COD and Dispersion analysis

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A few equations

- Magnetic field, radius and dispersion η in an ideal radial scaling FFAG (ignoring azimuthal variation)

$$B = B_0 \left(\frac{r}{r_0} \right)^k \qquad r = r_0 \left(\frac{p}{p_0} \right)^{\frac{1}{k+1}} \qquad \eta = \frac{r_0}{k+1} \left(\frac{p}{p_0} \right)^{\frac{1}{k+1}} = \frac{r}{k+1}$$

- Closed orbit distortion in the linear approximation caused by a dipole kick θ_j . Measured closed orbit the sum of the ideal closed orbit and COD.

$$r_i^{COD} = R_{ij}\theta_j, \quad R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi q)} \cos\left(\left| \psi_i - \psi_j \right| - \pi q \right)$$

$$r_i^{CO} = r_i + r_i^{COD}$$

COD versus momentum

- To maintain fixed tunes, the beta function (and response R_{ij}) and the radius must grow at the same rate with momentum.

$$\beta_{i,j} \sim \left(\frac{p}{p_0}\right)^{\frac{1}{k+1}} \Rightarrow R_{ij} \sim \left(\frac{p}{p_0}\right)^{\frac{1}{k+1}}$$

- The COD for two cases follow

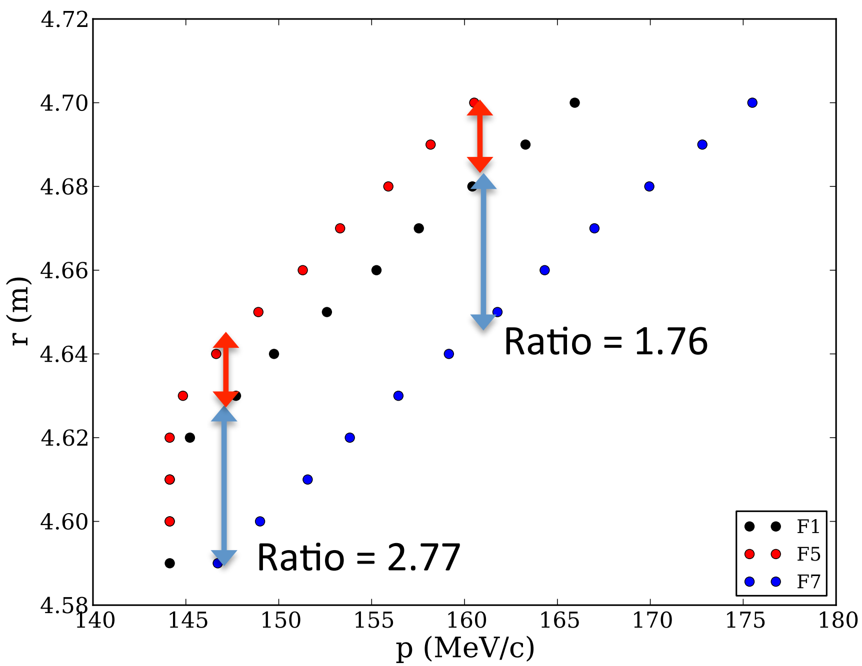
$$\dot{\theta} = 0, \quad r_i^{COD} \propto \left(\frac{p}{p_0}\right)^{\frac{1}{k+1}}, \quad \eta_i^{COD} = \frac{r_i^{COD}}{k+1}$$

