# Beam size and profile calculation from bunch monitor data

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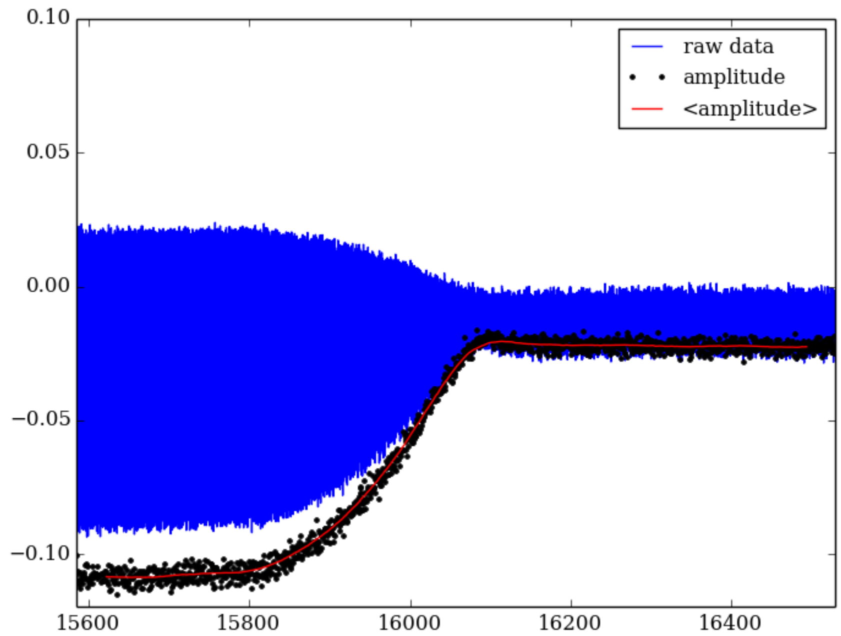
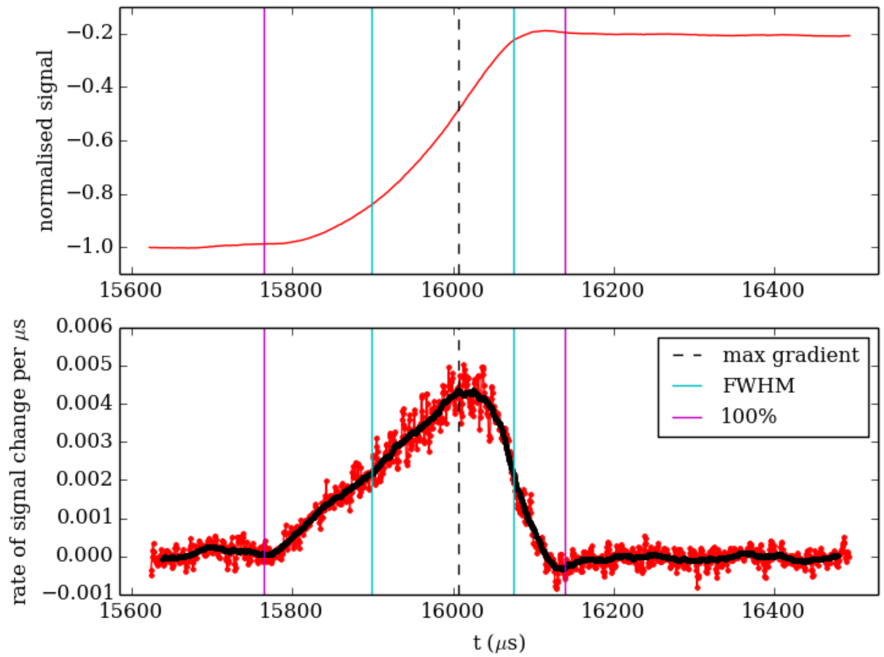
# Signal amplitude



