M. Haj Tahar

**Output from "KURRI FFAG" data treatment**

# 1 TOSCA map "TOSCA map f810 d1020.dat"

This section shows tracking results that describe the ring and check the correct behavior of the field reading and interpolation process. By ensuring the symmetry of the field experienced on each trajectory, from entrance

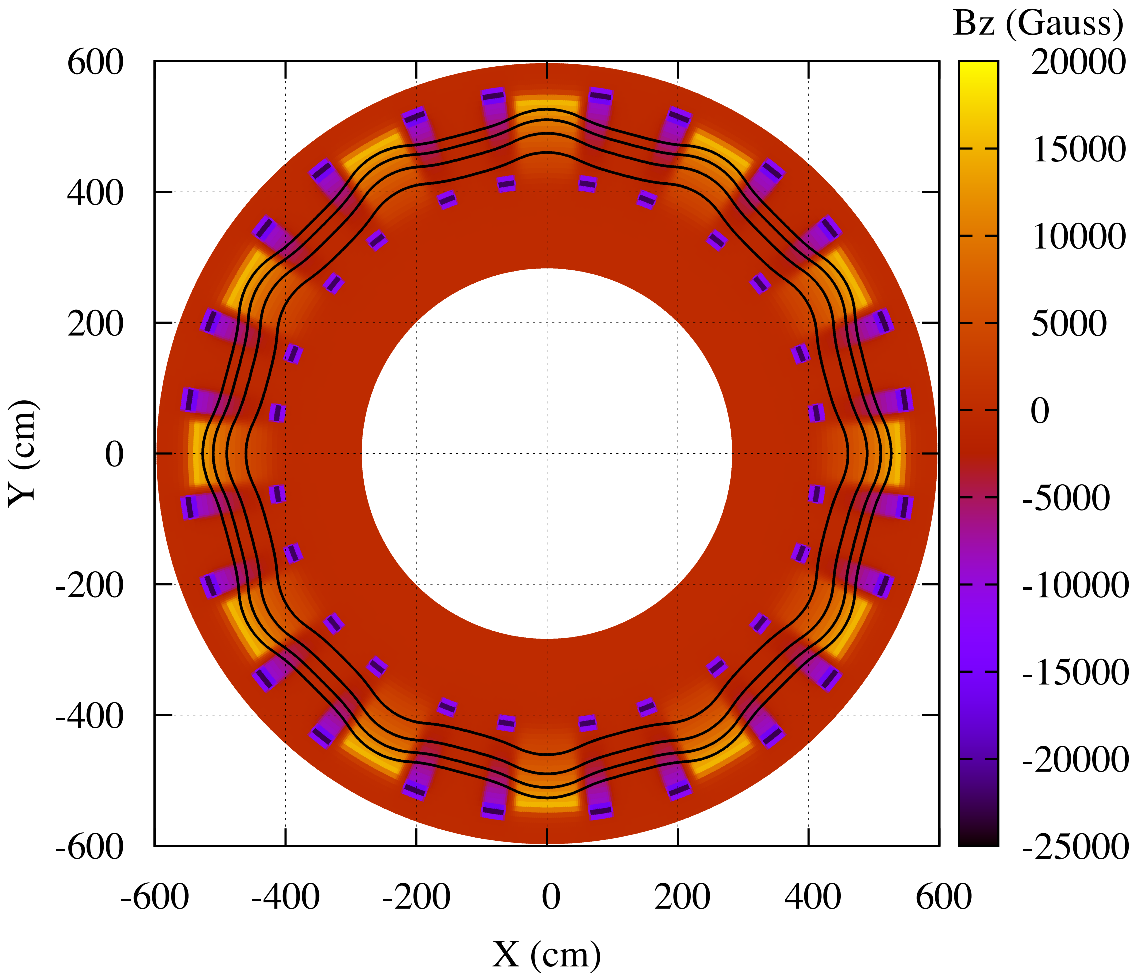


**Figure 1:** Plot of the z-component of the magnetic field in the median plane of the machine. Some of the Closed orbits (11, 30, 60 and 100 MeV) obtained from tracking are also shown here.

to exit of the cell, the Closed Orbits are obtained. Some of these orbits are shown in Fig. 1. Fig. 2 and Fig. 3 show the z component of the magnetic field along different trajectories from injection (11 MeV) to extraction (150 MeV).

# 2 Beam envelope

The axial and radial beam envelopes at injection are shown in Fig. 4 and 5 respectively. We can see that the symmetry of the orbit is ensured with respect to the azimuthal position $\theta = 0$ deg.