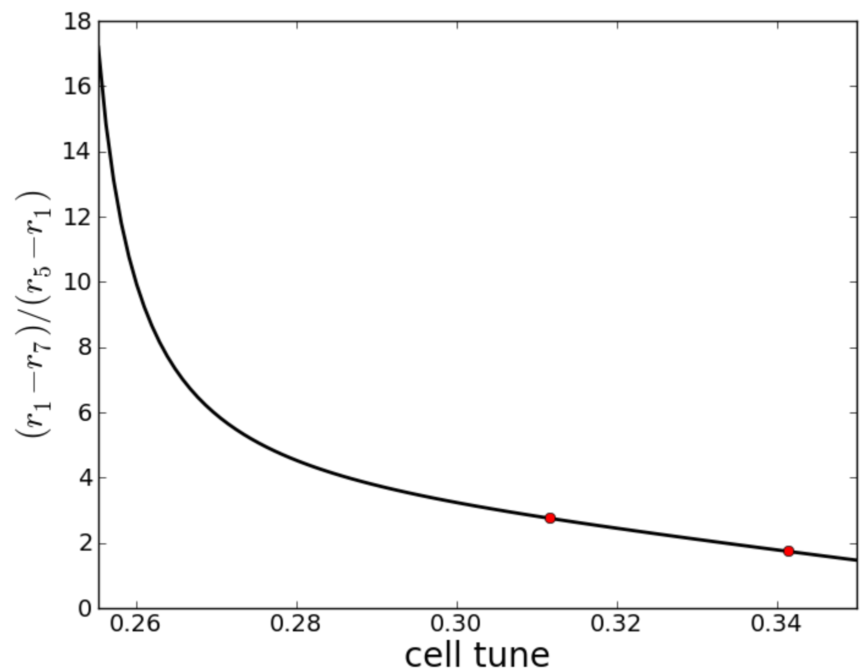
$$\theta \propto \frac{1}{p}, \quad r_i^{COD} \propto \left(\frac{p}{p_0}\right)^{-\frac{k}{k+1}}, \quad \eta_i^{COD} = -\frac{k}{k+1} r_i^{COD}$$

The data

- “QinBin” data involved moving probe to a radial position and using the bunch monitor to find the time taken for the beam to be lost.
- Time translated to momentum using Uesugi-san’s conversion file.
- Repeating for various radial positions provides a measure of the closed orbit versus momentum at the three probe locations.
- At the injection momentum it was found that the major source of COD is in the vicinity of the cavity.

