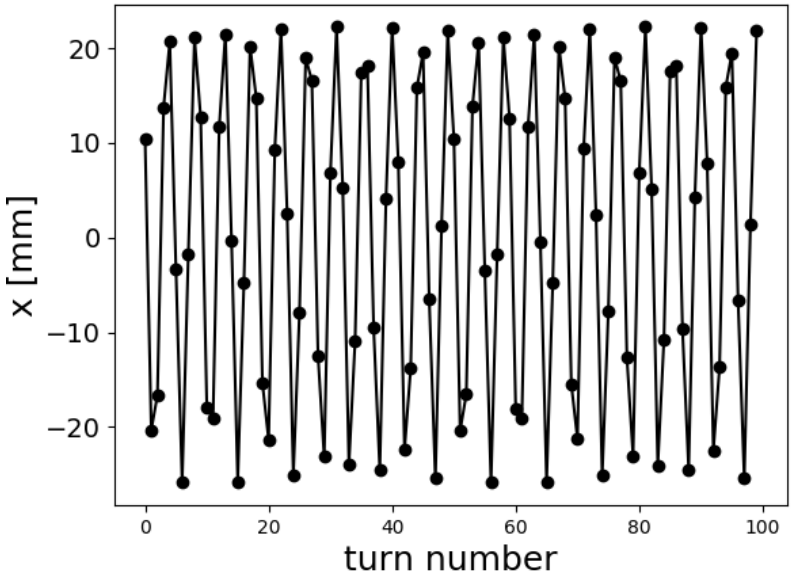
Frequency spectrum of coordinate data

David Kelliher

Goal

- Investigate what information can be extracted from the coordinate data using FFT.

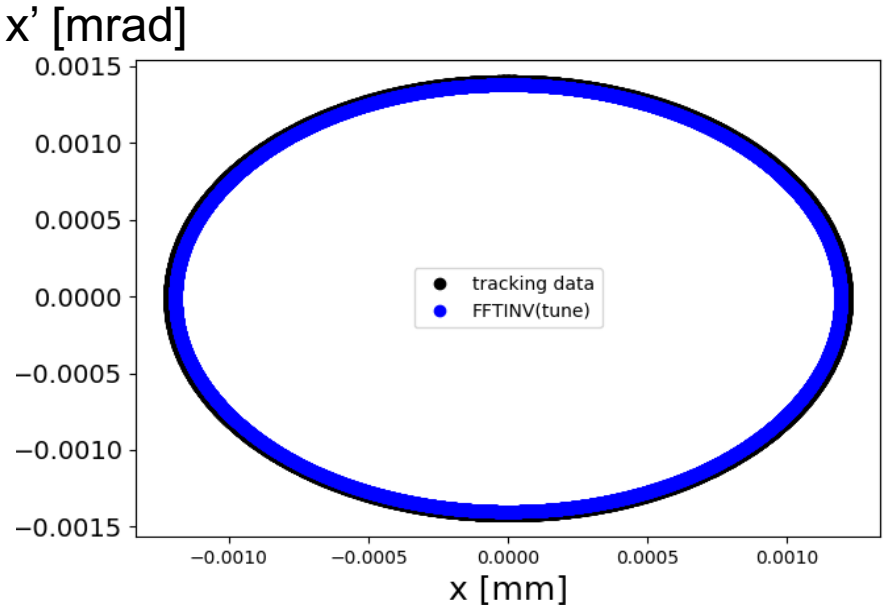


Method

- Single particle tracking using Zgoubi analytic radial FFAG model carried out.
- Used model for 12 cell 150 MeV KURRI ADS ring.

FFT/Inverse FFT

- Take FFT of cell-by-cell or turn-by-turn data to find peak at betatron tune.
- When the amplitude is close to zero, and FFT resolution (turn number) is sufficiently high, the phase space can be recovered by taking the inverse FFT, using data only at the peak from FFT(x) and FFT(x').

