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Recipe to estimate the distorted dispersion function from observed COD

v2: (add page 15)

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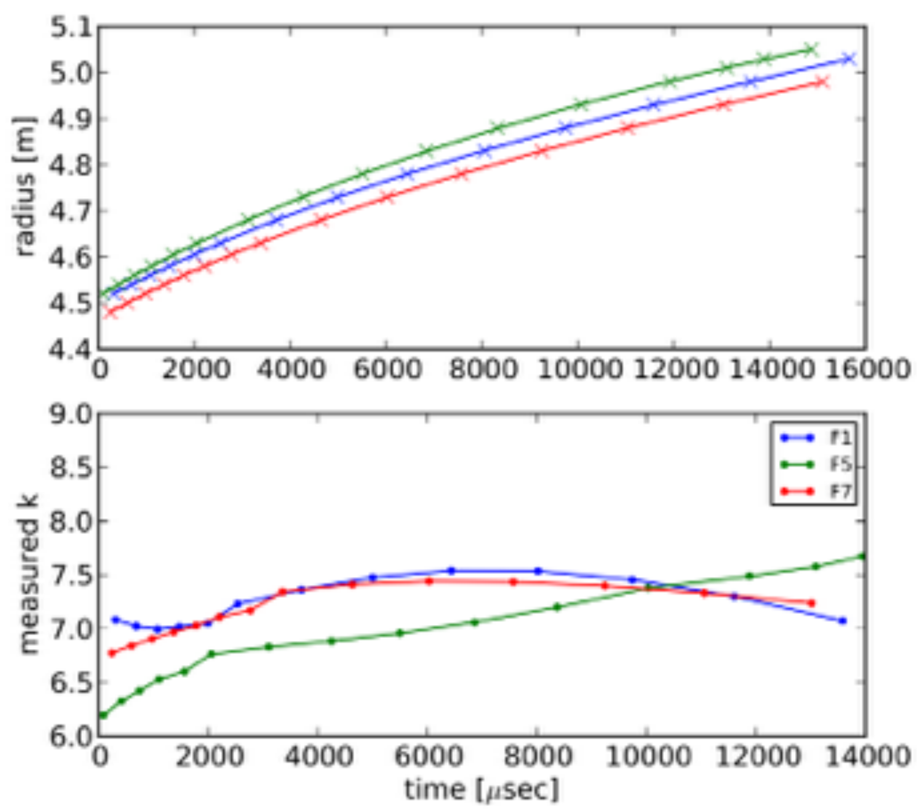
Observation

Observed dispersion function is different at each probe location.

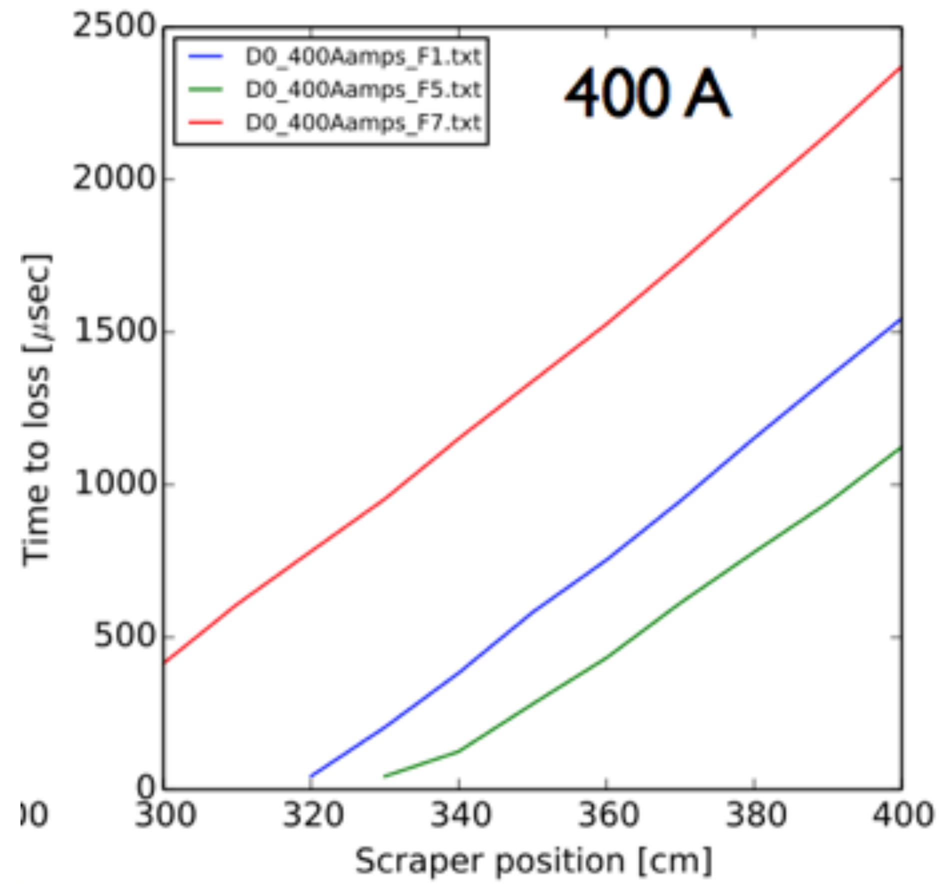
There seems no correlation between COD and the dispersion function.

F5 (green) behaves differently in dispersion measurement (left).

F1 (red) behaves differently in COD measurement (below).



upper: radius vs time
lower: k value vs time



Scaling FFAG

If all the magnet has the profile
$$\frac{B}{B_0} = \left(\frac{r}{r_0}\right)^k$$

orbit of momentum *p* locally satisfies
$$\frac{p}{p_0} = \left(\frac{r}{r_0}\right)^{k+1}$$