Single dipole kick scenario - Inferring tune from closed orbit data

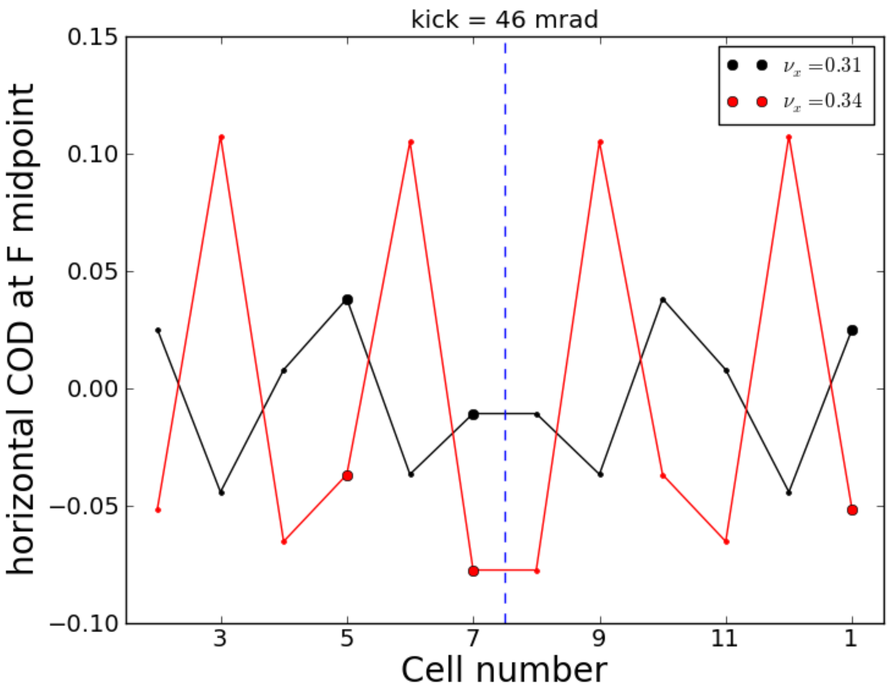


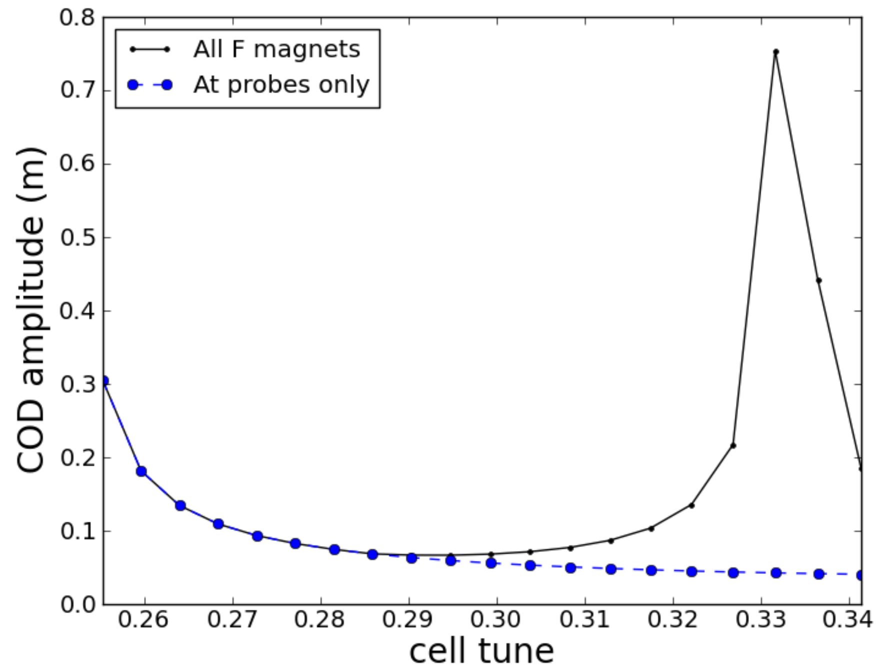


- Assuming a single error source and lattice symmetry, the ratio of closed orbit differences depends on the tune only.

$$\frac{r_1 - r_7}{r_5 - r_1} = \frac{\cos\left(\pi \nu_x \left(11 - nc\right)\right) - \cos\left(\pi \nu_x \left(1 - nc\right)\right)}{\cos\left(\pi \nu_x \left(5 - nc\right)\right) - \cos\left(\pi \nu_x \left(11 - nc\right)\right)}$$

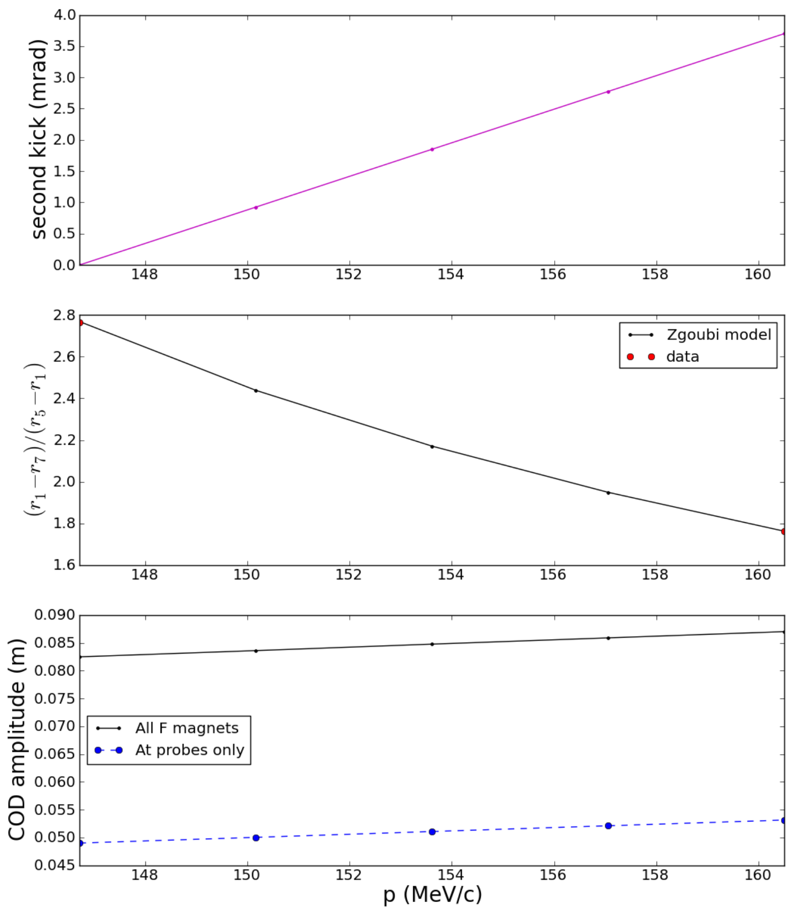
Single dipole kick scenario – Variation of COD amplitude with tune