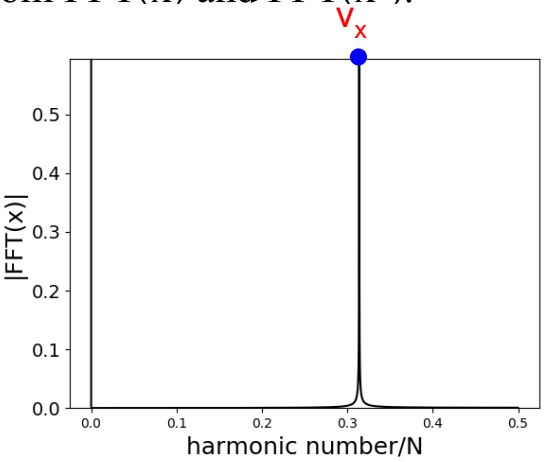


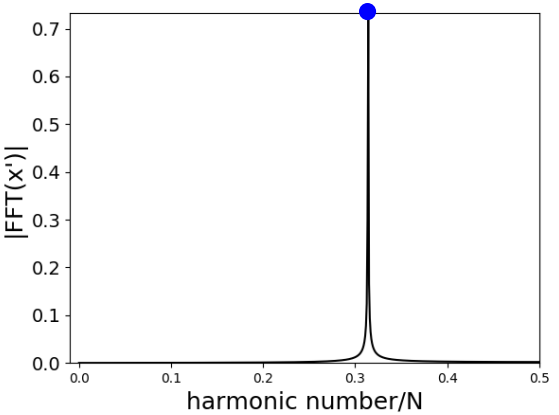
FFT(x)

iFFT(peak)

FFT(x')

iFFT(peak)

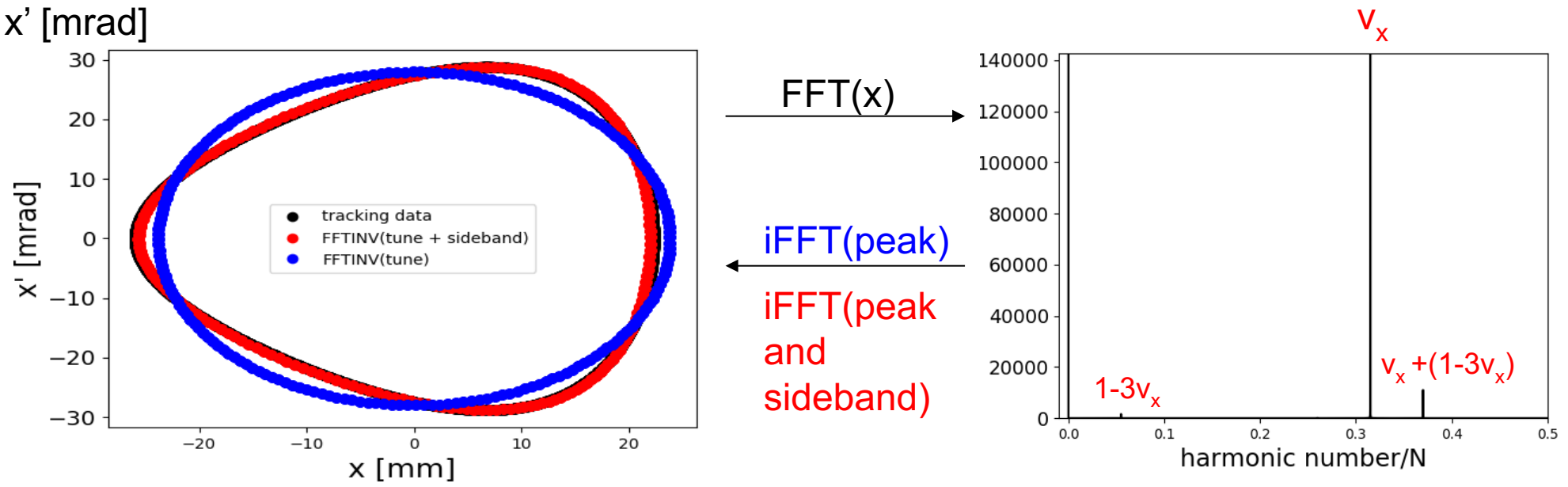




- $\epsilon_n = 10^{-6}$ π mm mrad. Zero vertical motion
- 1000 turns. Cell-by-cell coordiantes noted at midpoint of each long drift.
- Tune found by FFT: 0.3139.

