

Space charge simulation update (4)

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- Quick review of tune shift (spread).
- Multi-turn injection.
 - intensity dependence
 - with alignment errors
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- Conclusion

Space charge tune shift

- Gaussian distribution (cut at 2.5 sigma in simulation), the maximum tune shift is

$$\Delta Q_y = - \frac{r_p n_t}{2\pi \sqrt{\epsilon_{rms,y}} (\sqrt{\epsilon_{rms,x}} + \sqrt{\epsilon_{rms,y}}) \beta^2 \gamma^3 B_f}$$

- Beam size in horizontal is mainly determined by dispersion and momentum spread

$$\begin{aligned} \sqrt{\epsilon_x} &= \sqrt{\epsilon_{\beta,x} + (D_x \delta)^2 / \beta_x} \\ &= \sqrt{11 \times 10^{-6} + 133 \times 10^{-6}} \end{aligned}$$

$$\begin{aligned} b &= 1.3 \text{ m}, e = 8e-6 \\ D_x &= 0.87 \text{ m}, dp/p = 0.0132 \end{aligned}$$

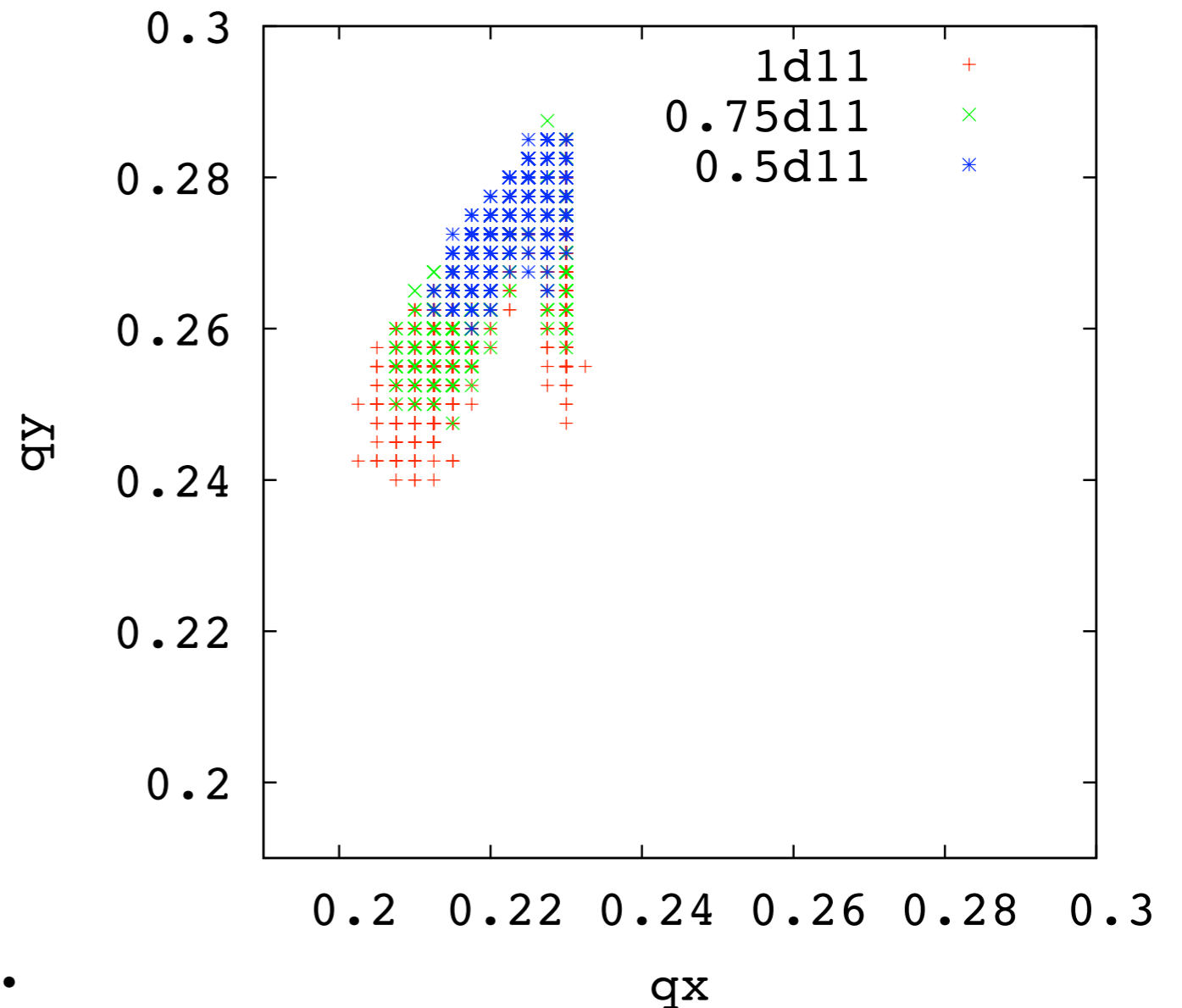
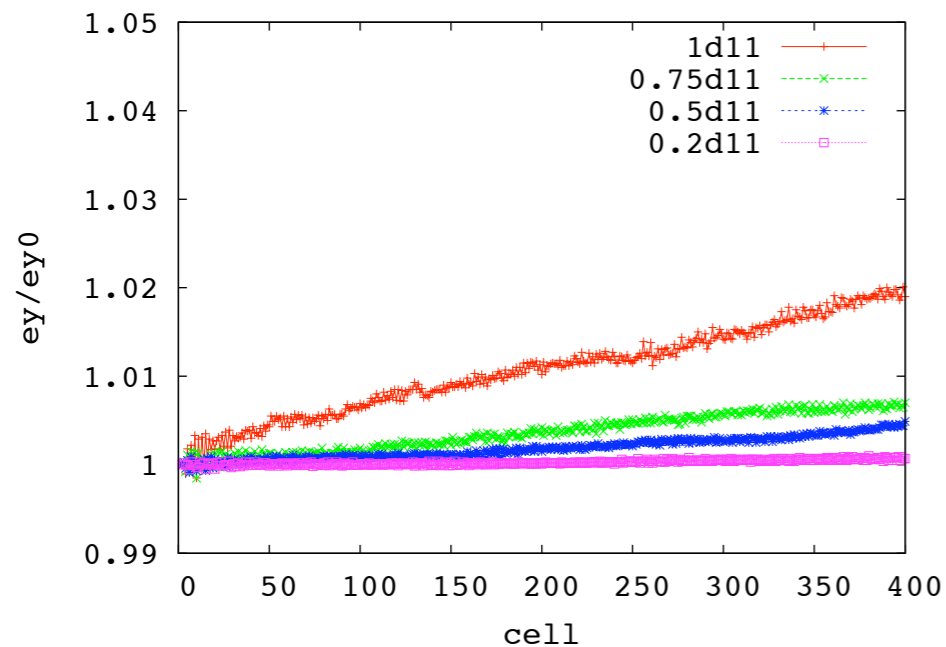
Intensity dependence