Therefore the dispersion function is, to the first order

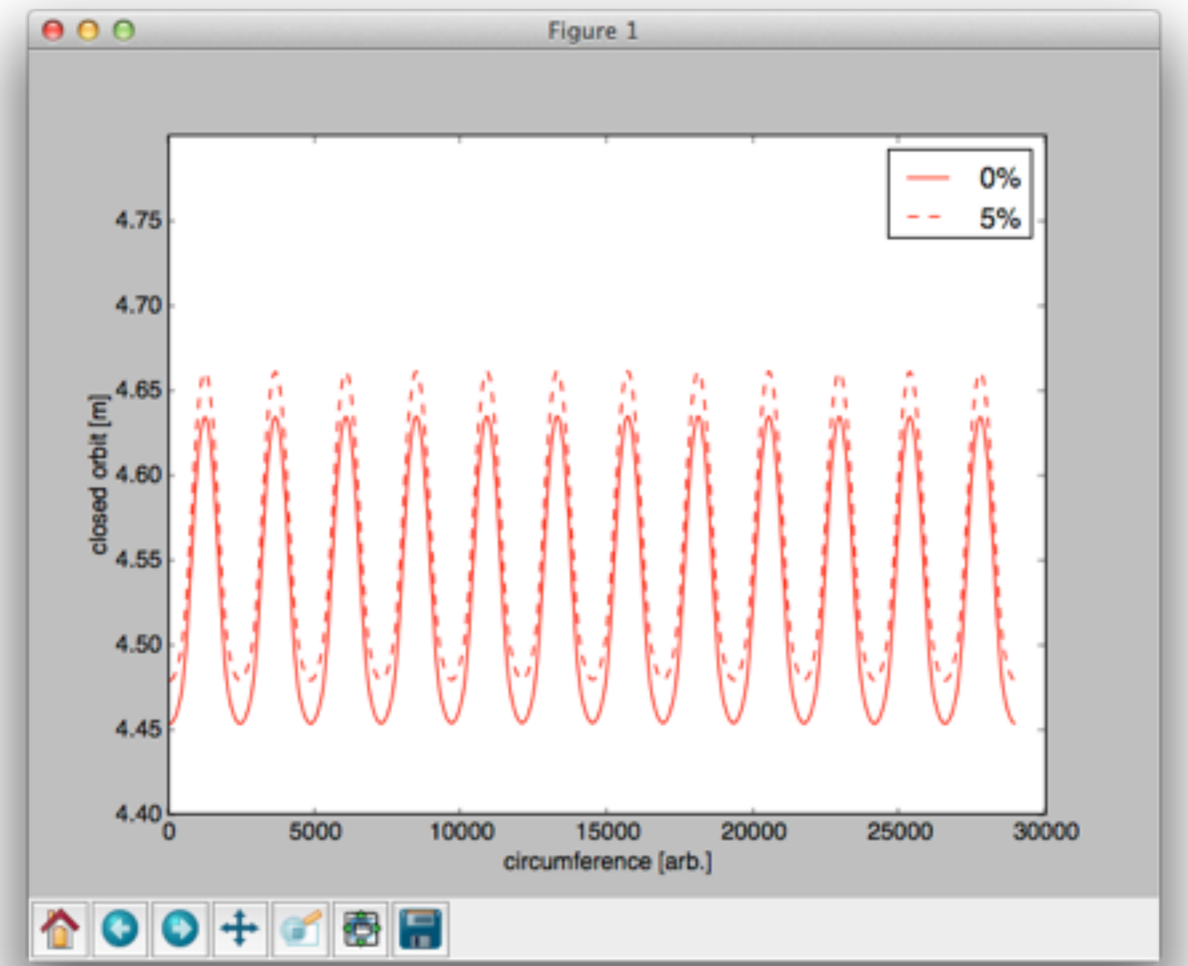
$$D(s) = \frac{r_0(s)}{k+1}$$

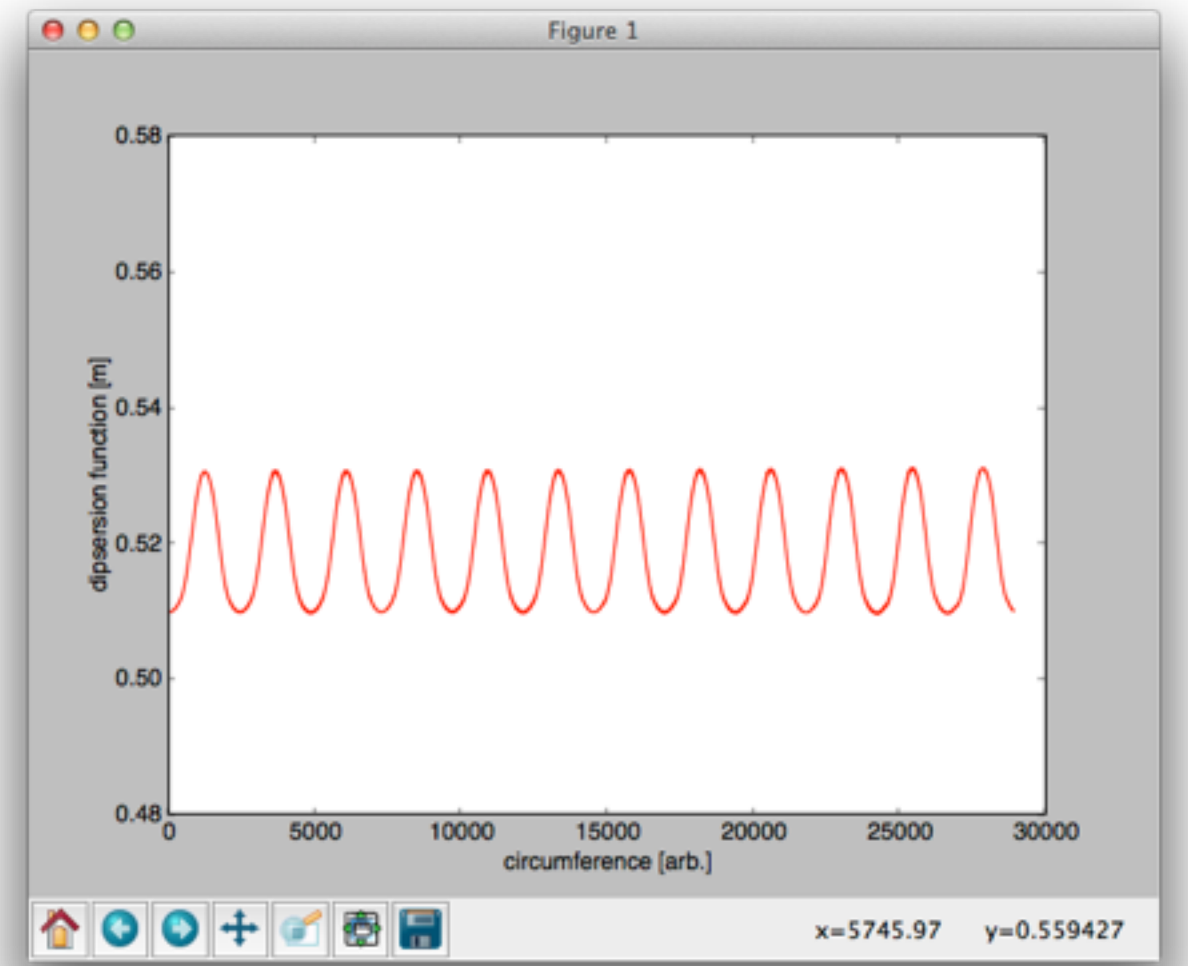
This means that the dispersion function follows the same shape of orbit $r_0(s)$.

