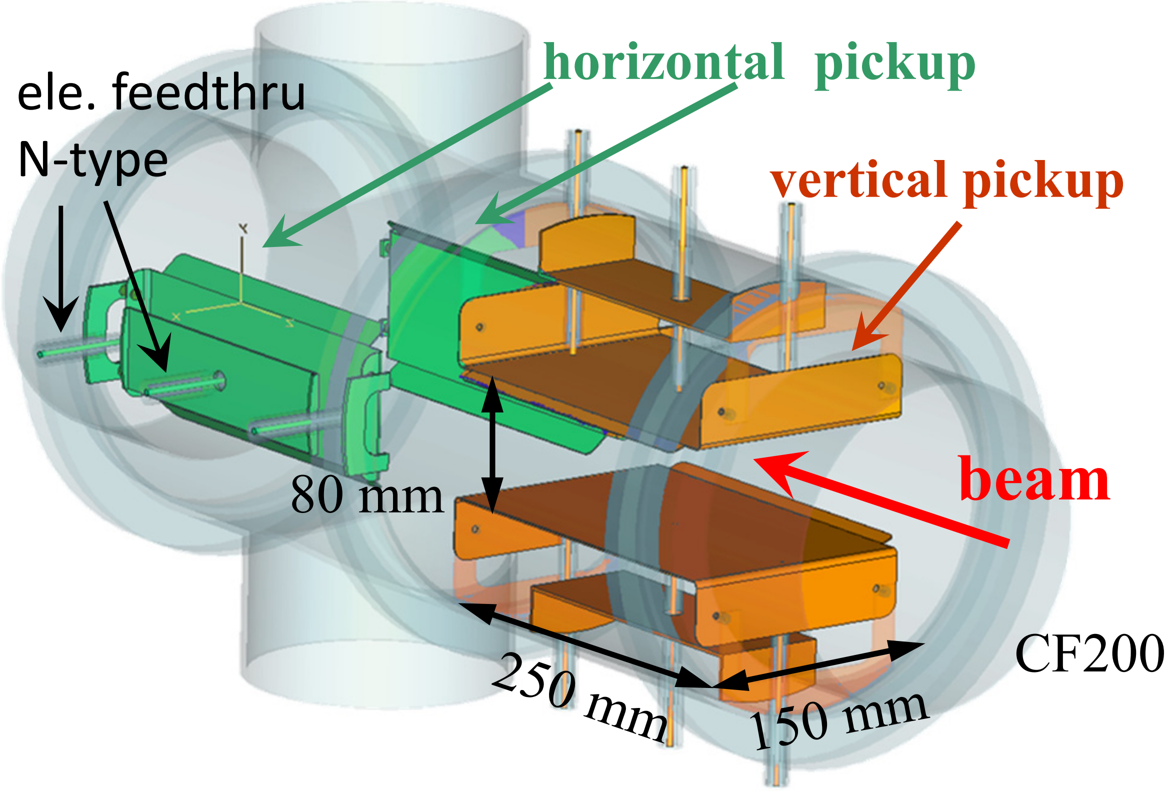
A Schottky pickup are e comparable to a capacitive BPM:

- Typ. 20 to 50 cm insertion length
- high position sensitivity for transverse Schottky
- Allows for broadband processing
- Linearity for position **not** important

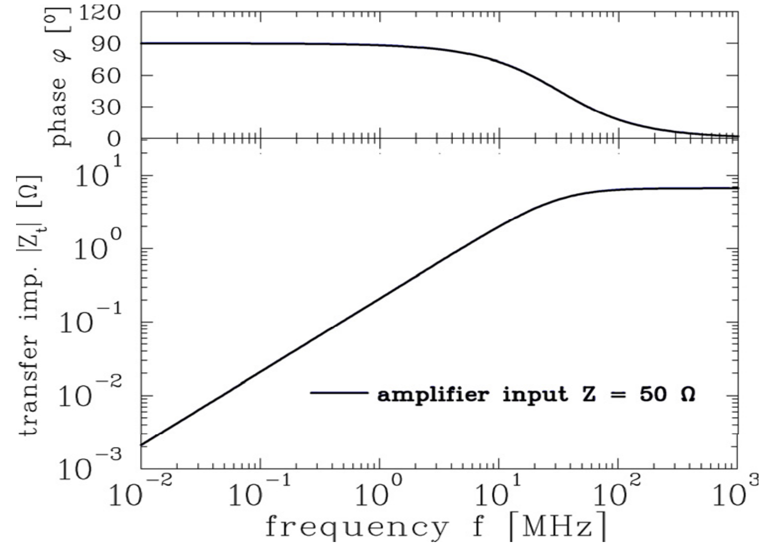
Example: Schottky pickup at GSI synhrotron



Example: Schottky for HIT, Heidelberg operated as capacitive (mostly) or strip-line



