KURRI FFAG COLLABORATION MEETING:

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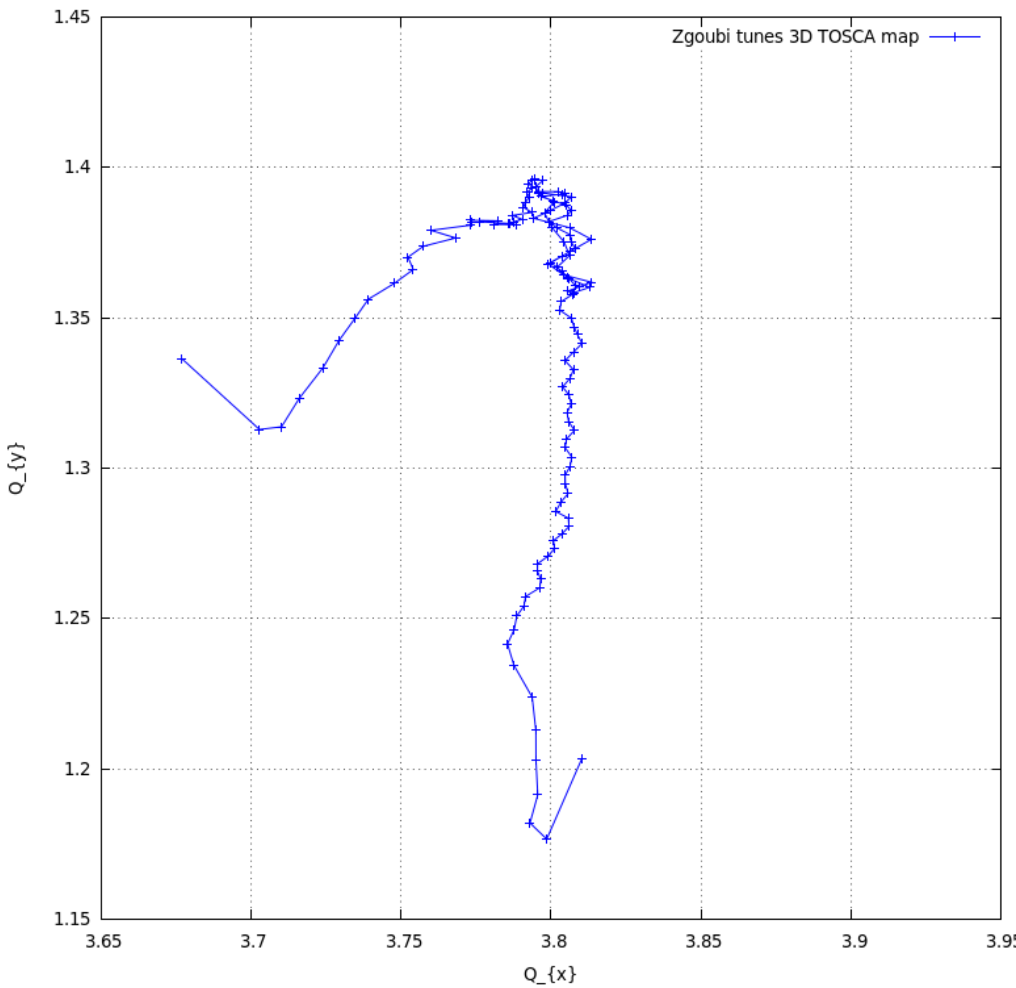
Part I: Oscillatory behavior of the tune

Oscillatory behavior of the tune

- "Is it possible to check the oscillatory region with different field map interpolation order & see if it changes the result?"



Interpolation method in Zgoubi

- In Zgoubi, there are both 2D and 3D field map optical elements:

3D = "TOSCA": only a 2nd order polynomial interpolation is available with a $3 \times 3 \times 3$ point grid used.

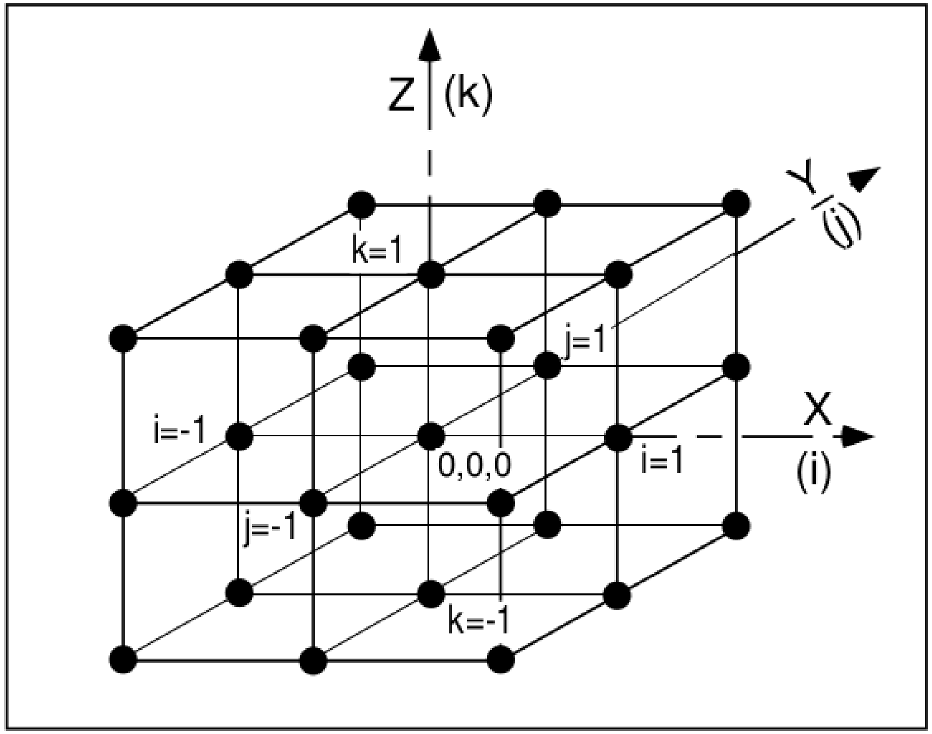


Figure 4: A 3-D 27-point grid is used for interpolation of magnetic or electric fields and derivatives up to second order. The central node of the grid ($i = j = k = 0$) is the closest to the actual position of the particle.