Symmetrical lattice

The dispersion functions follows the same shape as the orbit according to

$$D(s) = \frac{r_0(s)}{k+1}$$





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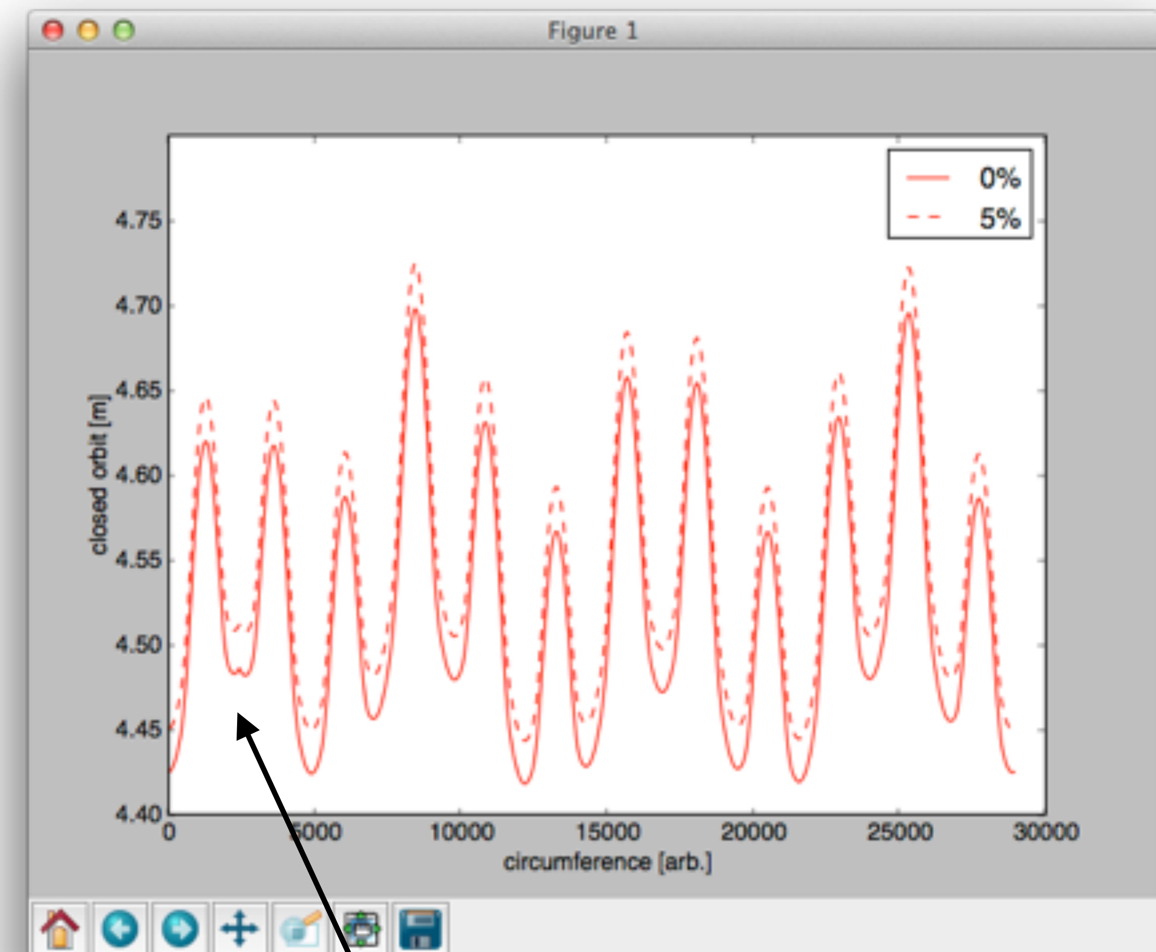
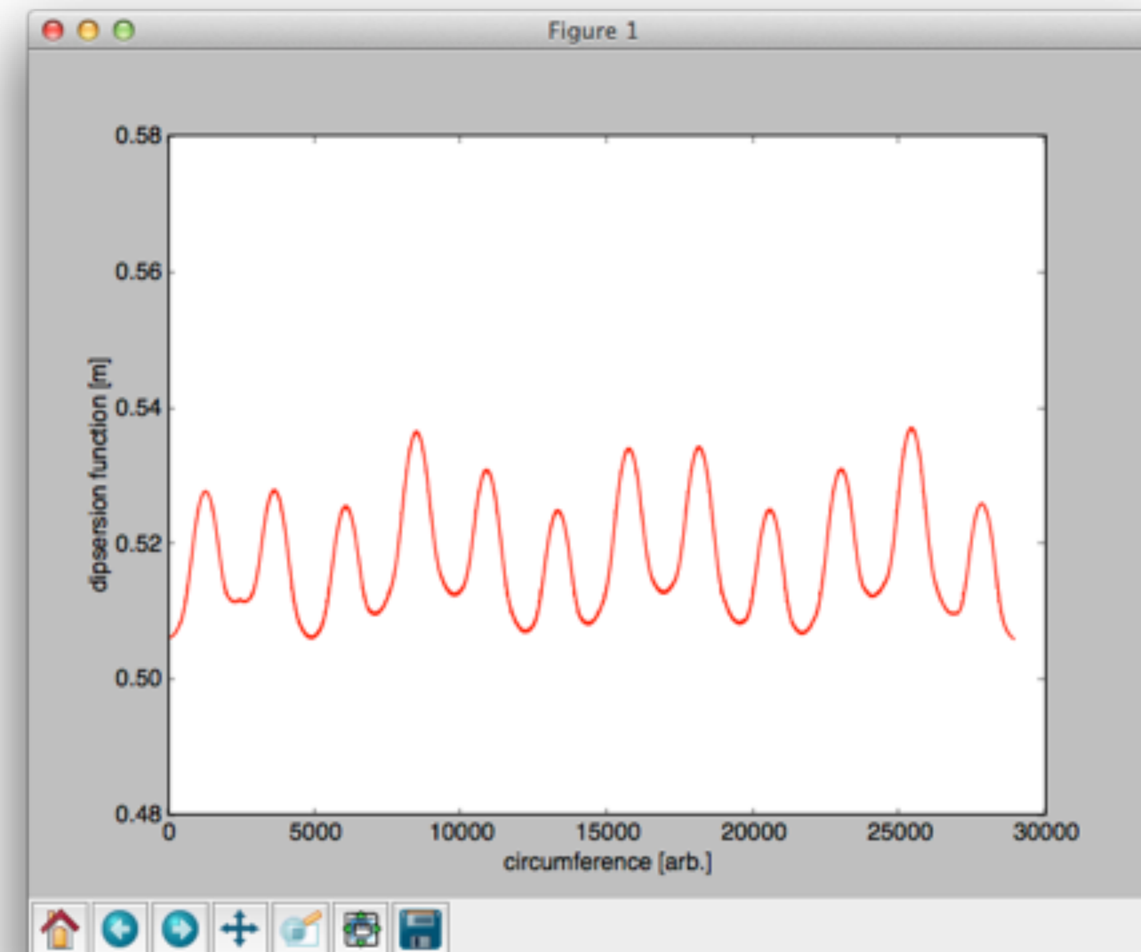
With perturbation

This is true with an extra magnet perturbation as long as it has the field profile of $\frac{B}{B_0} = \left(\frac{r}{r_0}\right)^k$

k=7.5

$$D(s) = \frac{r_0(s)}{k+1}$$





Position of perturbation

Question

Why observed COD and the dispersion function do not follow the same shape?

The answer is the perturbation which introduce COD does not have the field profile of

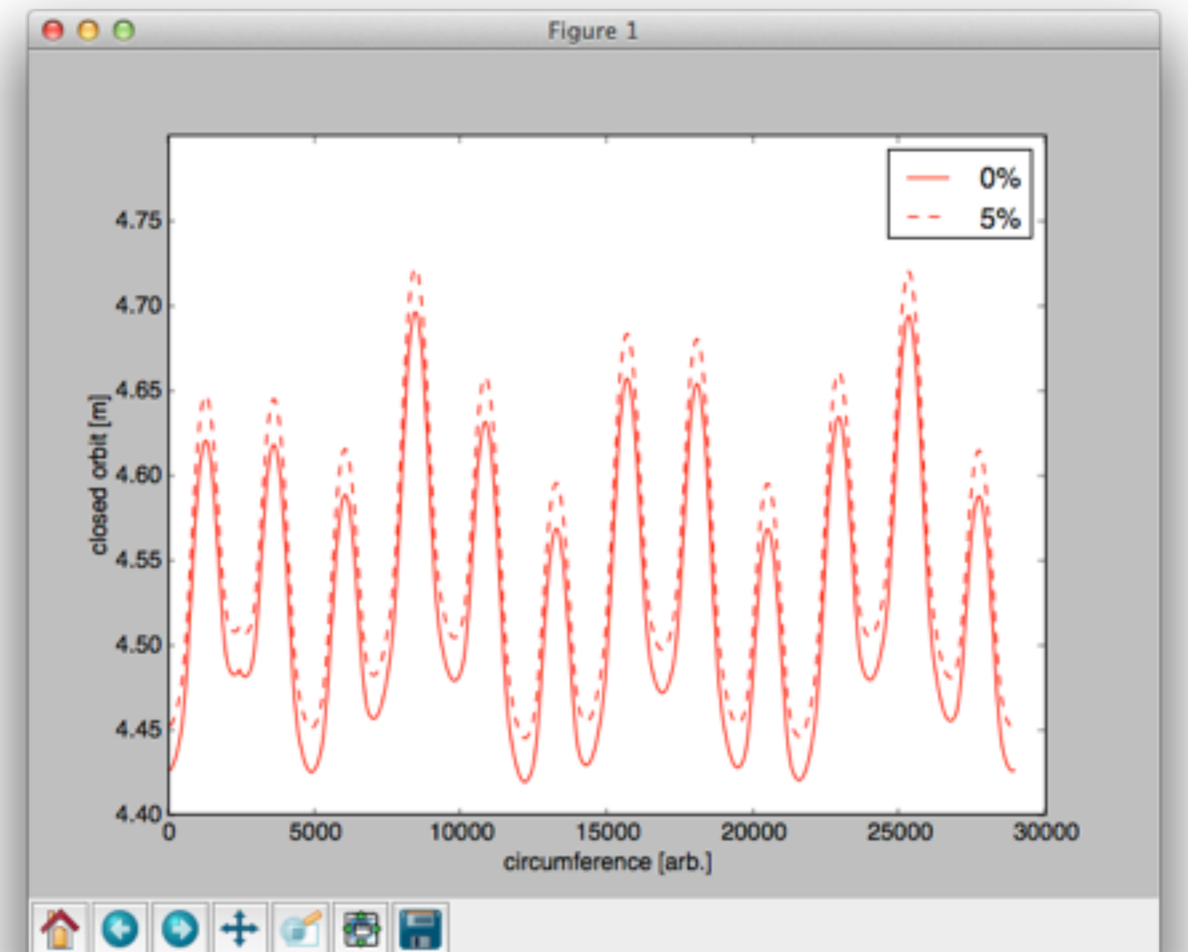
$$\frac{B}{B_0} = \left(\frac{r}{r_0}\right)^k$$

