



KURRI FFAG simulation update

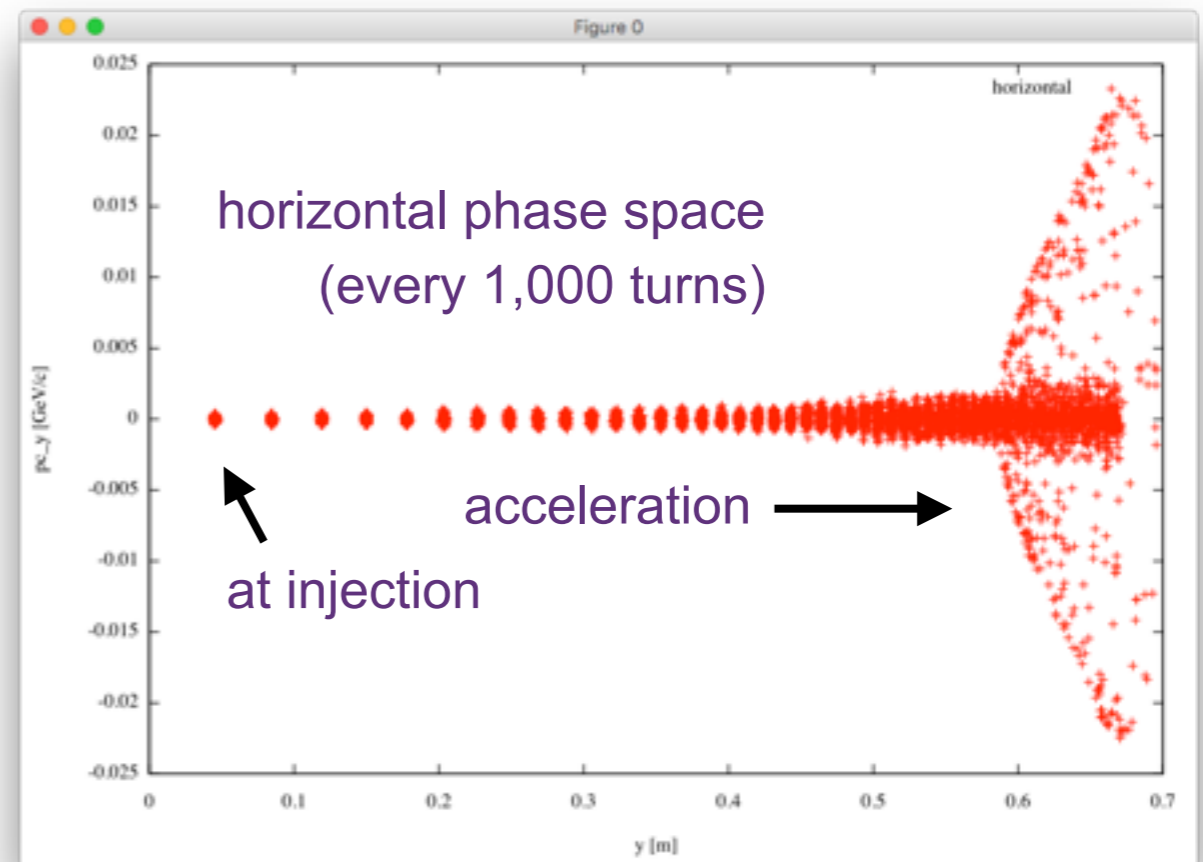
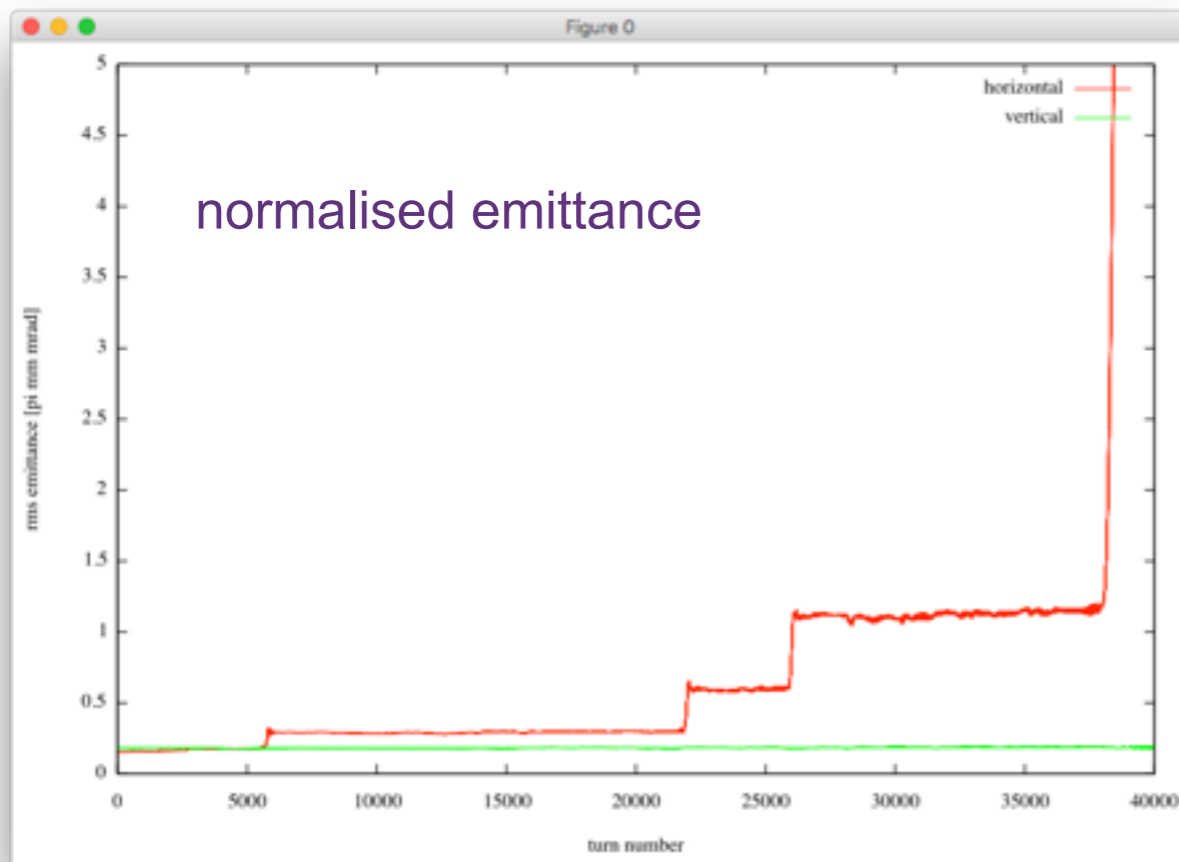
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Emittance evolution