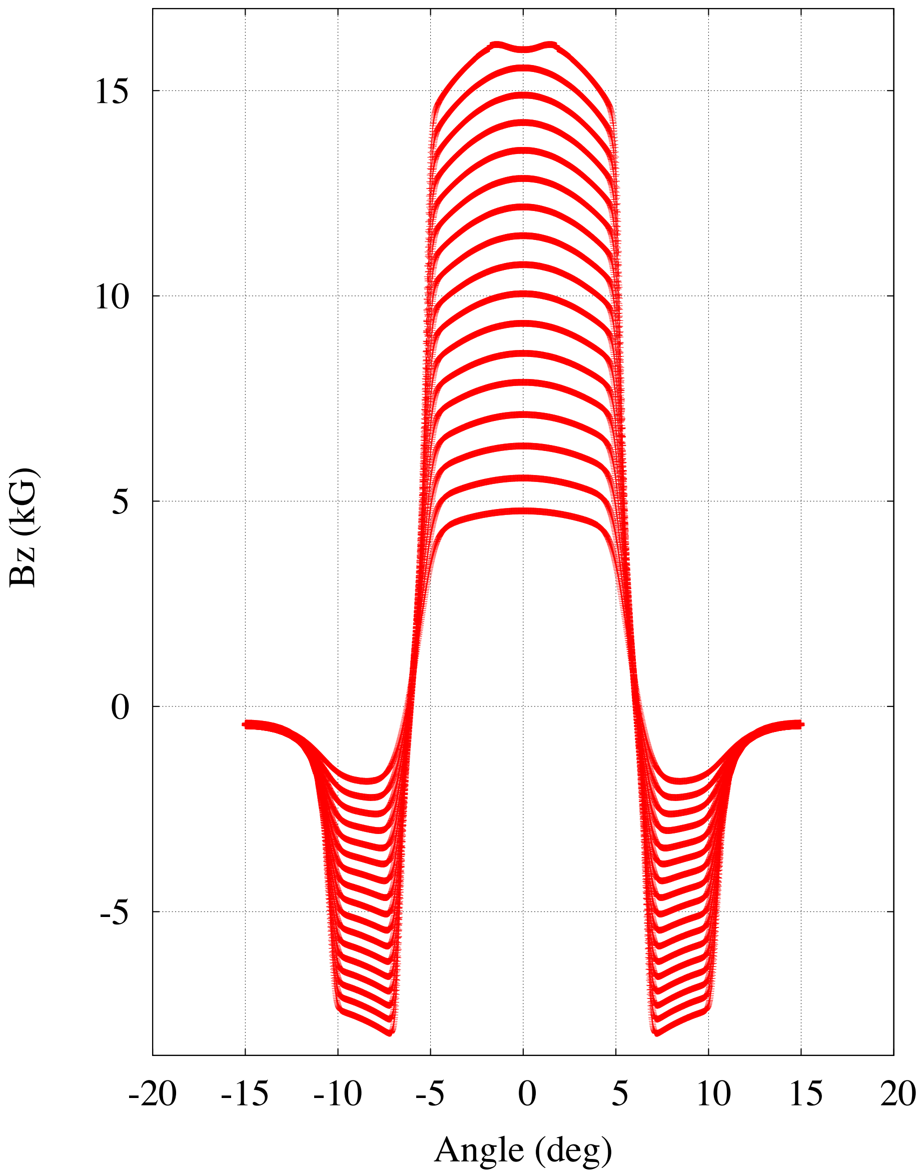


**Figure 2:** Magnetic field for one sector of the ring obtained for different trajectories (from injection (11 MeV) to extraction (150 MeV). Some discontinuities of the field are observed near the extraction (mainly for 150 MeV) at the azimuthal position $\theta = \pm 2$ deg (see Fig. 3)

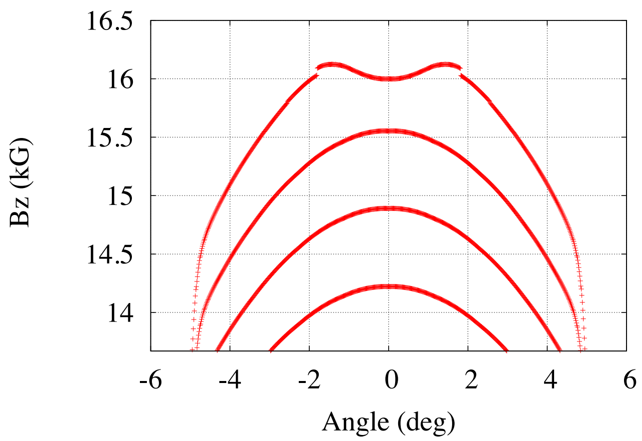


**Figure 3:** Zoom in the central region of the magnet for 4 different trajectories (CO): 113.38, 125.12, 137.32 and 150 MeV.