- Tune shift vs intensity

intensity in per bunch	cell dq_y ($q_y=0.287$)	cell dq_x ($q_x=0.227$)
0.2×10^{11}	-0.013 (0.274)	-0.004 (0.223)
0.5×10^{11}	-0.033 (0.254)	-0.009 (0.218)
1×10^{11}	-0.067 (0.220)	-0.019 (0.208)
1.5×10^{11}	-0.100 (0.187)	-0.028 (0.199)

Tune spread

- FFT of the first 50 turns (400 cells).



- Cell tune in the model lattice is $(0.227, 0.287)$.

Multi-turn injection (I)

- In reality, a beam current is accumulated with many injection turns.
- ~ 60 turns to accumulate $1\text{E}11$ ppb or $6\text{E}11$ ppp
- dp/p of linac beam is small (0.001) and rf bucket height is much larger (0.042).
- Mismatch in longitudinal phase space makes line density (or bunching factor) time dependent.

Multi-turn injection (2)

- Keep injecting the constant number of particles per turn.
- Increase intensity by injecting more number of turns (as in reality).
 - $1E11$ ppb ($6E11$ ppp) by 60 turns.
 - $2E11$ ppb ($12E11$ ppp) by 120 turns.
 - ...

Define “emittance” in the following way

- In vertical direction,

$$\epsilon_v = \sqrt{\langle y^2 \rangle \langle y'^2 \rangle - \langle yy' \rangle^2}$$

- In horizontal direction, it includes beam size from dispersion effects,

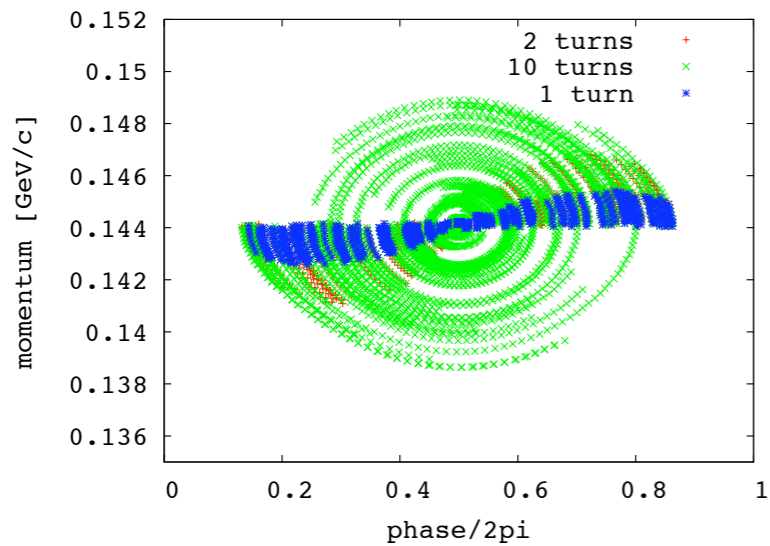
$$\epsilon_h = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