Zgoubi and Scode show similar emittance jump at some turns (energy).

No space charge. No error in the lattice.

No jump in vertical plane.



$$\text{Kinetic energy [MeV]} = 11.0 + 0.002 \times (\text{turn number})$$

Model lattice

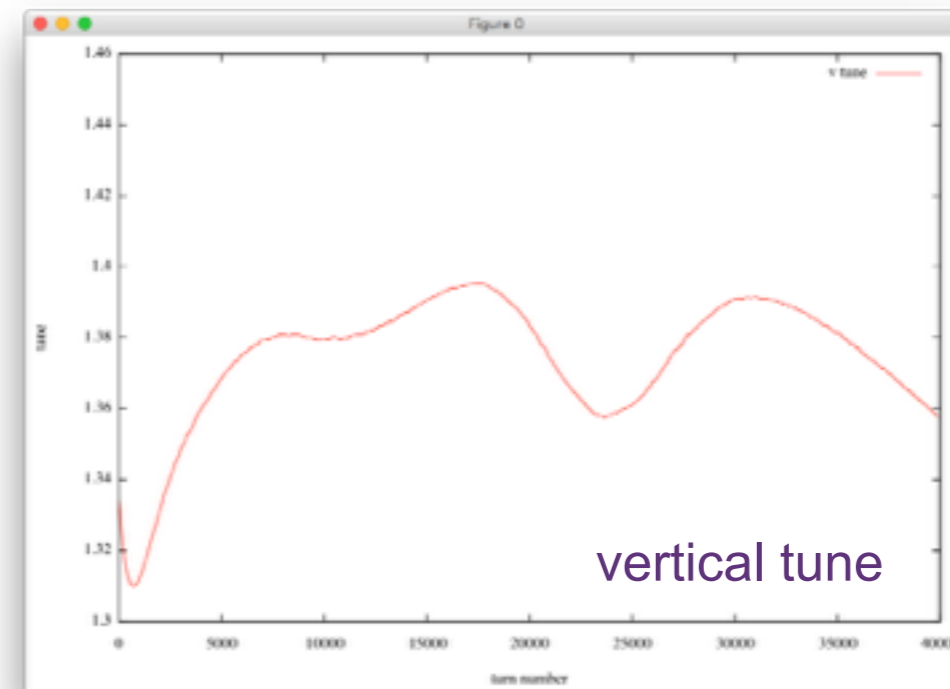
