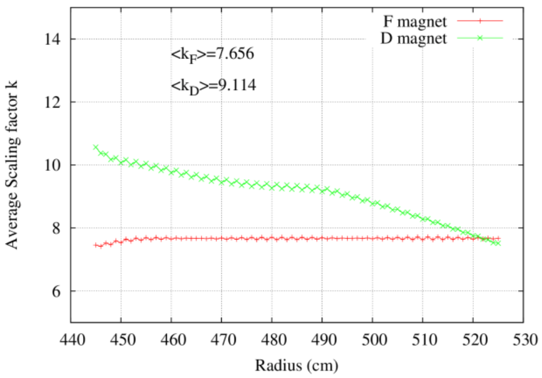
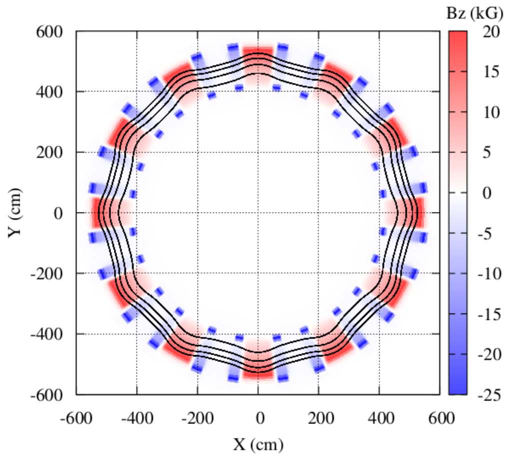
Tune calculation in non-scaling FFAG

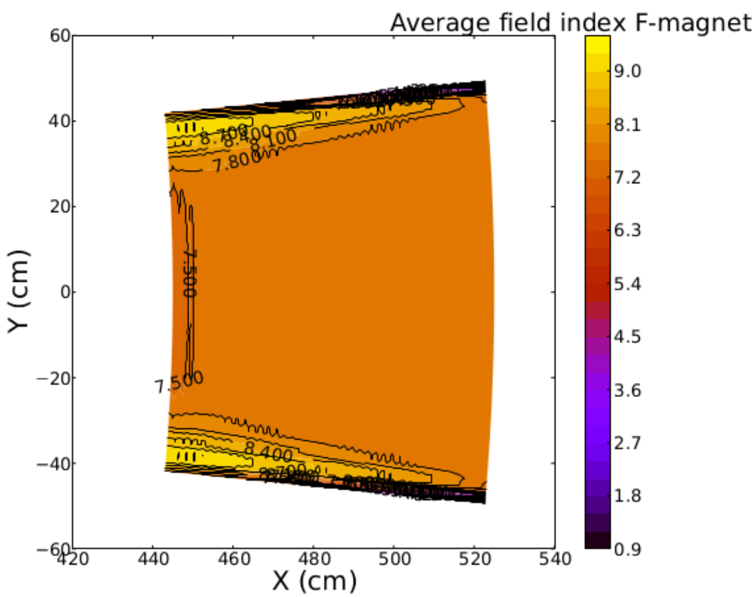
March 23rd , 2016

MALEK HAJ TAHAR
Doctoral student
UJF-Grenoble, BNL C-AD

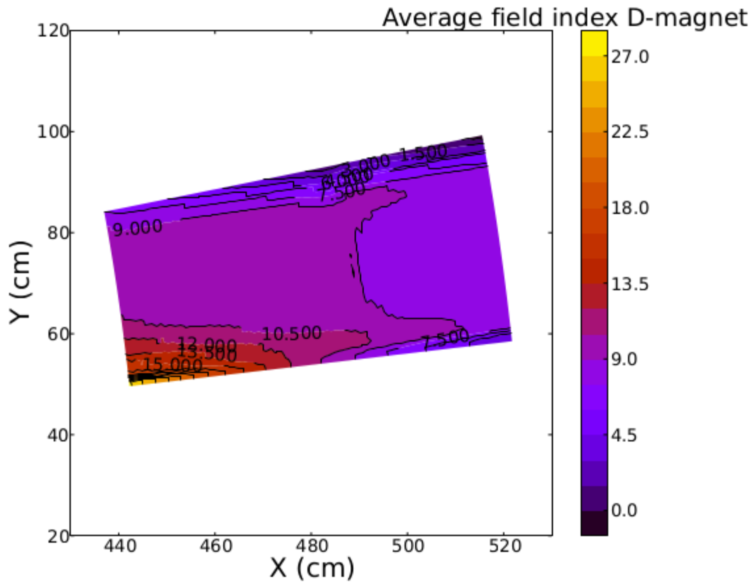
Scaling factors of the F and D magnet







(a) Average field index map of the focusing magnet (k_F)



(b) Average field index map of the defocusing magnet (k_D)

Beam stability analysis

- We use 3 different approaches to investigate the beam stability due to field errors in scaling FFAG:

1) Hard edge model
2) Bogoliubov method of averages
3) Zgoubi tracking model

The aim is to find a relationship between the tunes of the cell and the scaling factors k_F and k_D.