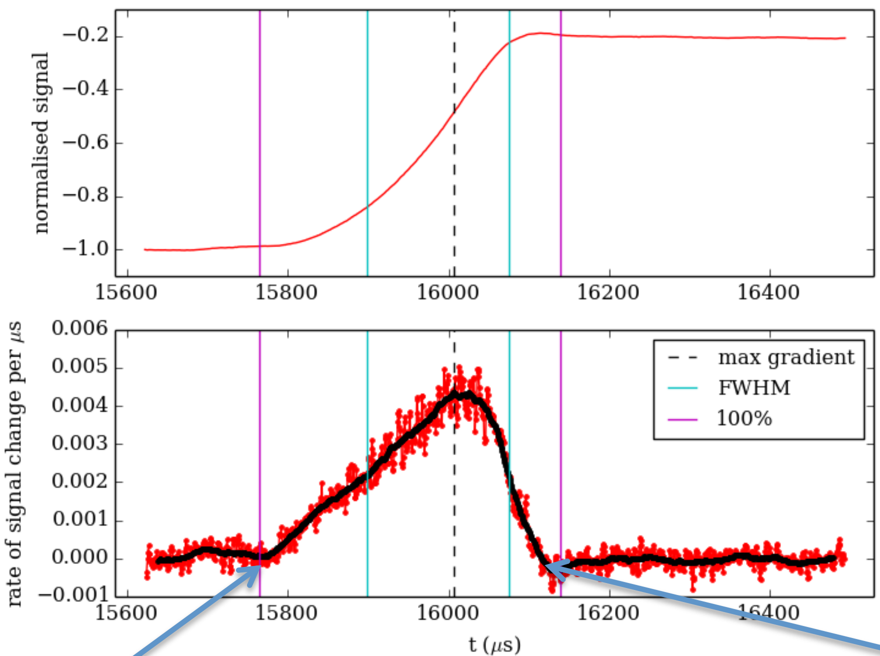

- Start with the bunch monitor data from the scope. Find the time range where the signal drops off (blue).
- In each time interval of about a revolution period duration, subtract the maximum from the minimum to get an amplitude (black points).
- Perform moving window average calculated over 20 consecutive points to smooth data (red curve).

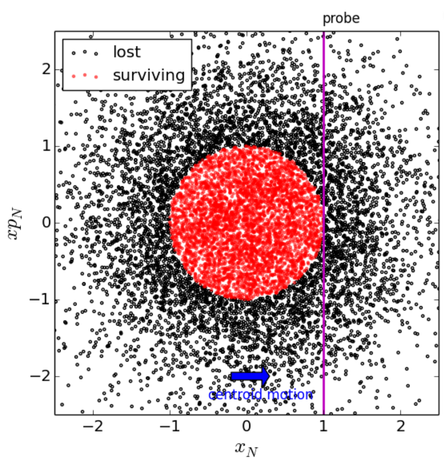
# Signal derivative

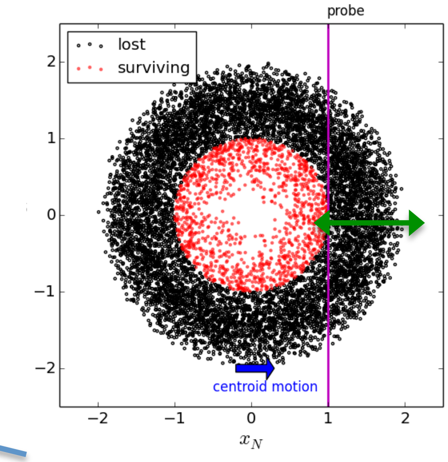



- Find derivative of normalised smoothed signal data (red points, lower figure), then window average a second time (black curve).
- Measure beam duration using the derivative of the data. Figure above shows the FWHM of the signal derivative (cyan) and the time where the signal drops to zero on either side of the maximum (magenta).