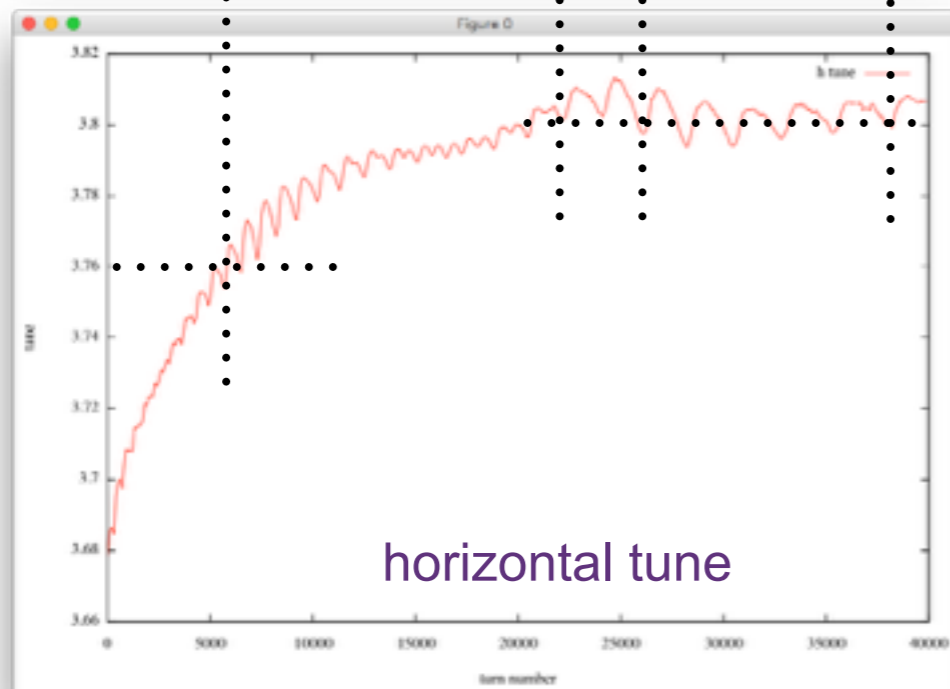
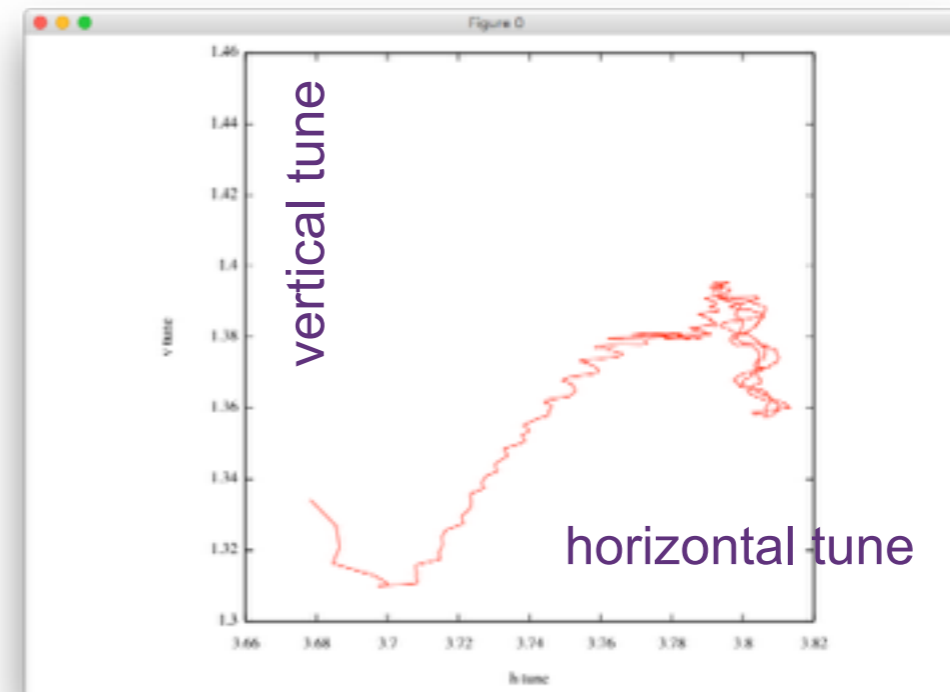
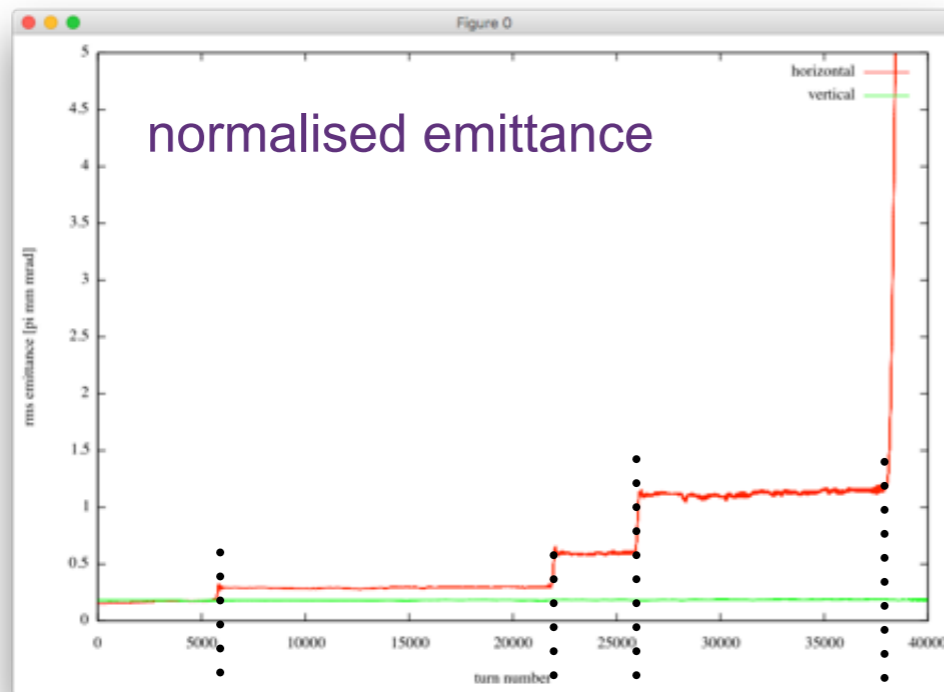
The model lattice preserves 12 fold symmetry.

Resonance driving term other than the multiple of 12 should not be excited.

$n Q_h = 12 h$ where n and h are integer.

