

Space charge simulation update (5)

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Contents

- Multiple scattering in charge exchange foil
 - Elastic scattering to see transverse emittance increase
- Simulation with both space charge and multiple scattering

Basic of elastic scattering

- Total elastic scattering cross section is

$$\sigma_T = 4\pi r_e^2 \left(\frac{Z_1 Z_2^{2/3}}{\alpha\beta} \right)^2$$

J.D.Jackson or

R. Fernow, "Intro to experimental particle phys."

r_e = classical electron radius (2.8E-13 cm)

Z_1 and Z_2 = atomic charge of the incident and the bound particle (1 and 6)

α = 1/137

β = v/c (0.152)

- Average number of single scattering event.

$$m = Nt\sigma_T$$

$$N = N_A \rho / A$$

N = the number of atom per unit volume, t = thickness of foil (1E-5 cm)

N_A = Avogadro constant (6E23)

ρ = density of matter (2 g/cm³ for carbon)

A = atom mass number (12)

Several models for “multiple” scattering

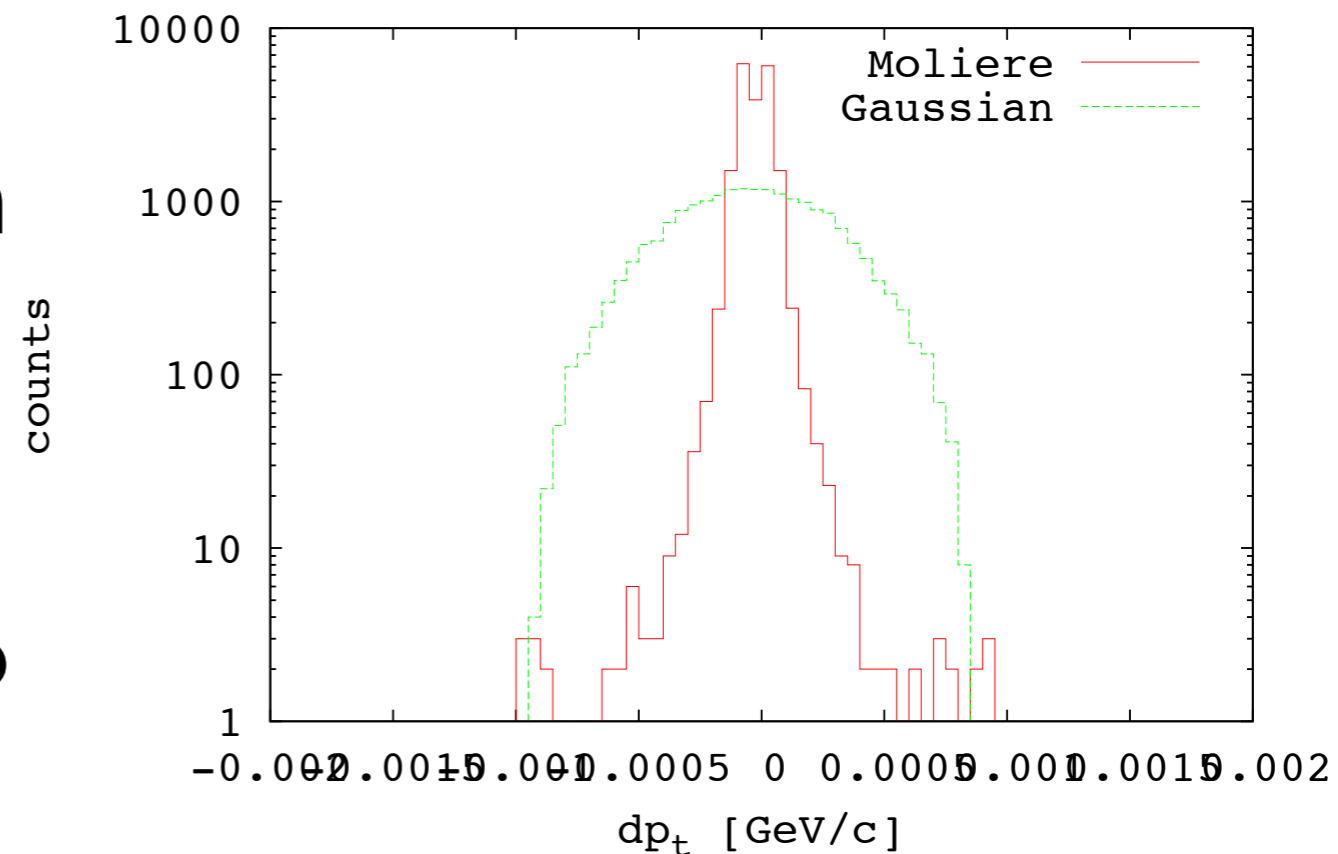
- Depending of average number of events
 - $m > 20$: multiple scattering
 - Moliere (Z. Naturforsch, 3A (1948) 78.)
 - Lewis (Phys. Rev. 78 (1950) 526.) c.f. Geant4
 - $m < 20$: plural scattering
 - Keil, Zeitler and Zinn (Z. Naturforsh, 15A (1960) 1031.)