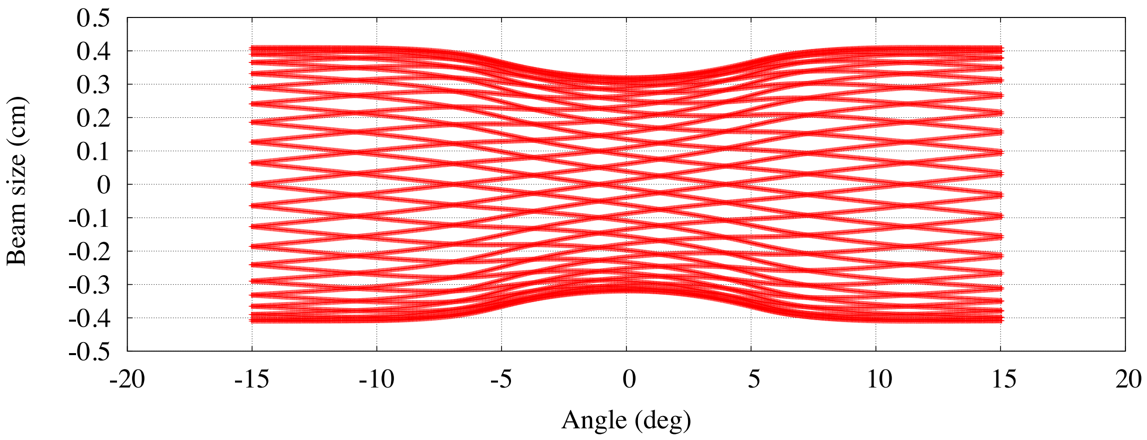


**Figure 4:** Axial beam envelope for one sector at injection energy (11 MeV). Normalized emittance $\epsilon_{norm}$ (90 %) = 0.630 mm.mrad) as specified in [3]

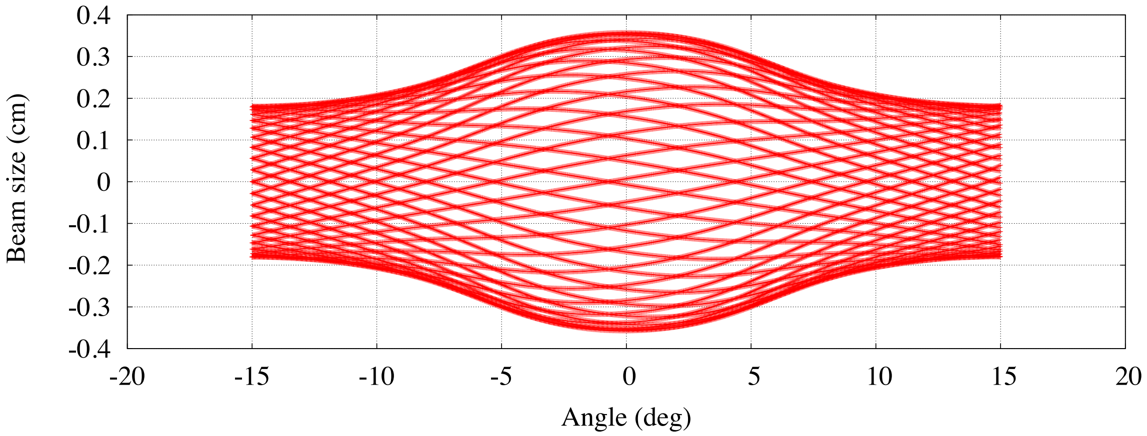


**Figure 5:** Radial beam envelope for one sector at injection energy (11 MeV). Normalized emittance $\epsilon_{norm}$ (90 %) = 0.680 mm.mrad)

## 3 Injection orbit

We search for the injection orbit: from backward tracking started at the foil position R = 460.146cm ; $\theta$=14.5 deg, we obtain the injection trajectory for the H- ions shown below in green.The field map is formatted in such a way that we start the backward tracking at the exact location of the foil. From that, an injection orbit is deduced.
The dispersion function is computed at the location of the foil: D = 0.5795 m. Comparing the particle position at the foil location with the expected values from the dispersion function there, we obtain the results shown in Table 1.

| Momentum change (Δp/p) | Expected values (D. Δp/p) | Zgoubi results (ΔR) |
|---|---|---|
| +1% | 0.579 cm | 0.572 cm |
| -1% | 0.579 cm | 0.563 cm |

**Table 1:** Comparison between particle position displacements at the foil location obtained from tracking and those obtained from computing the periodic dispersion function.