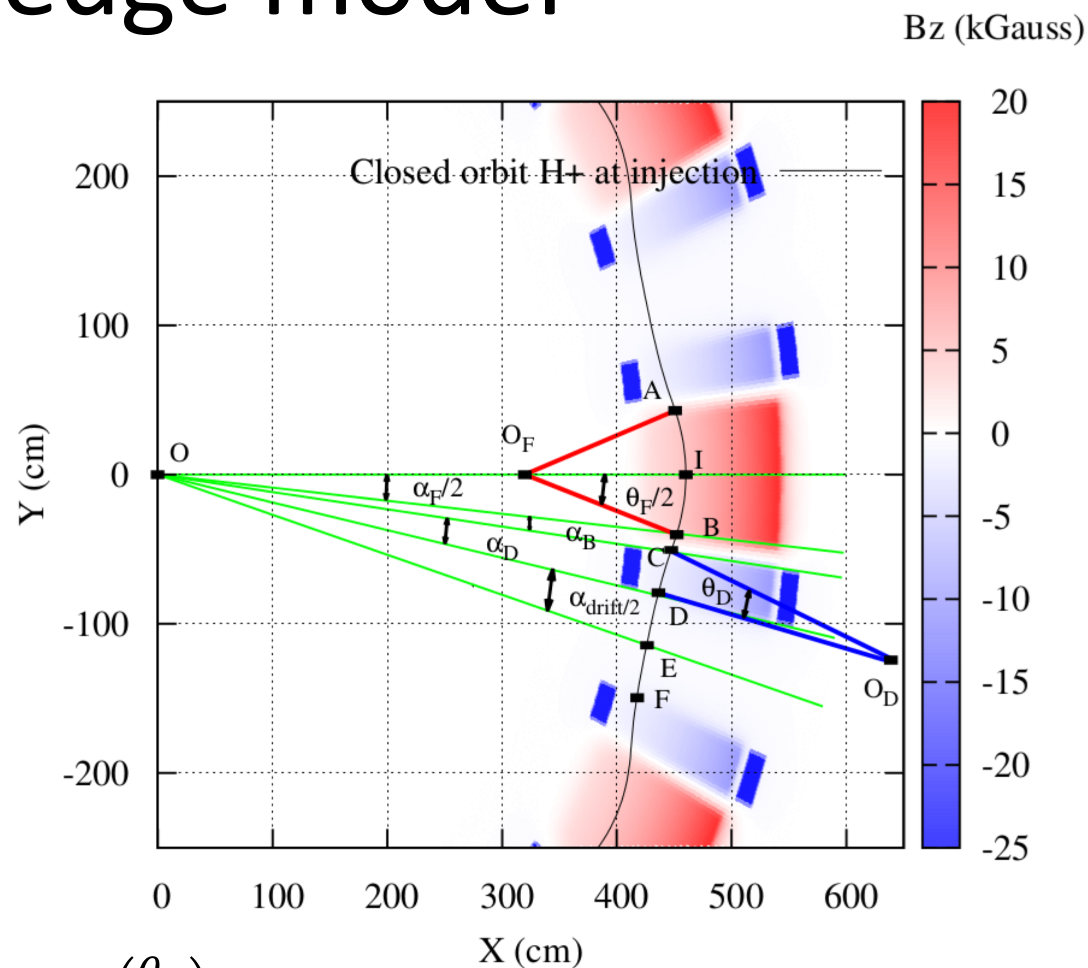
Hard edge model

$$\frac{\rho_F}{R_F} = \frac{\sin\left(\frac{\alpha_F}{2}\right)}{\sin\left(\frac{\alpha_F}{2}\right)+\sin\left(\frac{\theta_F-\alpha_F}{2}\right)}$$

$$\frac{L_B}{\rho_F} = \frac{\sin(\alpha_B)}{\cos\left(\frac{\theta_F-\alpha_F}{2}-\alpha_B\right)} \times \frac{\sin\left(\frac{\theta_F}{2}\right)}{\sin\left(\frac{\alpha_F}{2}\right)}$$

$$\frac{R_D}{L_B} = \frac{\cos\left(\frac{\theta_F-\alpha_F}{2}\right)}{\sin(\alpha_B)}$$

$$\frac{\rho_D}{R_D} = \frac{\sin(\alpha_D)}{cos\left(\alpha_D+\alpha_B+\frac{\theta_D-\theta_F+\alpha_F}{2}\right)} \times \frac{\cos\left(\frac{\theta_D}{2}\right)}{\sin(\theta_D)}$$



Hard edge model

- $\frac{d^2x}{ds^2} + \frac{1-n}{\rho^2} x = 0$
- $\frac{d^2z}{ds^2} + \frac{n}{\rho^2} z = 0$

where $$n(R) \approx -\frac{\rho}{B}\frac{dB}{dR} = -\frac{\rho}{R} k(R) \propto k(R)$$

$$\left\{ \begin{array}{l} \frac{d^2x}{ds^2} + \frac{1 + \frac{\rho}{R} k(R)}{\rho^2} x = 0 \\ \frac{d^2z}{ds^2} - \frac{k(R)}{\rho R} z = 0 \end{array} \right.$$

Conjecture:

- Back to Symon model:

We make the following conjecture:)

The Vertical tune of a DFD cell is given by:

