KURRI FFAG COLLABORATION MEETING:

JUNE 9th, 2016

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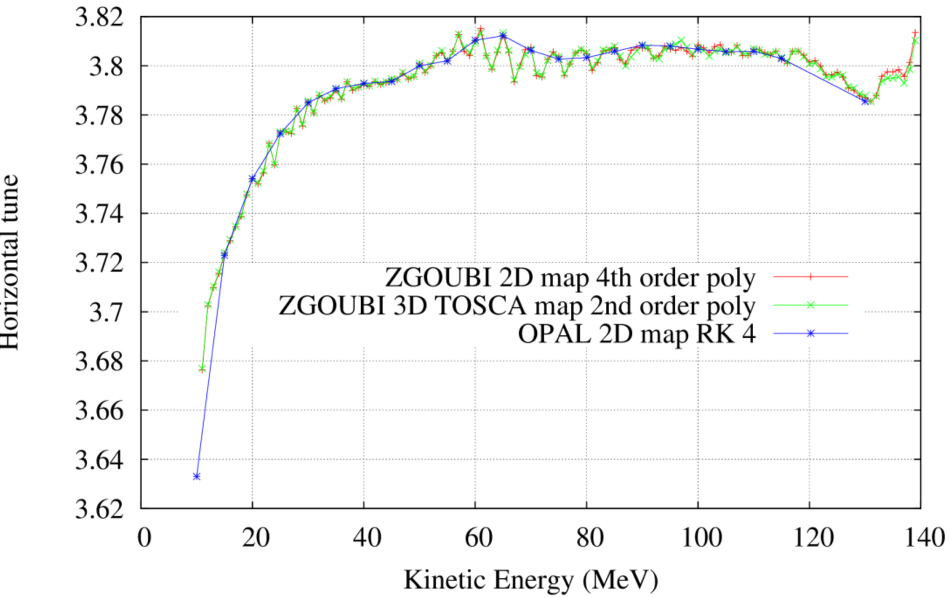
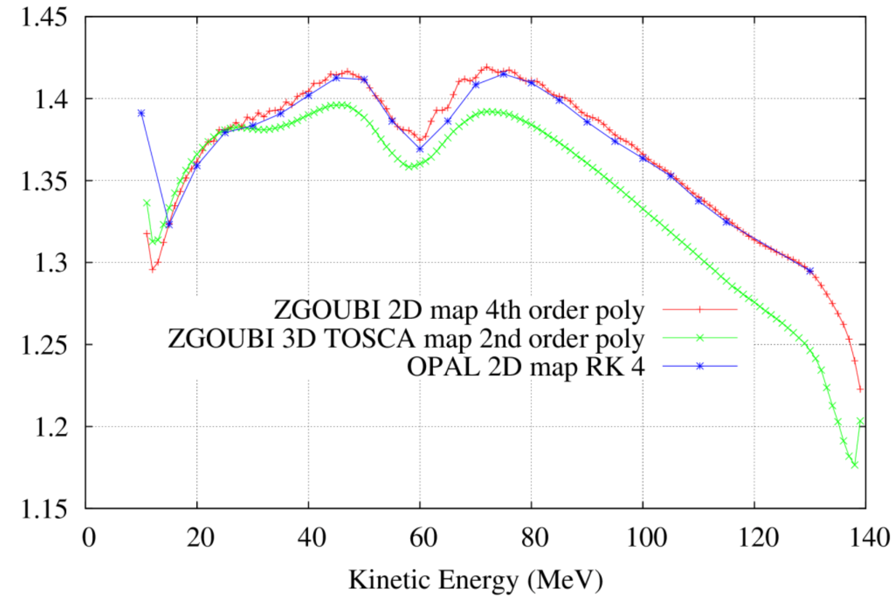
GRENOBLE 1 UNIVERSITE JOSEPH FOURIER SCIENCES.TECHNOLOGIE.SANTÉ