It can be seen that the orbit with higher momentum (Δp/p = +1 %) has lower radius, which can be understood from the formula:
$B\rho = \frac{p}{-q}$; q > 0 for H- ions at injection. This means that $p \nearrow \Longrightarrow B\rho \searrow$ or $\rho \searrow$ assuming that the field does not change much in that region (see Fig. 8).
Using the formula [4], $\frac{(B\rho)_2}{(B\rho)_1} = (\frac{r_2}{r_1})^{k+1}$, with $\frac{(B\rho)_2}{(B\rho)_1} = \frac{p_{+1\%}}{p_{inj}} = \frac{145.5598}{144.1187} = 1.01$,

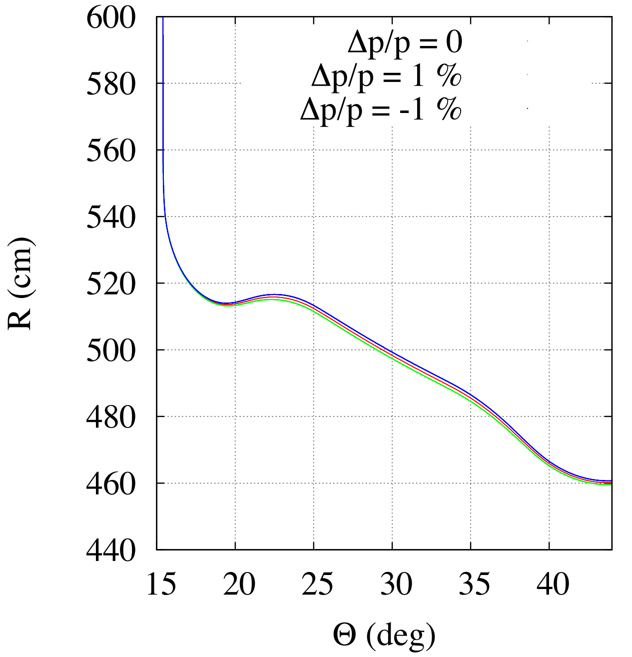


**Figure 6:** Injection trajectories for different set of $\Delta$p/p

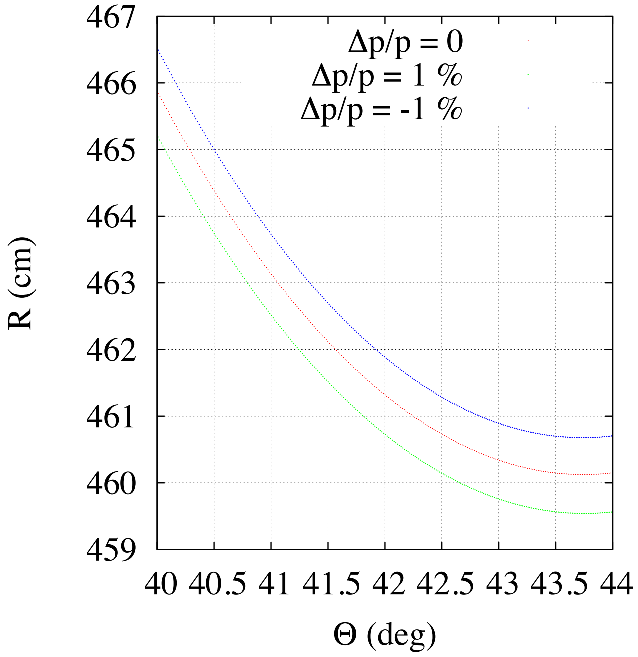


**Figure 7:** Zoom in Fig. 6: the red trajectory corresponds to $\Delta$p/p = 0, the green and blue trajectories display the result for $\Delta$p/p = +1 % and -1 % respectively.

$\Longrightarrow \Delta R = r_2 - r_1 = r_1 \times ((\frac{(B\rho)_2}{(B\rho)_1})^{\frac{1}{1+k}} - 1) = 0.5326$ cm ($r_1 = 460.146cm$ at the location of the foil and k=7.6), which is consistent with the results obtained in Table 1.