$$\nu_z^2 = x_1 k_F + x_2 k_D + x_3$$

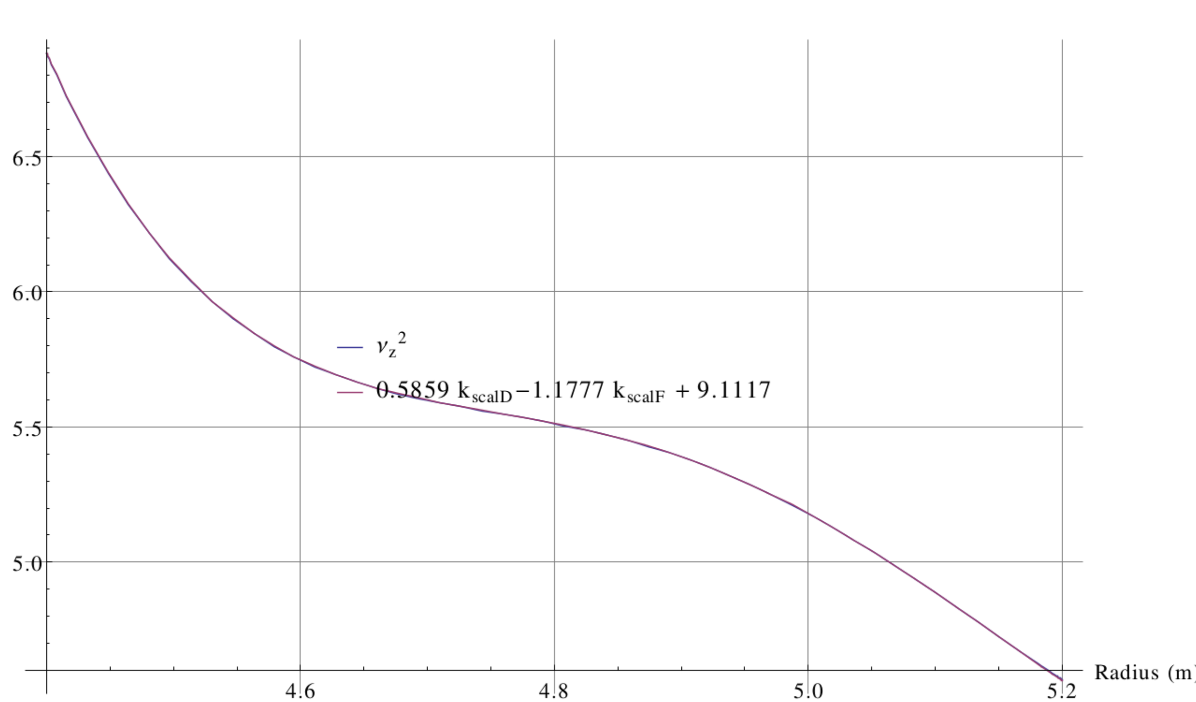
Analogy with $\nu_z^2 = -k + \frac{f^2}{2}$

The horizontal tune of a DFD cell is given by:

$$\nu_x^2 = x_1 k_F + x_2 k_D + (x_3 k_F + x_4 k_D + x_5)^2$$

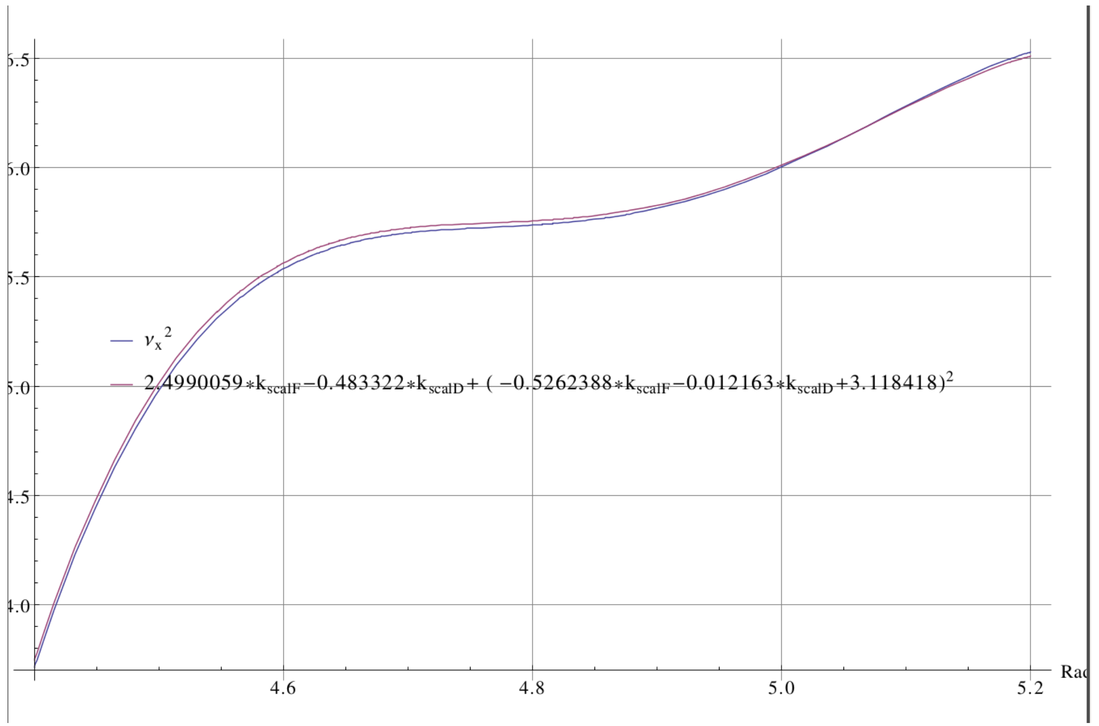
Analogy with $\nu_x^2 = k + 1 + A.(k+1)^2$

Solving the Vertical Tune equation



The conjecture is valid.

Solving the Horizontal Tune equation



The conjecture is valid.

- Can the previous conjecture be proven in the general case where $\frac{\rho}{R} \neq constant$?
- This would be a generalization to the case of a non-scaling FFAG.

- For that reason, the Bogoliubov method of averages has been applied.