Typical transfer impedance



Transfer impedance:

Coupling to beam $U_{signal} = Z_t \cdot I_{beam}$

Typically $Z_t = 1 \ldots 10\ \Omega$, C = 30 ... 100 pF $\Rightarrow f_{cut} \approx 30$ MHz

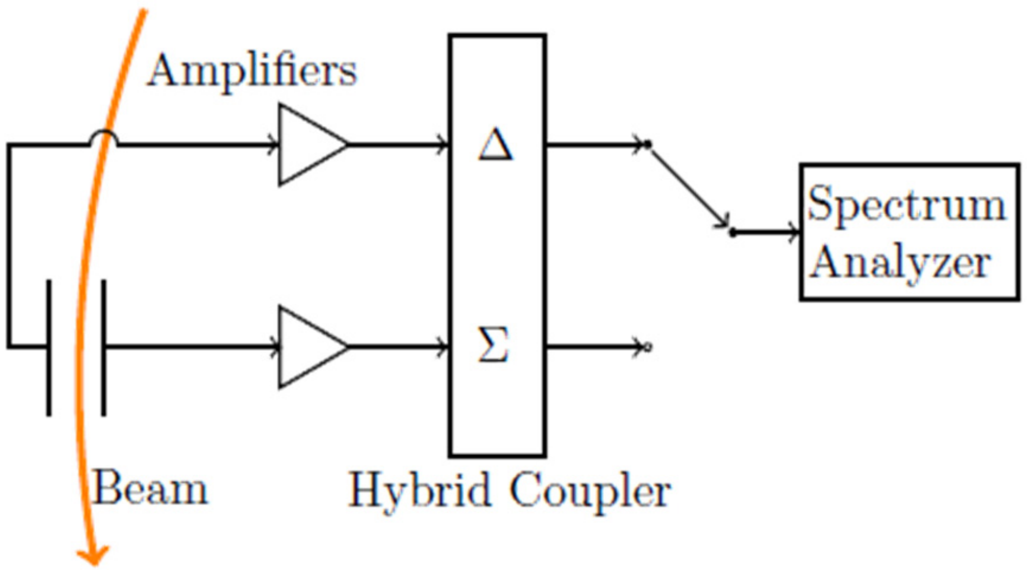
$\Rightarrow$ operation rang $f = 30 \ldots 200$ MHz

i.e. above f_{cut} but below signal distortion ≈ 200 MHz

Electronics for a typical broadband Pickup

Analog signal processing chain:

- Sensitive broadband amplifier
- Hybrid for sum or difference
- Evaluation by spectrum analyzer



Enhancement by external resonant circuit :

- Cable as $\lambda/2$ resonator
- Tunable by capacitive diode
- Typical quality factor Q ≈ 3 ... 10

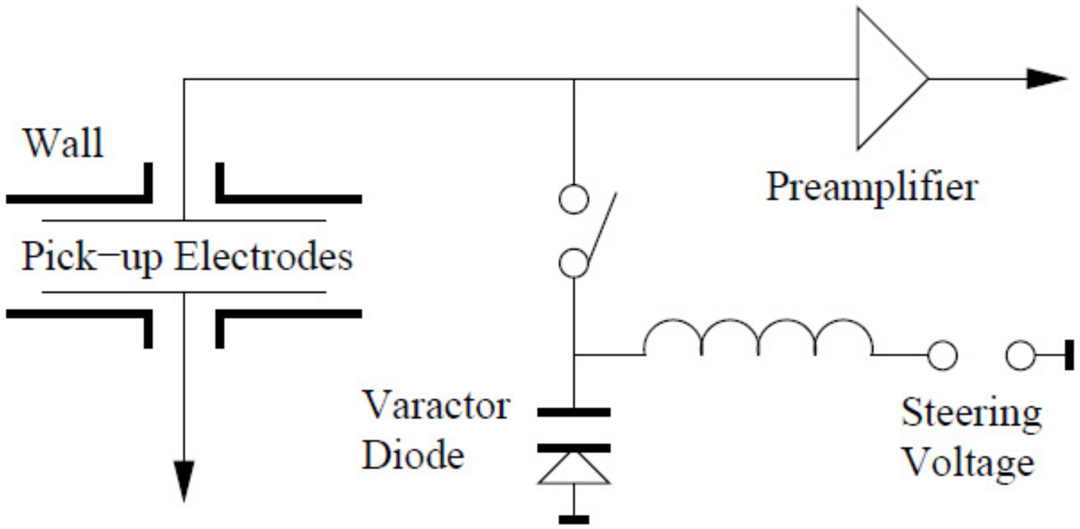
⇒ resonance must be broader than the beam's frequency spread

Challenge for a good design:

- Low noise amplifier required
- For multi stage amplifier chain: prevent for signal saturation