Model of foil in ERIT

- 20 micro g/cm² carbon foil with 11 MeV proton gives
 - $m = 9$
 - plural scattering region
- However, I use Moliere model because a look-up table (old CERN library: MLRL) was available
 - With a single scaling parameter to match the emittance growth by Mori&Okabe calculation.
 - Keep the distribution and do scaling of ampl.

Angular distribution

- Moliere model has extended tail
 - Comparison with a beam with Gaussian distribution.
- Assume emittance growth is mainly determined by the small angle scattering.
- Large angle contributes to the loss.



Adjust a single scaling parameter

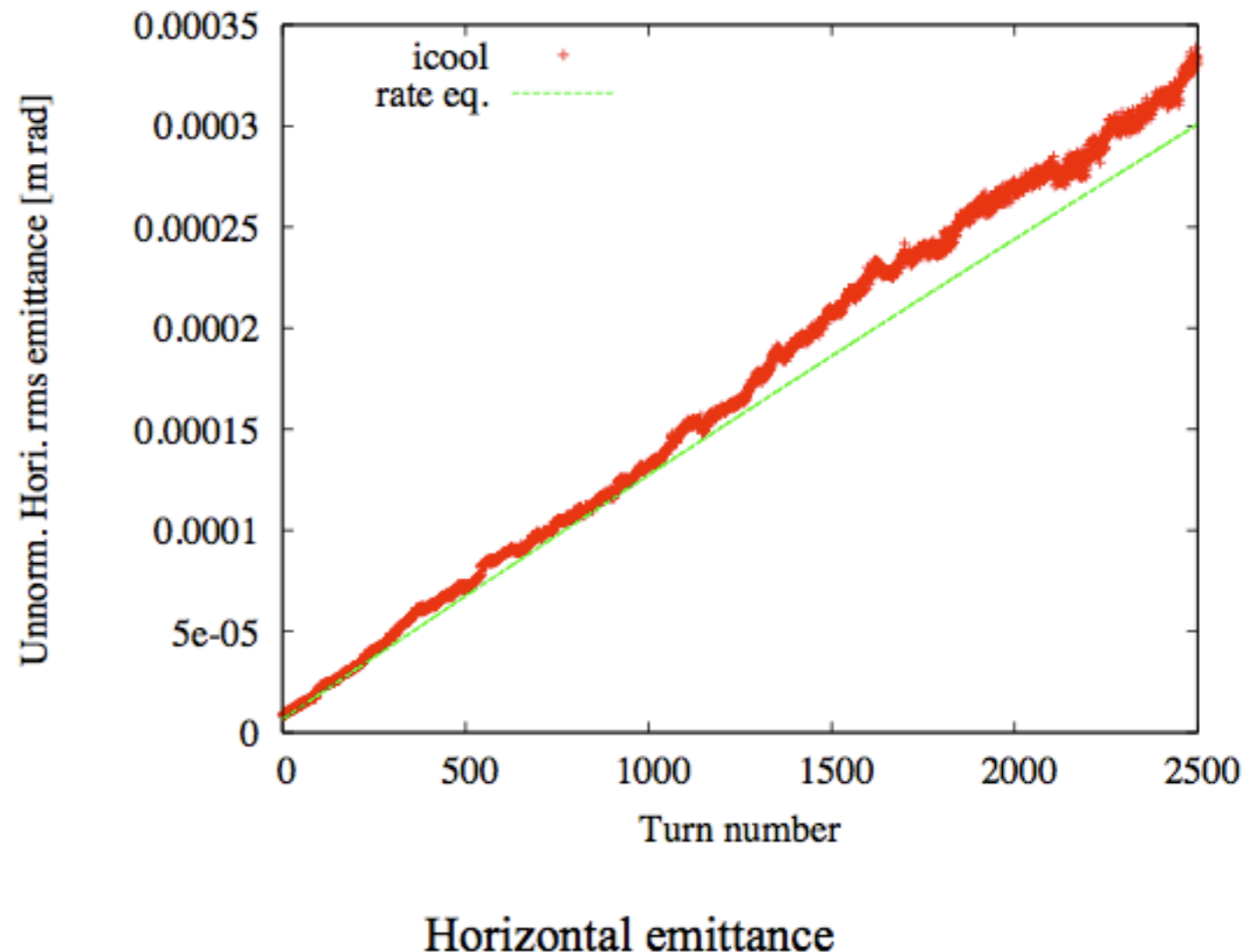
- Mori&Okabe calculation in 2011

$3E-4$ m rad/2500 turns (dE_s)
= $1.2E-7$ m rad/turn

Initial emittance (Ei)