Except the first jump, it happens at around $Q_h=3.8$

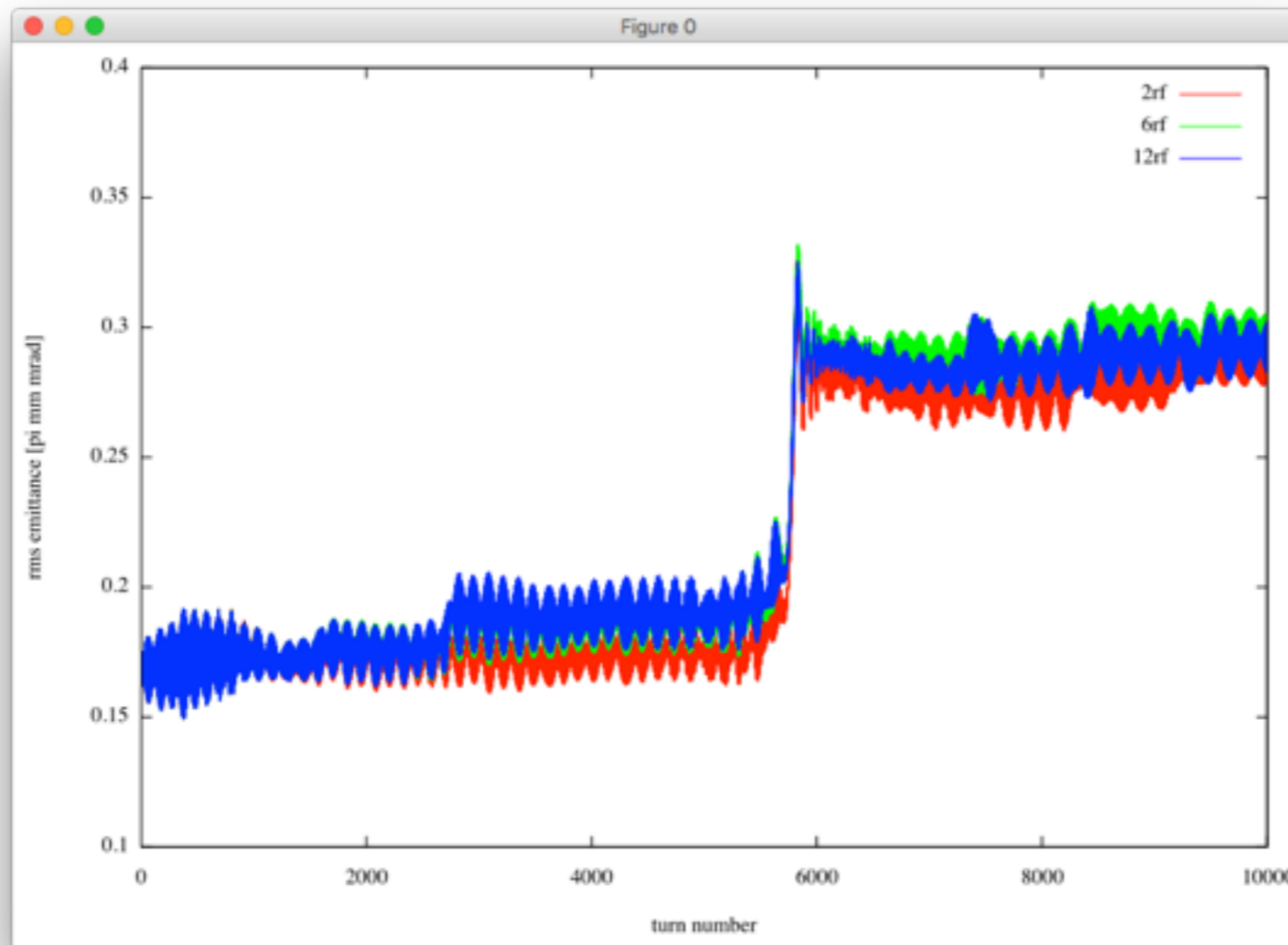
Tune evolution



Parameter dependence (1)

number of rf cavities

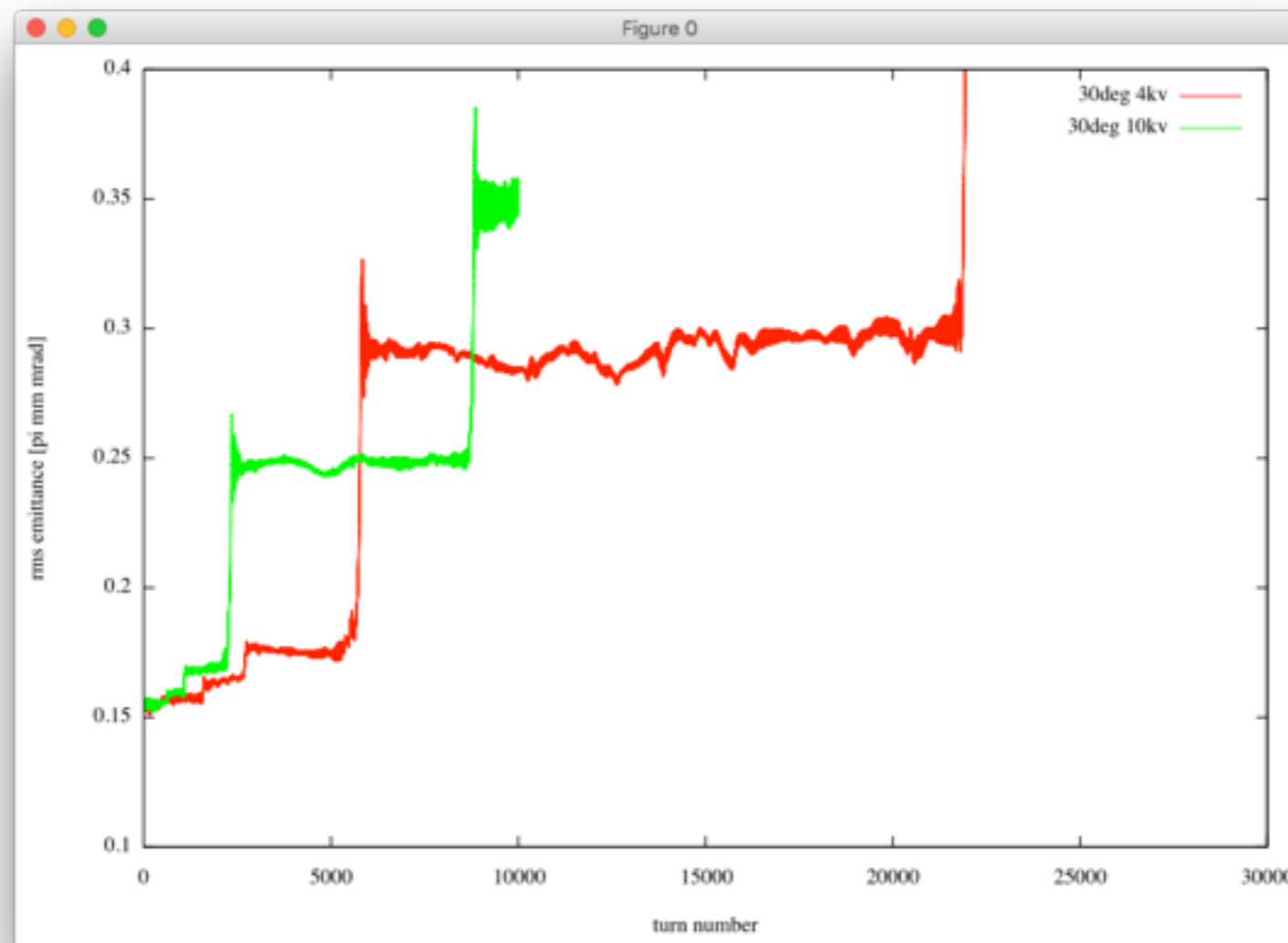
Exact location of rf cavities does not make difference.