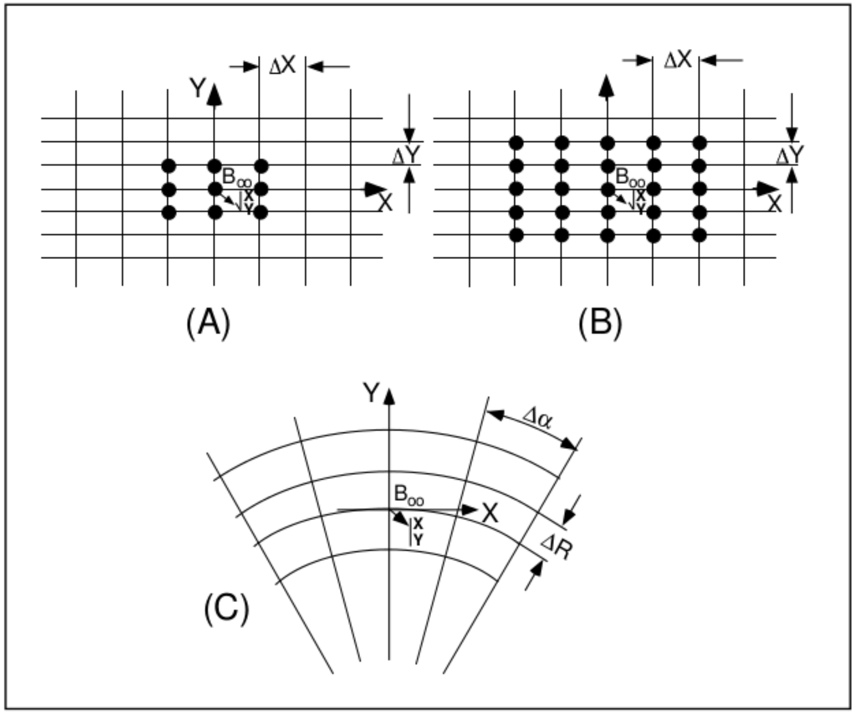
The tracking results produced so far are based on this method.

Interpolation method in Zgoubi

- <u>2D (POLARMES,...)</u>: three types of polynomial interpolation are provided:

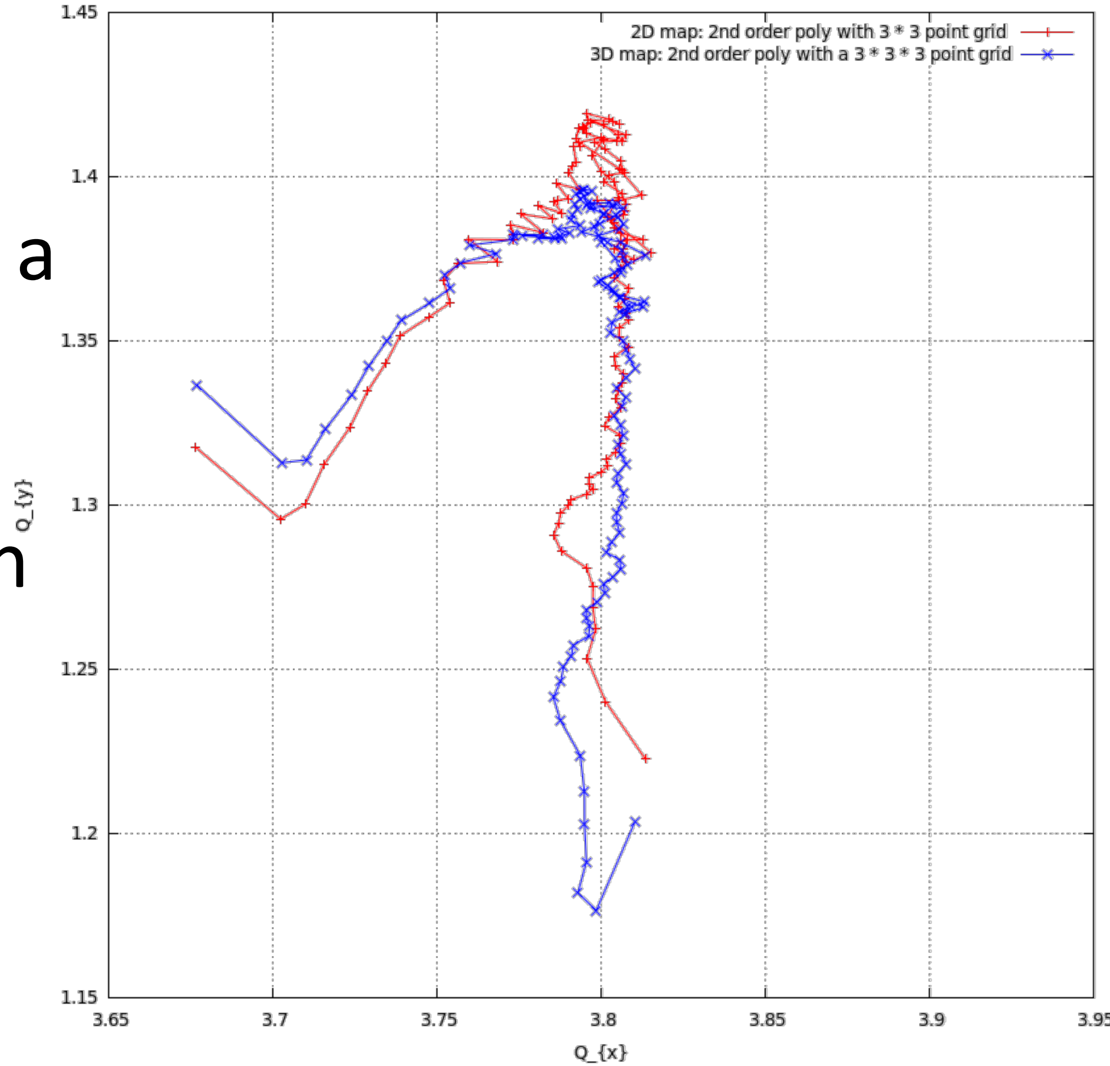
 - 2^{nd} order polynomial with, either a 3×3 point grid or a 5×5 point grid

 - 4^{th} order polyomial with a 5×5 point grid.