= $8E-6$ m rad

$$E = d(y_p) d(y)$$

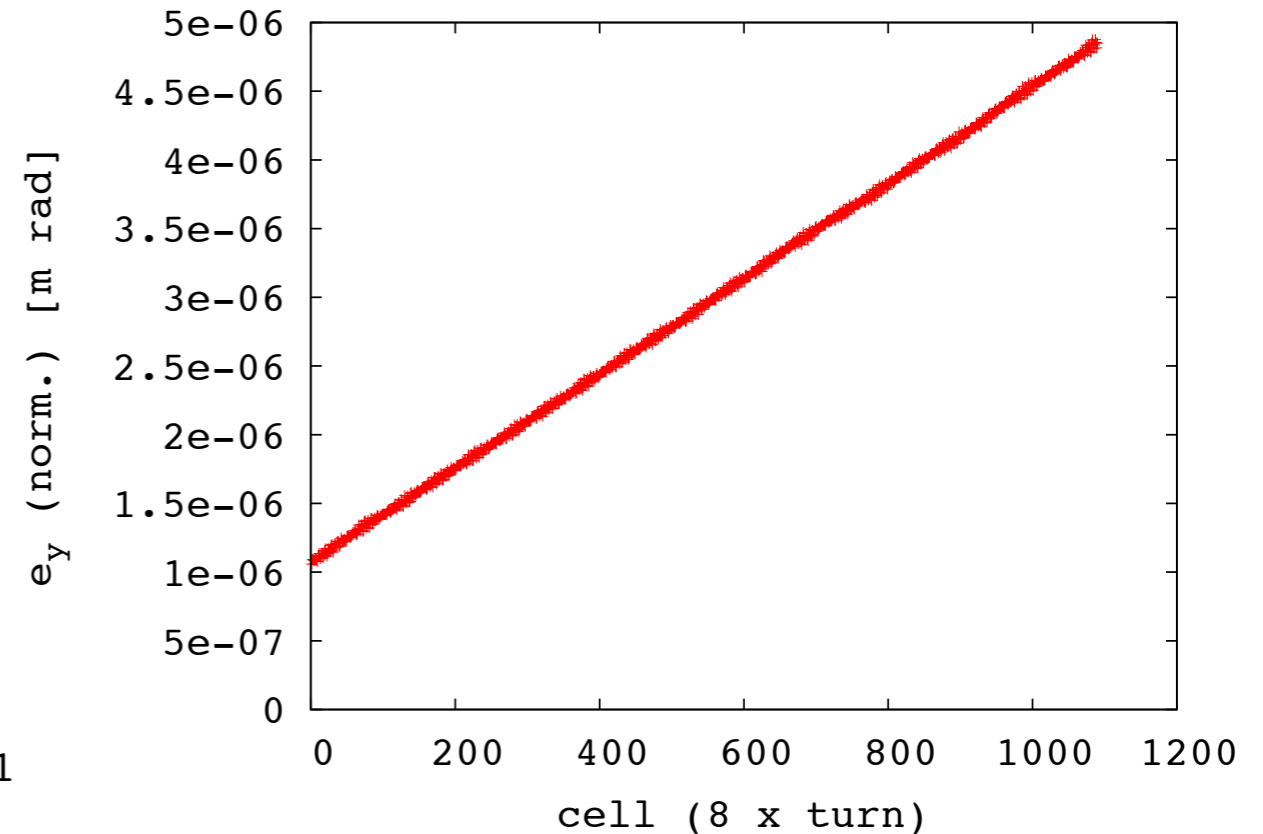
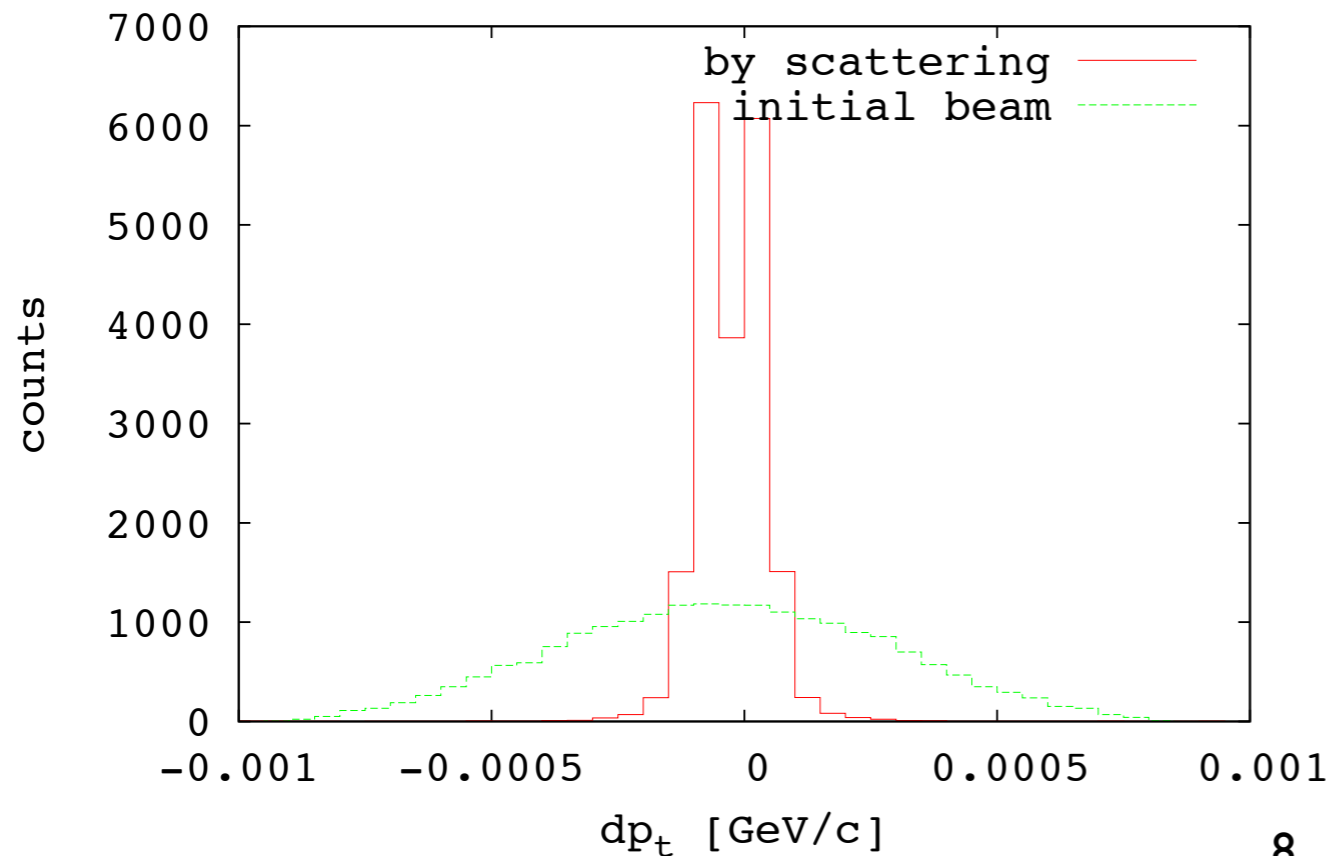
$$\begin{aligned} \text{Therefore, } d(y_p)_s / d(y_p)_i &= \text{Sqrt}[1.2E-7 / 8E-6] \\ &= 0.122 \end{aligned}$$



Emittance growth by foil scattering

- Scale the scattering angle.
- As a result, it gives the similar growth rate as Mori&Okabe.