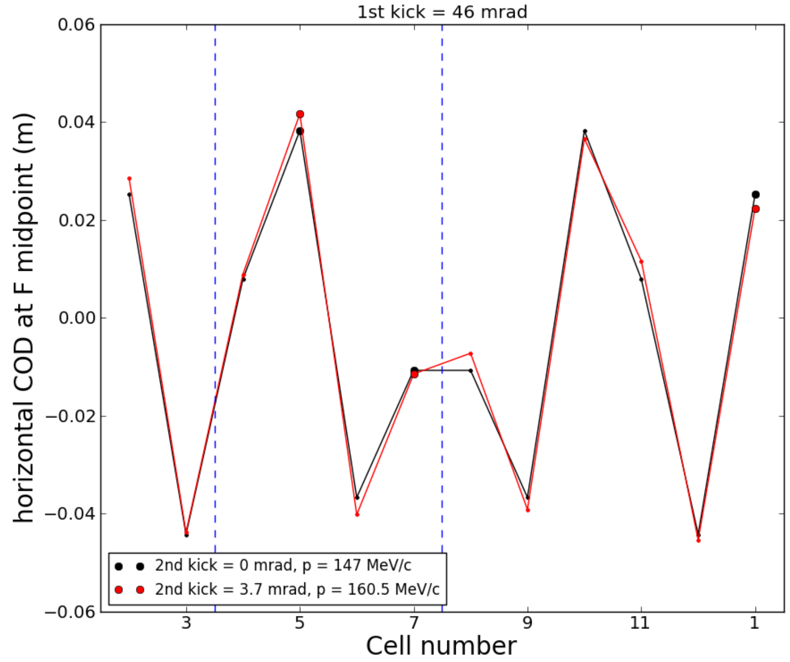




- COD obtained from Zgoubi analytic model zero amplitude optics, adjust k to set horizontal tune.
- COD measured at probes appears to fall as tune increases from 0.31 to 0.34. This is despite the increase in overall COD.
- However, the data indicates the COD between probes increases.