Phase space deformation

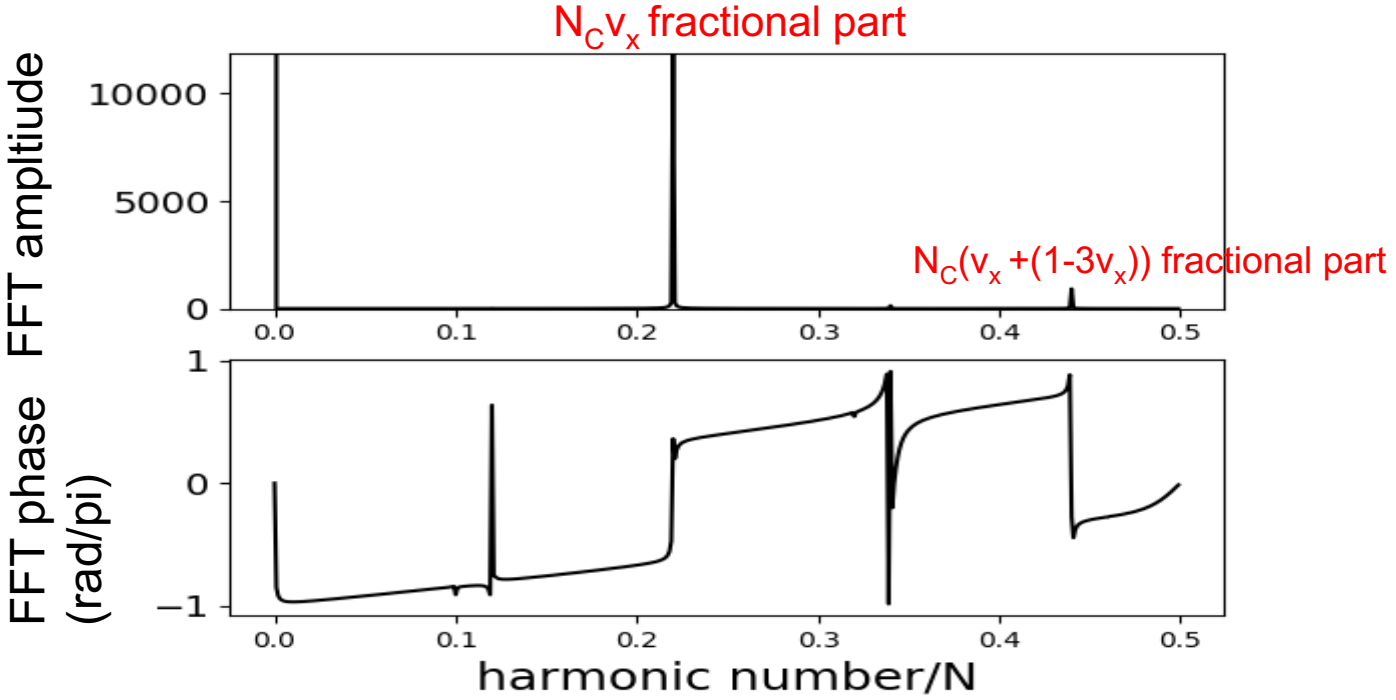
- Phase space becomes increasingly deformed with increasing amplitude.
- This is manifested by betatron tune sidebands separated by the third harmonic of the tune when close to 1/3.



- ϵ_n=500 π mm mrad.
- 1000 turns. Cell-by-cell coordinates noted at midpoint of each long drift.
- Tune found by FFT: 0.315.

Turn-by-turn data

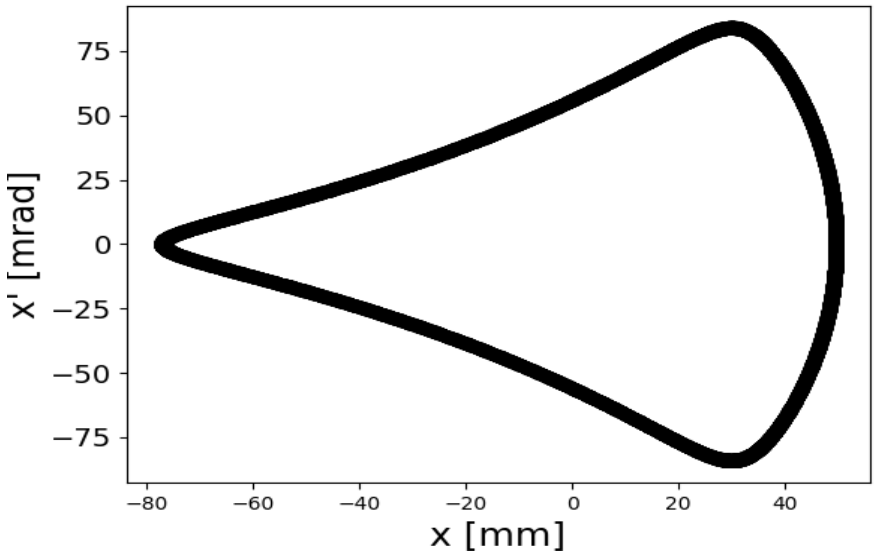
- Experimentally, we rely on turn-by-turn data recorded at a single BPM.
- FFT then measures the fractional part of the total tune.



