

# Space charge simulation update (3)

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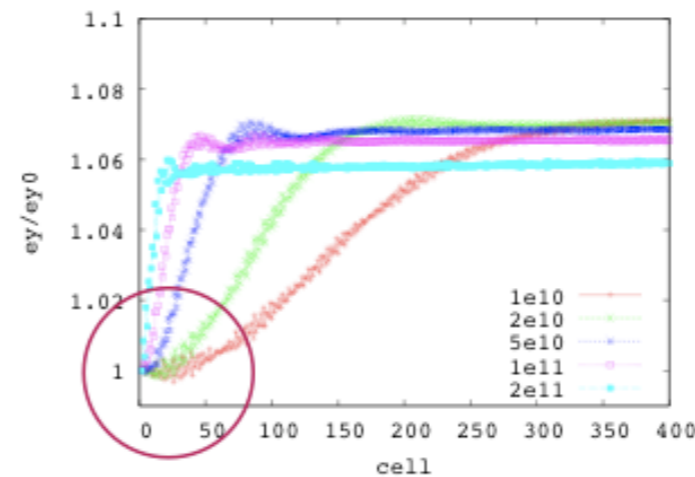
# Contents

- Calculation of tune shift and spread
- Emittance growth
  - ideal lattice
  - alignment error of 2 mm (rms)
- Multi-turn injection

# Slide from the last meeting

## Perfect lattice (2)

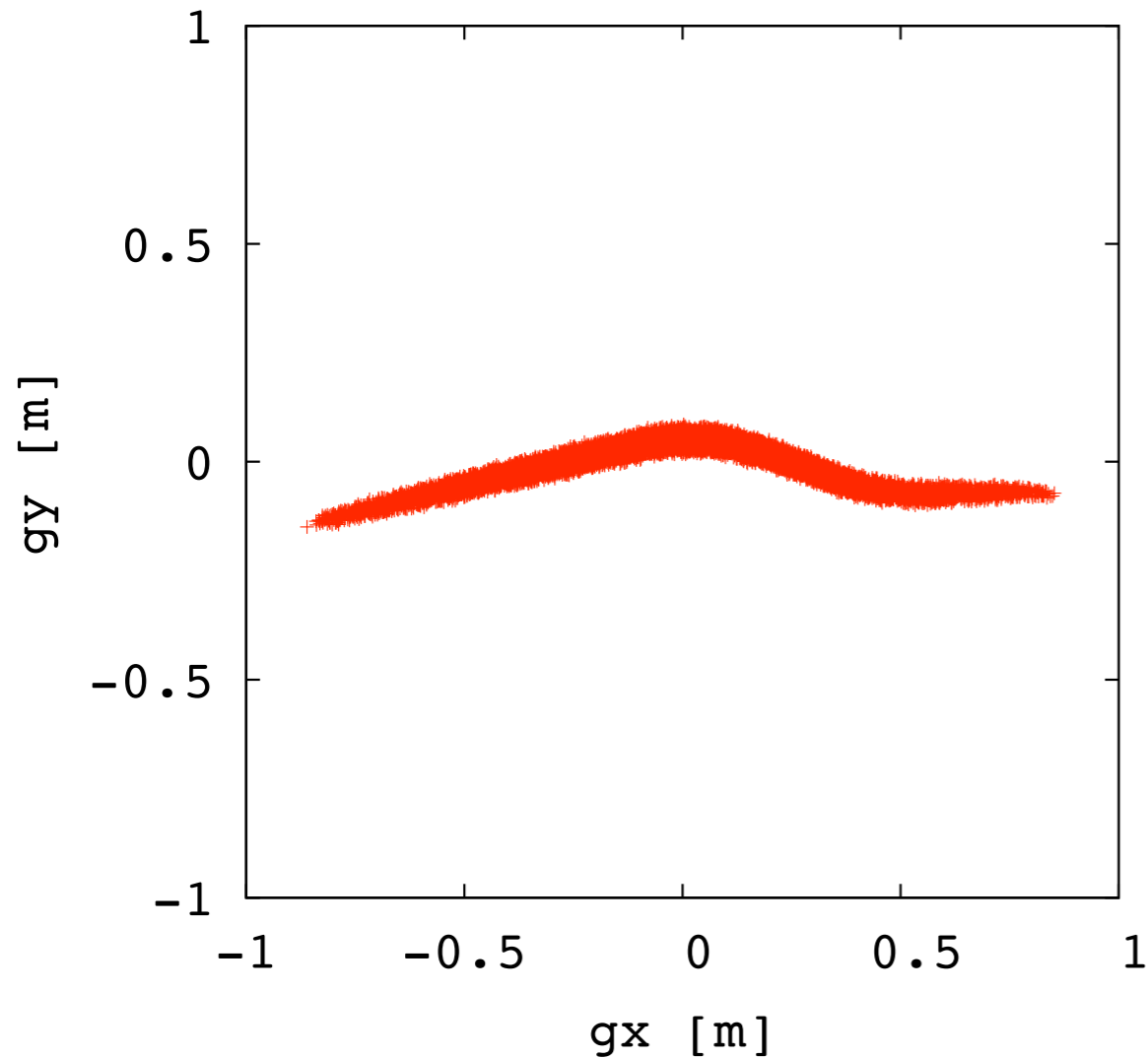
- Last time, I looked at only the first 5 turns (40 cells).
- “observation of parametric resonance at  $q_y=0.25$ ” may be wrong.