Two dipole kicks scenario

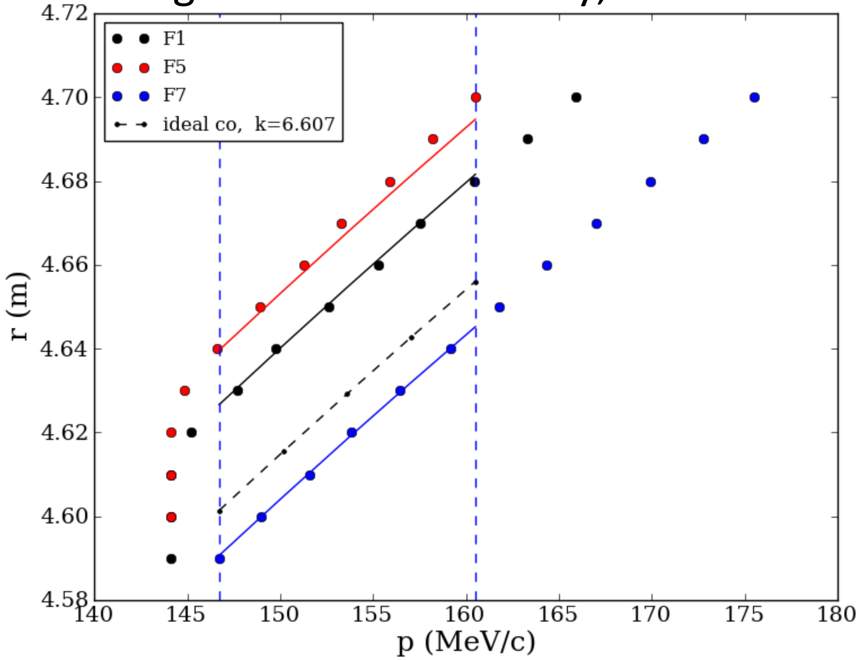




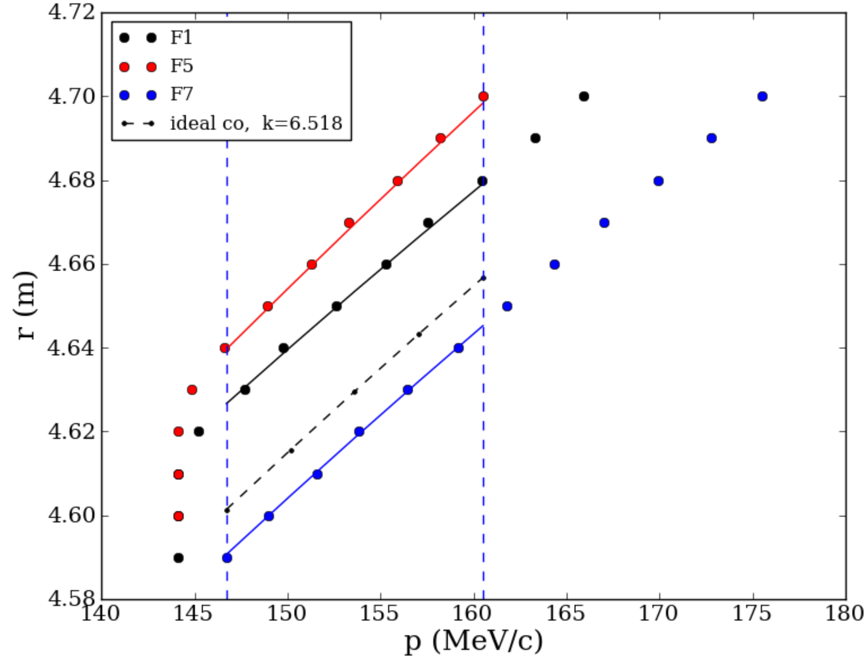
- Fix cell tune over momentum range at 0.311
- Fix dipole kick at cavity at 46 mrad.
- Introduce second dipole kick that increases linearly with momentum.
- Obtain reasonable agreement with both the observed change in ratio and increase in amplitude.