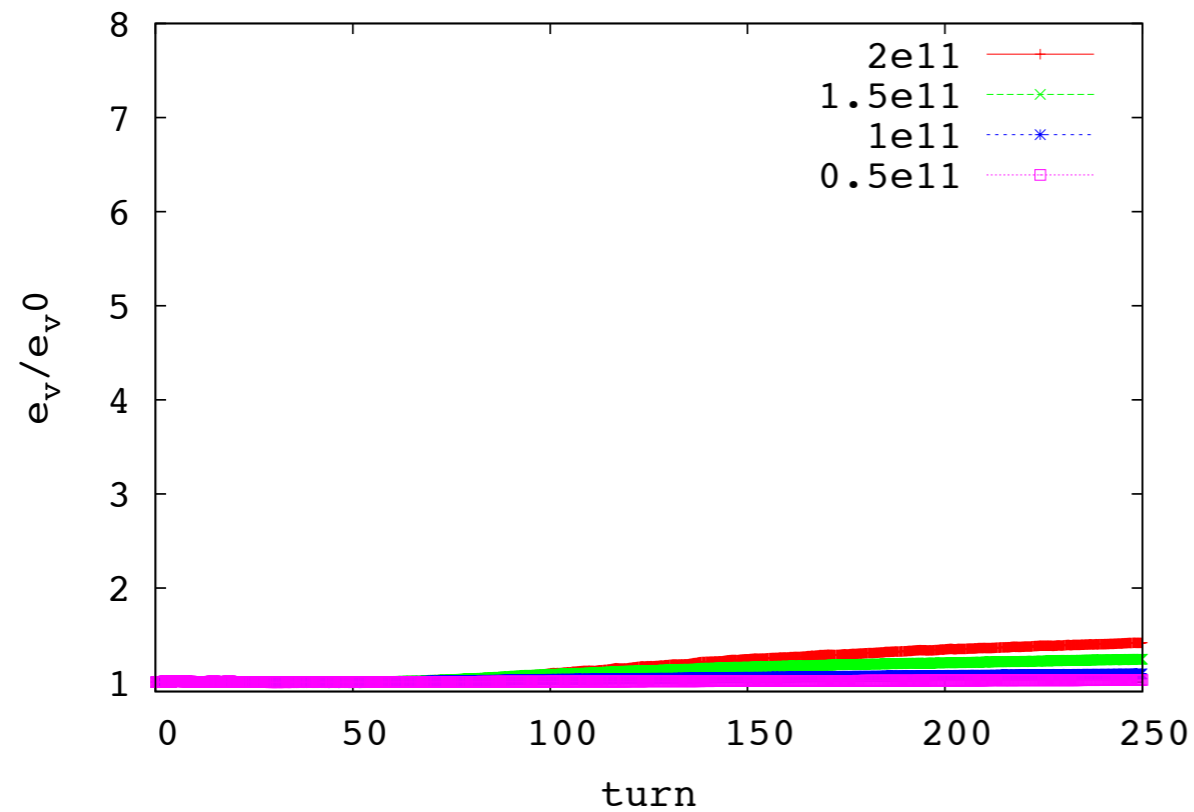
$$d(y_p)_s/d(y_p)_i \sim 0.1$$



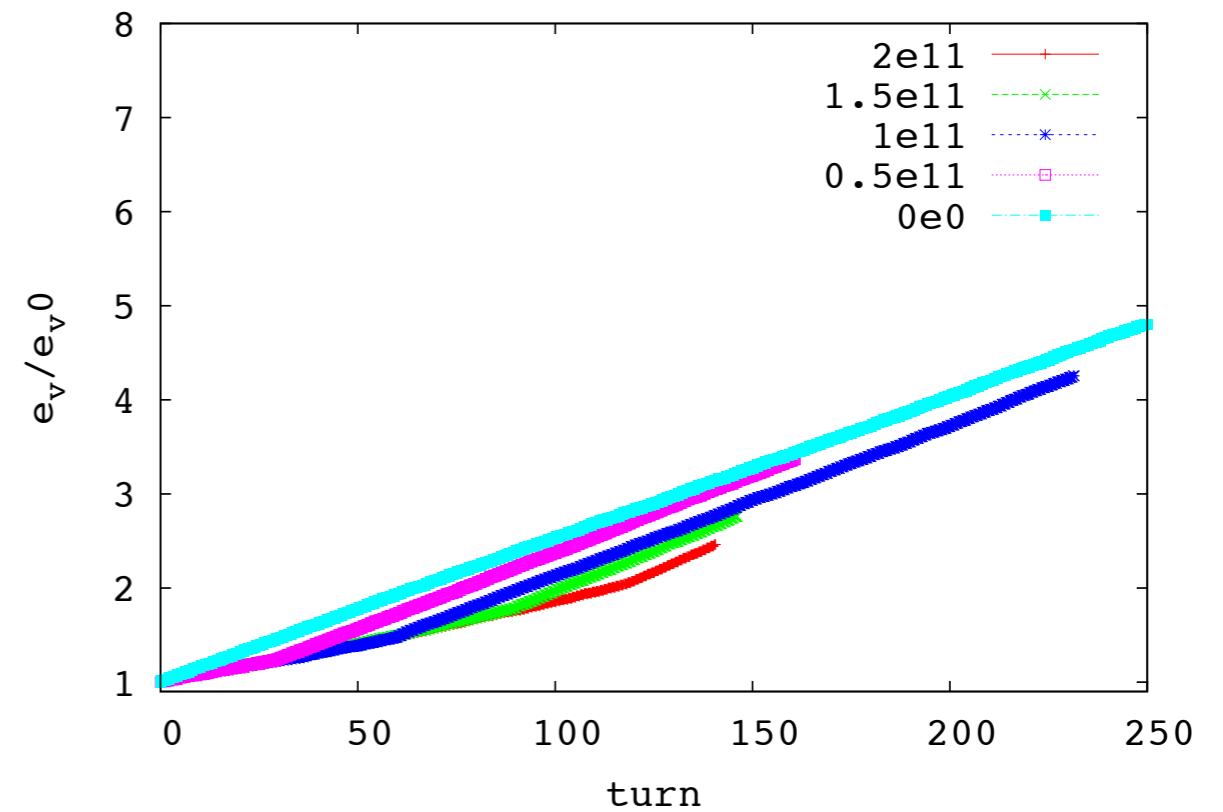
Preliminary tracking results with foil scattering and space charge