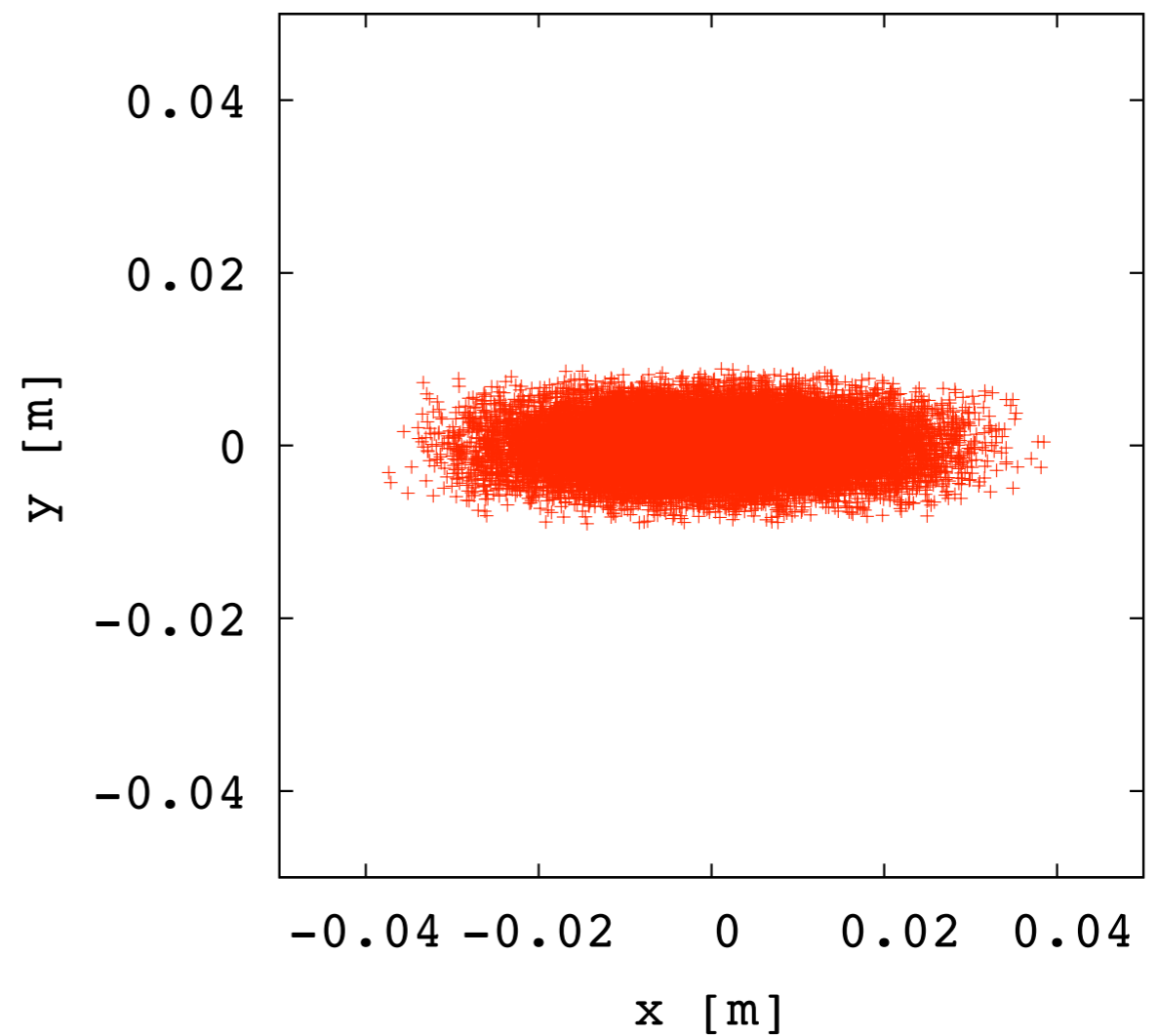
- Horizontal beam size was not calculated properly.
- bunch has a large curvature in a small ring.
- beam size due to dispersion and momentum spread.

# Bunch (beam) shape

- Top view: a bunch does not have an ellipsoidal shape.



- Different size in H and V although beta function are similar.



# Space charge tune shift

- Gaussian distribution in 3-D (cut at 2.5 sigma), the maximum tune shift is

$$\Delta Q_y = - \frac{r_p n_t}{2\pi \sqrt{\epsilon_{rms,y}} (\sqrt{\epsilon_{rms,x}} + \sqrt{\epsilon_{rms,y}}) \beta^2 \gamma^3 B_f}$$

- Beam size in horizontal is mainly determined by dispersion and momentum spread

$$\begin{aligned} \sqrt{\epsilon_x} &= \sqrt{\epsilon_{\beta,x} + (D_x \delta)^2 / \beta_x} \\ &= \sqrt{11 \times 10^{-6} + 133 \times 10^{-6}} \end{aligned}$$

$$\begin{aligned} b &= 1.3 \text{ m, } e = 8e-6 \\ D_x &= 0.87 \text{ m, } dp/p = 0.0132 \end{aligned}$$