It is likely a dipole without gradient or equivalently $k=0$.

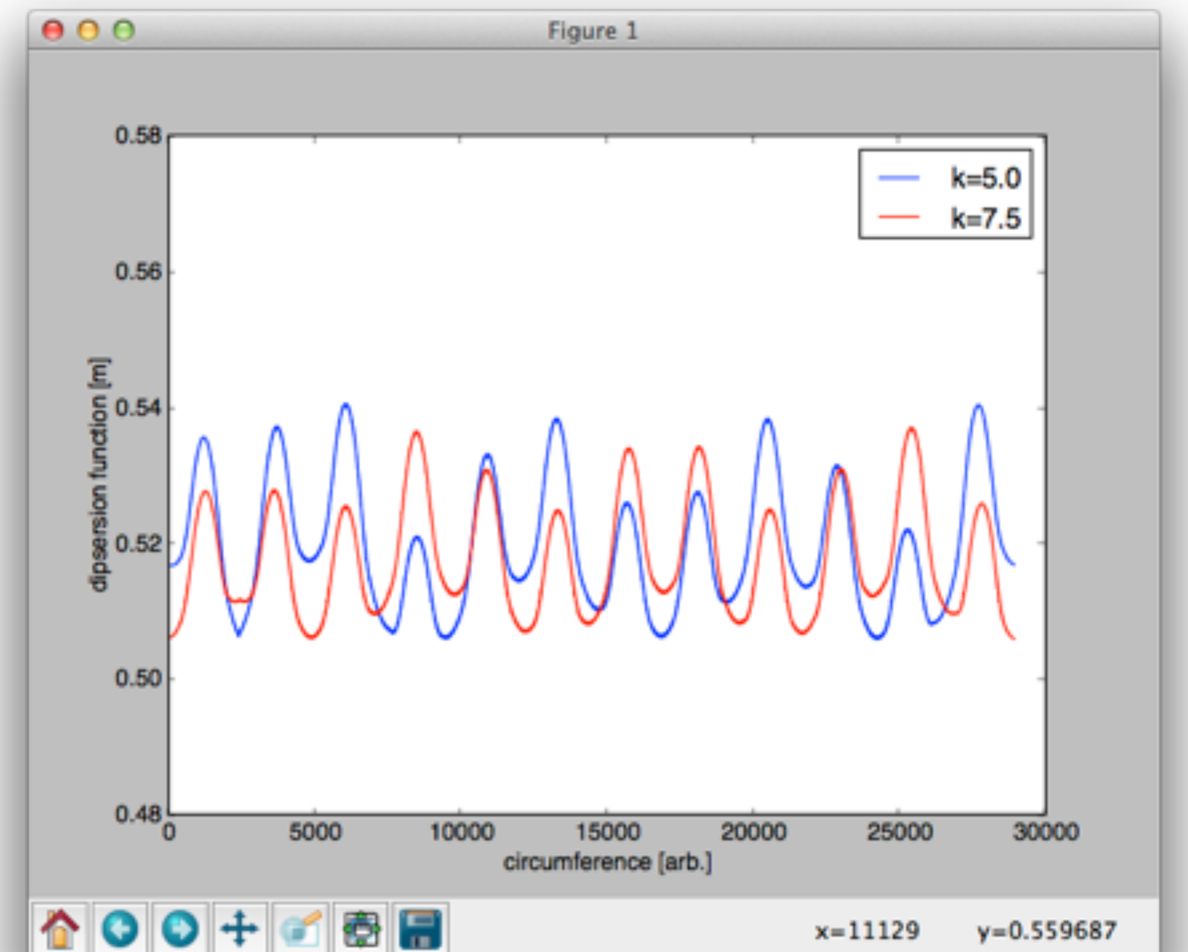
Perturbation

When the perturbation magnet has different *k* (=5.0), while the main magnets have *k*=7.5.

COD does not change much.



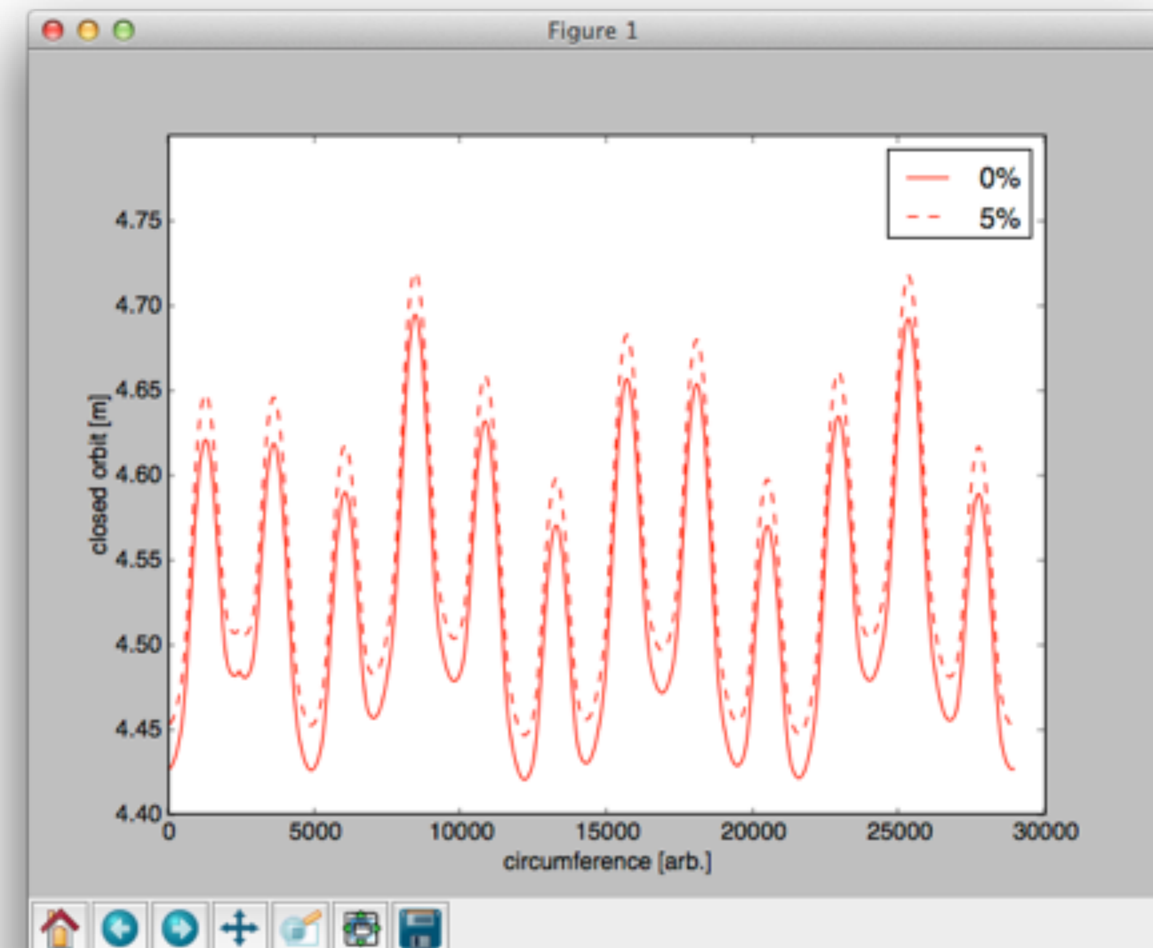
Dispersion is modulated.