2D vs 3D field map

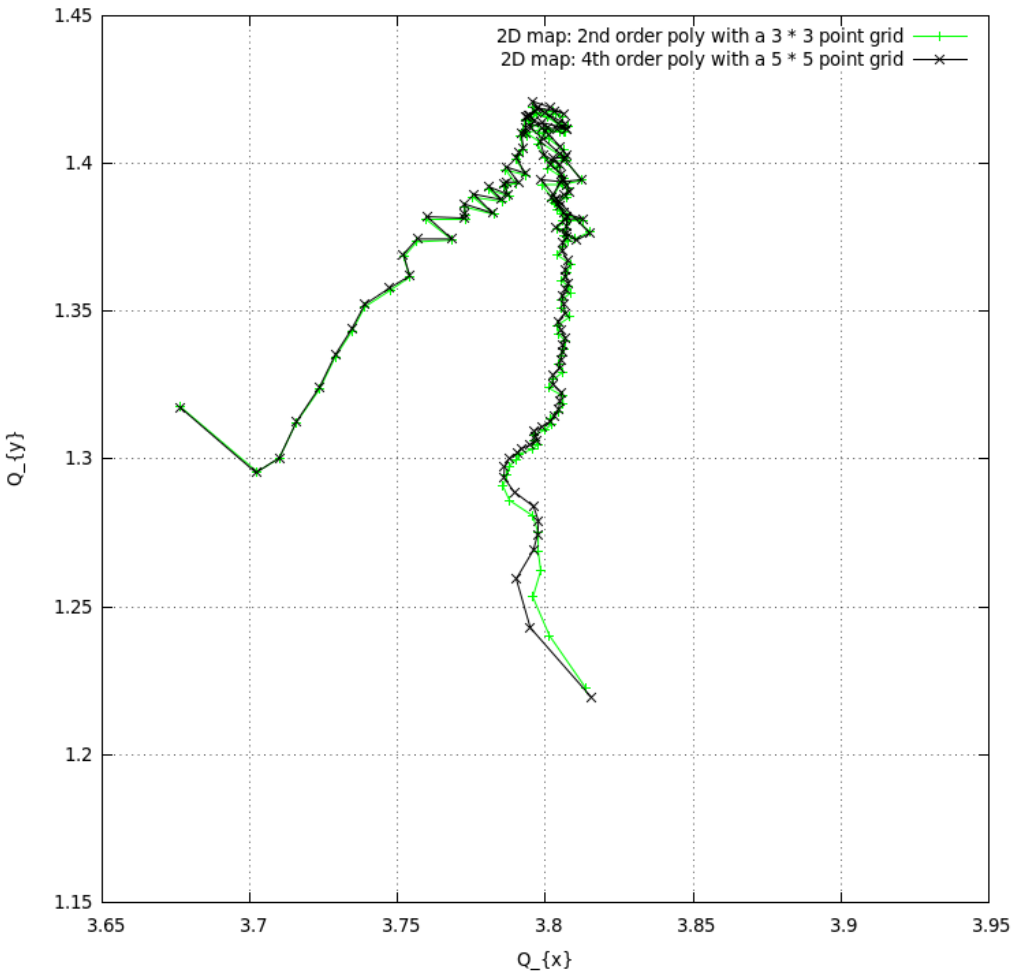
- Converted the 3D TOSCA field map into a 2D field map: the magnetic field is only defined in the median plane.
- The results are comparable.



⇨**It is quite accurate to use the 2D field map.**

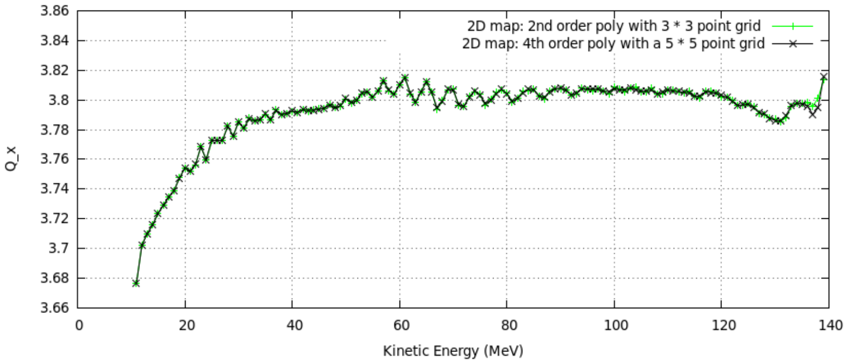
2^{nd} order vs 4^{th} order interpolation polynomial

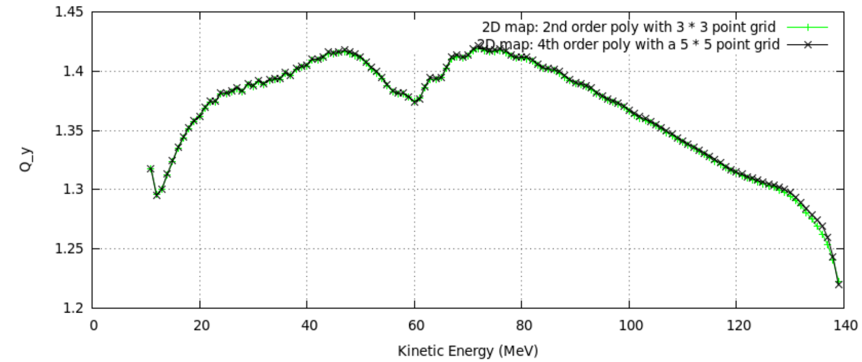
- Changed the order of the interpolation polynomial. Almost no change.
- The oscillatory behavior is still there.



This behavior could be intrinsic to the map.