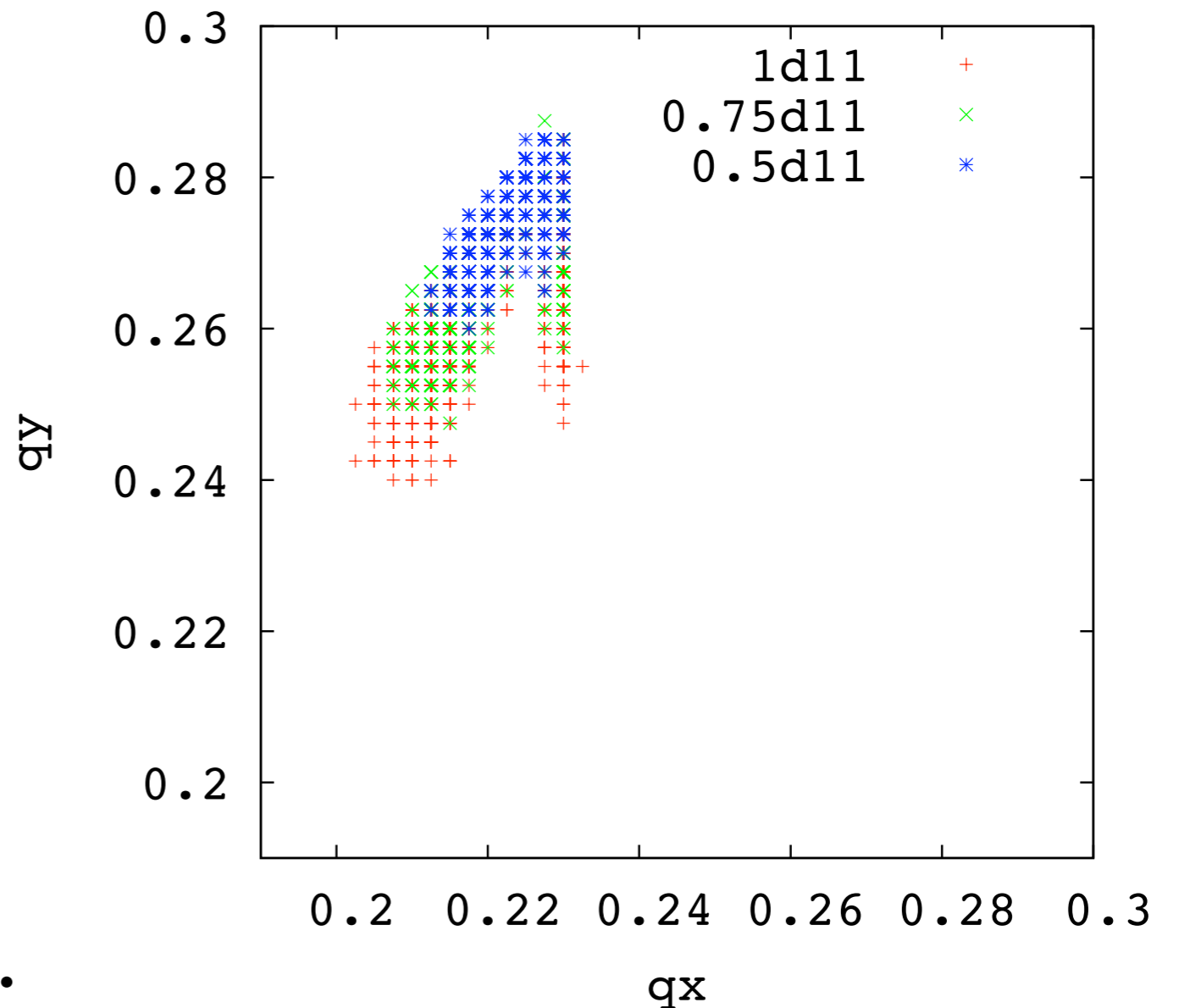
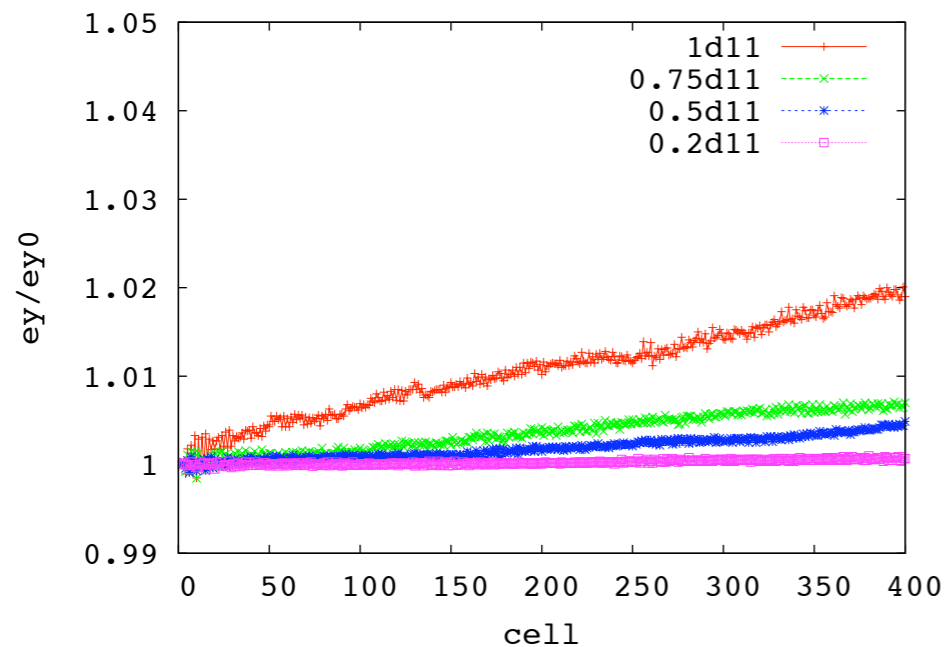
# Intensity dependence