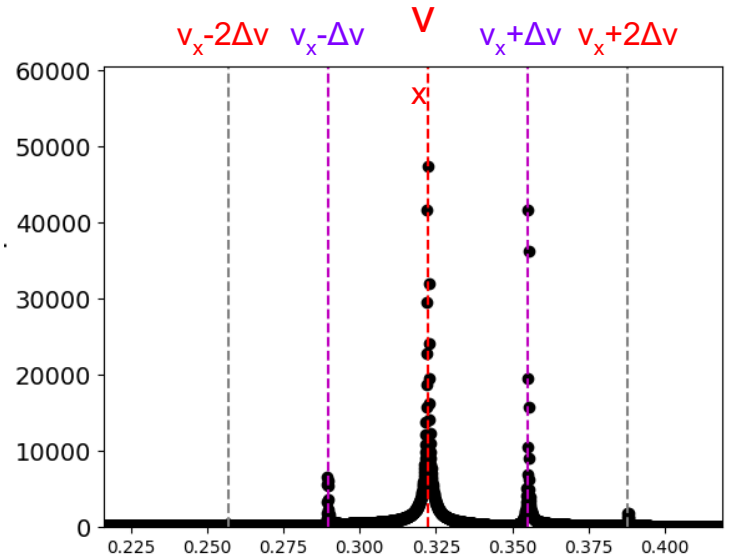
- N_C is cell number.
- ϵ_n=500 π mm mrad.
- 1000 turns. Turn-by-turn coordinates noted at midpoint of one long drift.
- Tune found by FFT: 0.315.

Higher harmonics

- As the DA is approached, higher harmonics of the tune appear the frequency spectrum.



FFT(x) →



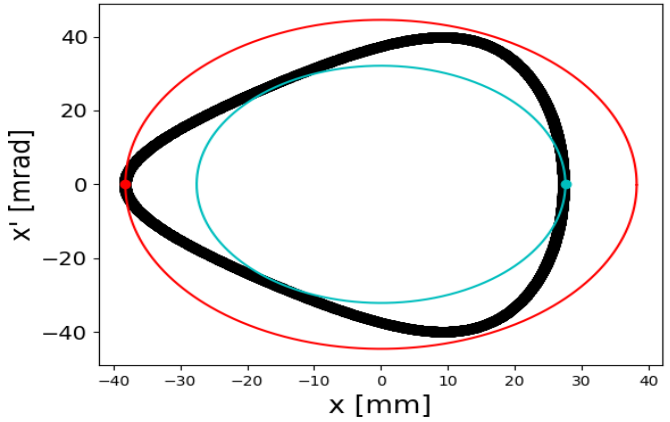
Zoomed view: FFT peak at tune is ~250000.