Bogoliubov method of averages

- Write the Hill's equation in cylindrical coordinates:

$$\begin{cases} \dfrac{d^2x}{d\theta^2} + \left[\mu(R,\theta)^2[1-n]\right] x = 0 \quad ; \quad \mu(R,\theta) = \dfrac{R}{\rho} \\ \dfrac{d^2y}{d\theta^2} + \left[\mu(R,\theta)^2 n\right] y = 0 \end{cases}$$

$$\frac{ds}{d\theta} = \sqrt{R^2 + (\frac{dR}{d\theta})^2} \approx R$$

Arclength in cylindrical coordinates

Bogoliubov method of averages

- This can be written in the standard form:

$$\frac{d^2x}{d\theta^2} + g(R,\theta)x = 0$$

$$\begin{aligned}
\nu^2(R) &= \langle g\rangle + \frac{1}{N^2}\left\langle \tilde{g}^2\right\rangle + \ldots \\
&\approx \langle g(R,\theta)\rangle + \left\langle \left[\int \{g(R,\theta) - \langle g(R,\theta)\rangle\} d\theta\right]^2 \right\rangle \\
&= g_1(R) + g_2(R)
\end{aligned}$$

Bogoliubov method of averages

Calculate the field index n:

$$n = -\frac{\rho}{B}\frac{\partial B}{\partial x} = -\frac{\rho}{B}\left[\frac{\partial B}{\partial R}\frac{\partial R}{\partial x} + \frac{\partial B}{\partial \theta}\frac{\partial \theta}{\partial x}\right]$$

$$\begin{cases} g_x(R,\theta) = \left(\frac{R}{\rho}\right)^2 \times (1-n) \approx \left(\frac{R}{\rho}\right)^2 \times \left[1 + \frac{\rho}{R}k(R)\right] \\ g_y(R,\theta) = \left(\frac{R}{\rho}\right)^2 \times n \approx -\frac{R}{\rho} \times k(R) \end{cases}$$

Introduce the azimuthal dependence of k in the following way:

$$k(R,\theta) = \begin{cases} k_F(R), & \text{if } \theta \in \theta_F \\ k_D(R), & \text{if } \theta \in \theta_D \\ k_{drift}(R), & \text{if } \theta \in \theta_{drift} \end{cases}$$

Bogoliubov method of averages

- This yields:

$$\boxed{\nu_x^2(R_E) = \sum_i \beta_i(R_E) - \sum_i \alpha_i(R_E) \times k_i(R_E)}$$

$$\boxed{\nu_y^2(R_E) = \sum_i \alpha_i(R_E) \times k_i(R_E) + \mathcal{F}^2 \left[1 + 2\tan^2(\xi)\right]}$$

where

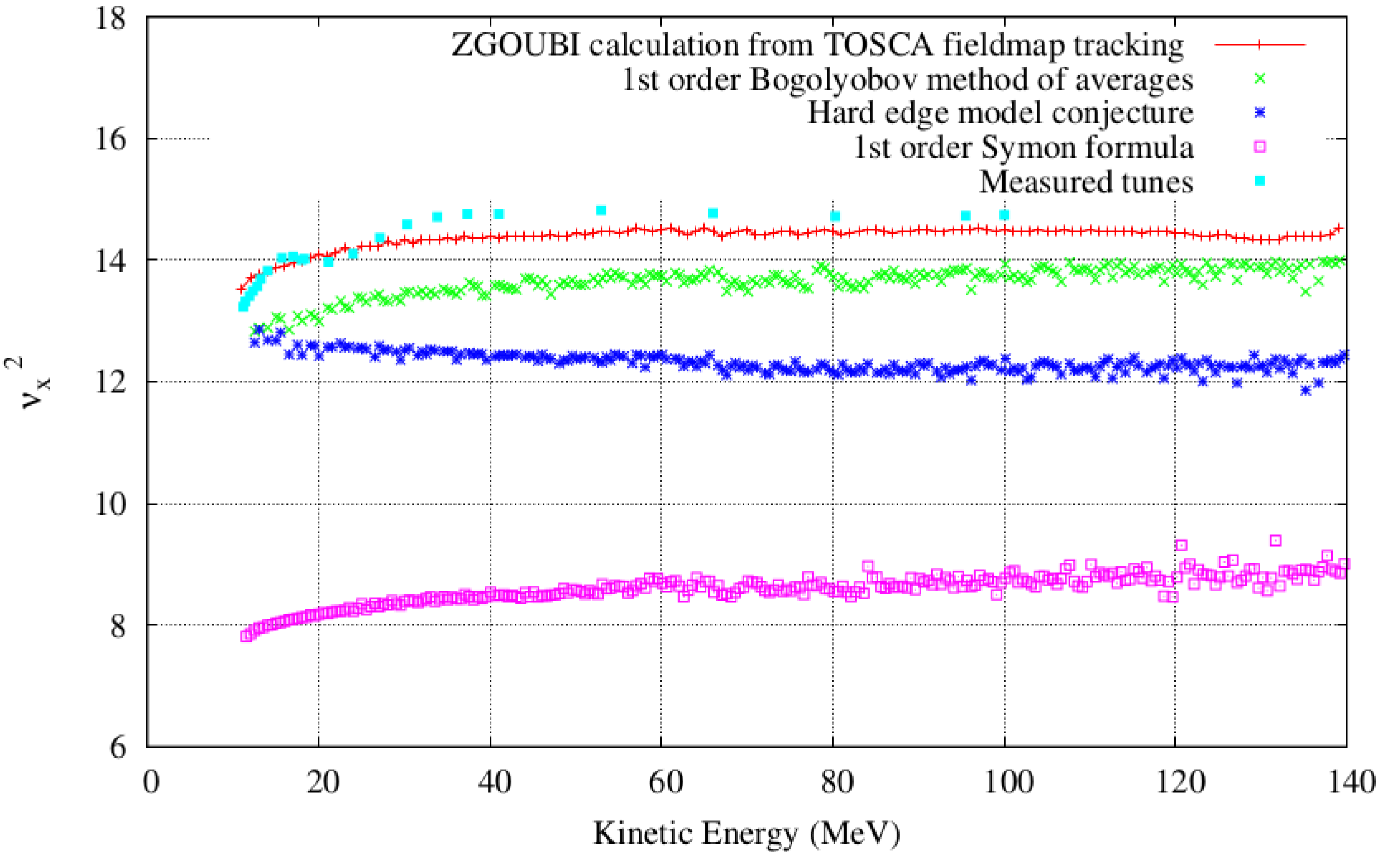
$$\alpha_i(R_E) = \frac{-1}{2\pi/N} \int_{\theta_i} \mu(R,\theta)d\theta = \frac{-1}{2\pi/N} \int_{\theta_i} \frac{R}{\rho} d\theta$$