- Tune shift vs intensity

intensity in per bunch	cell $dq_y$ ( $q_y=0.287$ )	cell $dq_x$ ( $q_x=0.227$ )
$0.2 \times 10^{11}$	-0.013 (0.274)	-0.004 (0.223)
$0.5 \times 10^{11}$	-0.033 (0.254)	-0.009 (0.218)
$1 \times 10^{11}$	-0.067 (0.220)	-0.019 (0.208)
$1.5 \times 10^{11}$	-0.100 (0.187)	-0.028 (0.199)

# Tune spread

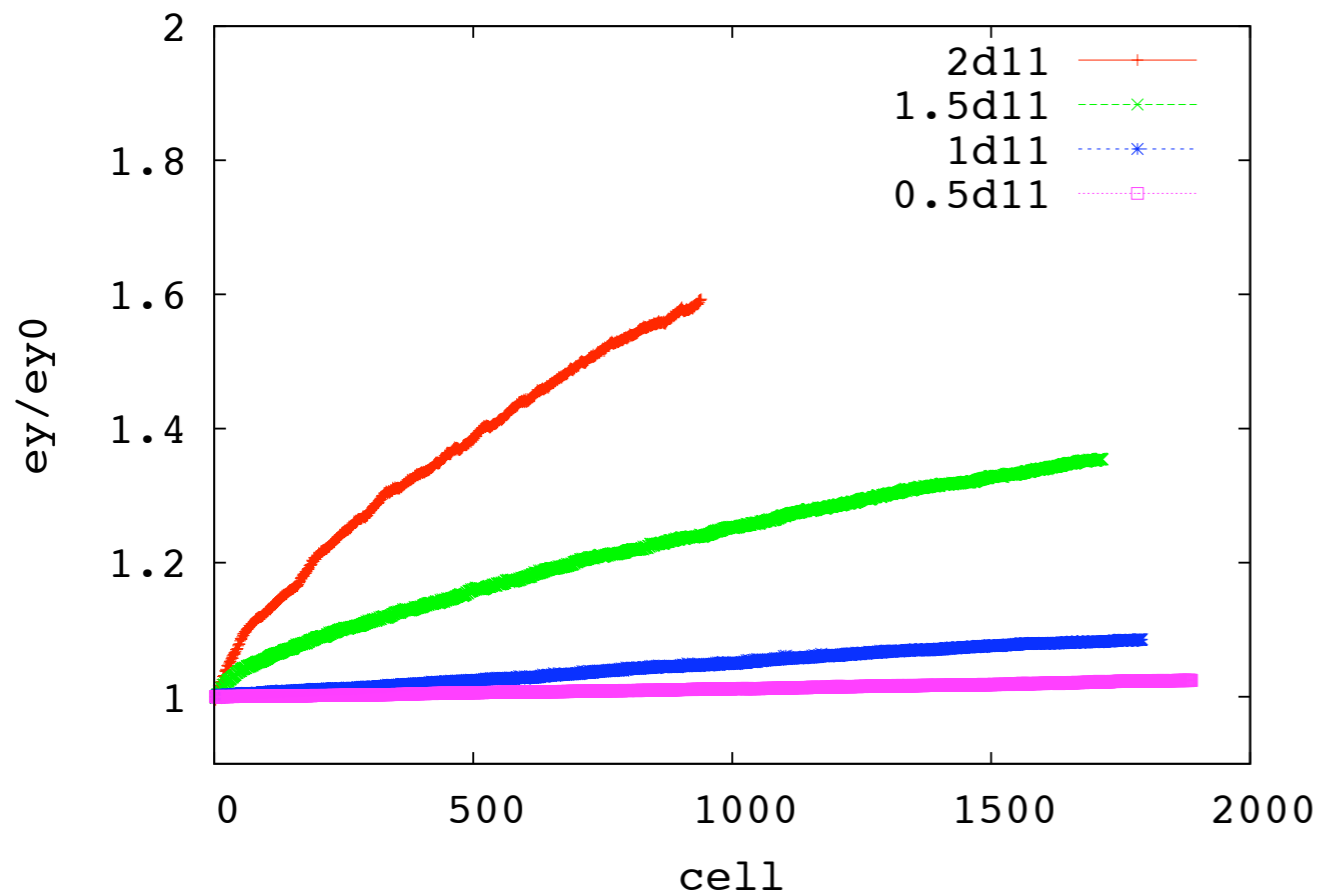
- FFT of the first 50 turns (400 cells).