Intensity dependence without alignment error

● without foil scattering



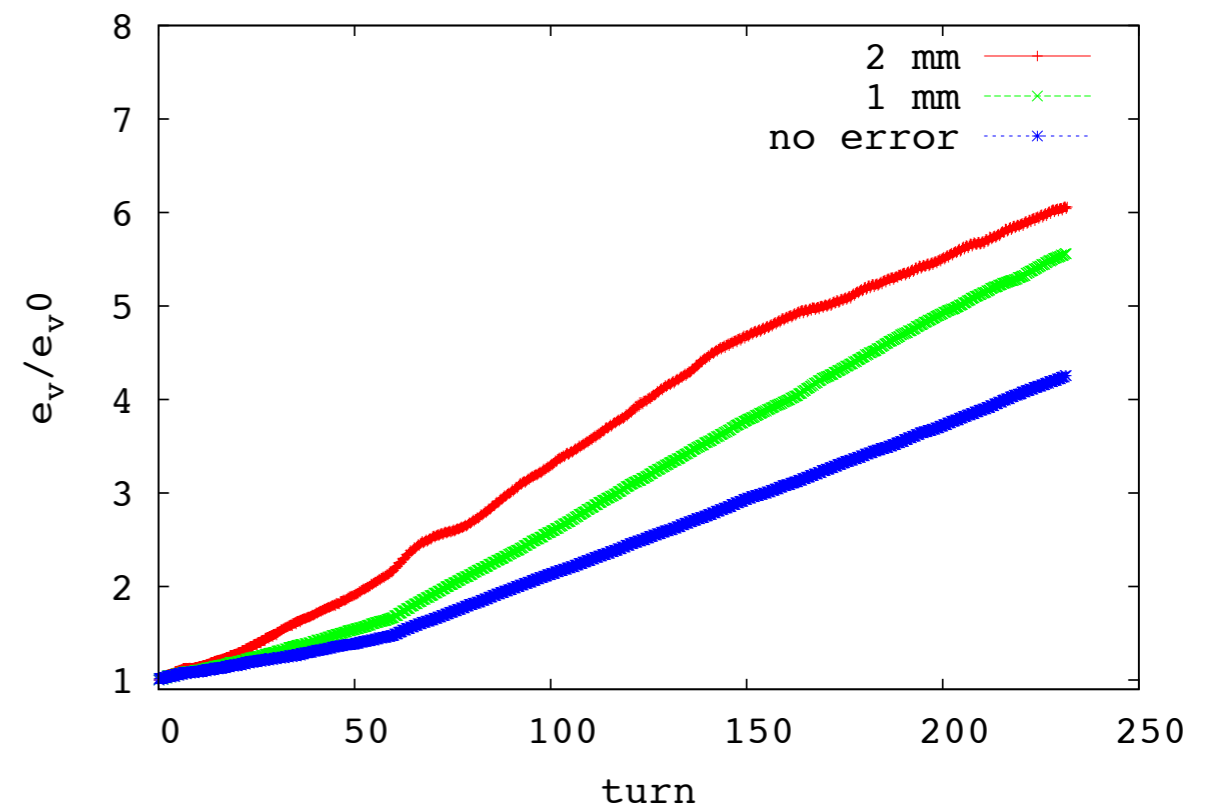
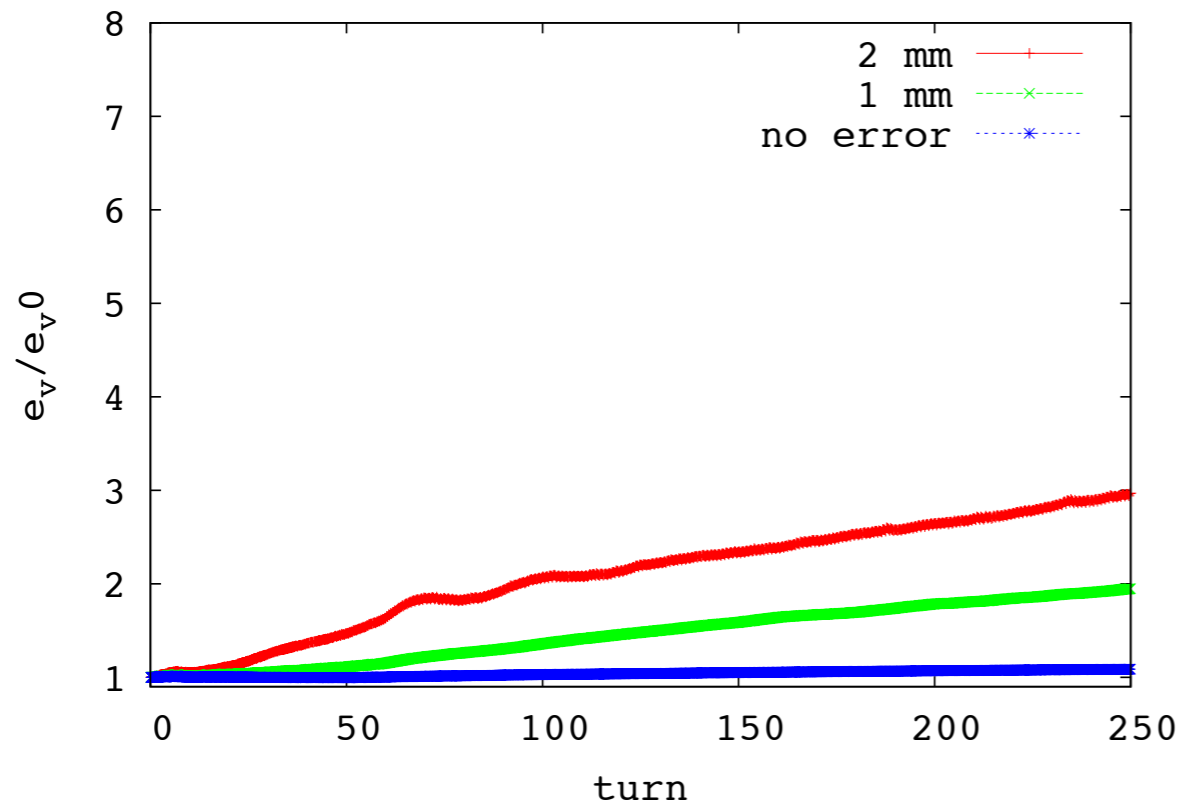
● with foil scattering



With alignment error intensity is fixed at $|E|$ ppb

- without foil scattering

- with foil scattering



Summary

- During multi-turn injection, emittance growth rate is about a half because hitting probability is $N_{inj}/2$.
- Emittance seems to be dominated by foil scattering with the present parameters.

Next step

- More accurate scattering model: one in Geant4, OPAL, etc.
- Does the measurement of charge distribution identify the source of emittance growth?
- RMS emittance growth by foil scattering may be overestimated by tail particles.