Ideal closed orbit predicted by model

Single fixed kick at cavity, fixed tune

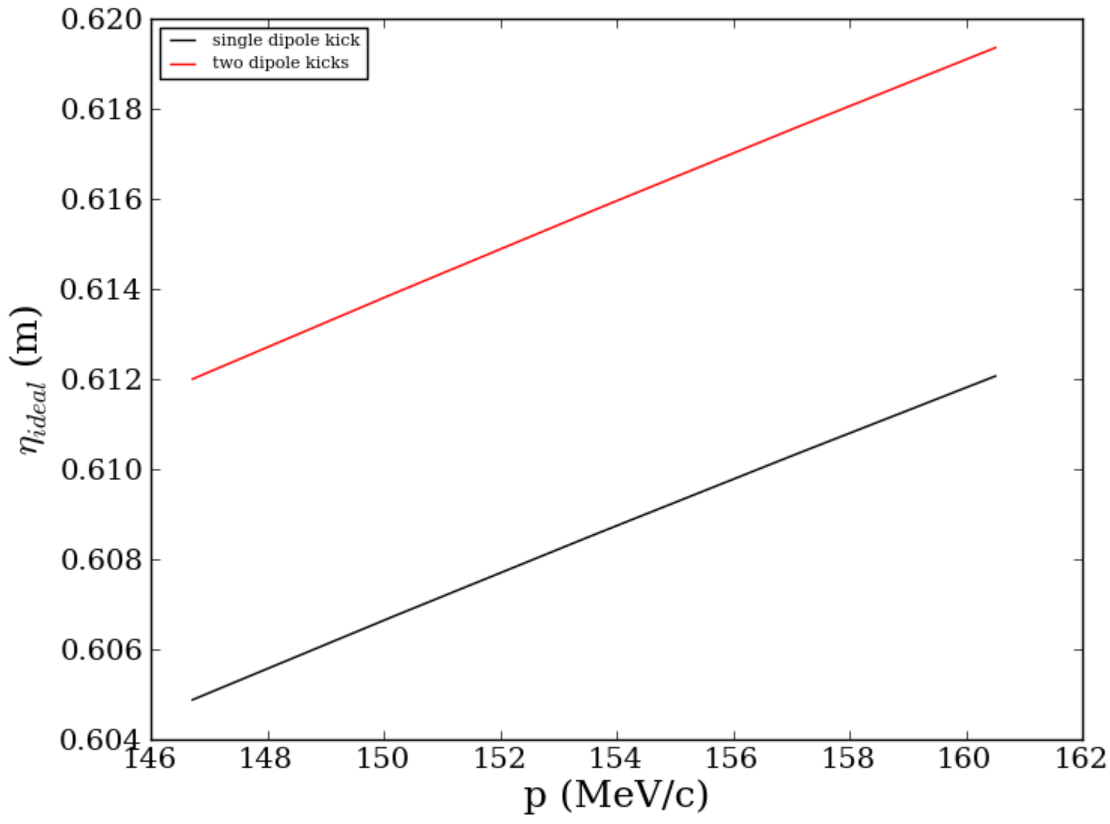


Additional linearly increasing kick



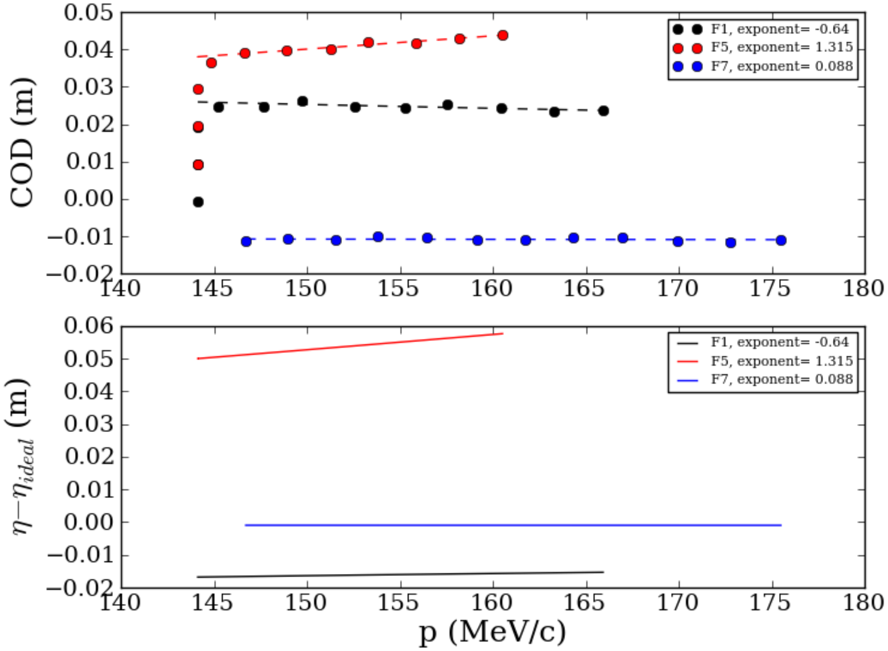
- Add COD produced by model to F7 radius data.
- Power law fit through COD zero point at each momentum to find scaling factor k.
- Note the scaling factor differs from that assumed by the model (k=7.45 in the Zgoubi analytic model).