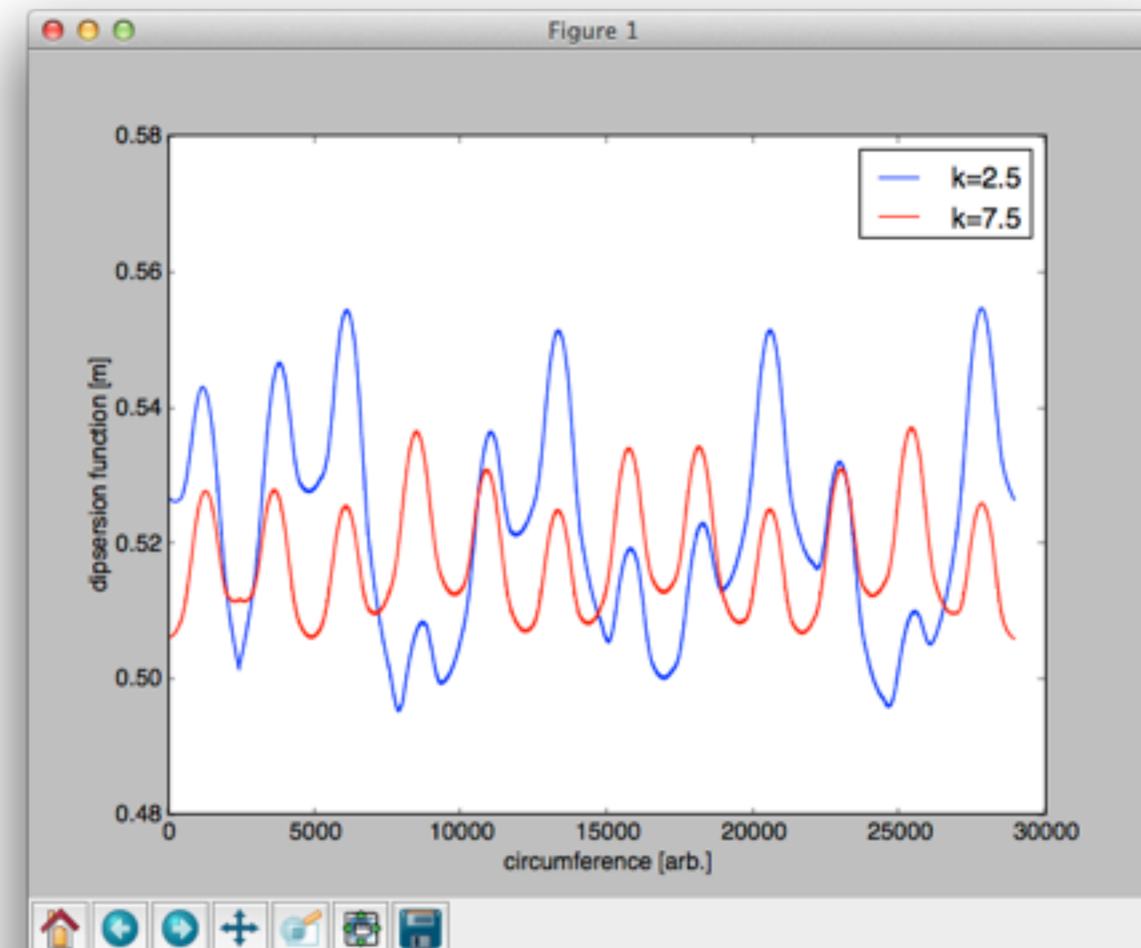
Perturbation

When the perturbation magnet has different *k* (=2.5),
while the main magnets have *k*=7.5.

COD does not change much.



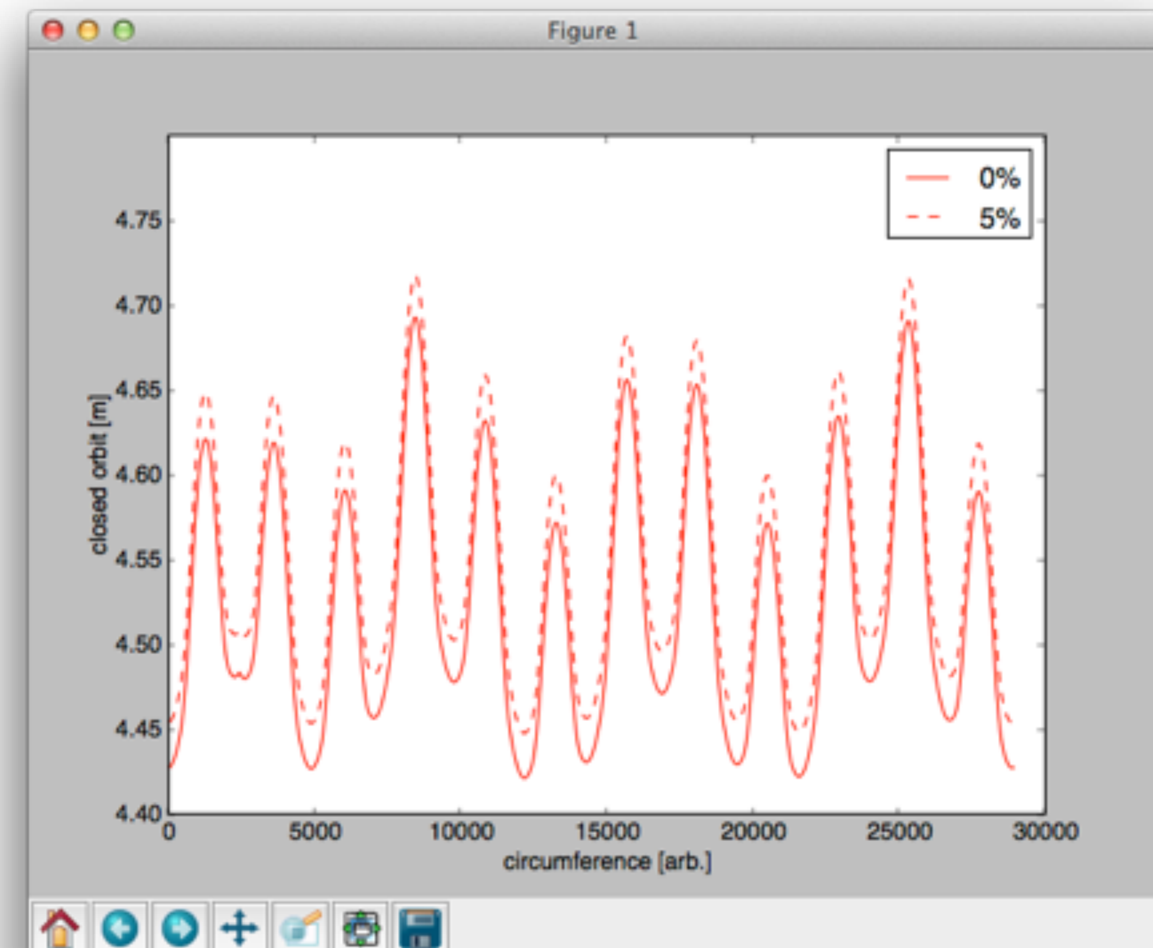
Dispersion is modulated more.