Parameter dependence (2)

acceleration speed

When acceleration is 2.5 times faster, the peaks appear at 1/2.5 turns.

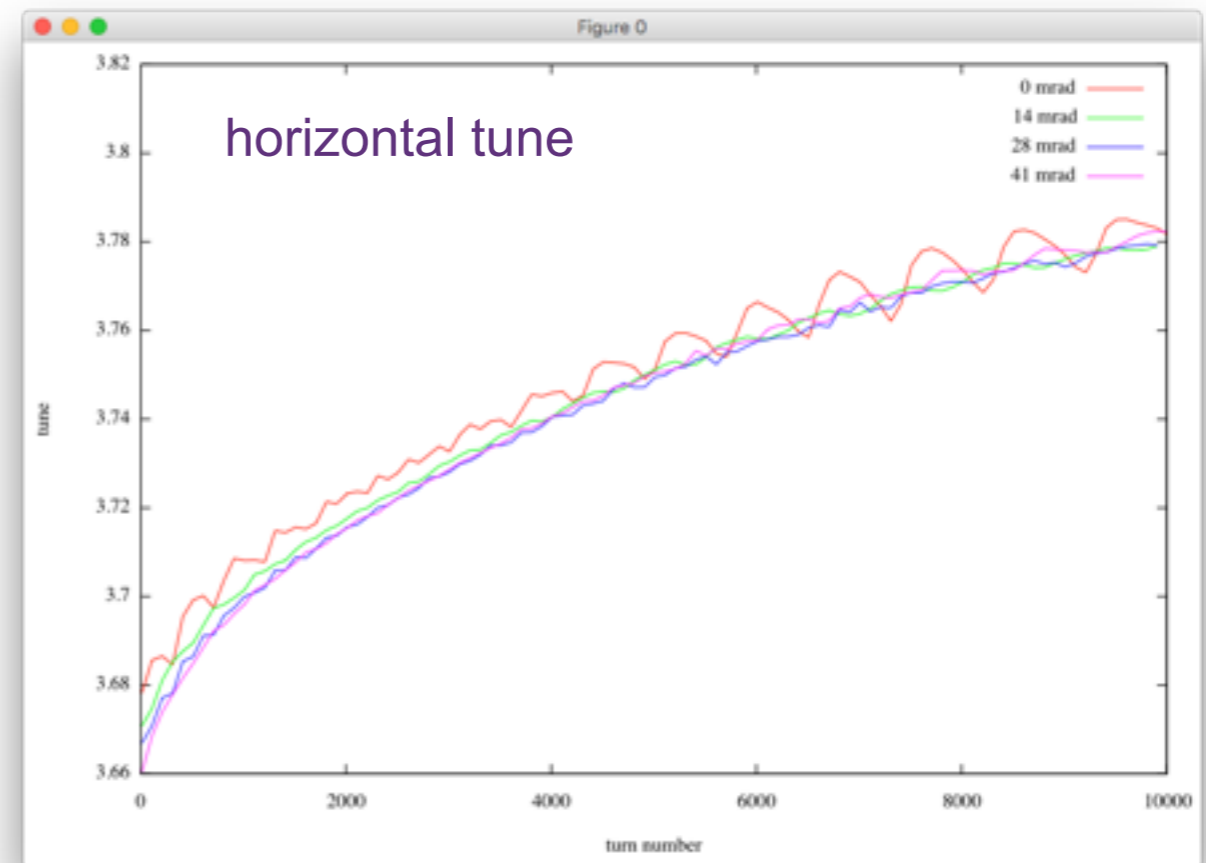
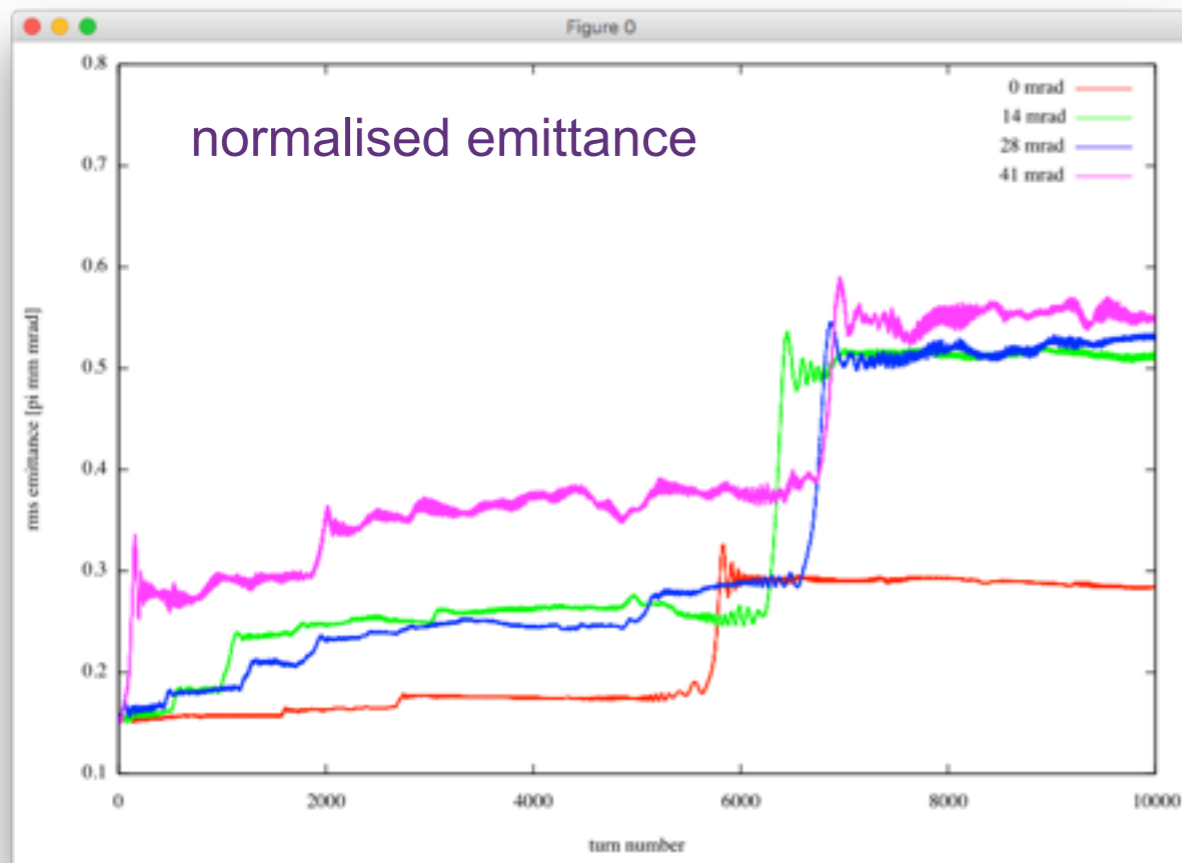