where

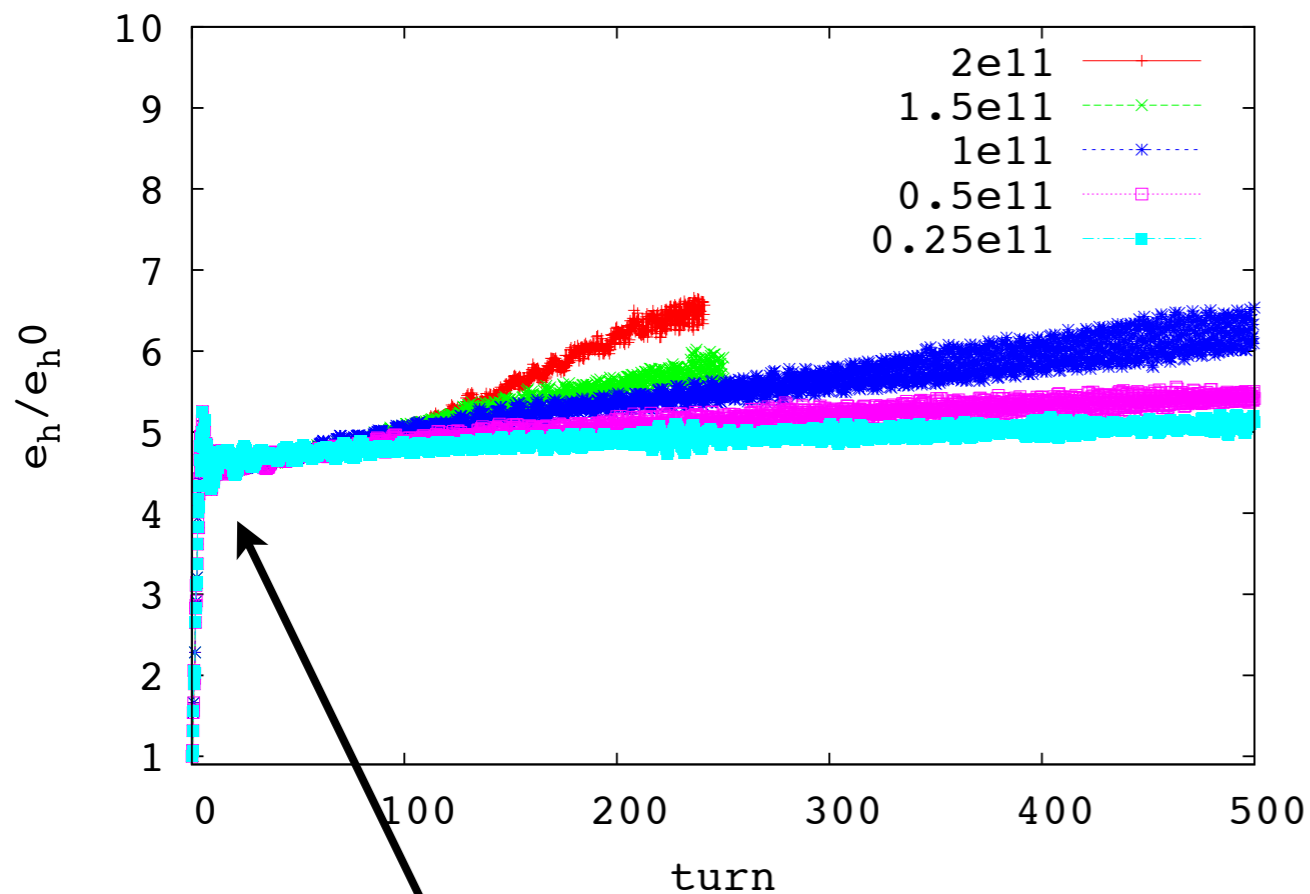
$$x = x_\beta + \delta D$$

$$x' = x'_\beta + \delta D'$$

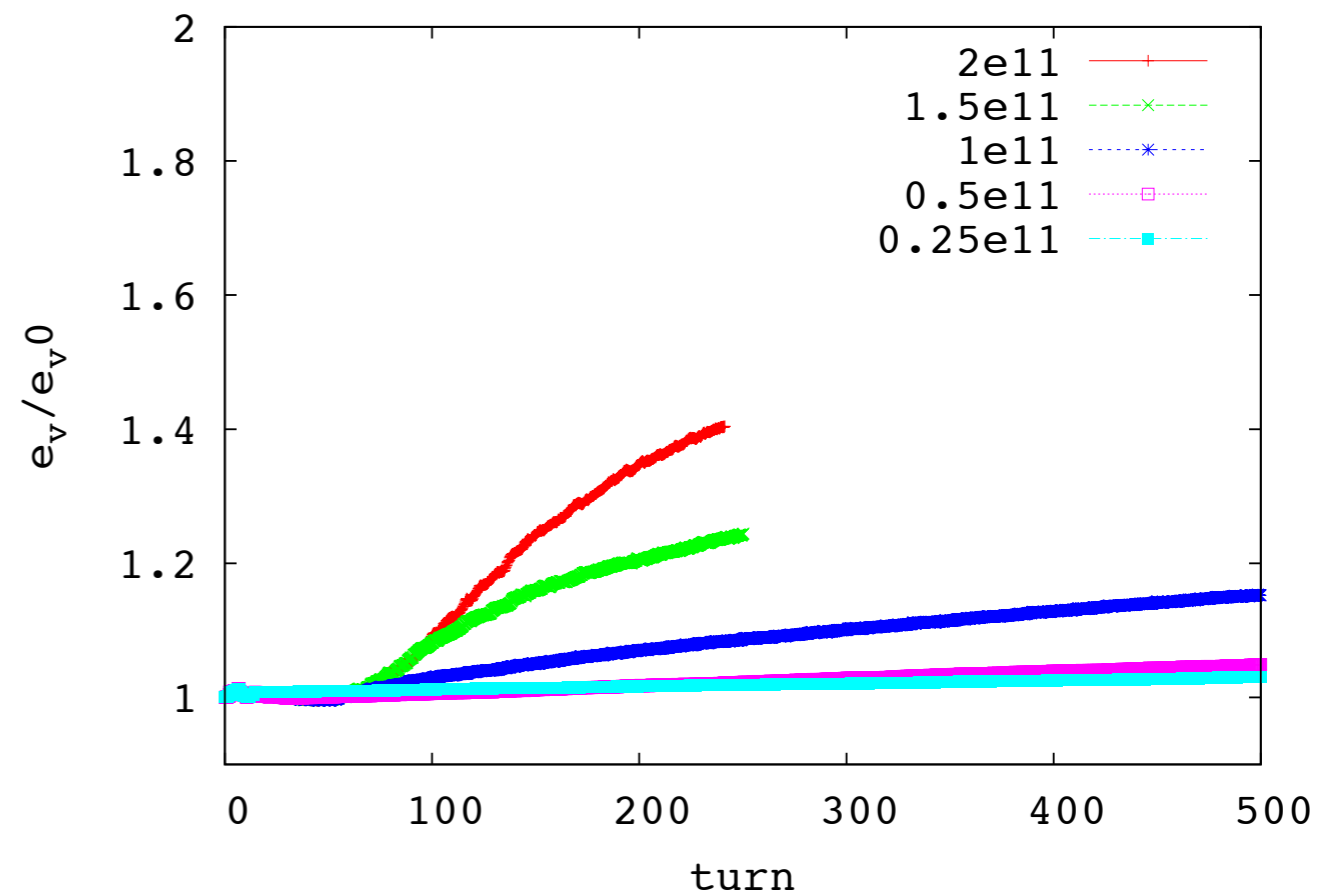
Intensity dependence