Tune comparison 2D map





Courtesy Andreas

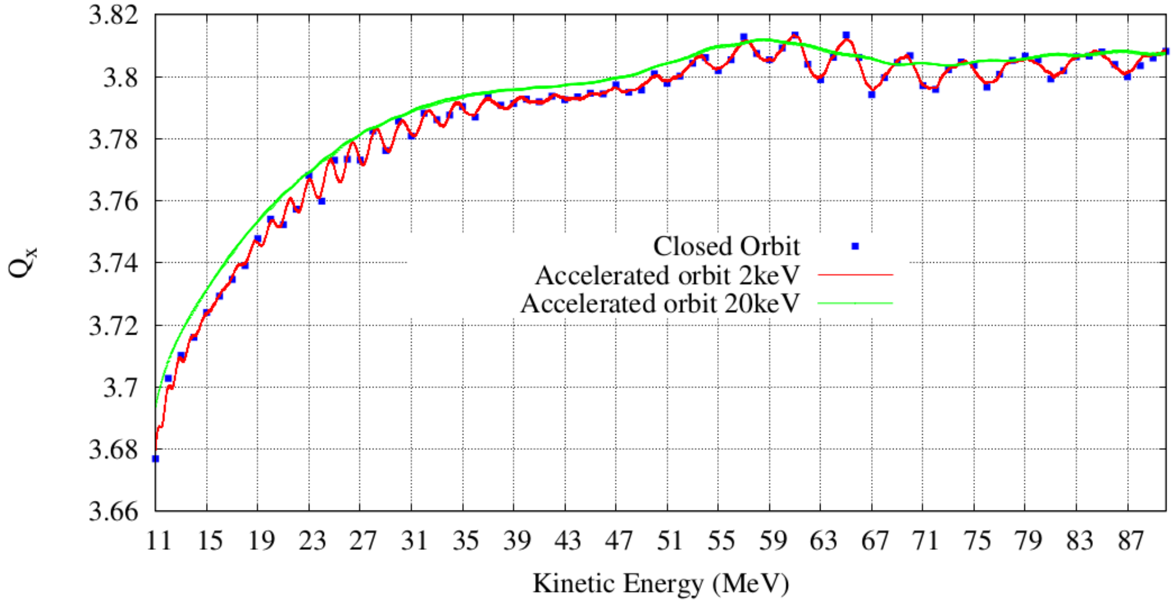
Tunes in OPAL obtained from the analysis of the constructed map.
Tracking also gives the same results provides one updates the revolution frequency for each closed orbit.

Space charge effects in FFAGs:

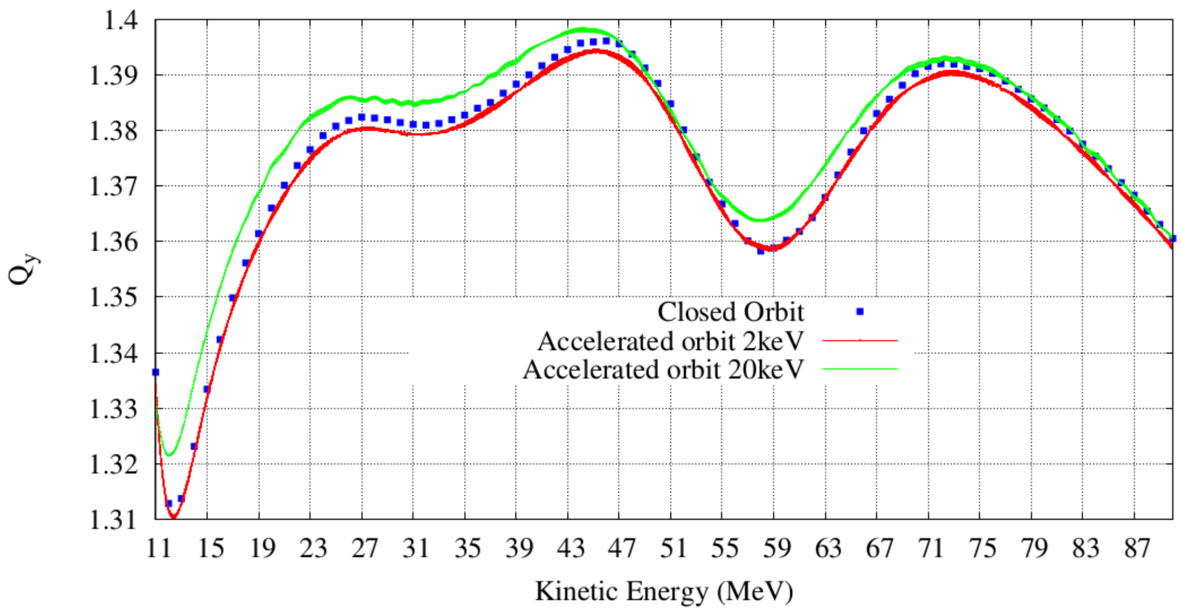
- The approach is based on the following procedure:

- Find the closed orbits for a given FFAG (kF,kD).
- Find the matching condition without space charge for each closed orbit (betatron functions calculated).
- Generate matched distribution around each closed orbit and track it over several turns in order to calculate the average as well as the rms values of the tunes (with and without space charge).
- The difference provides the average tune shift in presence of space charge and the calculation is based on the linear space charge kick approximation (KV beam assumed).

The assumption is that the lattice has slowly varying parameters, i.e the acceleration rate is small enough to assume that the matched beam distribution of the accelerated orbit is the same as that of the closed orbit (see next slide).