Parameter dependence (3)

COD

When COD is included, jump is larger and moves slightly.

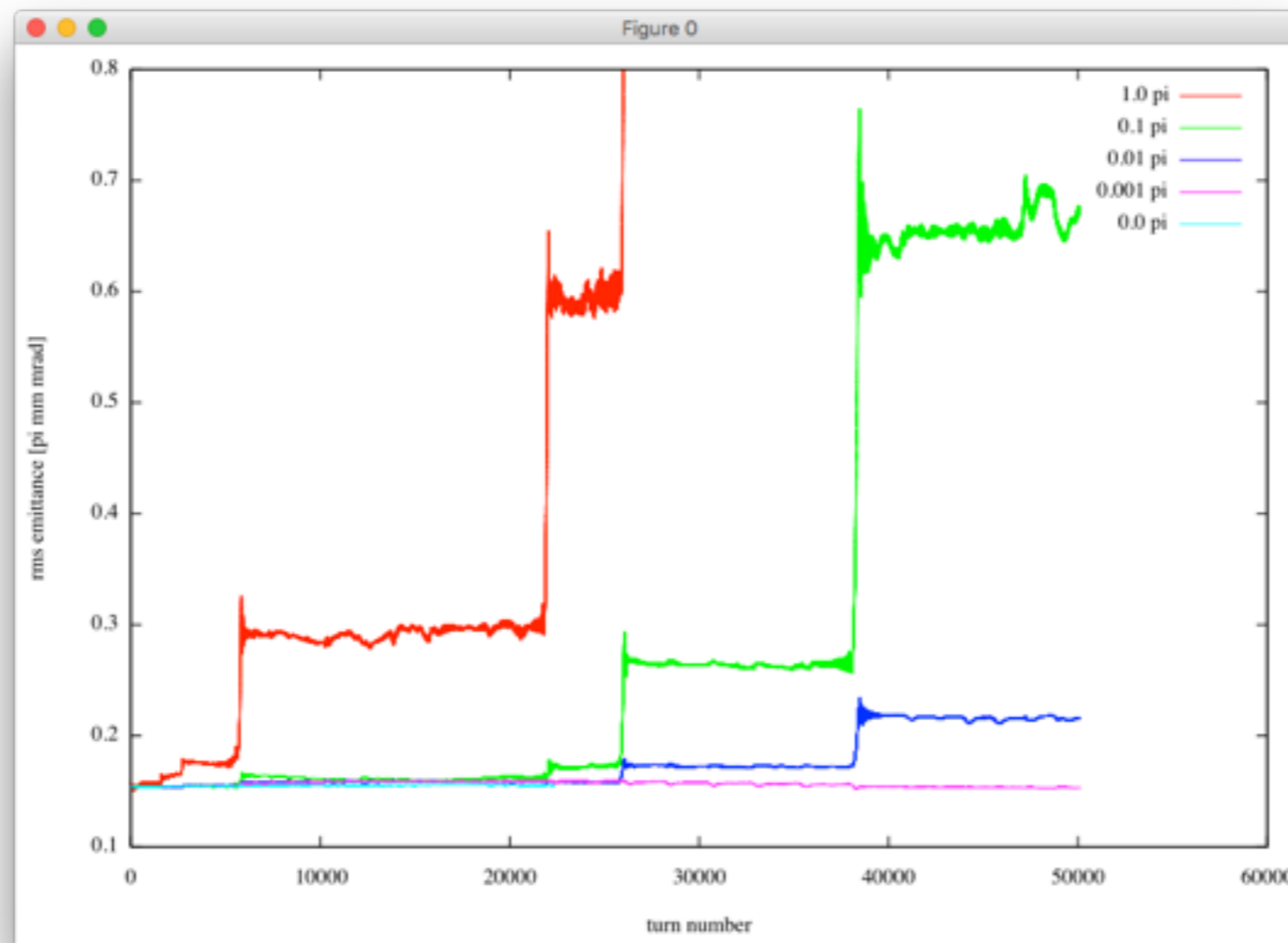
The shift is due to the slight change of tune.