$$\beta_i(R_E) = \frac{1}{2\pi/N} \int_{\theta_i} \mu(R,\theta)^2 d\theta = \frac{1}{2\pi/N} \int_{\theta_i} \left(\frac{R}{\rho}\right)^2 d\theta$$

Interpretation …

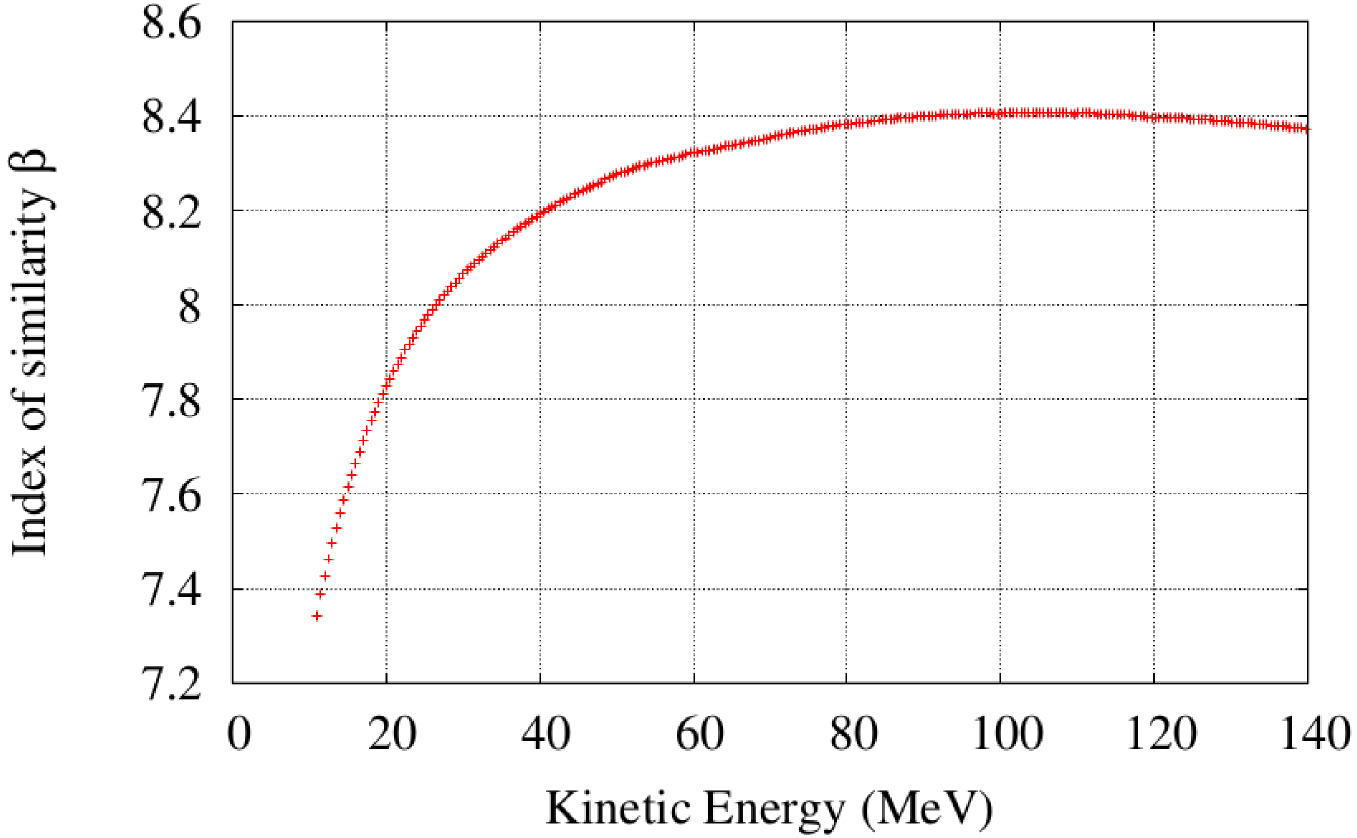
Application to the KURRI 150 MeV FFAG

$$\nu_x^2(E) = \beta_F(E) + 2\beta_D(E) + \beta_{drift}(E) - \alpha_F(E)k_F(E) - 2\alpha_D(E)k_D(E) - \alpha_{drift}(E)k_{drift}(E)$$
$$\nu_y^2(E) = \alpha_F(E)k_F(E) + 2\alpha_D(E)k_D(E) + \alpha_{drift}(E)k_{drift}(E)$$