3.82
3.8
3.78
3.76
3.74
3.72
3.7
3.68
3.66
Qx
11 15 19 23 27 31 35 39 43 47 51 55 59 63 67 71 75 79 83 87
Kinetic Energy (MeV)
Closed Orbit
Accelerated orbit 2keV
Accelerated orbit 20keV



1.4
1.39
1.38
1.37
1.36
1.35
1.34
1.33
1.32
1.31
Qy
11 15 19 23 27 31 35 39 43 47 51 55 59 63 67 71 75 79 83 87
Kinetic Energy (MeV)
Closed Orbit
Accelerated orbit 2keV
Accelerated orbit 20keV

Useful quantities

Average tune of a FFAG:

$$< \nu_{x,y} > = \frac{1}{N} \sum_{i=1}^{NCO} \nu_{x,y,i}$$; NCO = 30 closed orbits

RMS tune of a FFAG:

$$< \nu_{x,y}^{rms} > = \frac{1}{N} \sum_{i=1}^{NCO} (\nu_{x,y,i} - < \nu_{x,y} >)^2$$

= a measure of the tune spread

For a perfectly scaling FFAG, $< \nu_{x,y}^{rms} >$ = 0

Laslett tune shift

- Investigate the change in betatron oscillation frequency due to space charge forces. The linear Laslett tune shift is given by:

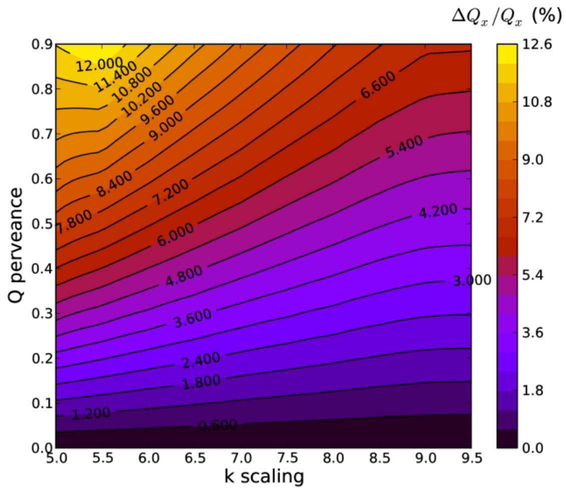
$$\Delta Q_x = \frac{1}{4\pi}\int_0^C \beta_x(s)\,\frac{2Q}{r_x(r_x + r_y)}\,ds$$

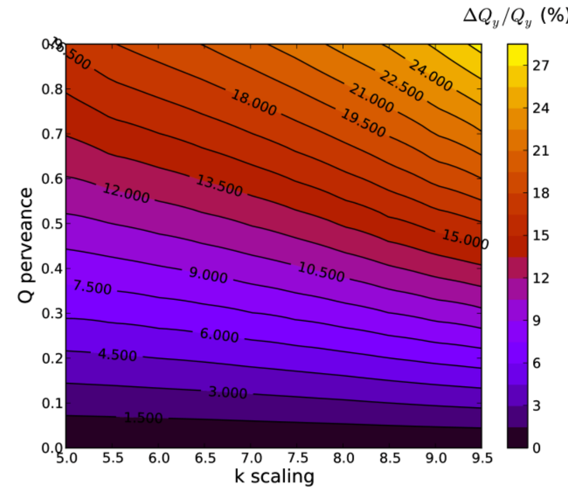
$$\Rightarrow \quad \frac{\Delta Q_x}{Q_x} \propto \frac{r_x^3}{r_x + r_y} Q \propto \frac{\nu_x^{-\frac{3}{2}}}{\nu_x^{-\frac{1}{2}} + \nu_y^{-\frac{1}{2}}} Q = f(k, Q)$$

where $$Q = \frac{q\lambda}{2\pi\epsilon_0 m \gamma_b^3 \beta_b^2 c^2} = \text{const}$$ = perveance term.