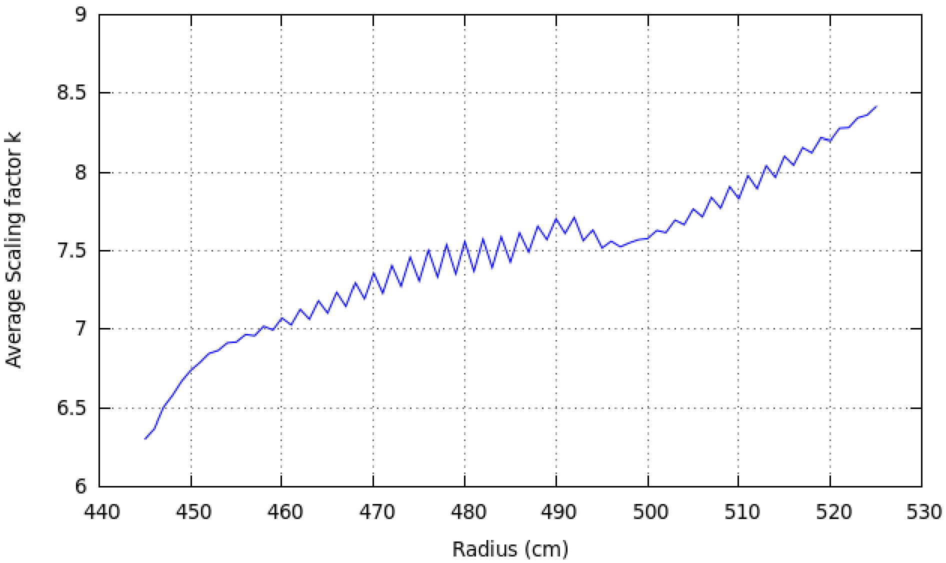
Scaling factor





⇨ This oscillatory behavior is more related to the horizontal tune oscillation.

Since $Q_x^2 \approx k+1$, where k is the scaling factor. Plotting k as a function of R, shows that k obeys to the same oscillatory behavior.



The average variation of the field as a function of R, which is imposed by the field map, seems to be responsible for this oscillatory behavior. A higher mesh size would help?

Part II: Space charge simulations

Transverse particle equations of motion

Courtesy S. Lund USPAS 2015

$$x'' + \frac{(\gamma_b\beta_b)'}{(\gamma_b\beta_b)}x' + \kappa(s)x = -\frac{q}{m\gamma_b^3\beta_b^2c^2}\frac{\partial\phi}{\partial x}$$

$$y'' + \frac{(\gamma_b\beta_b)'}{(\gamma_b\beta_b)}y' - \kappa(s)y = -\frac{q}{m\gamma_b^3\beta_b^2c^2}\frac{\partial\phi}{\partial y}$$

$$\kappa(s) = \frac{qG}{m\gamma_b\beta_b c} = \frac{G}{[B\rho]}$$

$$G = \frac{\partial B_x^a}{\partial y} = \frac{\partial B_y^a}{\partial x} = \frac{B_q}{r_p} \qquad [B\rho] = \frac{m\gamma_b\beta_b c}{q}$$

Now assuming that we have a uniform density elliptical beam in a periodic focusing lattice, we can calculate the free space self-field solution within the beam:

$$-\frac{\partial\phi}{\partial x} = \frac{\lambda}{\pi\epsilon_0}\frac{x}{(r_x+r_y)r_x}$$

$$-\frac{\partial\phi}{\partial y} = \frac{\lambda}{\pi\epsilon_0}\frac{y}{(r_x+r_y)r_y}$$

Linear Space Charge Correction term to the Hill's equation

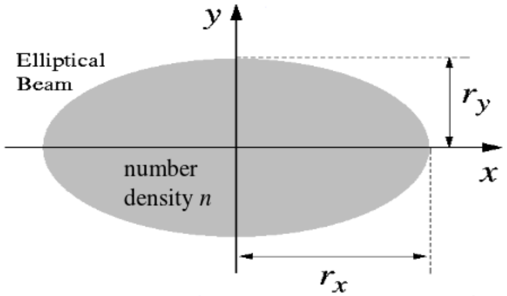
$$x''(s) + \left\{ \kappa_x(s) - \frac{2Q}{[r_x(s) + r_y(s)]r_x(s)} \right\} x(s) = 0$$

$$y''(s) + \left\{ \kappa_y(s) - \frac{2Q}{[r_x(s) + r_y(s)]r_y(s)} \right\} y(s) = 0$$

Dimensionless Perveance Q

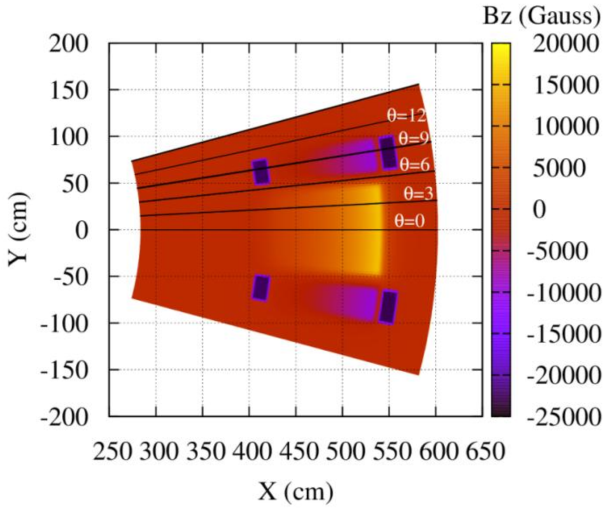
$$Q = \frac{q\lambda}{2\pi\epsilon_0 m \gamma_b^3 \beta_b^2 c^2} = \text{const}$$

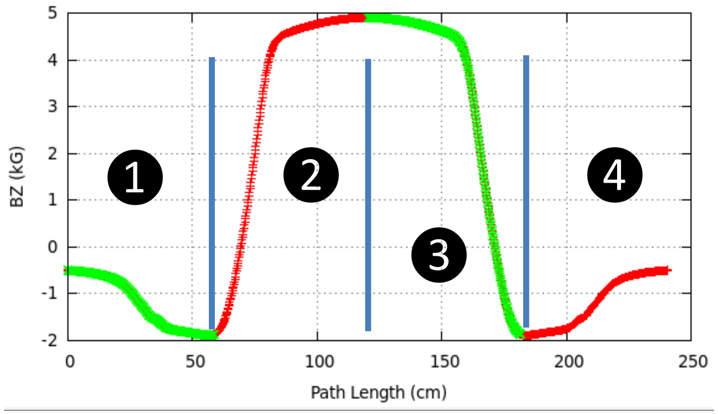
1) In order to solve for x(s), one has to know the beam radii $r_x(s)$ and $r_y(s)$.



2) If we cut the magnet into thin slices, we may assume that the beam radii do not change much within each slice.