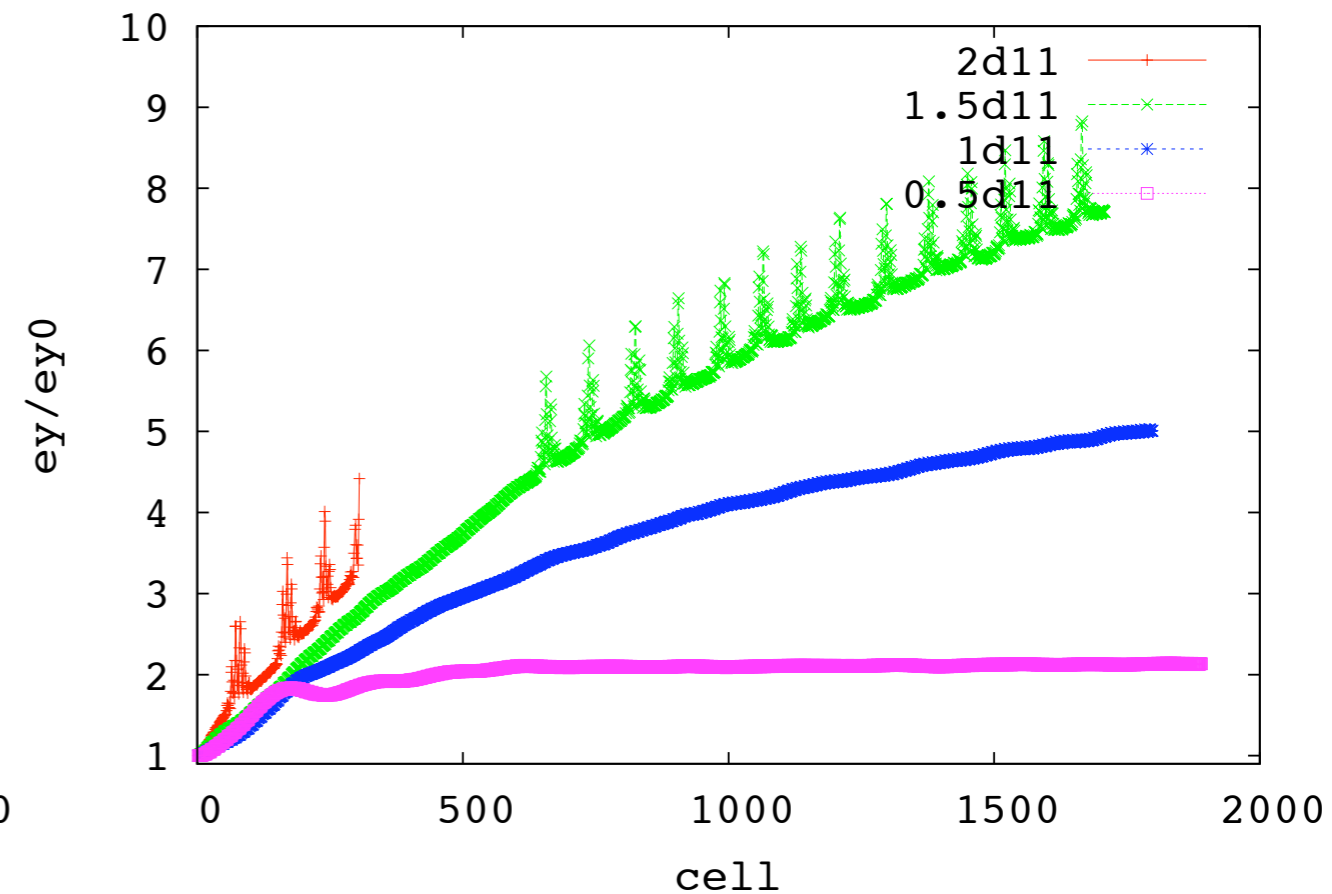
- Cell tune in the model lattice is  $(0.227, 0.287)$ .

# Long term behavior

- Without error, primary source is  $qy=0.25$ .