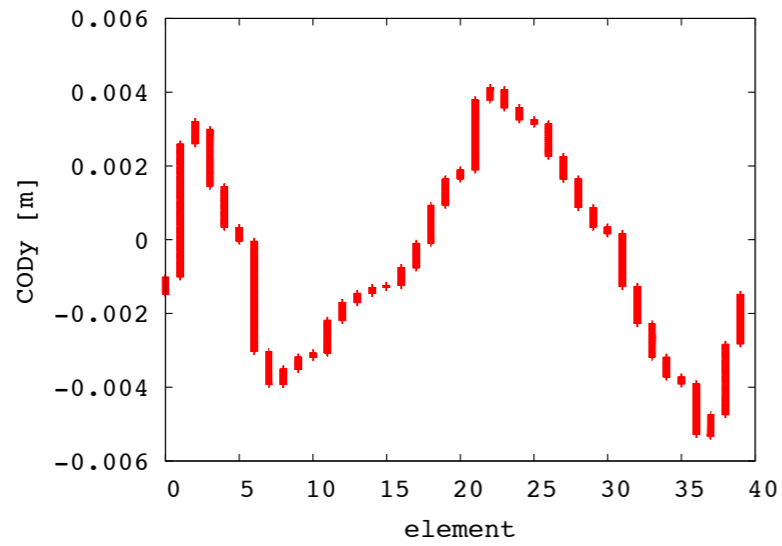
● horizontal



● vertical

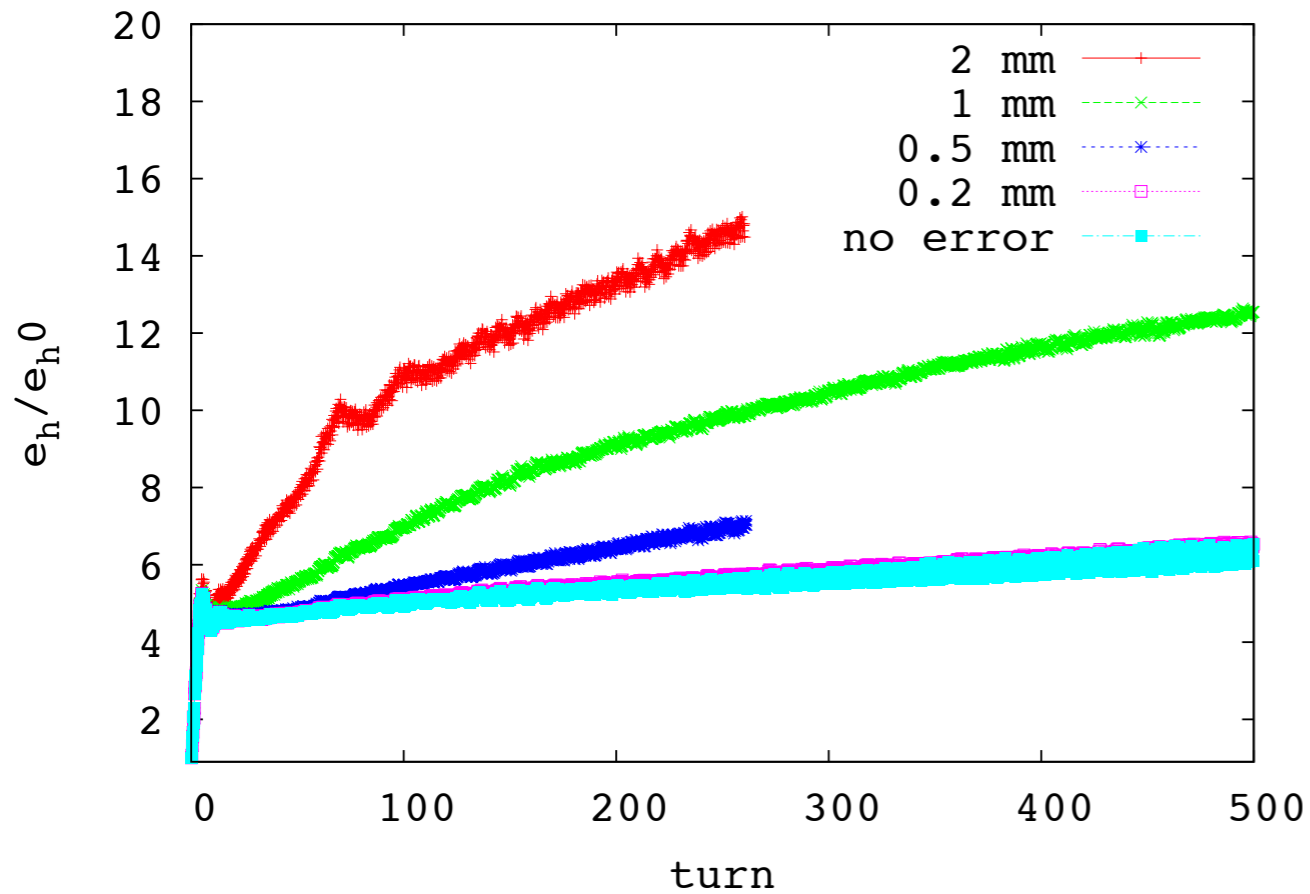


- Big jump in the first few turns in H is due to the increase of momentum spread in a rf bucket.