Choice of frequency range:

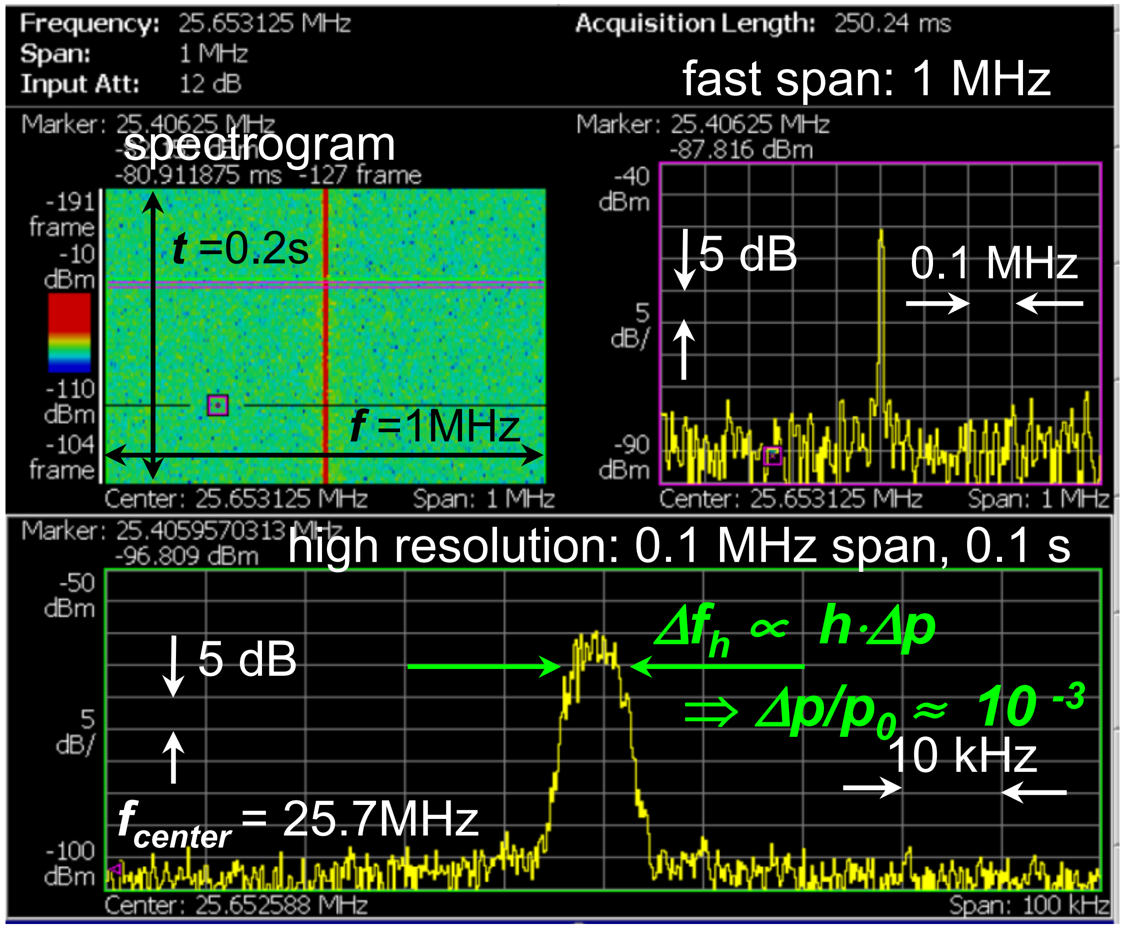
- At maximal pickup transfer impedance
- Lower ***f*** ⇒ higher signal
- Higher ***f*** ⇒ better resolution
- Prevent for overlapping of bands



Example of longitudinal Schottky Analysis for a coasting Beam

Example: **Coasting** beam at GSI synchrotron at injection

E_{kin} = 11.4 MeV/u ⇔ β = 15.5 %, harmonic number h = 119



KURNS hFFA:

$$\frac{\Delta P}{P_0} = -\frac{1}{\eta}\frac{\Delta f_h}{h f_0}$$

$$\Delta f_h = -\eta h f_0 \frac{\Delta P}{P_0}$$

$$= -\left(\frac{1}{k+1} - \frac{1}{\gamma_0^2}\right) h f_0 \frac{\Delta P}{P_0}$$

Assuming that ΔP/P=0.005, f0=2.985MHz and h=1 at 50MeV, Δf1 = 11.7kHz (k=7.5).

Application for coasting beam diagnostics:

- Injection: momentum spread via $\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{h\, f_0}$ as influenced by re-buncher at LINAC
- Injection: matching i.e. f_{center} stable at begin of ramp
- Dynamics during beam manipulation e.g. cooling
- Relative current measurement for low current below the dc-transformer threshold of ≈ 1μA