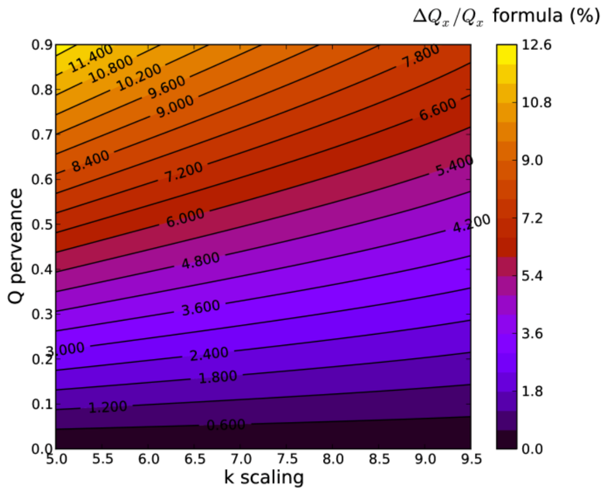
Scaling FFAG

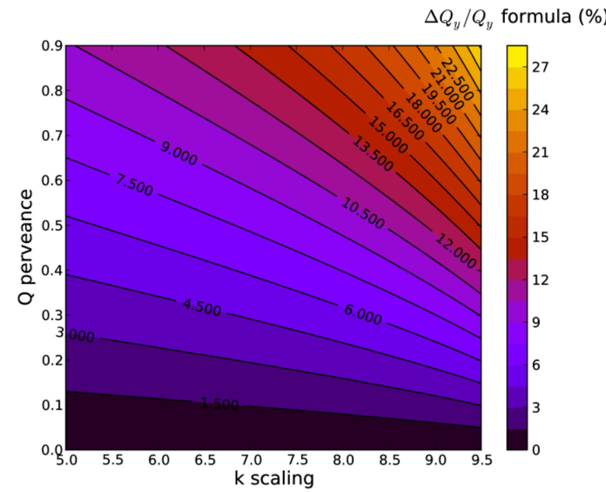
Tracking





Formula

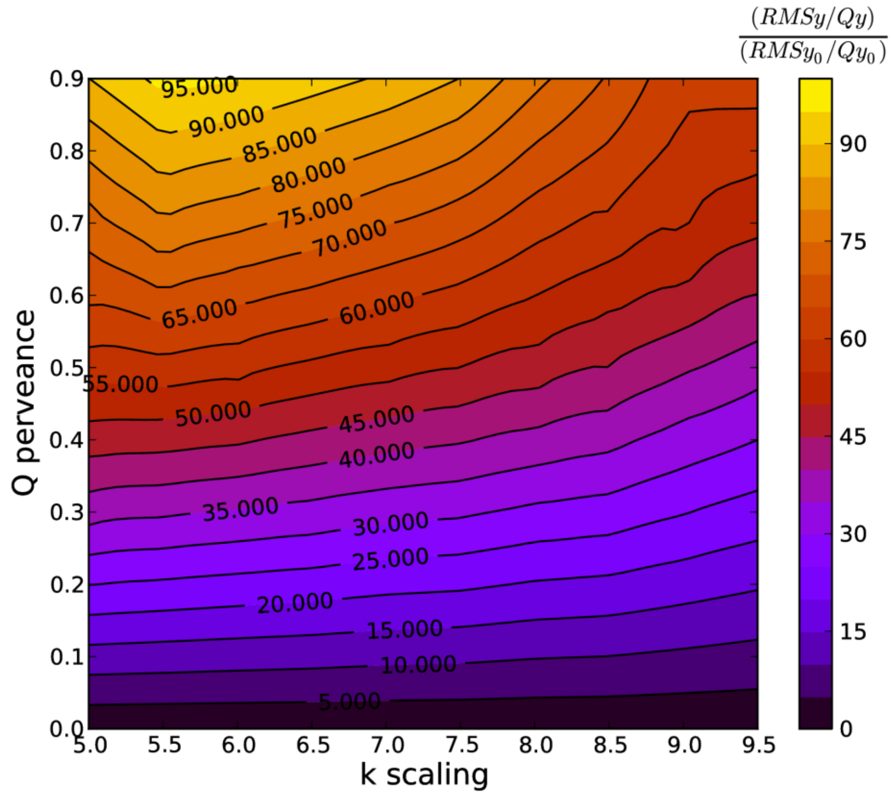




Horizontal

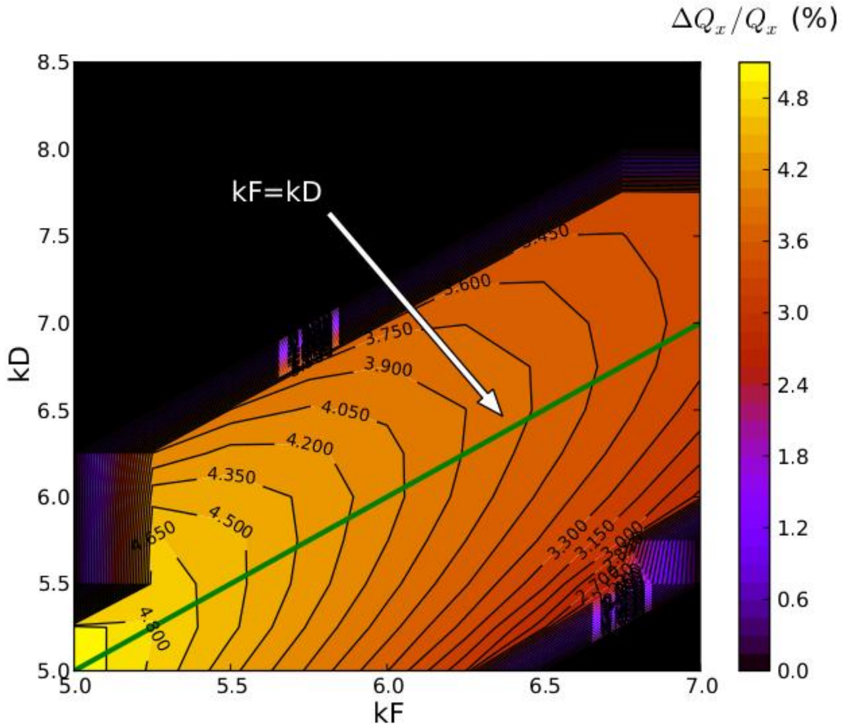