Slicing





Median plane Field map of the KURRI 150 MeV scaling FFAG

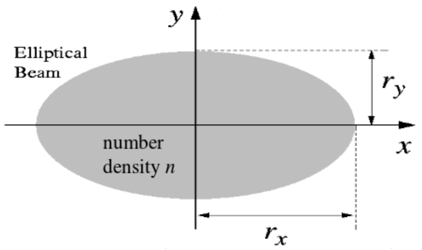
DFD triplet cut into 4 slices

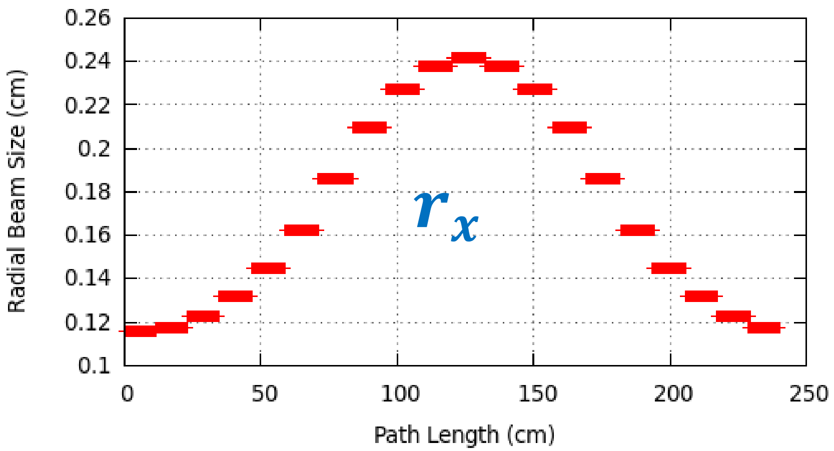
- It was found that by tweaking few parameters in the analytical model FFAG, it is possible to slice the magnet into thin elements.

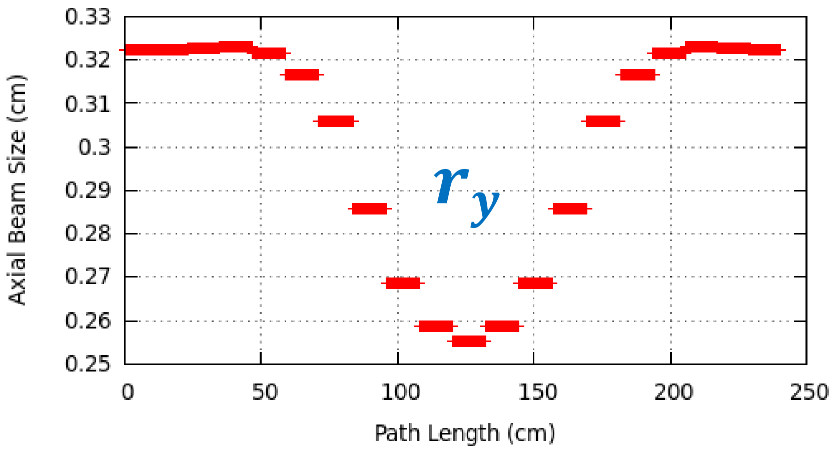
- With the field map, the results is straightforward.

A routine was implemented to achieve this.

RMS calculations:







At the entrance of each element (slice) the KV equivalent beam parameters are computed:

$$r_x = 2 < x^2 >^{\frac{1}{2}} \qquad \varepsilon_x = 4\left[< x^2 >< x'^2 > - < xx' >\right]^{\frac{1}{2}}$$

$$r_y = 2 < y^2 >^{\frac{1}{2}} \qquad \varepsilon_y = 4\left[< y^2 >< y'^2 > - < yy' >\right]^{\frac{1}{2}}$$

Requires lots of computation time: optimizing the number of slices may be a good investigation.

Space charge linear kick

Once the beam radii for each slice are computed, we can apply the space charge linear kick:

For each particle within the beam, the angles x’ and y’ are corrected after each integration step.

$$\begin{cases} \Delta x' = \dfrac{2Q}{(r_x+r_y)r_x} x.\Delta s \\ \Delta z' = \dfrac{2Q}{(r_x+r_y)r_y} y.\Delta s \end{cases}$$

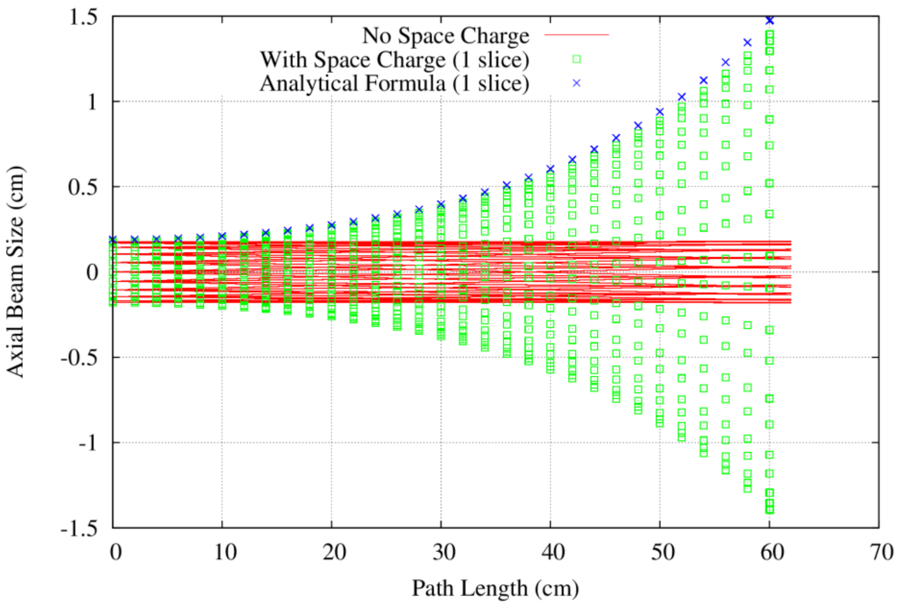
These equations are valid only inside the beam: the self electric field is non-linear outside the beam. So this model cannot describe the halo.