By varying the magnetic field strength by $\pm$ 1 %, it was shown (by comparing the trajectories obtained from tracking) that a change by +1 % of the field strength is equivalent to a change by -1 % of the momentum, and vice versa.

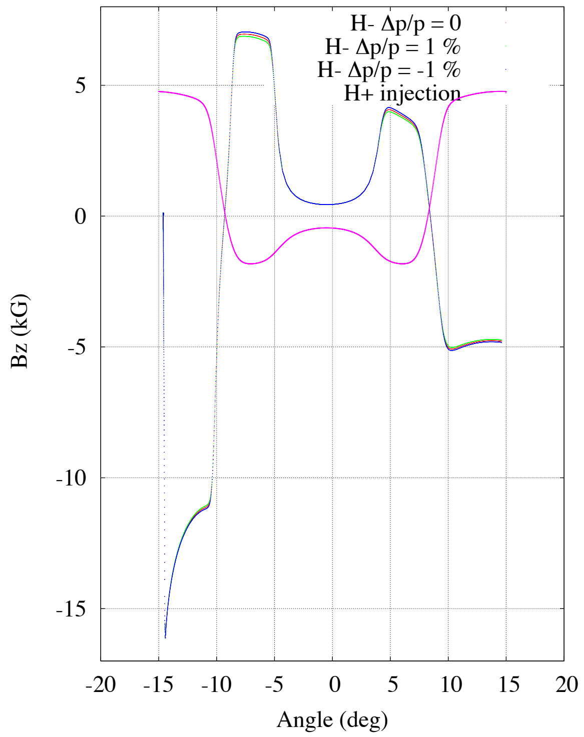


**Figure 8:** Magnetic field along different injection energies: in red, green and blue $\Delta p/p = 0$, 1 and -1 % respectively.

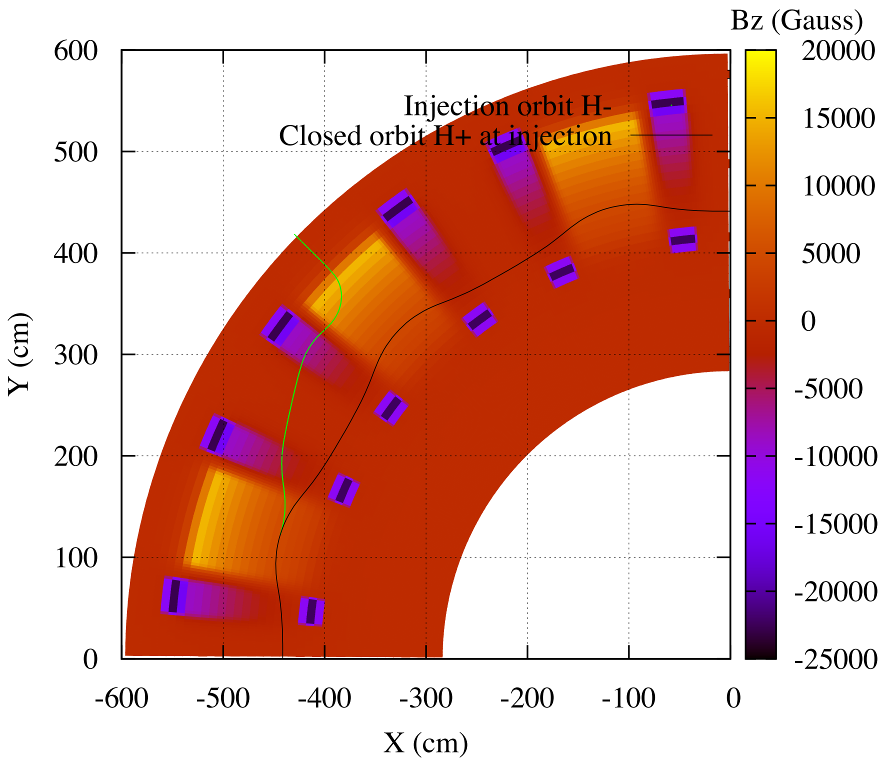


**Figure 9:** Injection trajectory for H- ions (shown in green).

## References


[1] EPAC2004 on KEK 150 MeV commissioning

[2] FFAG12 presentation on tune measurement

[3] H- injection at 150 MeV-FFAG in KURRI.

[4] Spiral FFAG lattice dsign tools. Application to 6-D tracking in a proton-therapy class lattice. http://www.sciencedirect.com/science/article/pii/S0168900208001228