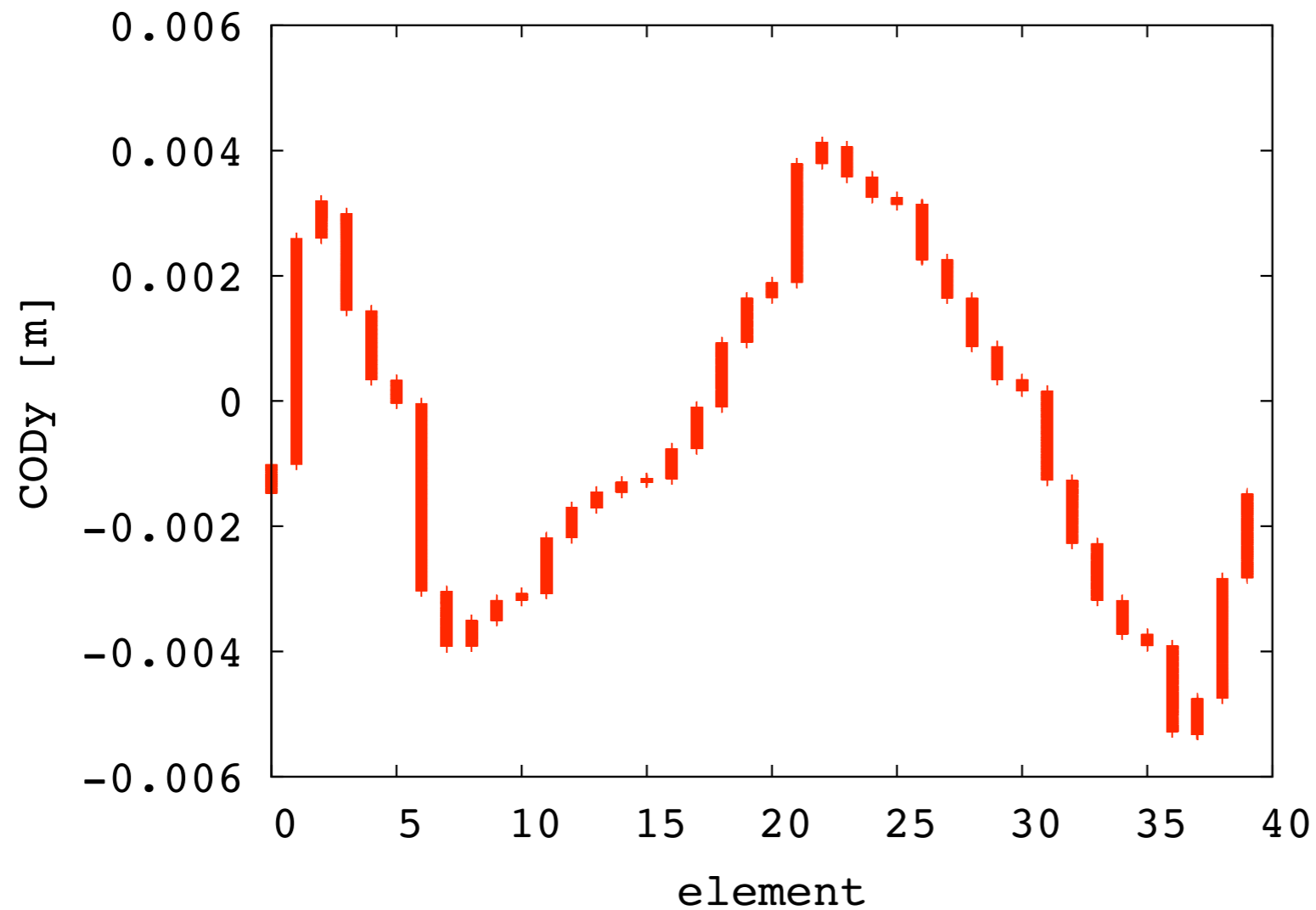


- With alignment error,  $Qy=2$  ( $qy=0.25$ ).