Validation/tests on-going

- Considering a drift, the transverse equations of motion simplify to:

$$x'' - \frac{2Q}{r_x(r_x + r_y)} x = 0$$

$$y'' - \frac{2Q}{r_y(r_x + r_y)} y = 0$$



- If we take: $x(s = 0) = x_0$

$x'(s = 0) = 0$

Then, the solution of this equation is:

$$x(s) = x_0 \cosh\left[\left(\frac{2Q}{r_x(r_x + r_y)}\right)^{\frac{1}{2}} s\right]$$

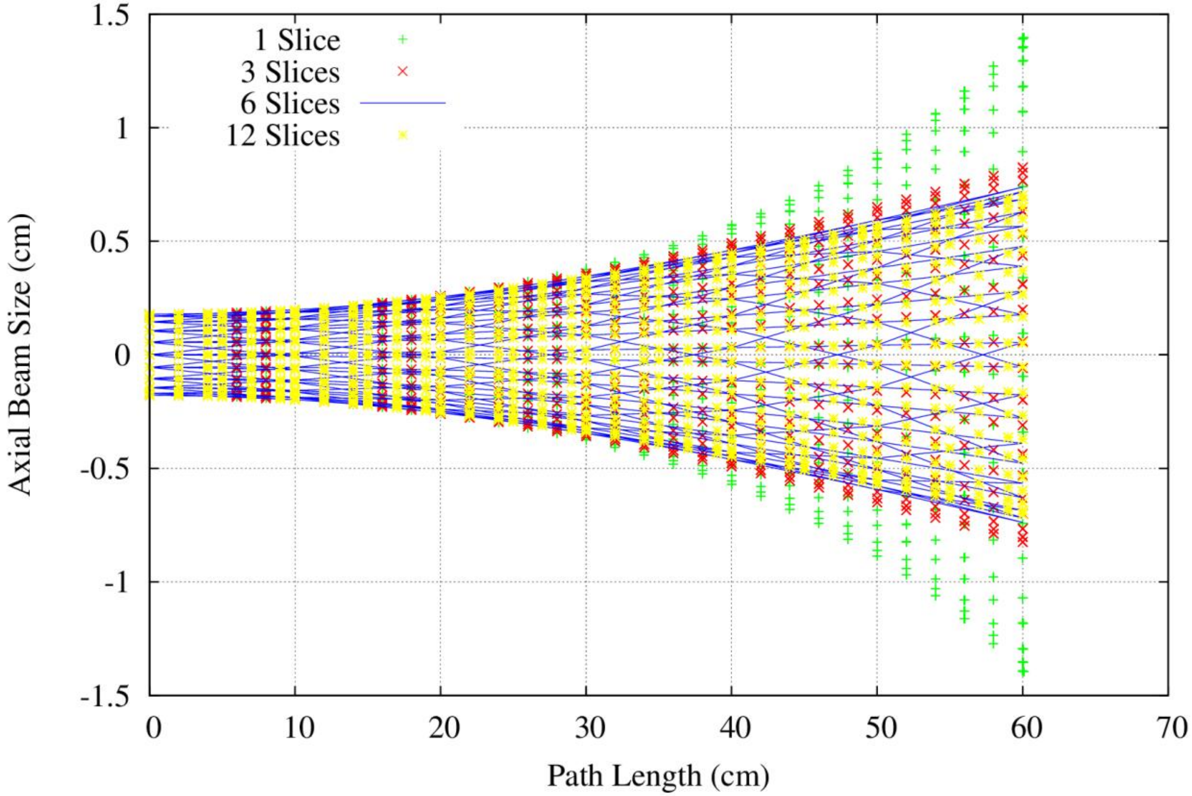
If we generate an initial distribution such as

$x_i'(s = 0) = 0 \quad \forall i$ then,

$$r_x(s) = x_0^{max} \cosh\left[\left(\frac{2Q}{r_x(r_x + r_y)}\right)^{\frac{1}{2}} s\right]$$

Convergence of the space charge calculation

How many slices before obtaining a numerical convergence of the space charge calculation?



⇨**Optimization problem.**

Adiabatic Damping

- Include the damping (acceleration) term in the transverse equations of motion:

$$\begin{cases} \dfrac{d^2x}{ds^2} + \dfrac{(\gamma_b\beta_b)'}{\gamma_b\beta_b}x' + \dfrac{1-n}{\rho^2}x = 0 \\[2ex] \dfrac{d^2z}{ds^2} + \dfrac{(\gamma_b\beta_b)'}{\gamma_b\beta_b}z' + \dfrac{n}{\rho^2}z = 0 \end{cases}$$

And $$n(R) \approx -\frac{\rho}{B}\frac{dB}{dR} = -\frac{\rho}{R}k(R)$$

Adiabatic Damping

- The previous equations become:

$$\begin{cases} \dfrac{d^2x}{ds^2} + \dfrac{(\gamma_b\beta_b)'}{\gamma_b\beta_b}x' + \dfrac{1+\frac{\rho}{R}k(R)}{\rho^2}x = 0 \\[2ex] \dfrac{d^2z}{ds^2} + \dfrac{(\gamma_b\beta_b)'}{\gamma_b\beta_b}z' - \dfrac{k(R)}{\rho R}z = 0 \end{cases}$$

Also, using the hard edge model it can be shown that $\frac{\rho}{R} = const$

$\frac{\rho}{R} = \alpha_F > 0$ for the F-magnet ; $\frac{\rho}{R} = \alpha_D < 0$ for the D-magnet

Solving the z-equation of motion:

- Using the WKB (Wentzel–Kramers–Brillouin) approximation, an approximate solution to this equation is (coeff slowly changing in time):

$$x(s) = x_0 A(s) \exp\left[\int ik_x(s)ds\right]$$

Solving for the vertical y- component, one obtain:

$$\begin{cases} k_z^2(s) = -\dfrac{k(R)}{\rho R} = -\dfrac{k(R)}{\alpha R^2} \\ A(s) = \dfrac{1}{\sqrt{\beta_b \gamma_b}} \times \dfrac{1}{\sqrt{|k_z(s)|}} = \dfrac{\sqrt{R}}{\sqrt{\beta_b \gamma_b}} \times \left(\dfrac{|\alpha|}{k(R)}\right)^{\frac{1}{4}} \end{cases}$$

Conclusions:

- Damping law for scaling FFAG:

$$\varepsilon_{norm} \propto \frac{\beta\gamma}{R}\left(\frac{k(R)}{|\alpha|}\right)^{\frac{1}{2}} \times \varepsilon$$

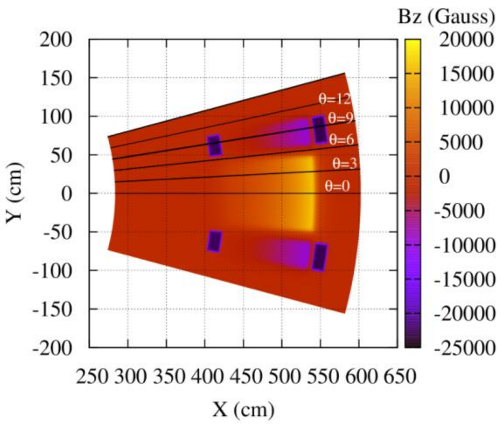
- For a KV beam, the space charge kick is:

$$K_{z,sc} = \frac{2Q}{(r_x + r_z)r_z} \propto \frac{q\lambda}{m_0 c^2 \gamma_b^2 \beta_b R}\left(\frac{k(R)}{|\alpha|}\right)^{\frac{1}{2}}$$

It seems that, compared to a synchrotron, the relativistic space charge effects in scaling FFAG could be lower ..