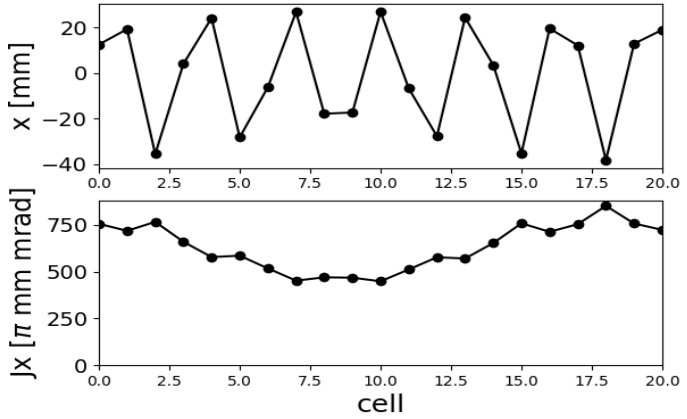
Define $\Delta v_x = (1-3v_x)$

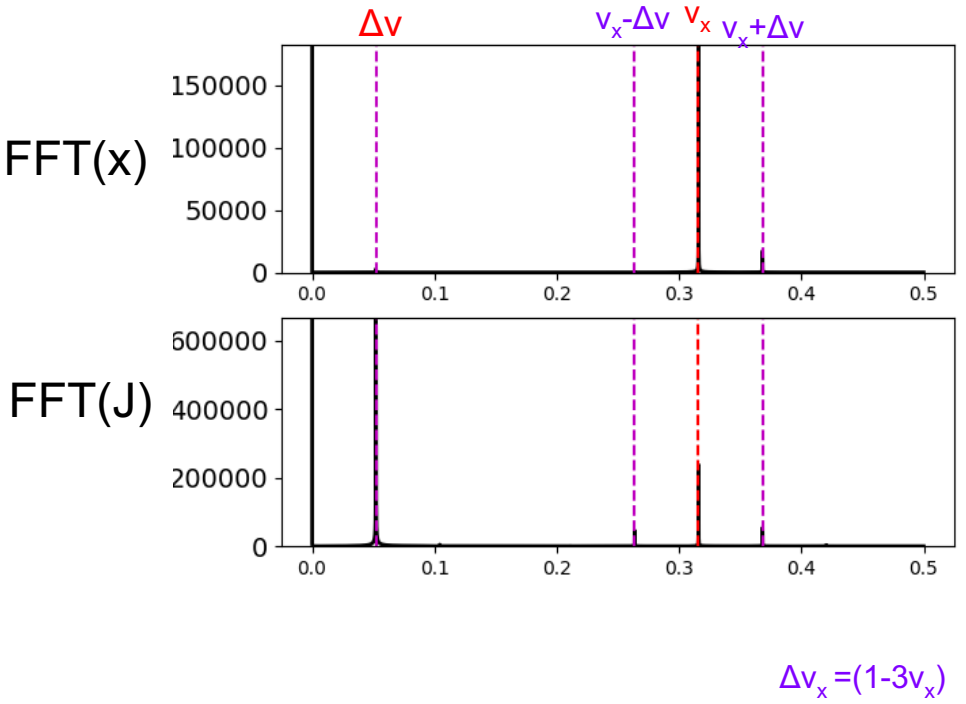
- ϵ_n=5000 π mm mrad.
- 1000 turns. Cell-by-cell coordinates noted at midpoint of each long drift.
- Tune found by FFT: 0.322.

FFT of invariant

- Suppress peak at betatron tune by taking FFT of invariant (action) instead of coordinate.
- First convert (x,x') to action J (using zero ampltiude optics). Then take FFT of J.