# COD with alignment error of +/- 2mm



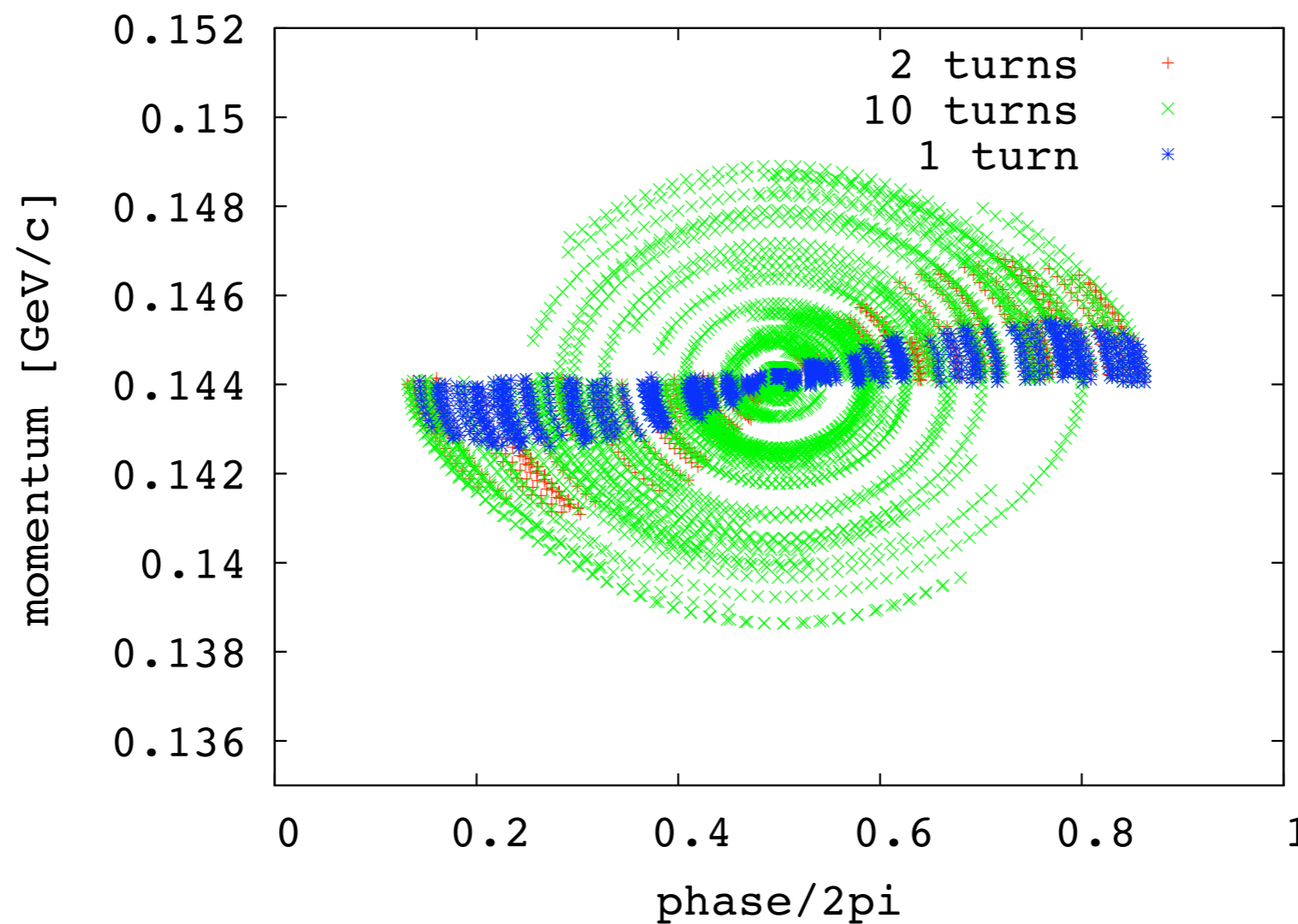
5 elements in one cell

# Multi-turn injection

- In reality, a beam current is accumulated with many injection turns ( $\sim 50$  turns).
- $dp/p$  of linac beam is small (0.001) and rf bucket height is much larger (0.042).
- Mismatch in longitudinal phase space makes line density (or bunching factor) time dependent.

# Longitudinal phase space

- Synchrotron tune is about  $1/13$ .
- Keep injecting new particles.