Tests of the space charge module on the KURRI 150 MeV FFAG

Analytical model of the 150 MeV KURRI FFAG machine implemented in Zgoubi.




```
 'FFAG'               #START                                                                     3
20
5   30.     440.48836355                         NMAG, AT=tetaF+2tetaD+2Atan(XFF/R0), R0
0.000  0.  -0.4412832  0.239148                 mag 0 : ACNT, dum, B0, K                        MAGNET 1
6.3  03.                               EFB 1 : lambda, gap const/var=0/.ne.0
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
4.7 0.    1.E6  -1.E6  1.E6  1.E6
6.3  03.                               EFB 2
4  -3.07033892e+00, 8.59656096e+00,  -1.04829407e+01,   5.80500507e+00   0. 0.  0.
-3.6  0.    1.E6  -1.E6  1.E6  1.E6
0. -1                                  EFB 3 : inhibited by iop=0
0  0.       0.       0.       0.       0. 0.  0.
0. 0.       0.       0.       0.       0.
6.465  0.  -1.6151060  9.426756             mag 1 : ACNT, dum, B0, K                            MAGNET 2
8.3  03.                               EFB 1 : lambda, gap const/var=0/.ne.0
5  -4.12200913e-01   2.22904985e+00  -6.80512267e-01 1.23609453e-01 -7.87155179e-03  0.  0.
+2.5  3.1    1.E6  -1.E6  1.E6  1.E6
6.3  03.                               EFB 2
4  -8.23066935e-01, 2.36019103e+00,  -3.84298625e-01,   2.43560489e-01  0. 0.  0.
-1.765  0.    1.E6  -1.E6  1.E6  1.E6
0. -1                                  EFB 3 : inhibited by iop=0
0  0.       0.       0.       0.       0. 0.  0.
0.  0.   0.    0.    0. 0.
15.   0.   3.400519, 7.707476                    mag 2 : ACNT, dum, B0, K,dummies                MAGNET 3
6.3  03.                               EFB 1
4   -4.13707399e-01,   2.14307057e+00, -4.26620705e-01,   1.70354587e-01  0. 0.  0.
5.37   0.    1.E6  -1.E6  1.E6  1.E6
6.3  03.                               EFB 2
4   -4.13707399e-01,   2.14307057e+00, -4.26620705e-01,   1.70354587e-01   0.  0.  0.
-5.37   0.    1.E6  -1.E6  1.E6  1.E6
0. -1                                  EFB 3
0  0.       0.       0.       0.       0. 0.  0.
0.  0.   0.    0.    0. 0.
23.535  0.0  -1.615106,  9.426756              mag 3 : ACNT, dum, B0, K                         MAGNET 4
6.3  03.                               EFB 1
4   -8.23066935e-01, 2.36019103e+00,  -3.84298625e-01,   2.43560489e-01  0. 0.  0.
1.765  0.    1.E6  -1.E6  1.E6  1.E6
8.3  3.                               EFB 2
5   -4.12200913e-01   2.22904985e+00  -6.80512267e-01  1.23609453e-01   -7.87155179e-03  0.   0.
-2.5  -3.1    1.E6  -1.E6  1.E6  1.E6
0. -1                                  EFB 3
0  0.       0.       0.       0.       0. 0.  0.
0.  0.   0.    0.    0. 0.
30.  0.  -0.4412832  0.239148                mag 4 : ACNT, dum, B0, K                           MAGNET 5
6.3  03.                               EFB 1 : lambda, gap const/var=0/.ne.0
4  -3.07033892e+00, 8.59656096e+00,  -1.04829407e+01,   5.80500507e+00  0. 0.  0.
3.6 0.    1.E6  -1.E6  1.E6  1.E6
6.3  03.                               EFB 2
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-4.7  0.    1.E6  -1.E6  1.E6  1.E6
0. -1                                  EFB 3 : inhibited by iop=0
0  0.       0.       0.       0.       0. 0.  0.
0.  0.   0.    0.    0. 0.
2  2  125.                    KIRD anal/num (=0/2,25,4), resol(mesh=step/resol)
```

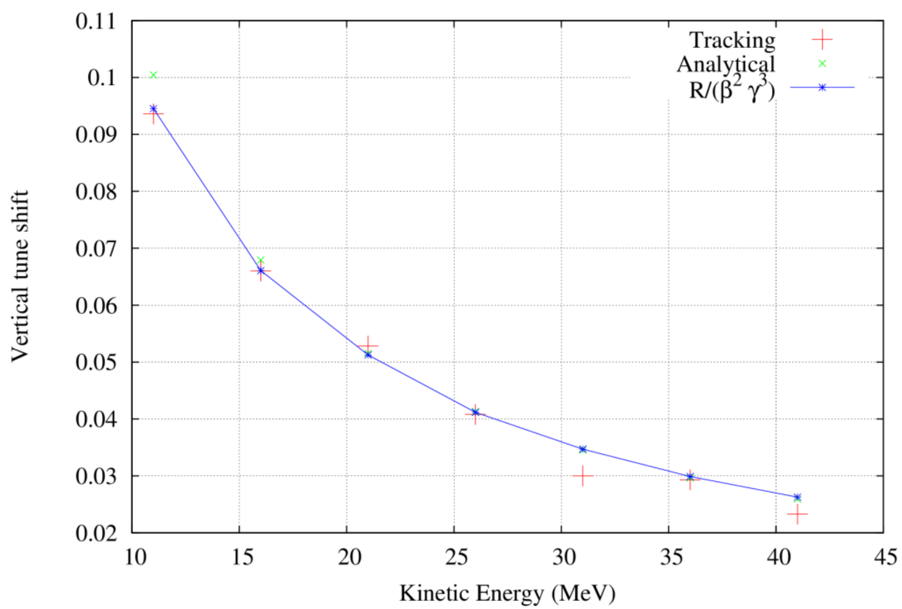
Laslett tune shift

- We investigate the change in betatron oscillation frequency due to space charge forces. The linear Laslett tune shift is given by:

$$\Delta Q_x = \frac{1}{4\pi}\int_0^C \beta_x(s)\frac{2Q}{r_x(r_x+r_y)}ds \quad \Rightarrow \quad \Delta Q_x \propto \frac{R}{\beta^2\gamma^3}$$

If the emittance is kept the same for all energies

⇨**Tracking results are consistent with the Scaling law of the Laslett tune shift.**



$Q \approx 6.4 \times 10^{-8}$ at injection.