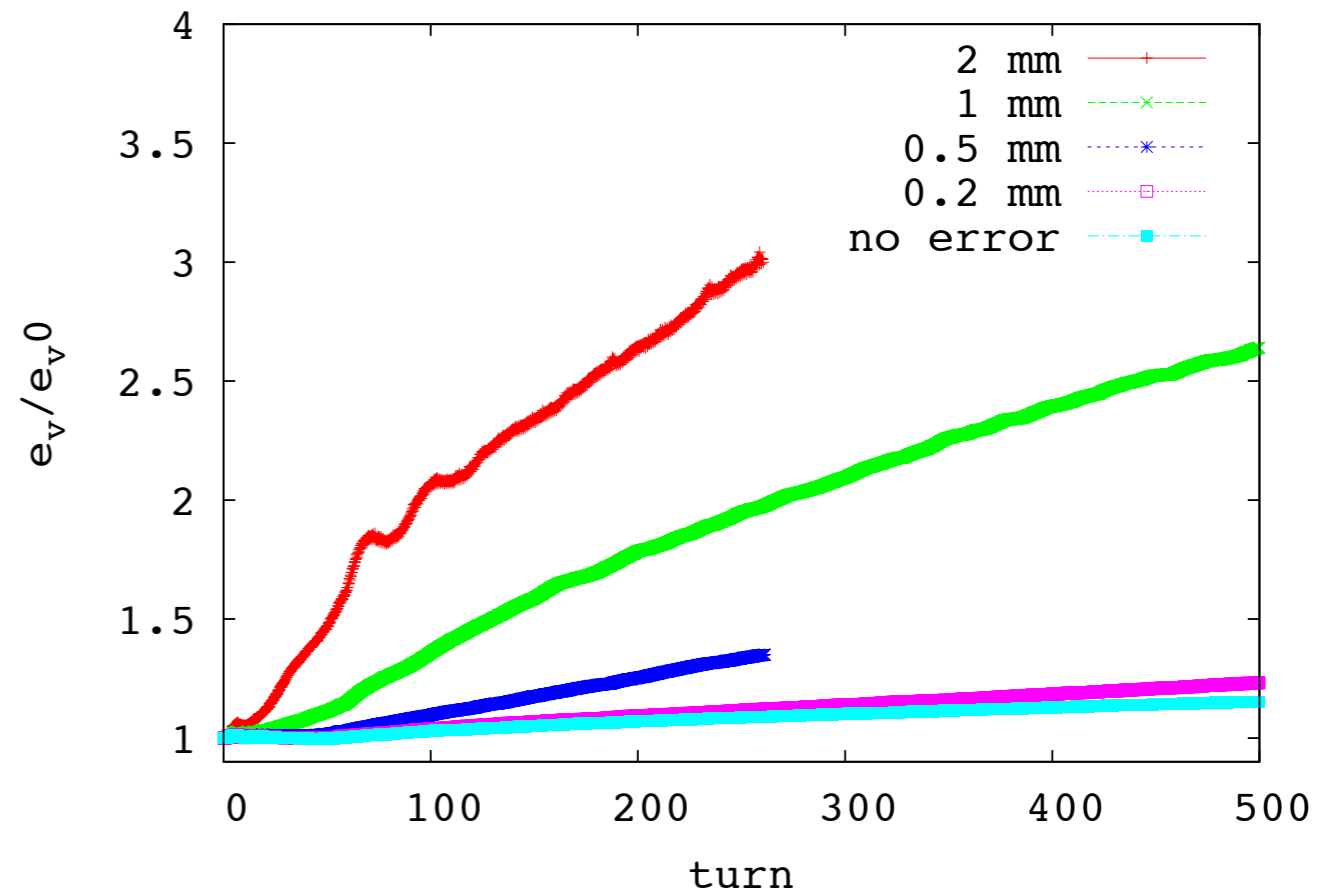


Alignment error with $|E|$ ppb

● horizontal

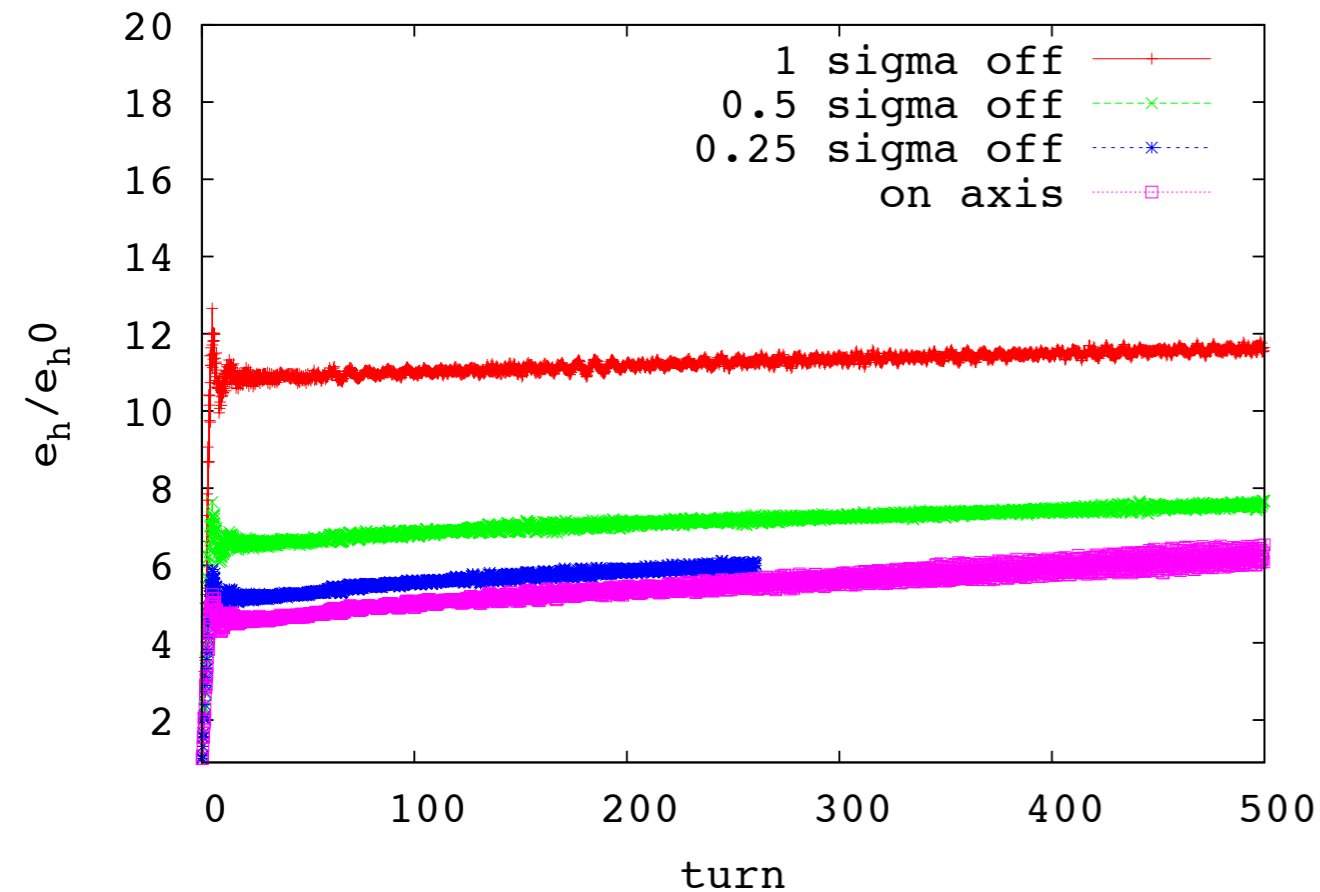


● vertical

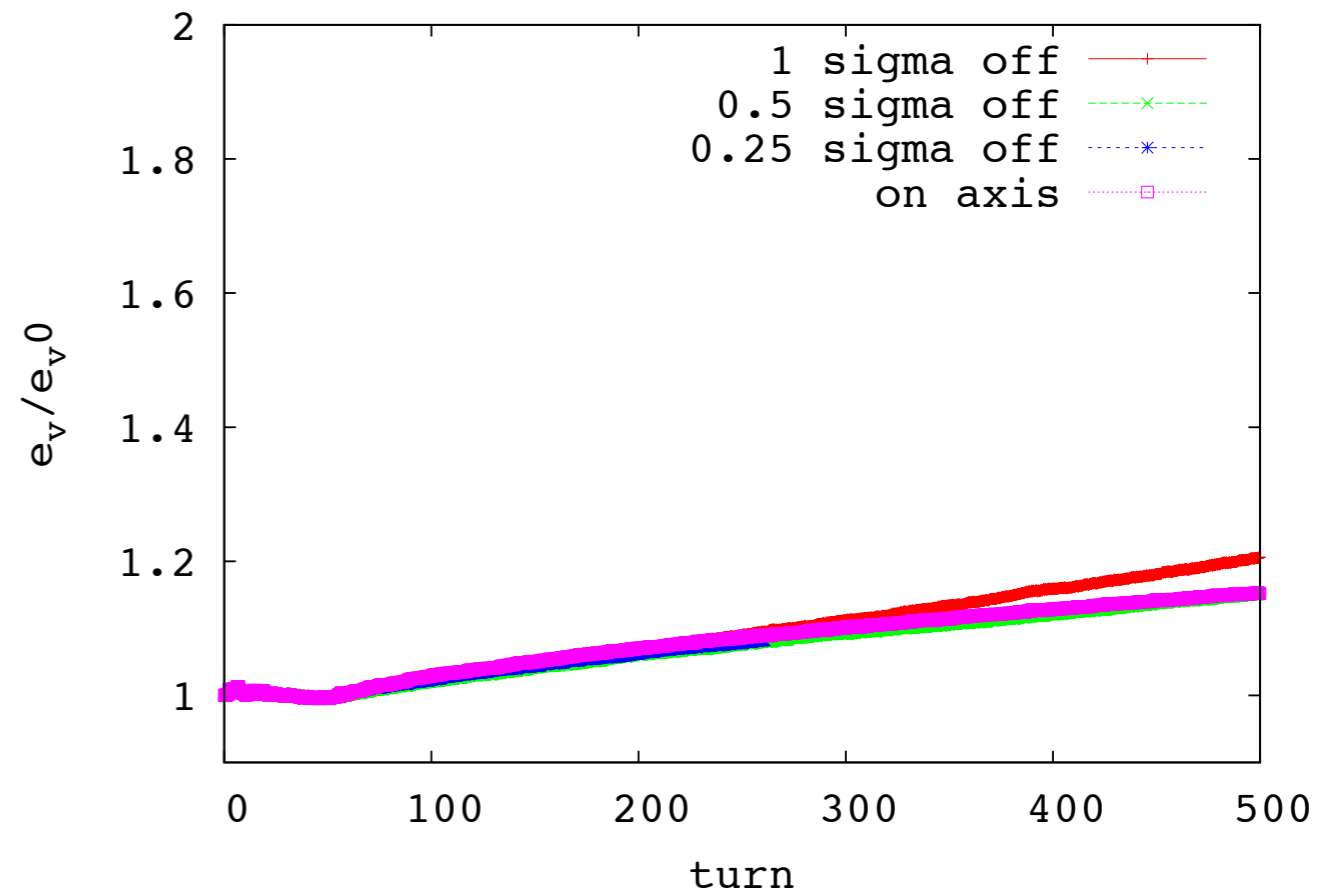


Injection orbit error in horizontal with $|E|$ ppb

● horizontal