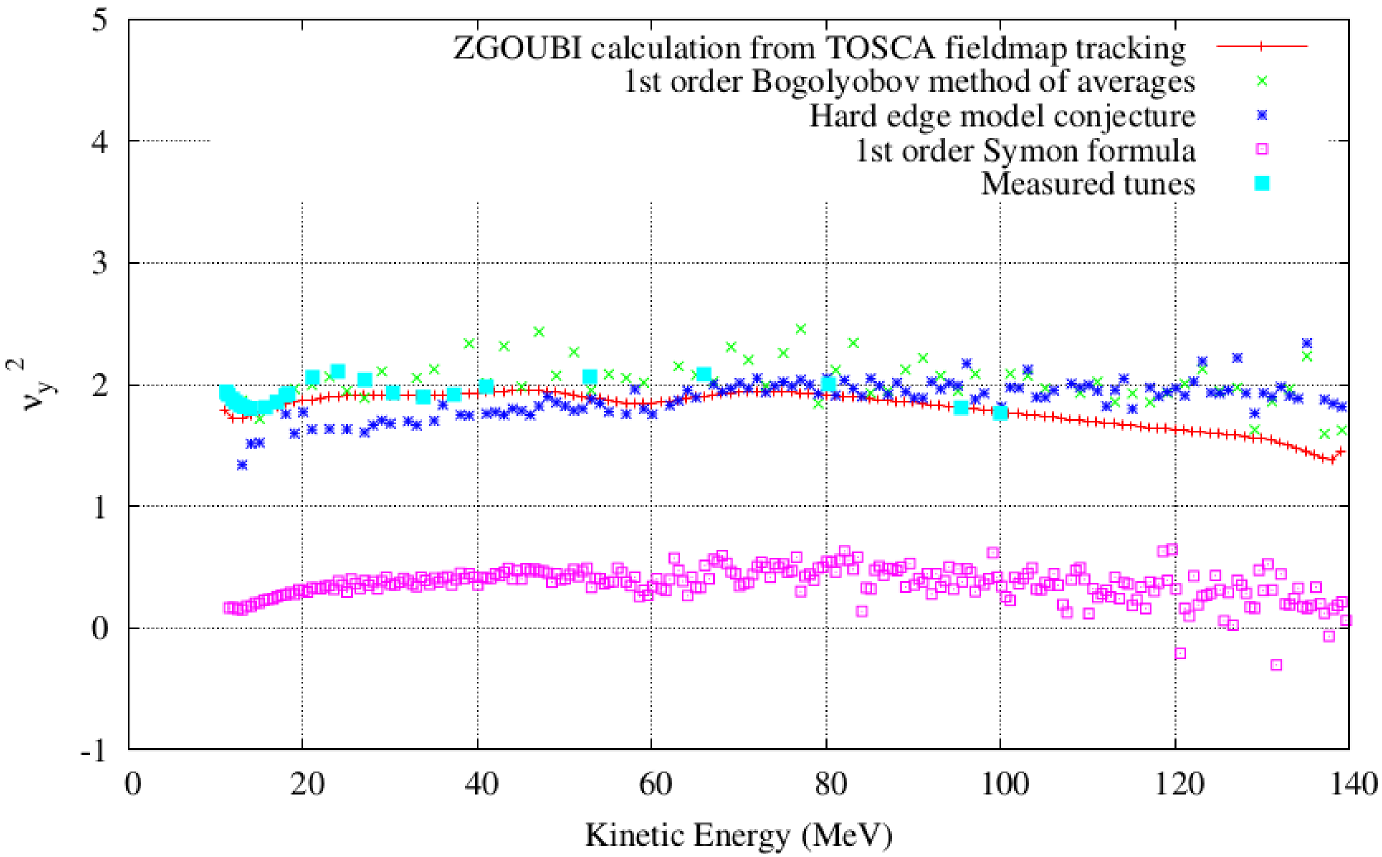
Index of similarity β

$$\beta_i(R_E) = \frac{1}{2\pi/N}\int_{\theta_i} \mu(R,\theta)^2 d\theta = \frac{1}{2\pi/N}\int_{\theta_i} \left(\frac{R}{\rho}\right)^2 d\theta$$