Vertical

Scaling FFAG

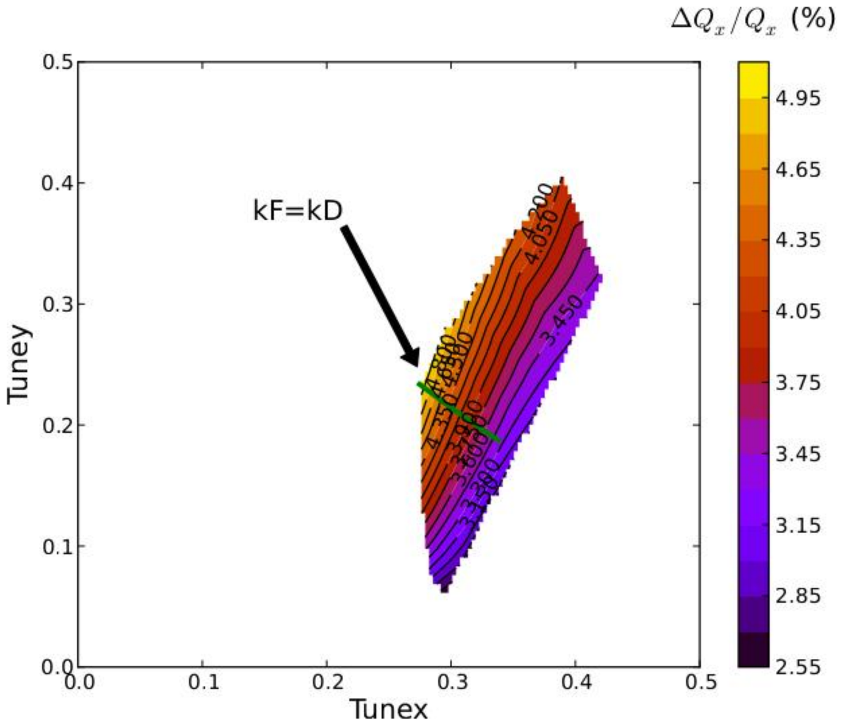


The tune variation for a scaling FFAG in presence of space charge increases by two orders of magnitude. **Can this be cured?**

Non scaling FFAG (1)