Parameter dependence (4)

vertical emittance

When vertical emittance is reduced (1 pi to 0 pi), the jump disappears.



Possible explanation of jump

Because of finite betatron oscillations in vertical direction, 12 fold symmetry in horizontal direction within one turn is broken.

e.g. sextupole field: x^2-y^2 in horizontal
 xy in vertical

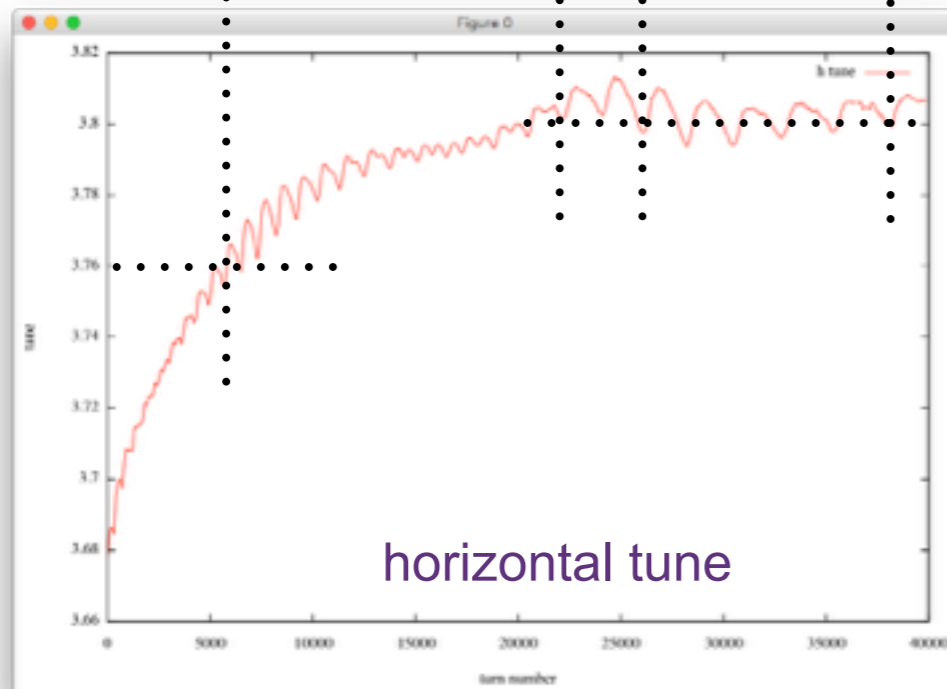
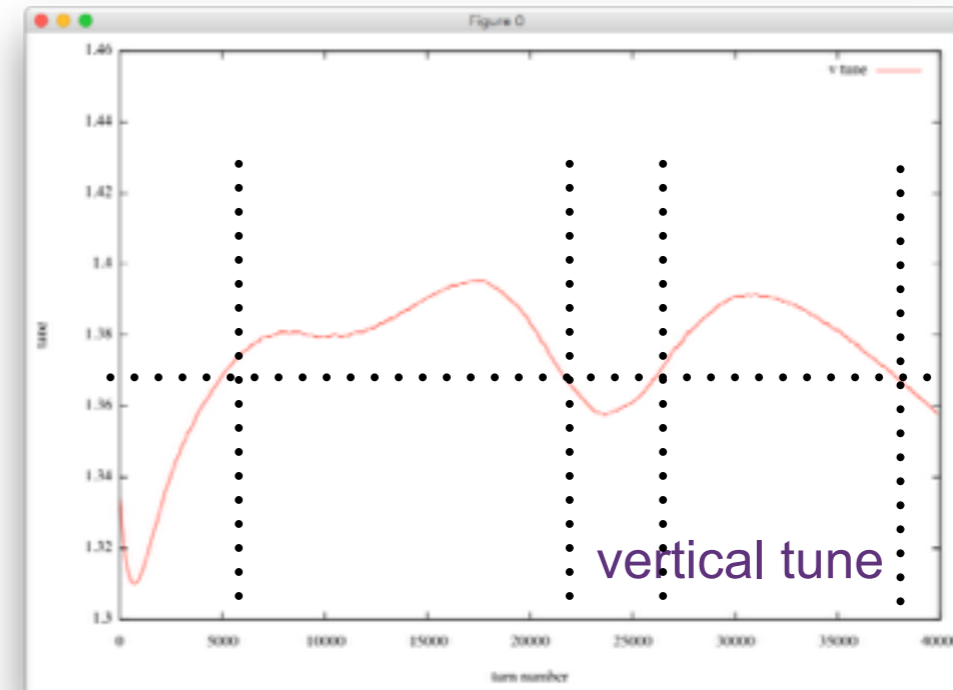
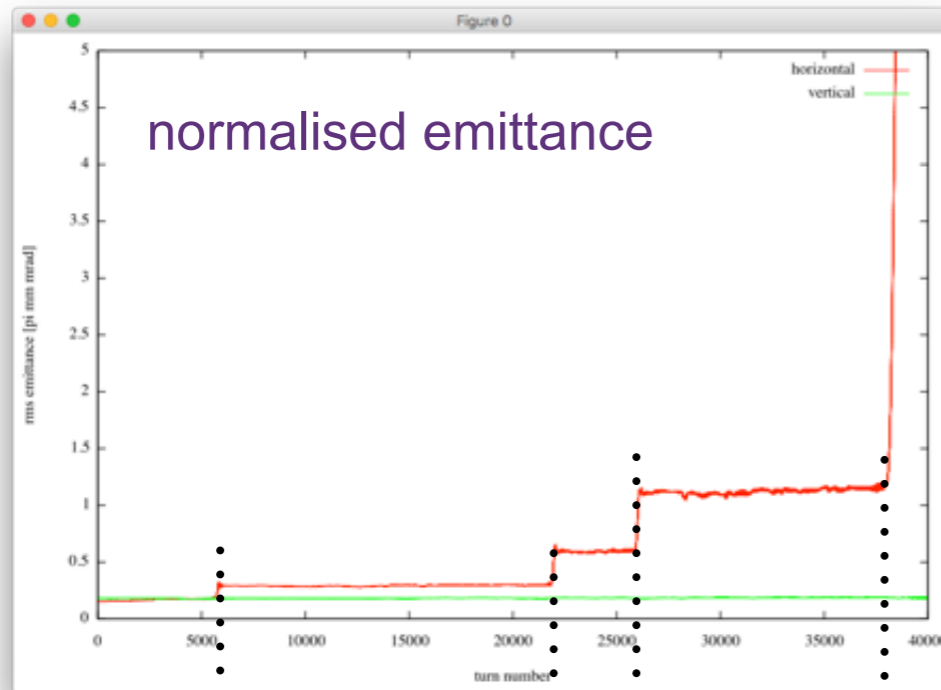
But why no jump in vertical?