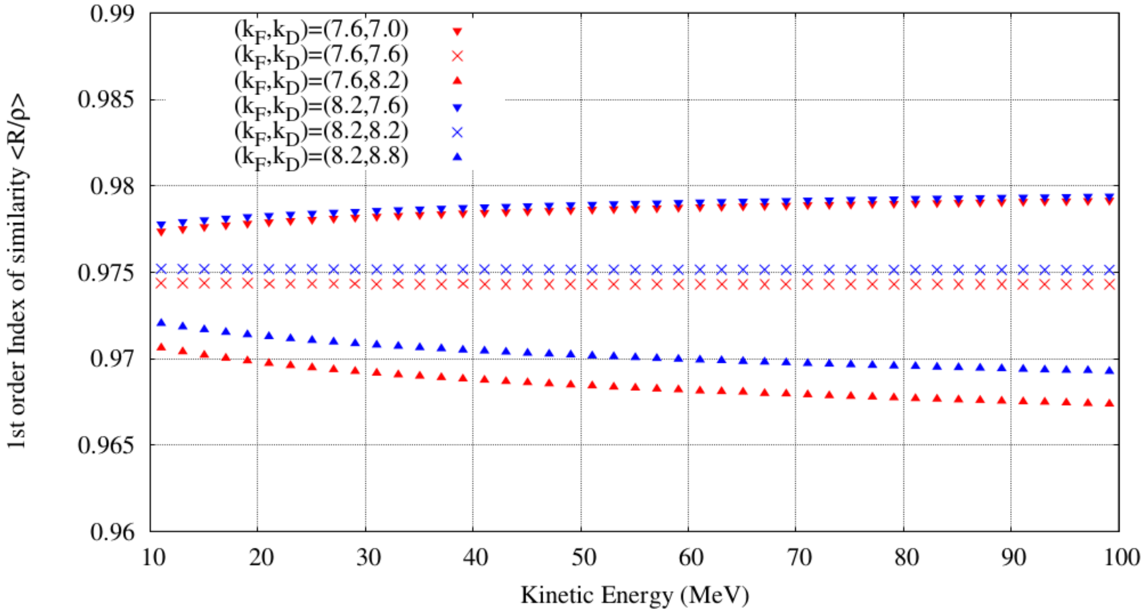


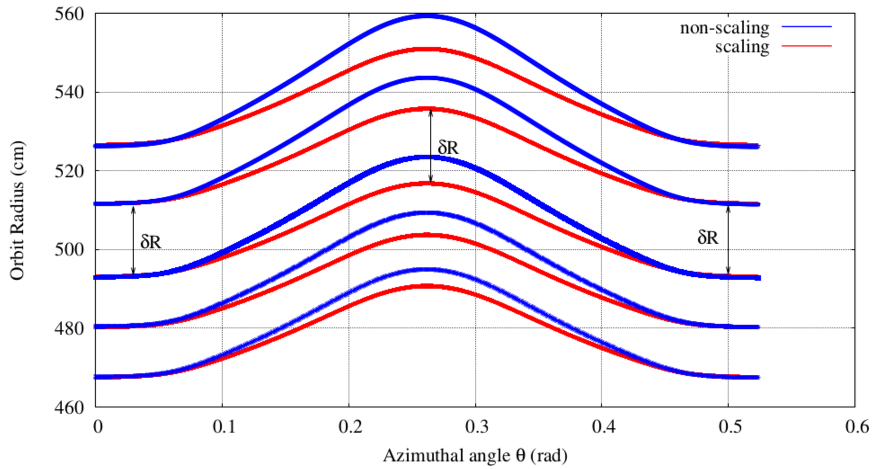
Application to the KURRI 150 MeV FFAG

$$\nu_x^2(E) = \beta_F(E) + 2\beta_D(E) + \beta_{drift}(E) - \alpha_F(E)k_F(E) - 2\alpha_D(E)k_D(E) - \alpha_{drift}(E)k_{drift}(E)$$
$$\nu_y^2(E) = \alpha_F(E)k_F(E) + 2\alpha_D(E)k_D(E) + \alpha_{drift}(E)k_{drift}(E)$$



Scaling ($k_F = k_D$) vs non-scaling ($k_F \neq k_D$)