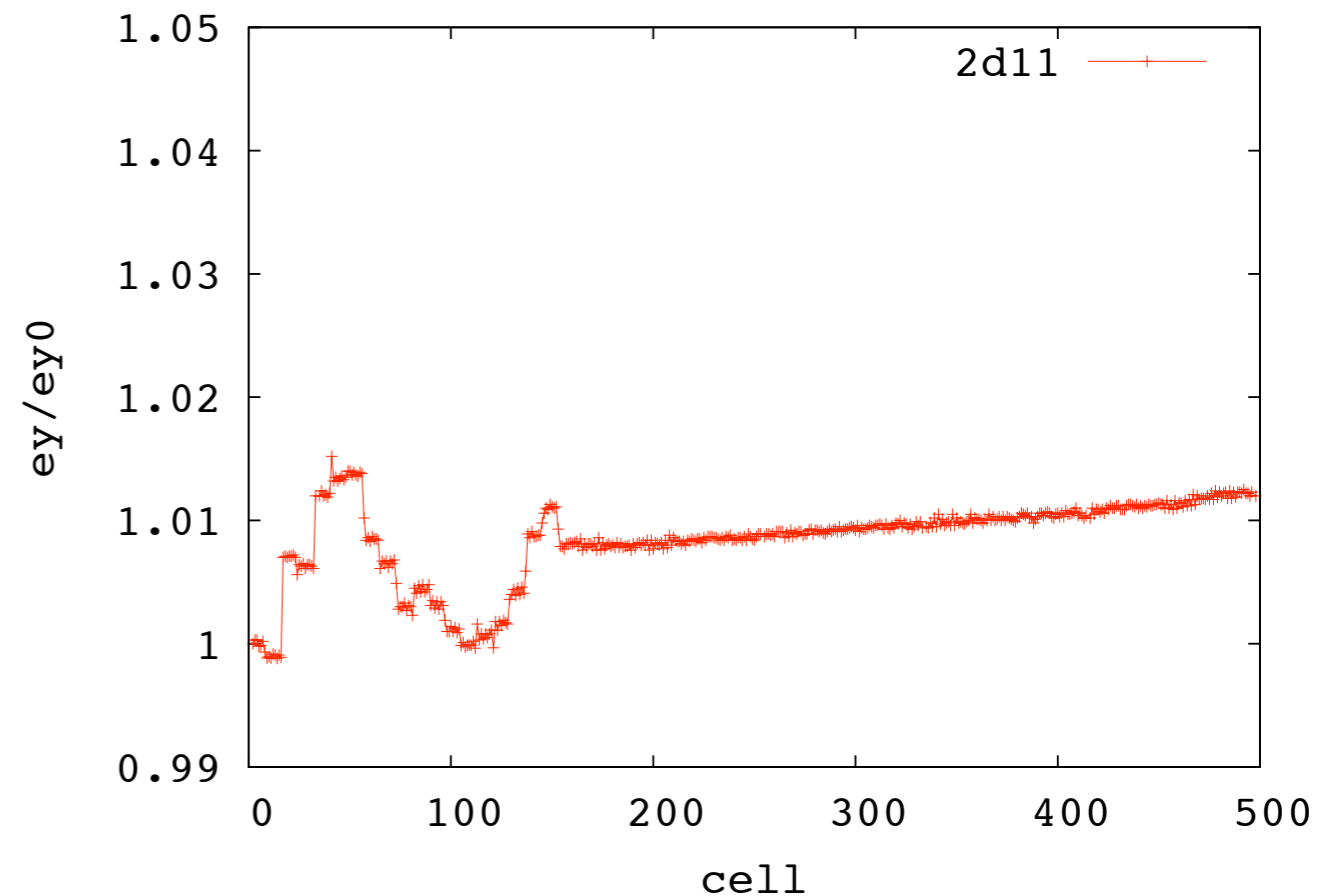
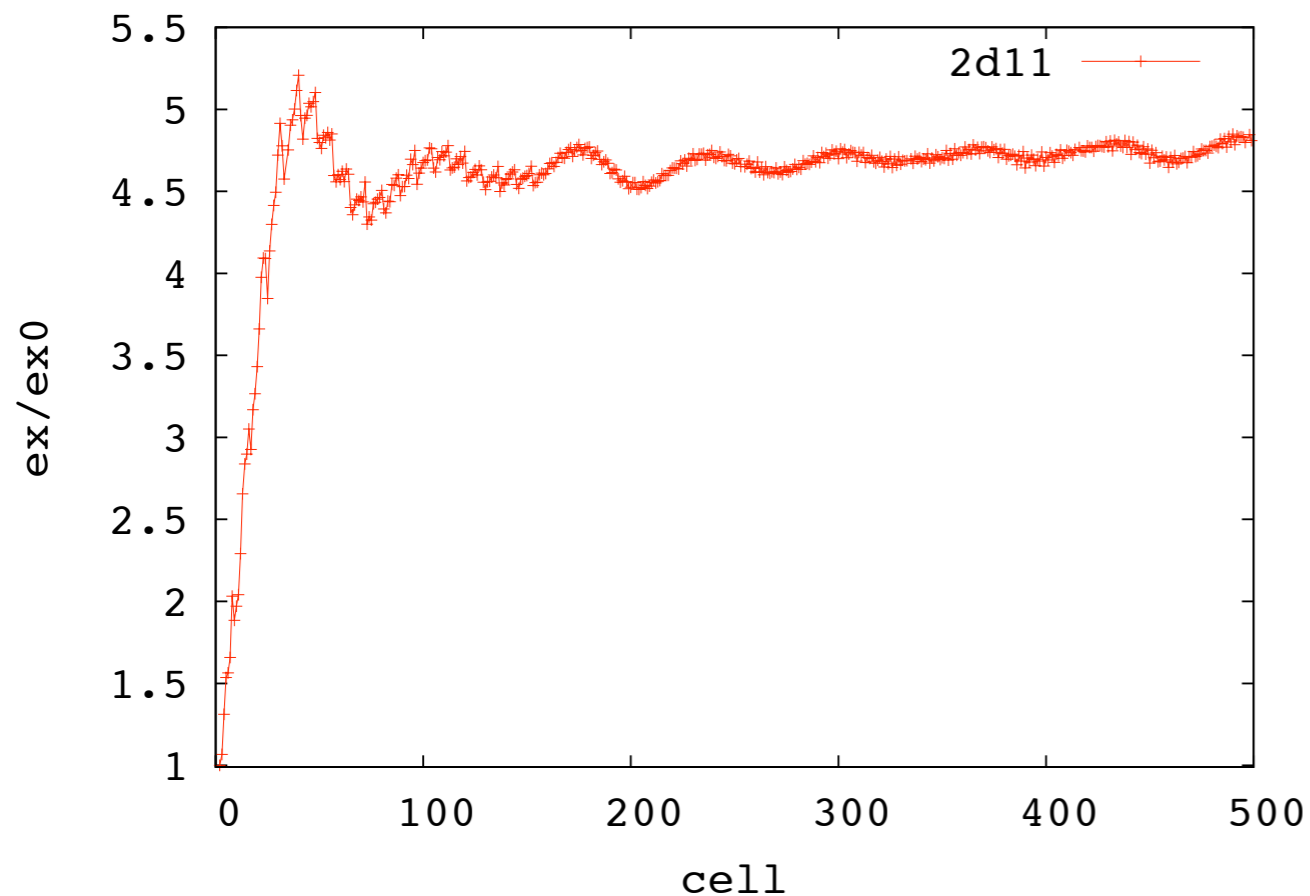


# Multi-turn injection simulation

- 1st: inject  $N$  particles.
- 2nd: inject  $N$  particles and throw randomly selected  $N$  particles away. Increase charge per particle by 2.
- 3rd: inject  $N/2$  particles and throw randomly selected  $N/2$  particles away. Increase charge per particle by 3.
- $n$ -th: inject  $N/(n-1)$  particles and throw randomly selected  $N/(n-1)$  particles away. Increase charge per particle by  $N$ .

# Results with 30 turns injection

- Horizontal has rapid increase of ‘beam size’ due to dispersion.
- Not much ‘emittance’ growth in vertical.
- Bunching factor is larger.



# Summary

- Tune spread calculated in simulation seems reasonable.
- Larger emittance growth with higher current. Similar results in a synchrotron.
- Simulation with multi-turn injection shows
  - horizontal beam size is determined by dispersion and hard to separate from emittance growth.
  - space charge effects are weaker due to larger bunching factor at least at the beginning.