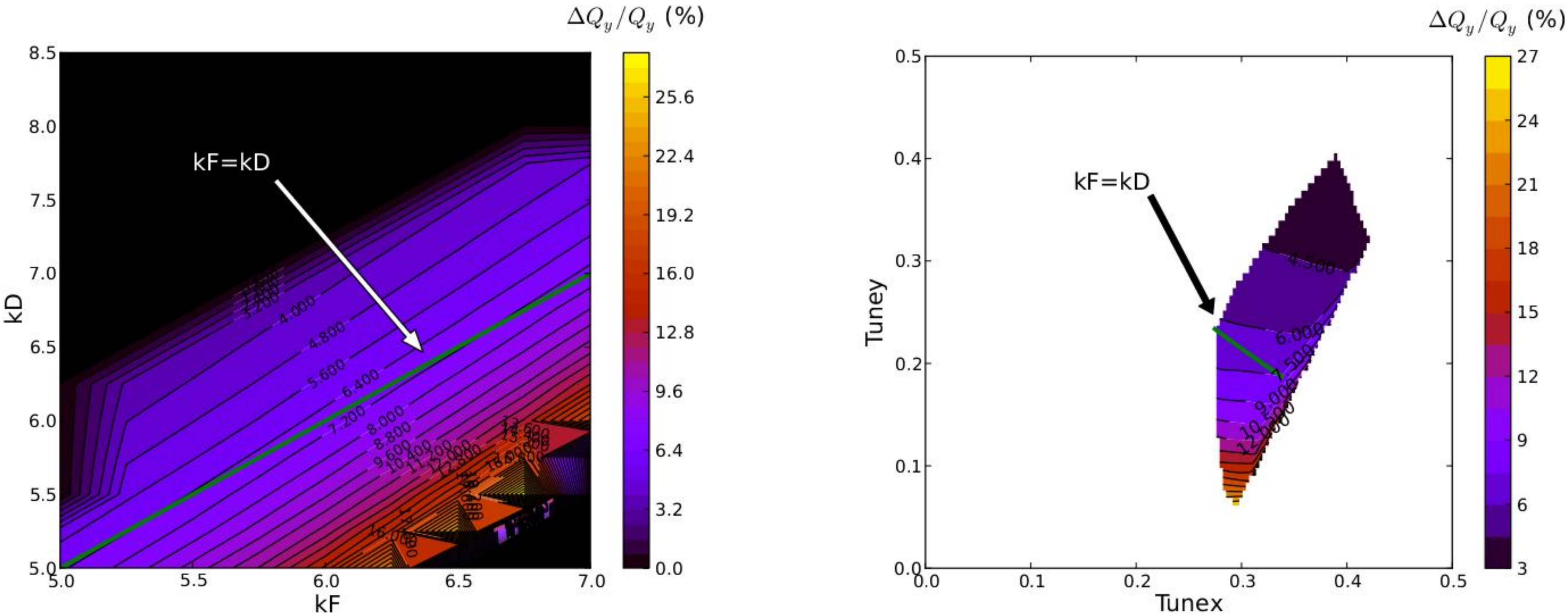


(a) (k_F, k_D) map.



(b) Tune diagram.

Non scaling FFAG (2)



(a) Horizontal plane. (b) Vertical plane.

How to cure the scaling imperfections induced by space charge

- The idea is to play either with kF or kD in such a way to counterbalance the space charge effects which induce a larger tune shift at the injection.
- In other words, could we find a configuration in which, perturbing either kF or kD or both, it would be possible to introduce a tune variation countering the space charge forces, i.e

$$\begin{cases} \nu_x = \nu_{x0} + \delta\nu_x(\delta k_F, \delta k_D) + \delta\nu_x(space\ charge) \\ \nu_y = \nu_{y0} + \delta\nu_y(\delta k_F, \delta k_D) + \delta\nu_y(space\ charge) \end{cases}$$

How to cure the scaling imperfections induced by space charge

This implies

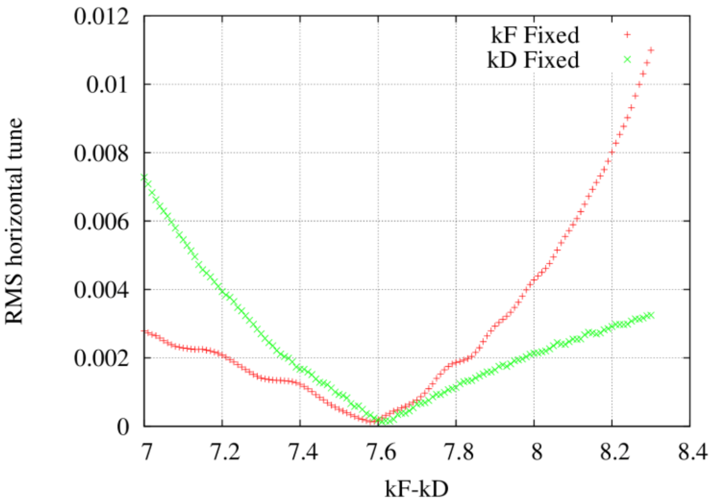
$$\begin{cases} \delta\nu_x(\delta k_F, \delta k_D) = -\delta\nu_x(space\ charge) \\ \delta\nu_y(\delta k_F, \delta k_D) = -\delta\nu_y(space\ charge) \end{cases}$$

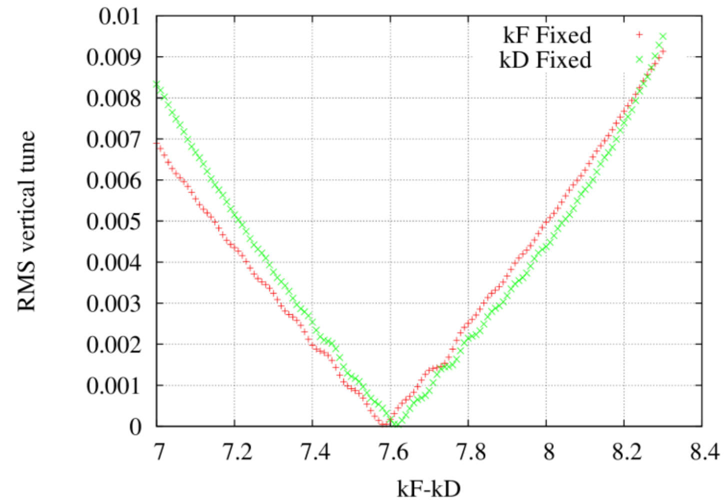
Therefore, the tune shift (introduced deliberately through scaling imperfections) to counterbalance space charge forces must be a **decreasing function of the radius in both planes**.

Reminder: it was shown previously that if $k_F > k_D$, the tune exhibits such a behavior. The question that remains is how to determine $(\delta k_F, \delta k_D)$ to counterbalance the exact space charge effect?