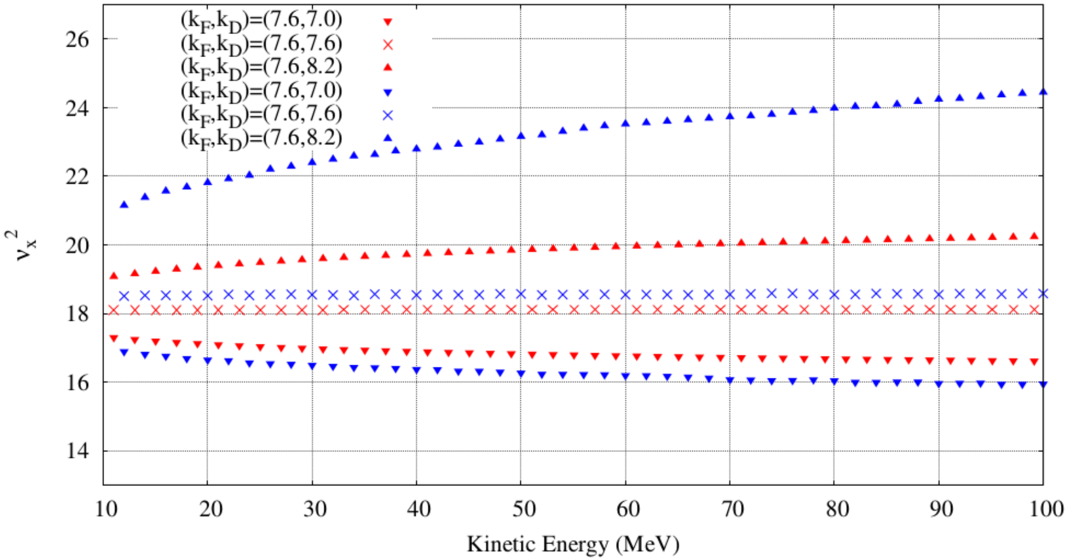


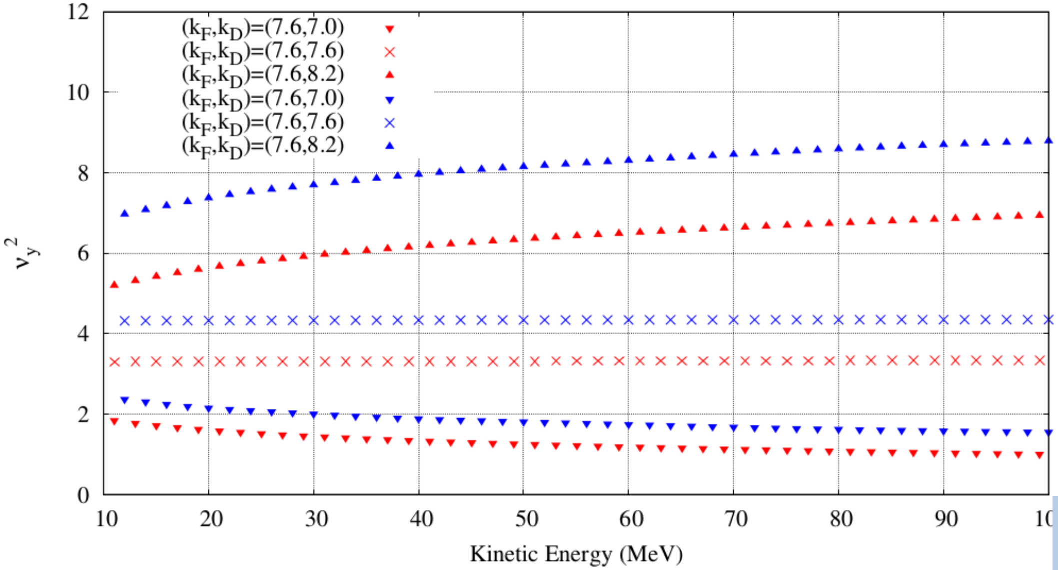
The F and D magnets show antagonistic behavior

Comparison of the Zgoubi results (blue) with the 1st order bogoliubov method of averages show good agreement.

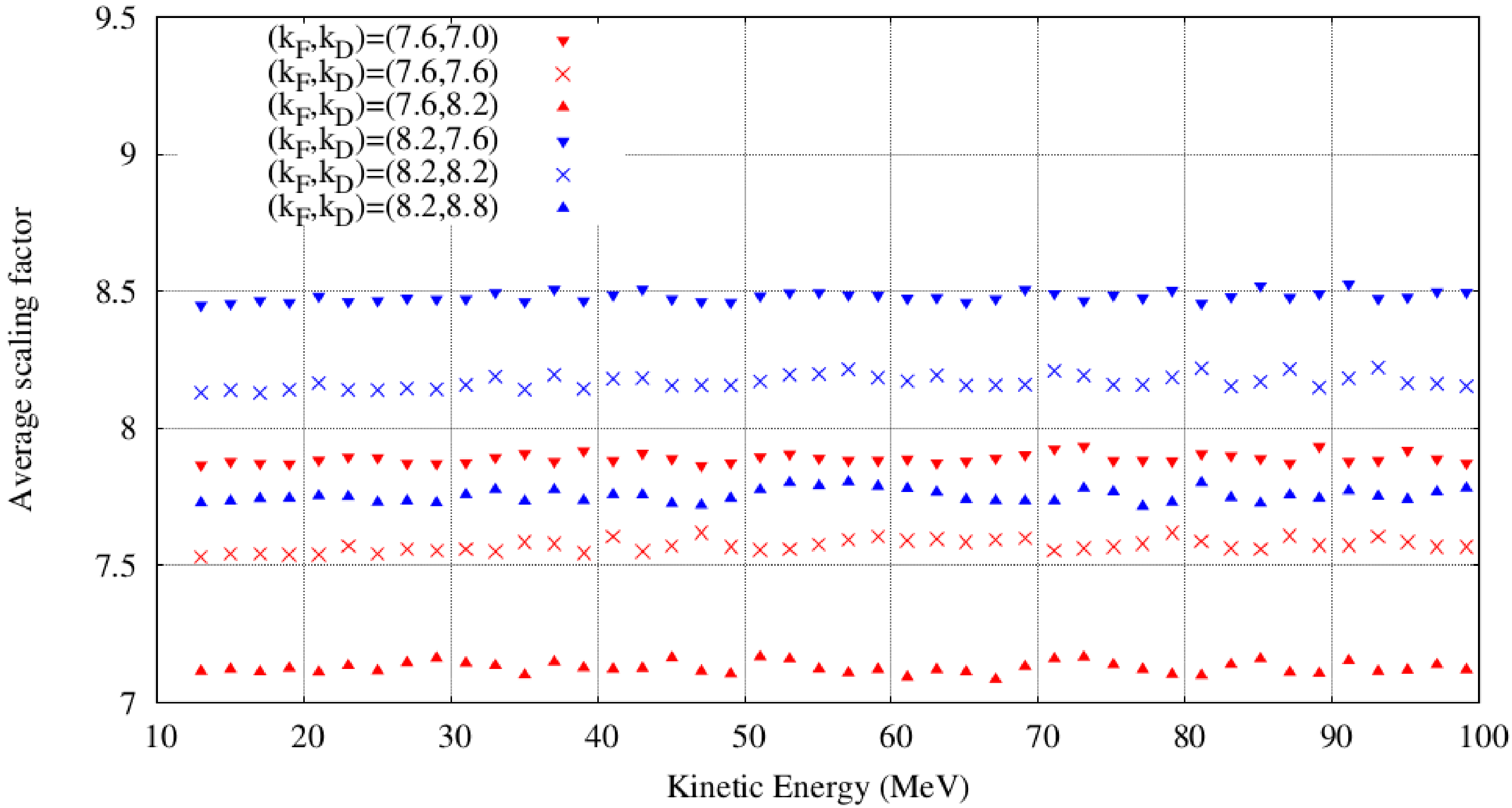
Dpending on whether $k_F > k_D$ or the opposite, the tune exhibits antagonistic behavior.

Thus, in presence of systematic errors of the field, i.e $k_D > k_F$ for instance, the idea would be to introduce the opposite error by implementing trim coils every two sectors. The superperiod becomes DFD-DFD





Average scaling factor



Having the average scaling factor k constant is not a sufficient condition to obtain a fixed tune machine

Thank you