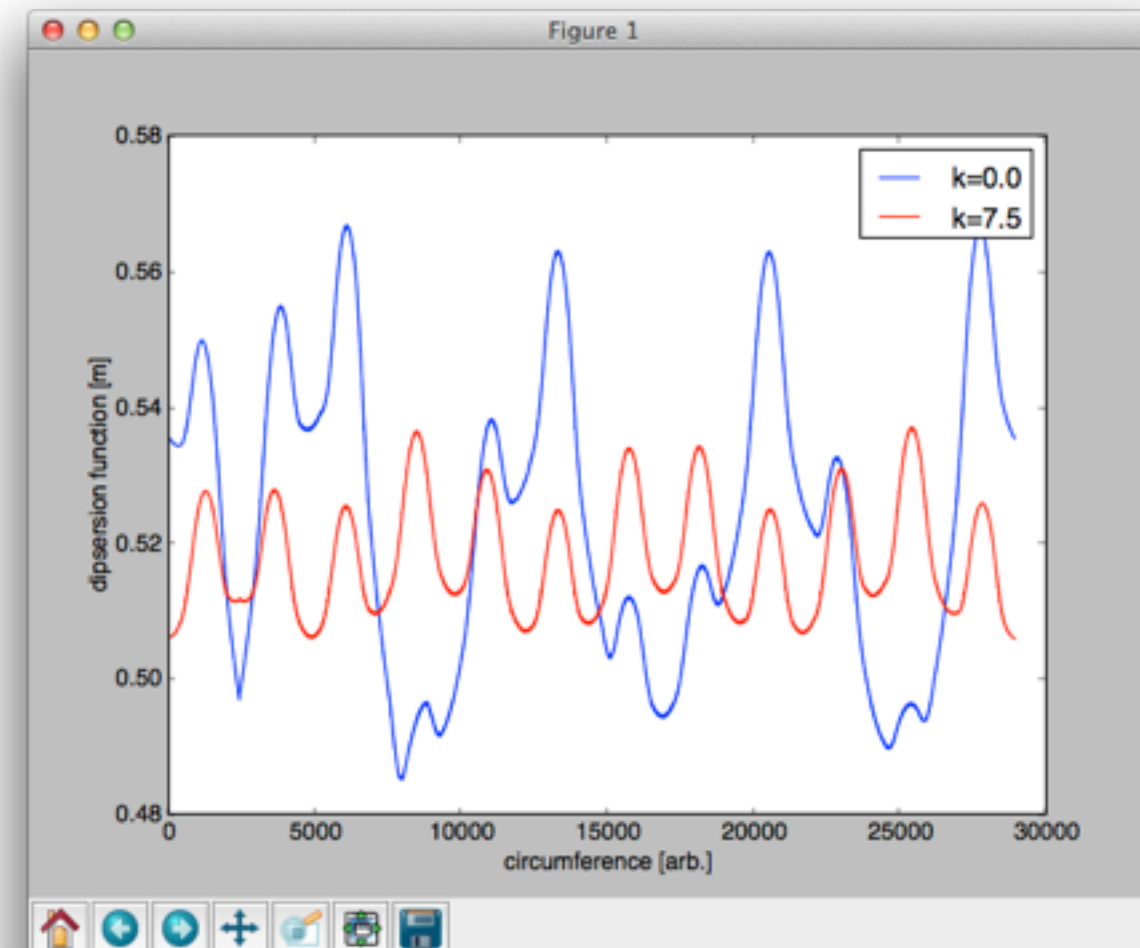
Perturbation

When the perturbation magnet has different *k* (=0.0), while the main magnets have *k*=7.5.

COD does not change much.



Dispersion is modulated most.



Next question

How is this modulation of the dispersion function understood?

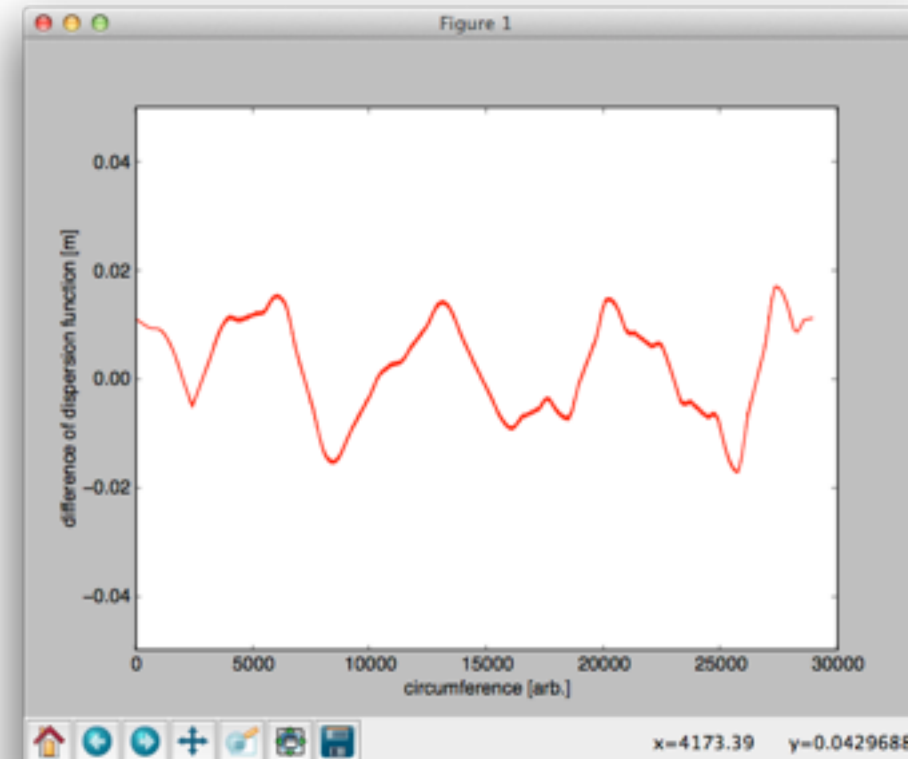
The dispersion function is the gradient of orbit with respect to momentum.

Different k or zero k (no gradient) introduces perturbation which destroys scaling property (such as the same shape of COD and the dispersion).