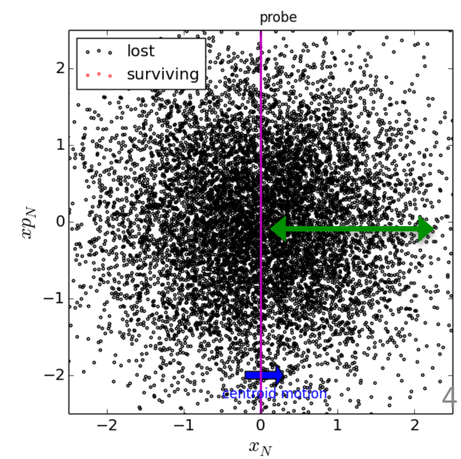
# Bunch crossing model




$$\Delta x = \int_{p(t_1)}^{p(t_2)} \frac{\eta(p(t))}{p(t)} dp$$



# Conversion to distance algorithm

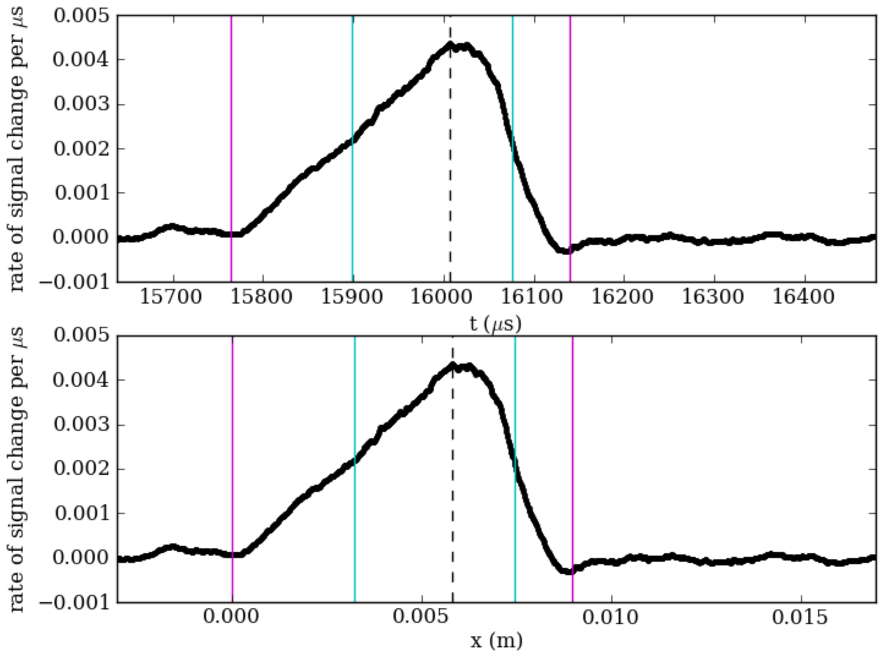
- Use table by Uesugi-san to convert time to momentum
- Assume we know dispersion $\eta_0$ at some reference momentum $p_0$. Assume we also know the scaling index k and assume it is fixed over the momentum range. In addition, assume the beam size doesn't change during the signal fall off.
- For every time and time increment Δt in bunch monitor data
    - ➢ Find p and Δp from lookup table.
    - ➢ Calculate dispersion at momentum p

$$\eta = \eta_0 \left( \frac{p}{p_0} \right)^{\frac{1}{k+1}}$$

    - ➢ Calculate closed orbit shift, $\Delta r = \eta \Delta p / p$
    - ➢ Repeat at every time step to incrementally find total closed orbit shift over time range of interest