How to cure the scaling imperfections induced by space charge

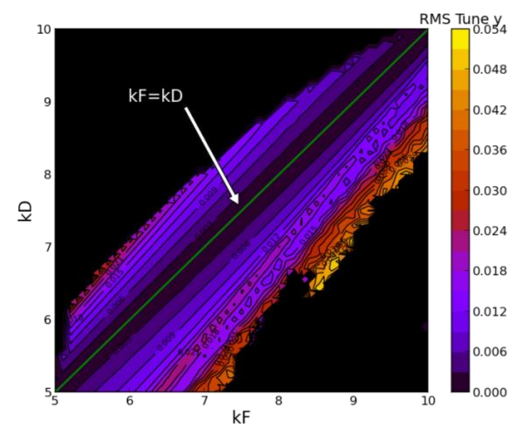
Reminder : it was previously established that the rms tune variations of the bare tune in the vicinity of the central line of the stability diagram (kF=kD), are proportional to $|k_F - k_D|$





Therefore, one can write:

$$\begin{cases} < \nu_{x,bare}^2 >^{1/2} \approx a_x \,.\, |k_F - k_D| \\ < \nu_{y,bare}^2 >^{1/2} \approx a_y \,.\, |k_F - k_D| \end{cases}$$



How to cure the scaling imperfections induced by space charge

Thus, equating the rms tune variations of the bare tunes (with scaling introduced imperfections) with the rms tune variations of the space charge tune, one can write:

$$\begin{cases} <\nu^2_{x,bare}>^{1/2} \approx a_x\,.\,|k_F - k_D| = <\nu^2_{x,spach}>^{1/2} \\ <\nu^2_{y,bare}>^{1/2} \approx a_y\,.\,|k_F - k_D| = <\nu^2_{y,spach}>^{1/2} \end{cases}$$

If $k_F > k_D$, then one obtains (we choose to maintain k_D fixed) ,

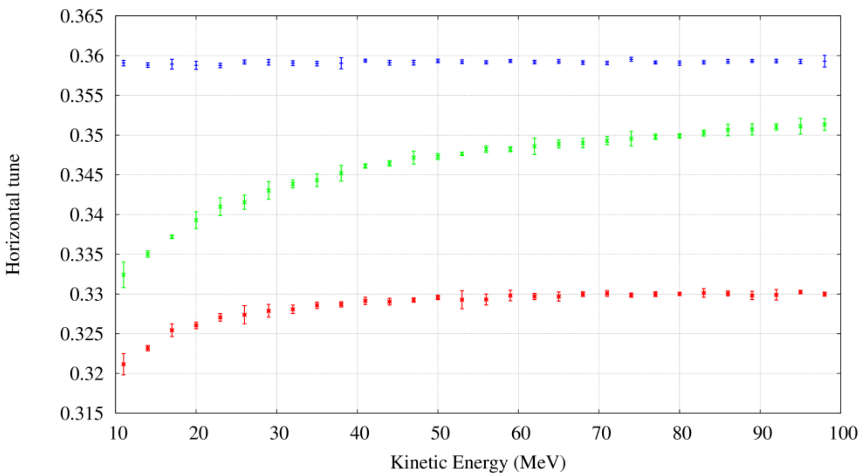
$$k_F \in \mathrm{k_D} + \left[\frac{<\nu^2_{x,spach}>^{1/2}}{a_x} : \frac{<\nu^2_{y,spach}>^{1/2}}{a_y}\right]$$

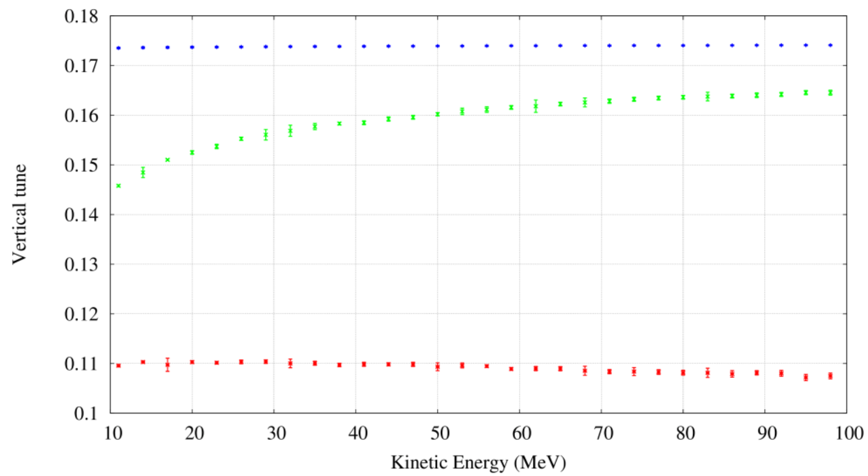
Main assumption: the space charge effects are not much different when scaling imperfections introduced.