Difference of the dispersion

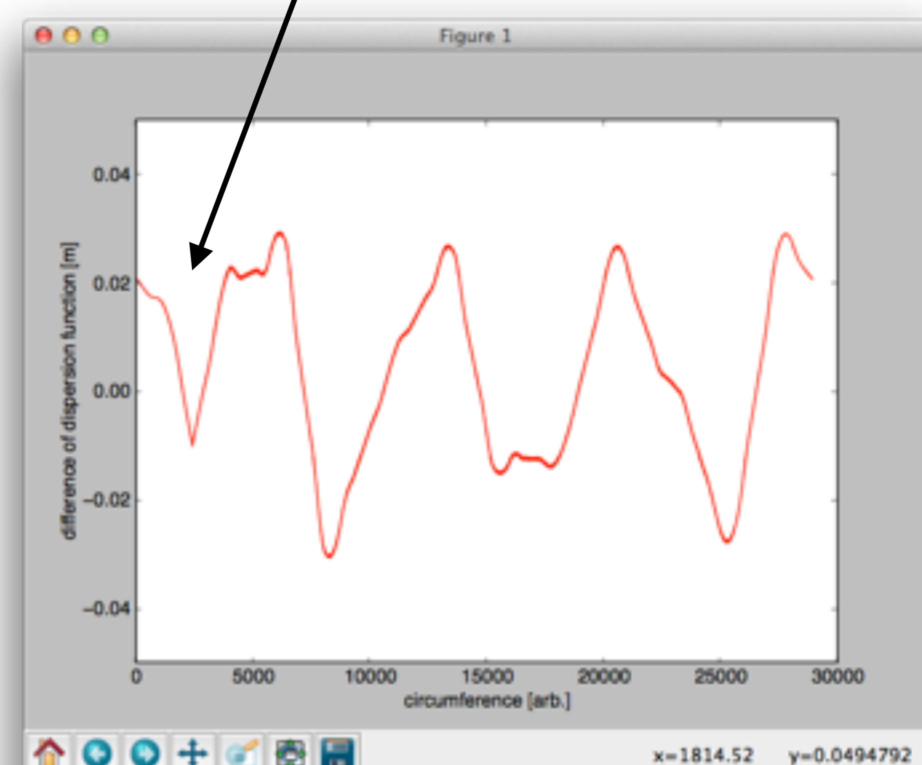
If we take the difference of the dispersions with respect to the one of *k*=7.5, we can identify usual COD whose amplitude is proportional to *k-k'*, where k' is of the perturbation.

k-k'=2.5

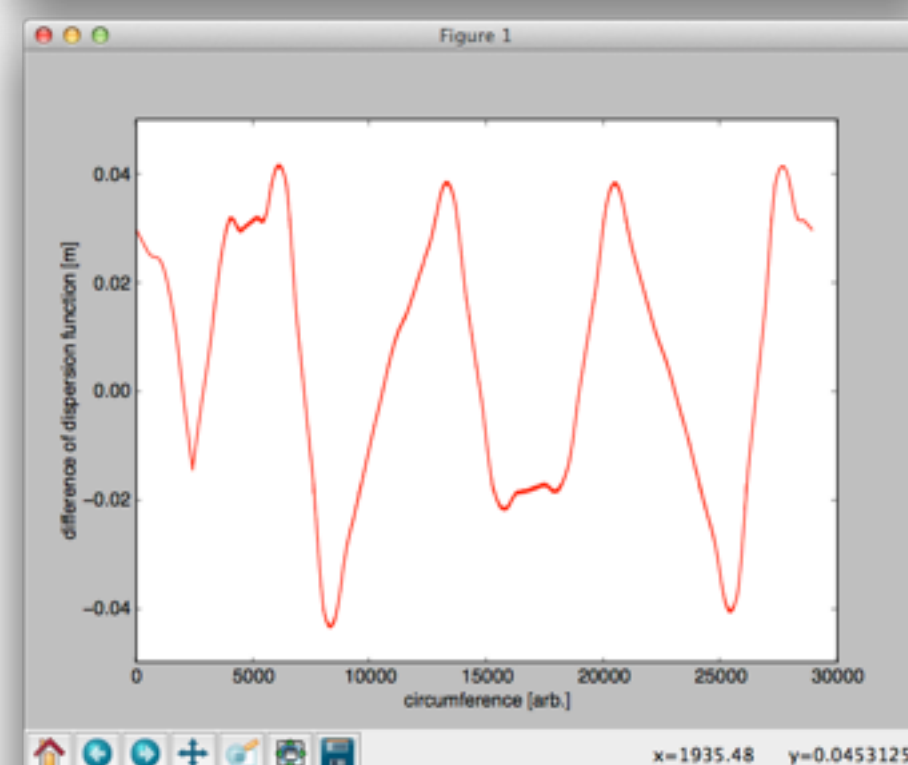


Position of perturbation

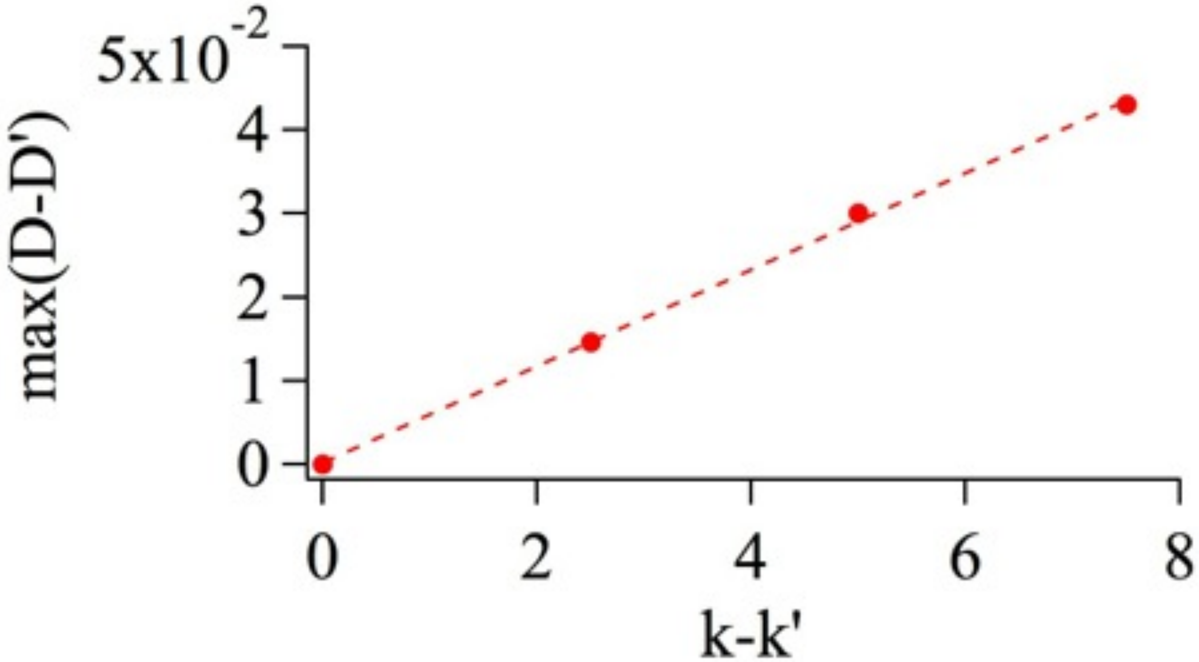
k-k'=5.0



k-k'=7.5



Difference of the dispersion



Amplitude of the difference of the dispersion function is proportional to the difference of *k-k'*.

From the graph, *max(D-D')=0.006(k-k')*.