$$r_n = \sum_{i=1}^{n} \Delta r_i$$

# Conversion to distance example

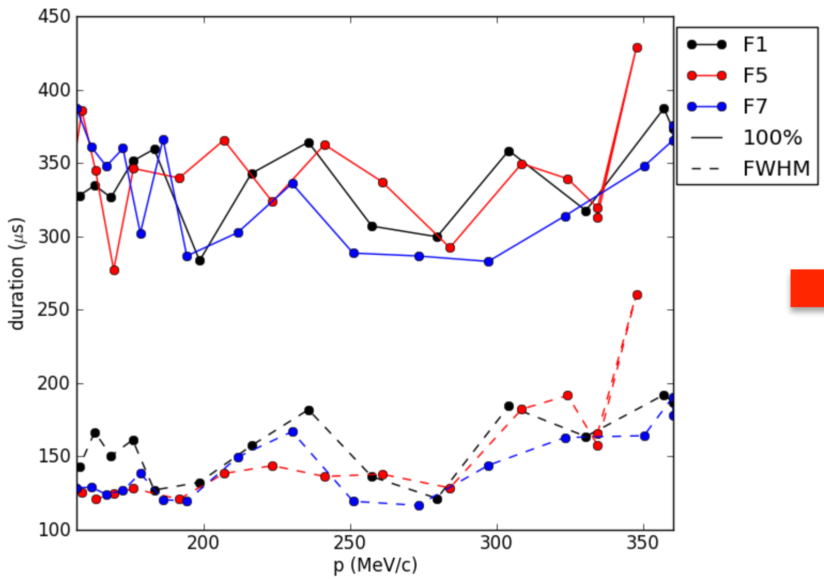


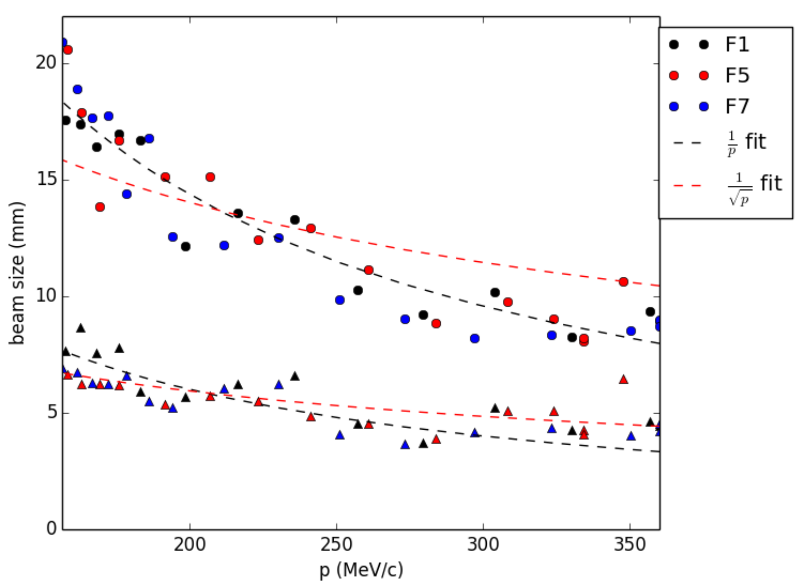

- Shape of signal derivative looks the same when plotted against distance rather than time
- Momentum varies linearly with time in this case.
- Grid spacing ratio between two points is given by

$$\frac{\Delta x_2}{\Delta x_1} = \frac{\eta_2}{\eta_1}\frac{p_1}{p_2}\frac{\Delta t_2}{\Delta t_1} = \frac{\eta_2}{\eta_1}\frac{p_1}{p_2}$$

- Ratio reaches 1-3% over the signal fall-off time range, in the cases looked at.

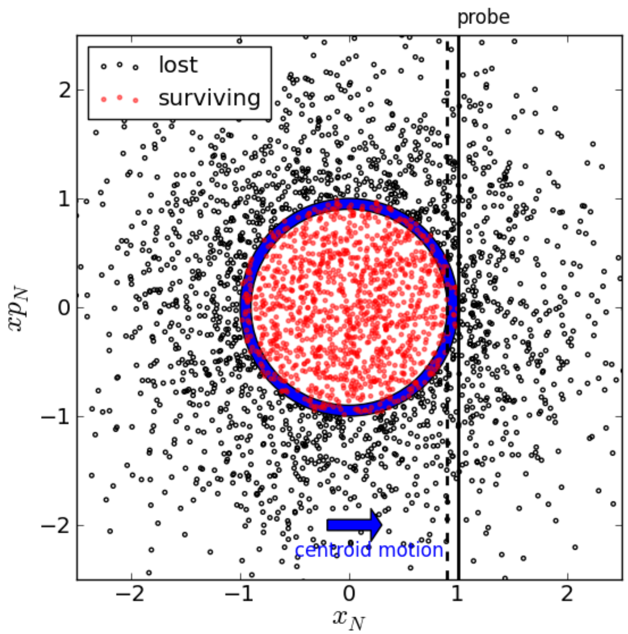
# Signal fall-off duration and beam radius




- No clear trend in variation of signal fall off duration with momentum.
- The resulting beam size tends to decline with momentum. The 100% beam size appears to decline with 1/p, i.e. faster than adiabatic damping.

# Beam distribution model




- The entire beam moves across the probe on a much slower time scale (~500 turns) than betatron oscillations (~3 turns).
- The signal drop off at any time, is proportional to the number of particles outside some amplitude (given by the distance from the probe to the beam centroid).
- It follows that the derivative of the bunch monitor signal represents the number of particles within a ring in phase space. In effect we are measuring the phase averaged distribution as a function of amplitude.
- In normalised phase space coordinates

$$f(r) = \int_r \oint f(r,\theta)\, d\theta\, dr = 2\pi r f\left(r,\theta\right)\Delta r$$

- Note, this picture ignores the "hollow beam" case where there is some region with no particles (or an undetectable number) in the centre of the distribution.