Ideal dispersion predicted by model

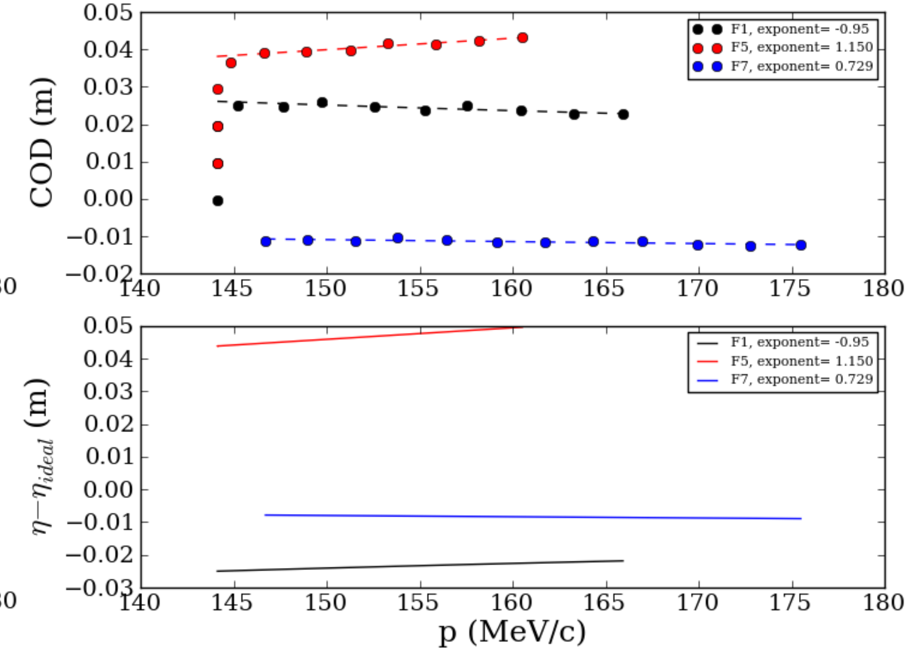


COD and dispersion distortion

Single fixed kick at cavity, fixed tune

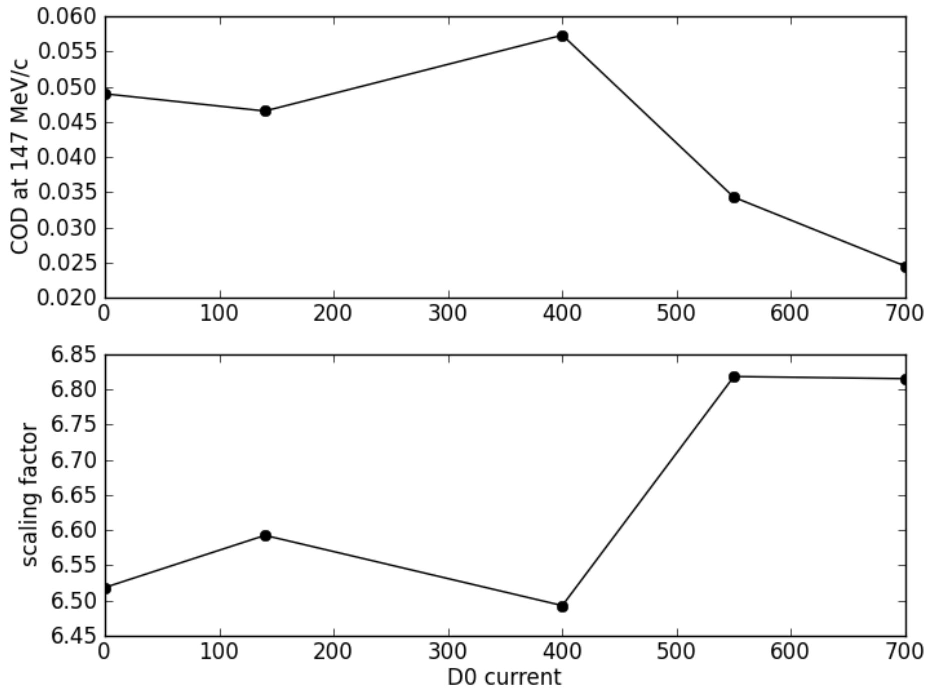


Additional linearly increasing kick



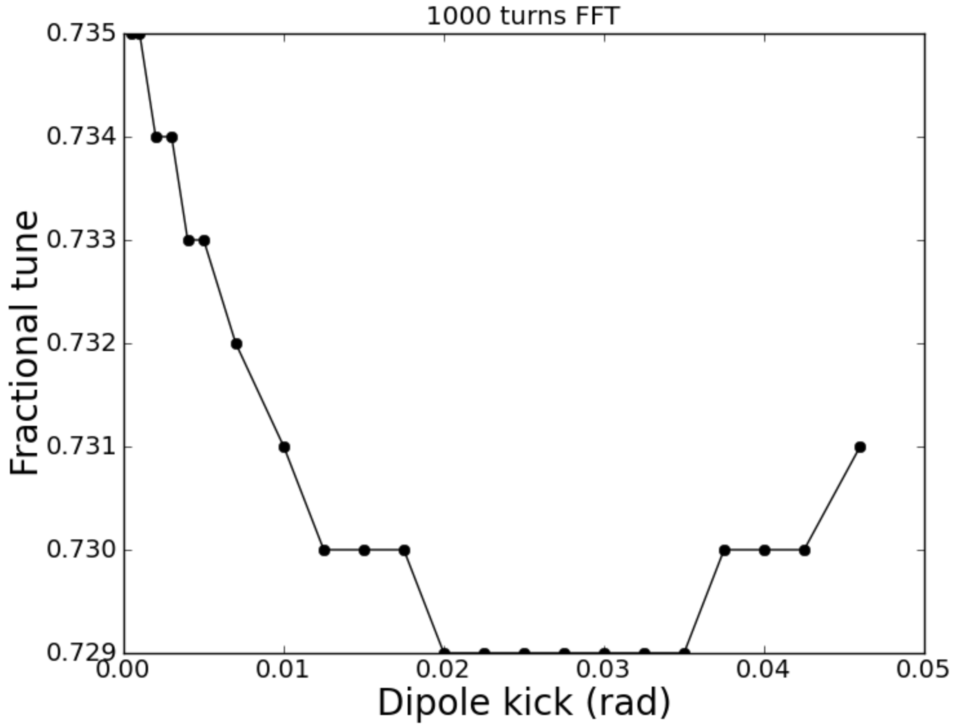
- Subtract ideal closed orbit from data to find COD
- Powerlaw fit to COD data, r_{COD} = a*p^b.
- The dispersion distortion follows from the fit, η-η_{ideal} = a*b*p^b