Difference of the dispersion

Difference of the dispersion means $$D - D' = \frac{\Delta r}{\Delta p/p_0} - \frac{\Delta r'}{\Delta p/p_0}$$

where \Delta r' contains the (lack of) kick due to *k'* index dipole

and this extra kick was applied at \Delta r_kick $$\frac{\Delta r_{kick}}{r_0} = \frac{1}{k+1}\frac{\Delta p}{p_0}$$

Therefore, the kick angle is $$\left(\frac{\Delta BL}{(B\rho)}\right)_k - \left(\frac{\Delta BL}{(B\rho)}\right)_{k'} = (k-k')\frac{\Delta r_{kick}}{r_0}\frac{B_0 L}{(B\rho)} = \frac{k-k'}{k+1}\frac{B_0 L}{(B\rho)}\frac{\Delta p}{p_0}$$

COD due to this kick $$y_{co}(s) = \frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin\pi Q}\cos(\pi Q - |\psi(s) - \psi(s_0)|)\frac{k-k'}{k+1}\frac{B_0 L}{(B\rho)}\frac{\Delta p}{p_0}$$

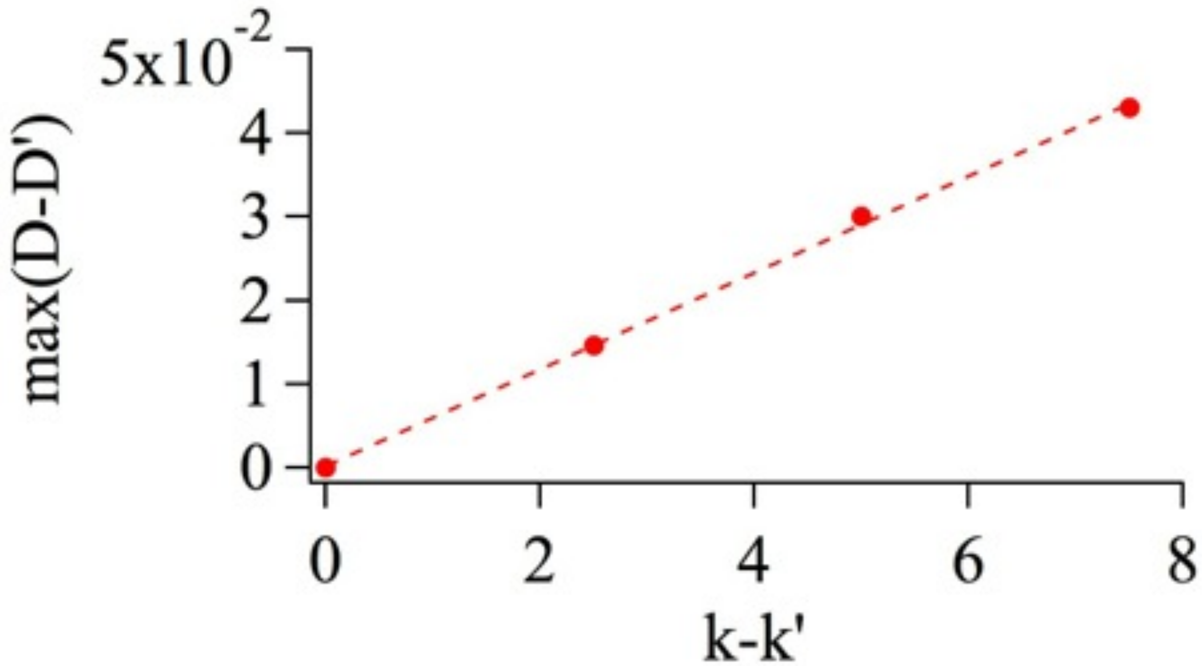
should account for the difference of the dispersion.

$$max(D_1 - D_2) = max(\frac{\Delta r_1 - \Delta r_2}{\Delta p/p}) = \frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin\pi Q}\frac{k-k'}{k+1}\frac{B_0 L}{(B\rho)}$$

After putting some numbers,

From the equation, *max(D-D')=0.011(k-k').*

Difference of the dispersion



From the graph, *max(D-D')=0.006(k-k')*.

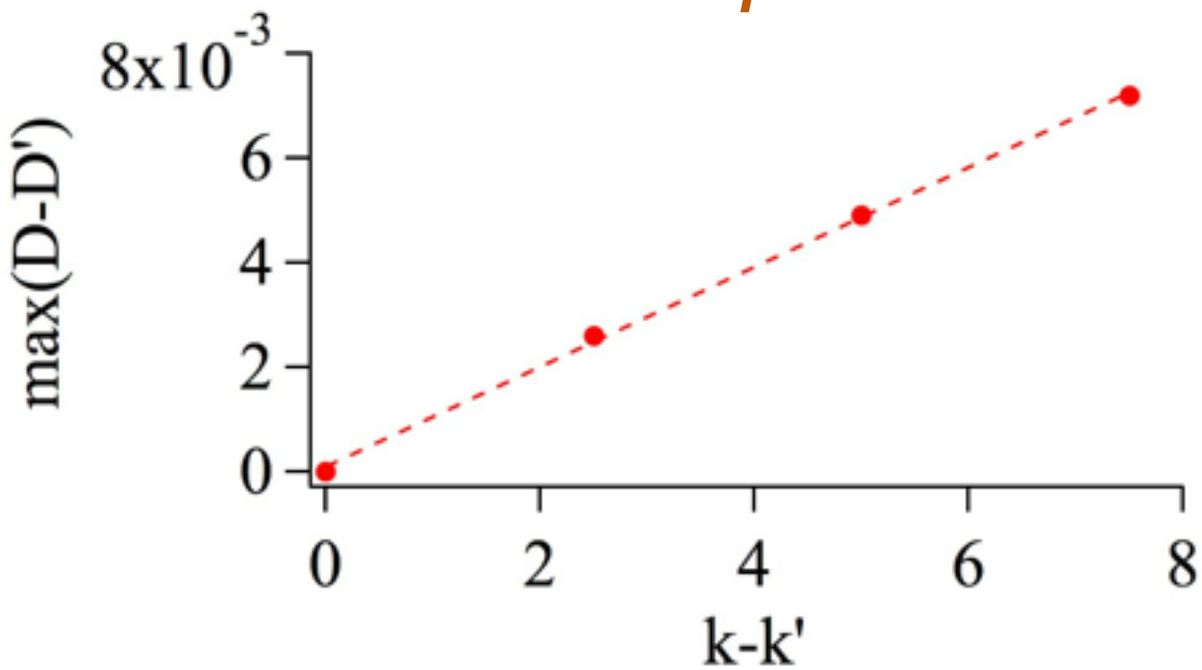
$$max(D_1 - D_2) = max(\frac{\Delta r_1 - \Delta r_2}{\Delta p/p}) = \frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin\pi Q}\frac{k-k'}{k+1}\frac{B_0 L}{(B\rho)}$$

From the equation, *max(D-D')=0.011(k-k')*.

Difference of factor 2 could be due to uncertainly of beta function, assumption of small perturbation to the orbit, etc.

Difference of the dispersion

comparison with 10 times smaller perturbation



From the graph, *max(D-D')=0.0010(k-k').*

$$max(D_1 - D_2) = max(\frac{\Delta r_1 - \Delta r_2}{\Delta p/p}) = \frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin\pi Q}\frac{k-k'}{k+1}\frac{B_0 L}{(B\rho)}$$

From the equation, *max(D-D')=0.0011(k-k').*

Smaller perturbation makes the agreement better.

Recipe

1. Find the amplitude of COD.
2. Assume the source of COD is a pure dipole, or with different k.
3. Add extra kick at the source of COD which comes from the difference between *k* and *k'* (=0 if pure dipole).
4. Calculate the addition of the dispersion function to the one described as

$$D = \frac{r_0}{k+1}$$