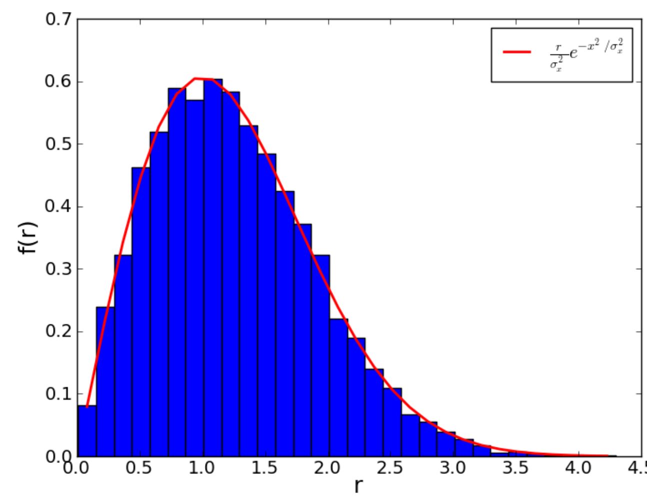
# Gaussian distribution

- Assuming uncorrelated distributions in x,x'.

$$f(x,x') = \frac{1}{2\pi\sigma_x\sigma_{x'}} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{x'^2}{\sigma_{x'}^2}\right)}$$

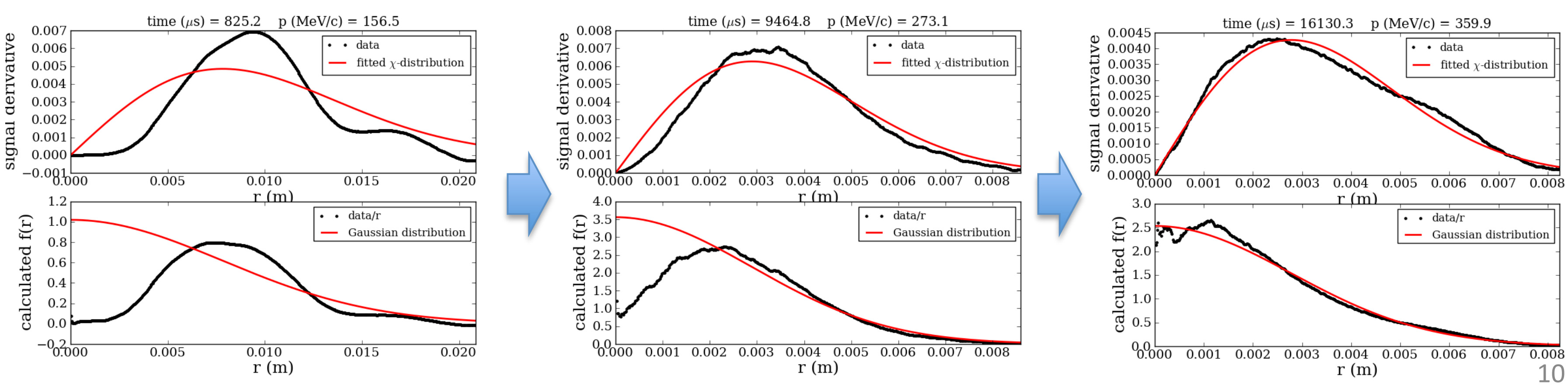
- The distribution of amplitudes $r=\sqrt{x^2+x'^2}$ is given by a chi distribution of order 2. Assuming $\sigma_x=\sigma_{x'}$ the distribution becomes