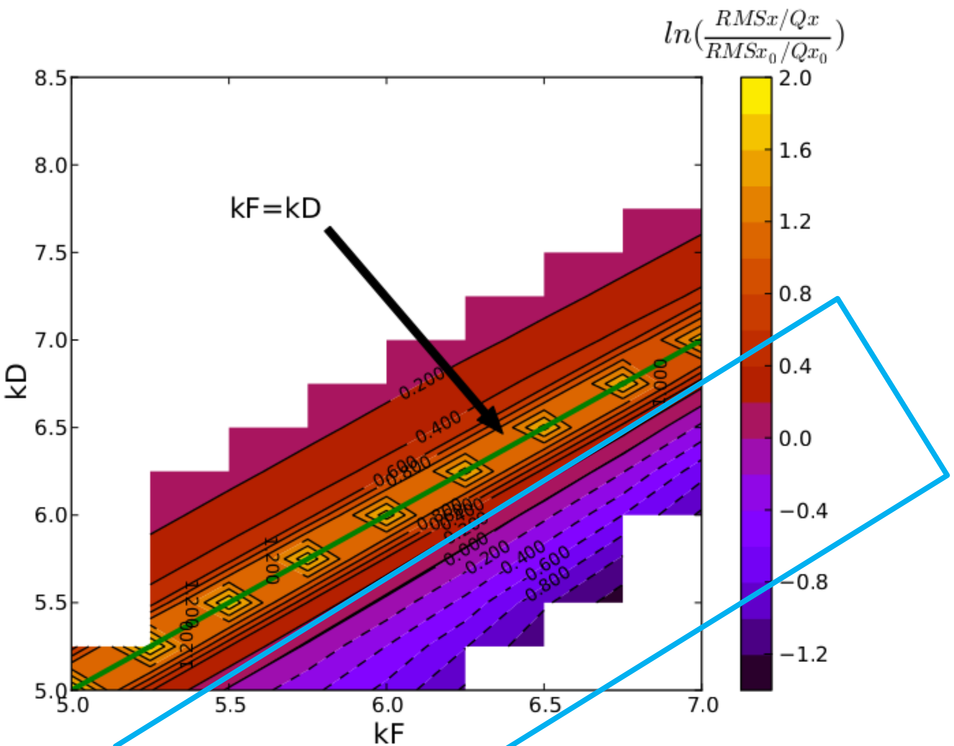
Test example:

We choose the lattice corresponding to $k_F = k_D = 7.6$.
Based on the previous results, one obtains,

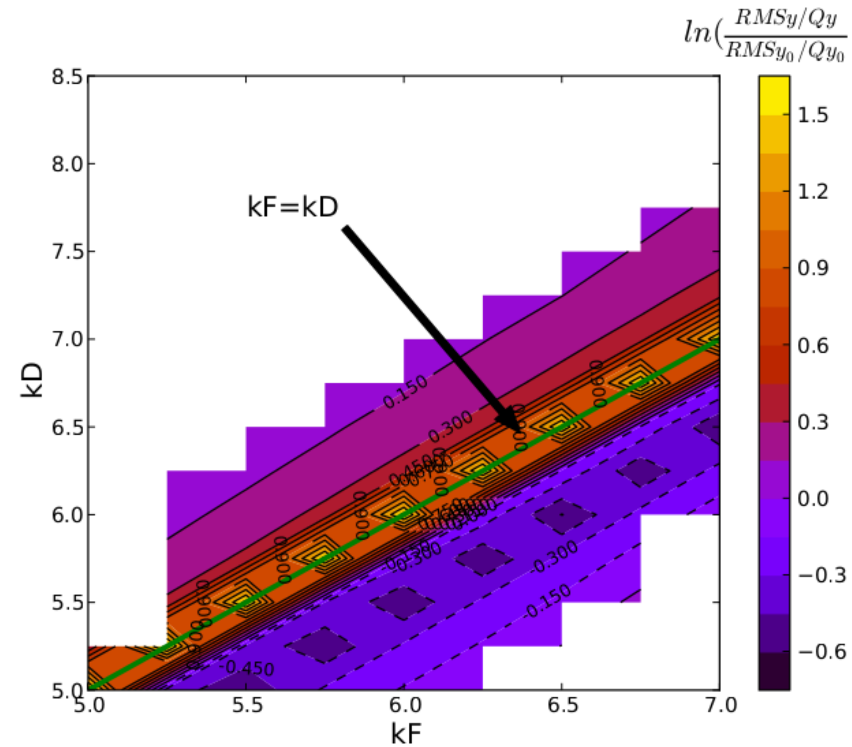
$$k_D \in \mathrm{k_F} - [0.49 : 0.6] = [\,7.0 : 7.11]$$







(a) Horizontal plane.



(b) Vertical plane.

Conclusion

- Combining the effects of space charge with scaling imperfections allows to minimize the tune excursion, provided one is located at the bottom region of the stability diagram (kF>kD)

- For the KURRI FFAG, kF<kD (this is why the tune is essentially an increasing function of the energy). Therefore, the effect of space charge on the rms tune variation will be more detrimental.

- The previous results assume a KV beam. Need to complete further tests with different distribution functions.