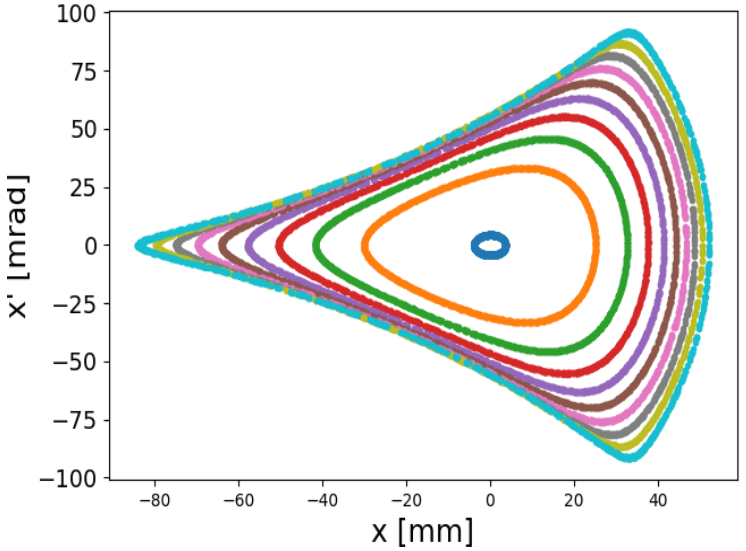


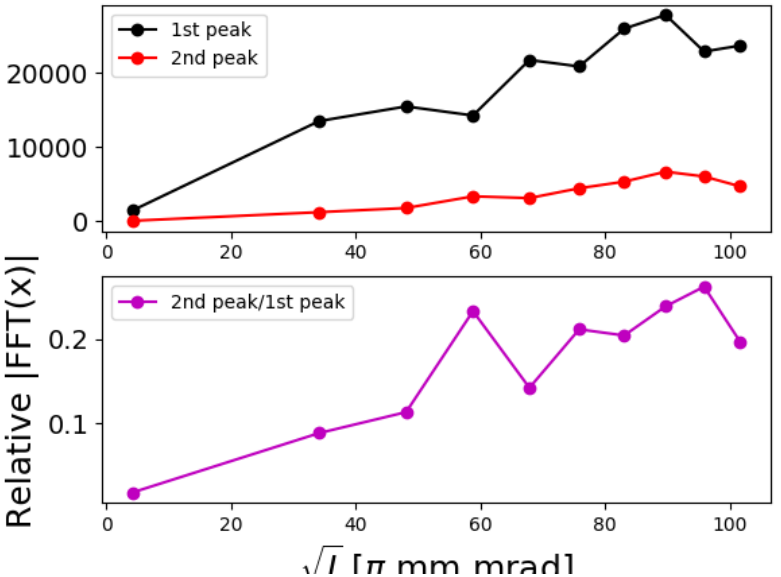




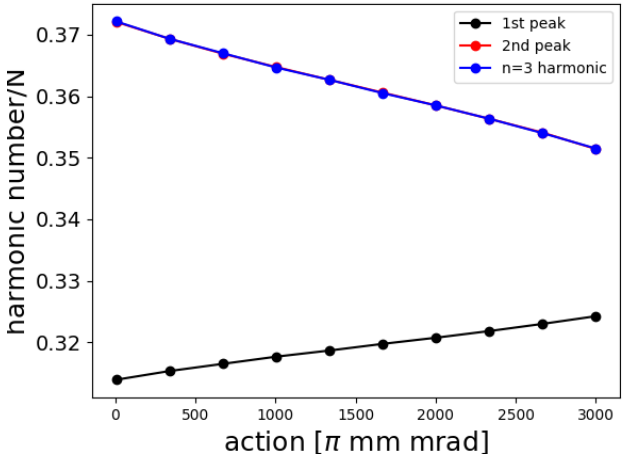
- ϵ_n=1000 π mm mrad.
- 1000 turns. Cell-by-cell coordinates noted at midpoint of each long drift.
- Tune found by FFT: 0.3159

Amplitude scan

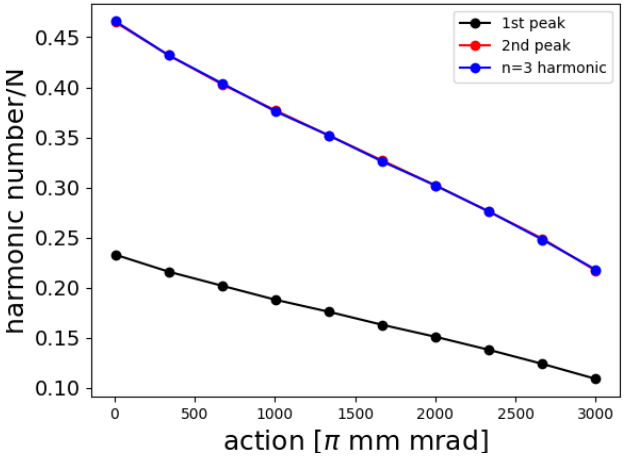




Cell-by-cell measurements.



Turn-by-turn measurements.



Discussion

- Peak in Fourier spectrum at the betatron tune may provide a measurement of tune variation with amplitude.
- FFT of coordinates may show peaks associated with the deformation of phase space or transverse coupling. Jumps in Fourier phase may be easier to measure.
- Such information should be useful for DA and coupling studies.
- Single particle study only. Momentum spread in bunch will lead to decoherence of signal. No pencil beam available in reality – how much of the tranverse amplitude can be explored by kicking the bunch before the beam is lost?