$$f(r) = \frac{r}{\sigma_x^{\ 2}} e^{-\frac{r^2}{2\sigma_x^2}}$$



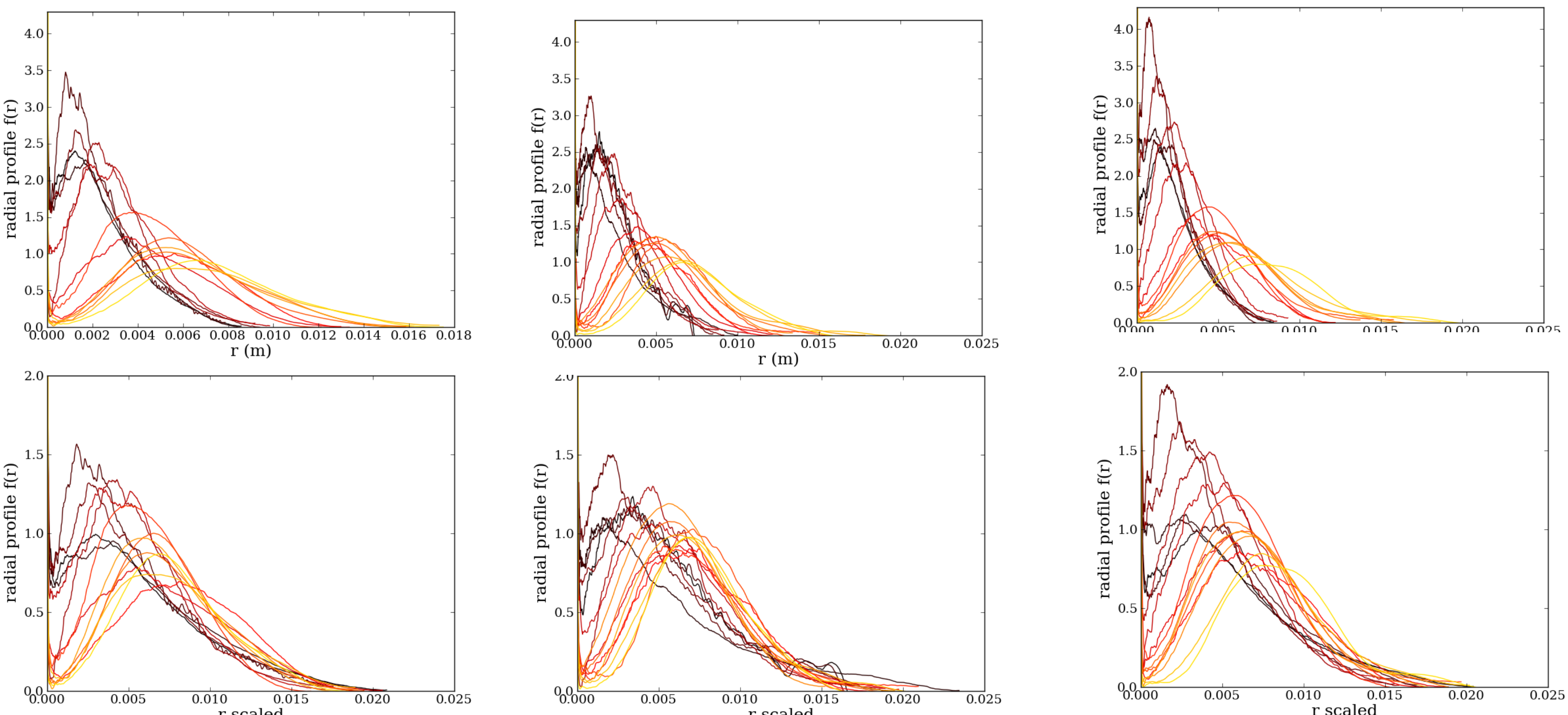
# Radial profile calculation (1)

- The origin below corresponds to the the beam centroid (closed orbit), i.e one is moving outwards with increasing x.
- The above statement relies on the assumption that there is no zero charge density core which cannot be detected.
- The radial profile f(r)) is found by dividing the signal derivative by the r coordinate (black dots in lower subfigures). This sometimes results in spurious large values near the origin.
- Fit signal derivative with chi-distribution (red line). Dividing this distribution by the r coordinate yields a Gaussian profile that can be compared with the data.
- The results indicates an initially hollow radial profile which begins to fill in after some momentum until it eventually approaches a Gaussian-like profile.



# Profile evolution (1)

- Similar profile evolution seen in all three probes (momentum increases light yellow to dark red).
- For clarity, scale r-axis by momentum to conserve beam size (lower figures)




F1 F5 F7

# Hollow beam profile

- Since the beam distribution may have a hollow core of unknown extent, an arbitrary offset can be added to the radius data.
- This offset acts to depress the distribution, particularly for low radii.
- In the example below a 5mm offset is added. If the arbitrary offset is unchanged with momentum, then the trend for increasing charge density close to the origin persists.