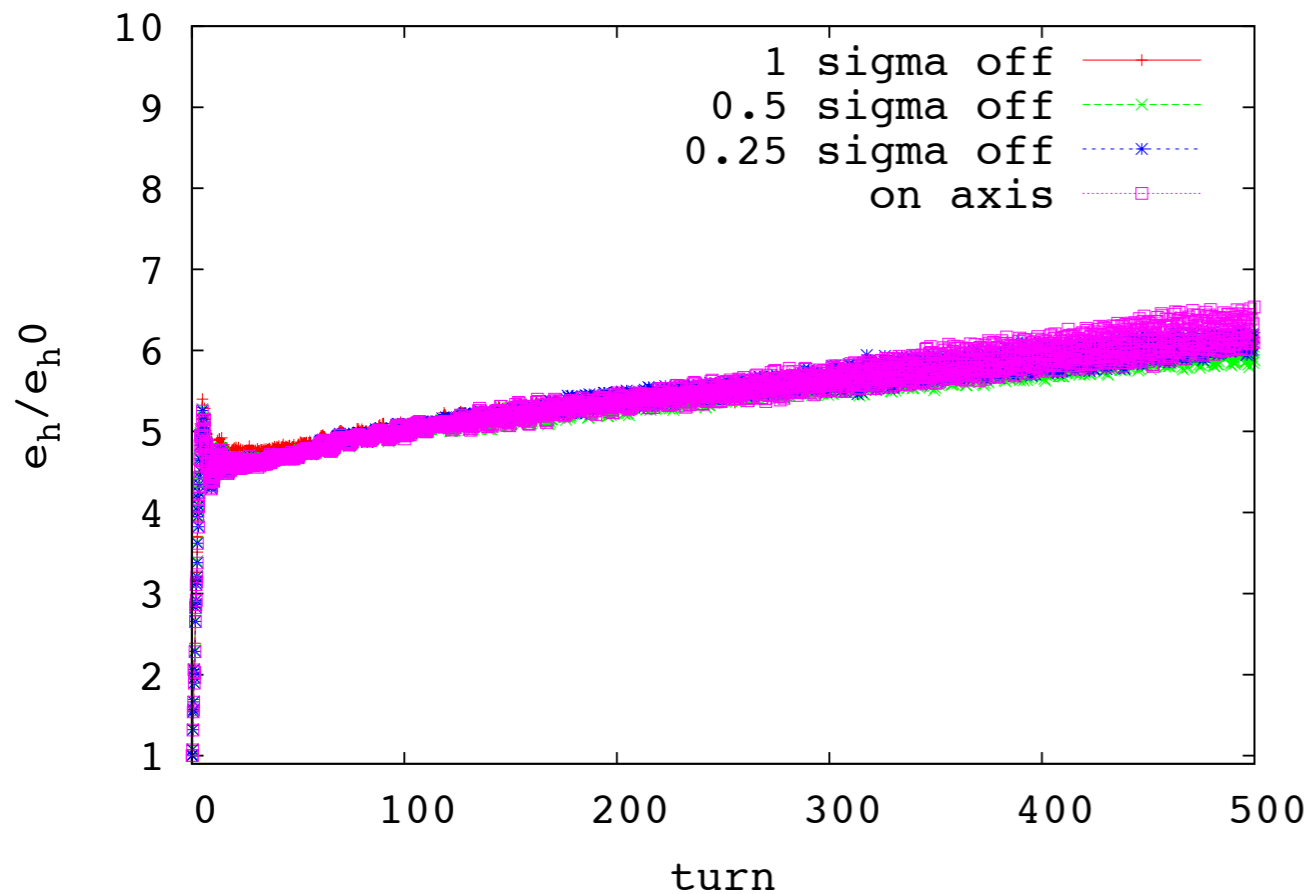
● vertical



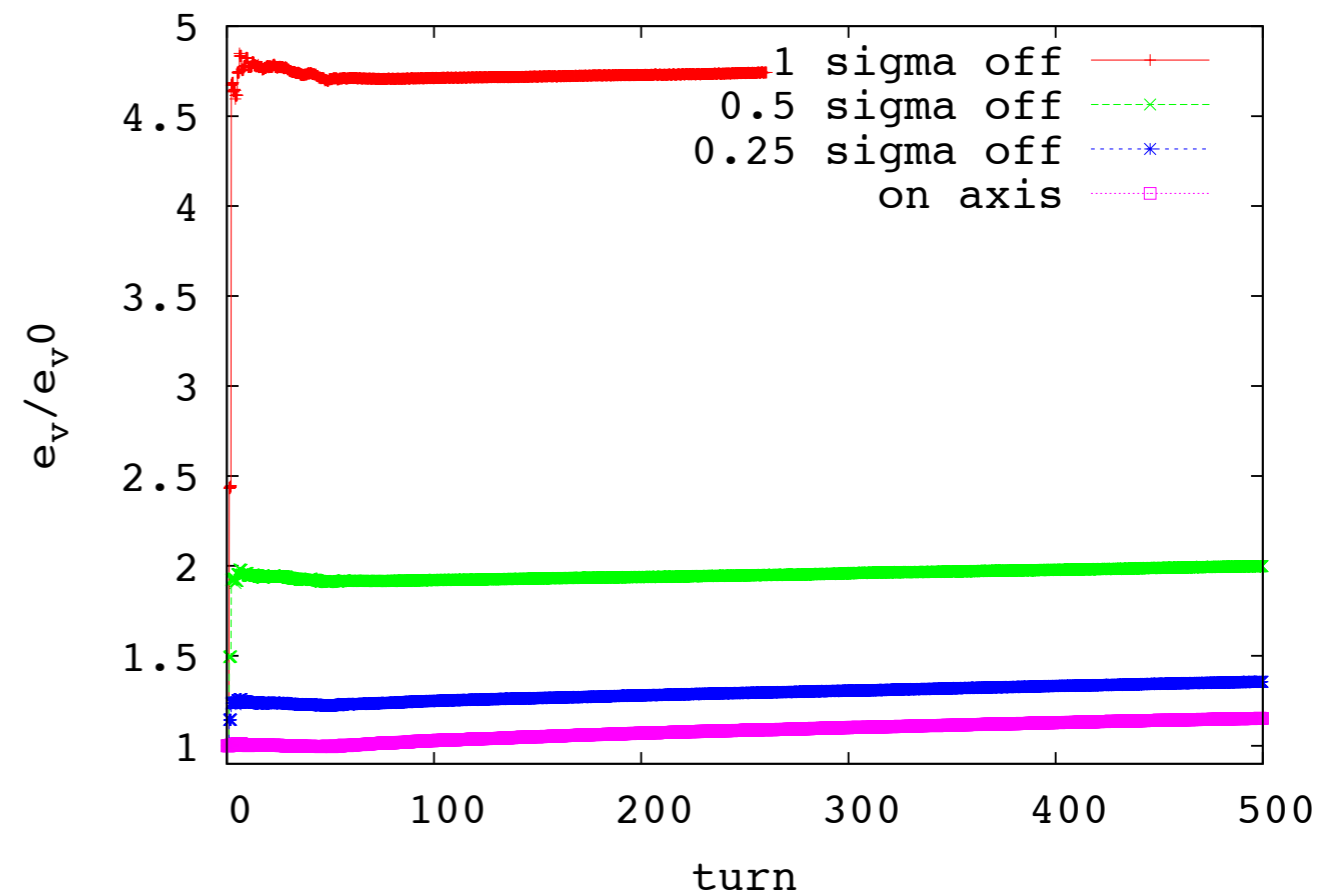
- We should repeat this with alignment error.

Injection orbit error in vertical with $|E|$ ppb

● horizontal



● vertical



- We should repeat this with alignment error.

Conclusion

- As expected, emittance increase appears beyond $1\text{E}11$ ppb.
- Alignment error of ± 0.5 mm makes the growth faster.
- 50% growth, or 20% in beam size, with ± 1 mm alignment error after ~ 120 turns (first 60 turns are injection period).
- Horizontal beam size is dominated by dispersion. We should use vertical beam size to see the space charge effects.

Next step

- Now we have a good idea how much and how fast the emittance growth by space charge occurs.
- Question is whether it is less or more than that by foil scattering.