Dispersion effect:

Investigate the effect of dispersion in presence of space charge.

We know that: $$p = p_0 \left(\frac{r}{r_0}\right)^{k+1} = p_0 \left(\frac{r_0 + x}{r_0}\right)^{k+1} = p_0 \left(1 + \frac{x}{r_0}\right)^{k+1}$$
$$\approx p_0 \left[1 + (k+1)\frac{x}{r_0}\right] \quad ; \qquad x \ll r_0$$

$$\Rightarrow \quad \frac{\Delta p}{p_0} = \frac{p - p_0}{p_0} \approx \frac{k+1}{r_0} x \quad \Rightarrow \quad D \approx \frac{r_0}{k+1}$$

And more generally, it can be shown (by solving the inhomogeneous x-equation of motion), that the dispersion function can be written in the general form:

$$D \approx \frac{r_0}{\nu_x^2}$$

where r_0 is the average radius of the orbit of reference momentum p_0

Dispersion effect

- The presence of space charge reduces the net focusing effect which would increase the dispersion effect:

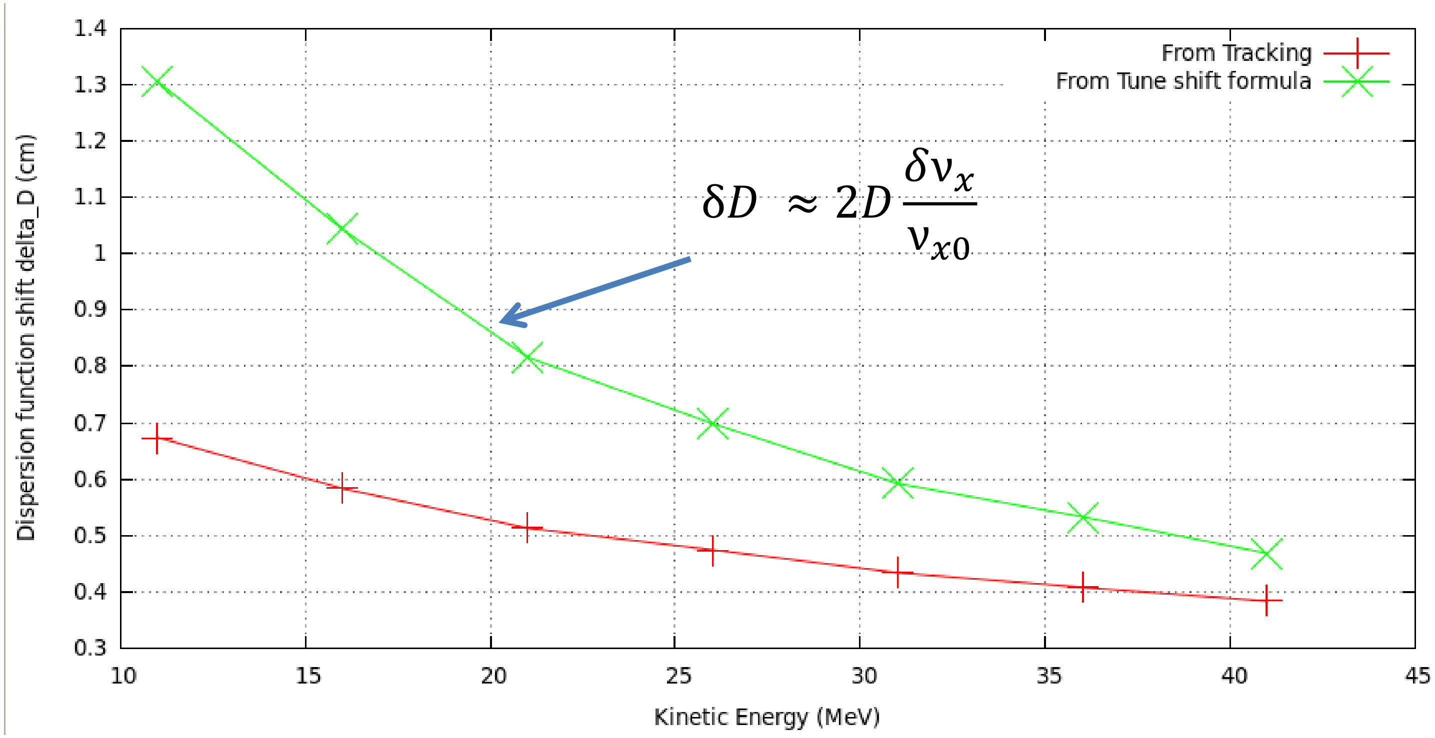
$$D \approx \frac{r_0}{\nu_x^2} = \frac{r_0}{(\nu_{x0} - \delta\nu_x)^2} = \frac{r_0}{\nu_{x0}^2 \left(1 - \frac{\delta\nu_x}{\nu_{x0}}\right)^2} \qquad ; \qquad \delta\nu_x \ll \nu_{x0}$$

$$\approx \frac{r_0}{\nu_{x0}^2}\left(1 + 2\frac{\delta\nu_x}{\nu_{x0}}\right)$$

Thus, the dispersion function is modified due to the presence of space charge by:

$$\delta D \approx 2D \frac{\delta\nu_x}{\nu_{x0}}$$

Dispersion effect



The formula above overestimates the shift of the dispersion function due to space charge.

This is expected, since there is an interplay between the dispersion and the space charge effects: the dispersion increases the beam size ⇨ reduces the space charge kick
⇨ the tune becomes less depressed than in the case with no dispersion.

The tracking contains this interplay between dispersion and tune shift and so is more accurate.

Nuclear-The Foundation of Clean Energy
Thank you