D0 current



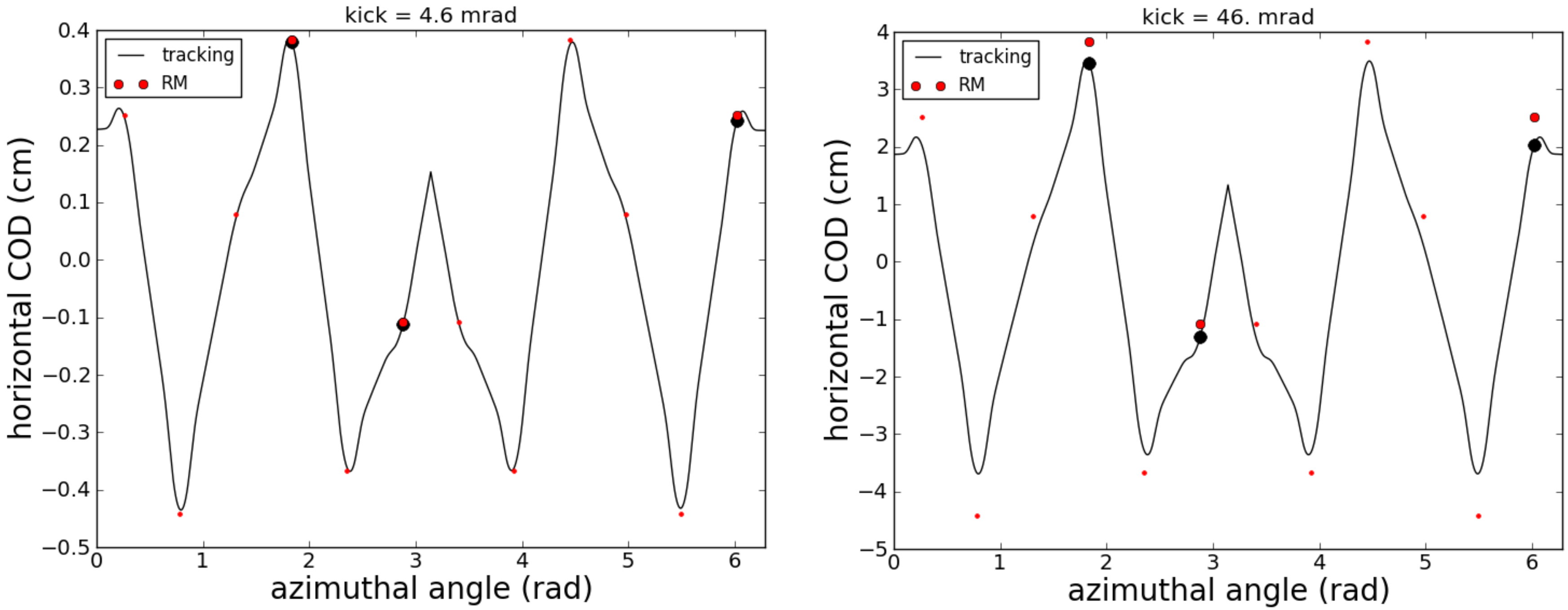
- Apply analysis to data with non-zero D0 coil current. The tune is assumed to be unchanged, two dipole kicks included as before. Scale kick at cavity to match measured COD at 147 MeV/c.
- COD seems to decrease with D0 current, except at 400 A
- Scaling factor found by fitting model to data increases with momentum (apart from 400A). This result implies the tune increases with D0 current.

Detuning with COD amplitude



- FFT of turn-by-turn tracking (using analytic model) shows shift in tune with amplitude.
- It might be worth measuring tune at various D0 currents, though the resolution of results may be insufficient to detect any variation.

Model COD: Tracking versus RM



- The COD was calculated using the response matrix obtained from low amplitude optics .
- This increasingly diverges from tracking result as amplitude is increased, probably due to the contribution of nonlinear components.

Future Work

- Calculate model COD by tracking rather than using RM.
- Track field map model to see if scaling factor agrees with measurements.
- If possible, model D0 coil more accurately to see if its excitation might have any effect on tune.
- Check if the second error source ideal holds up to further scrutiny.