In Scode, linear interpolation between field grids only gives quadrupole term.

Vertical tune is close to 1 so that the same harmonics component stays longer?

Except the first jump, it happens at around $Q_h=3.8$

Tune evolution



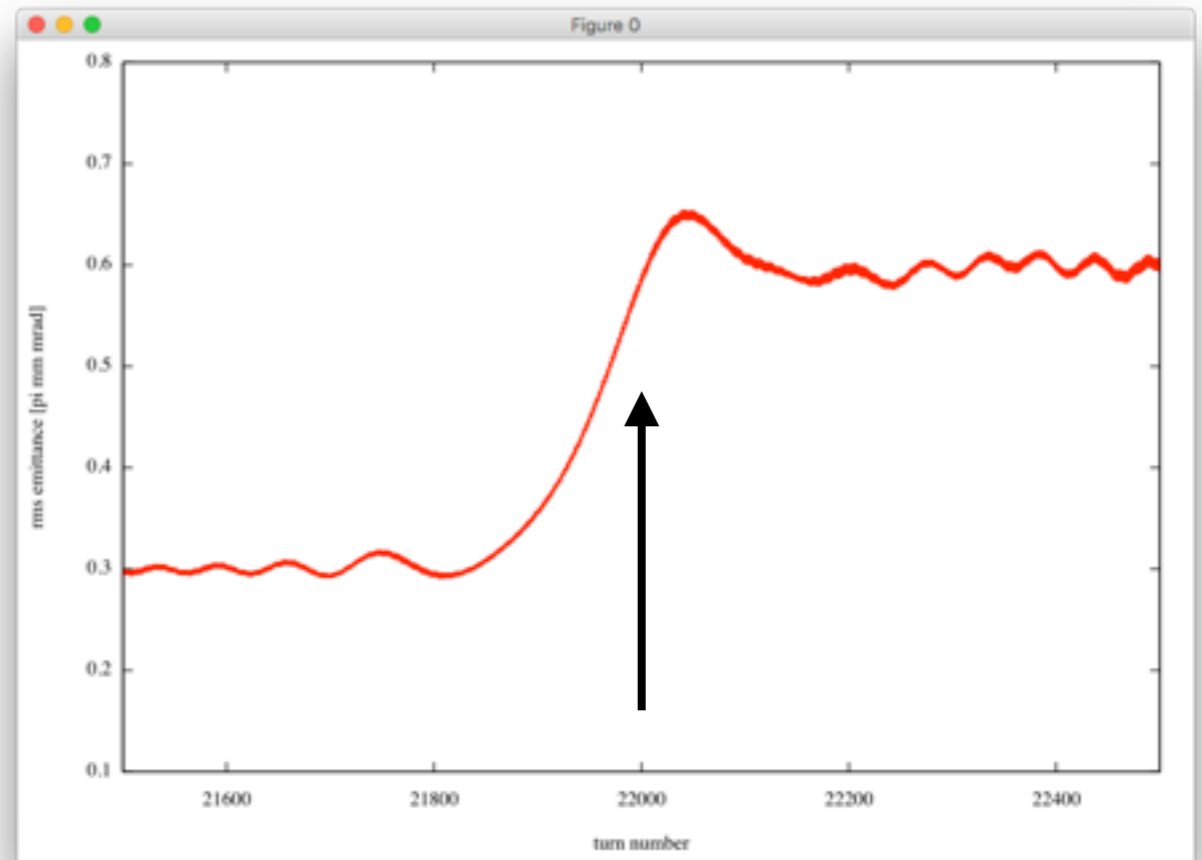
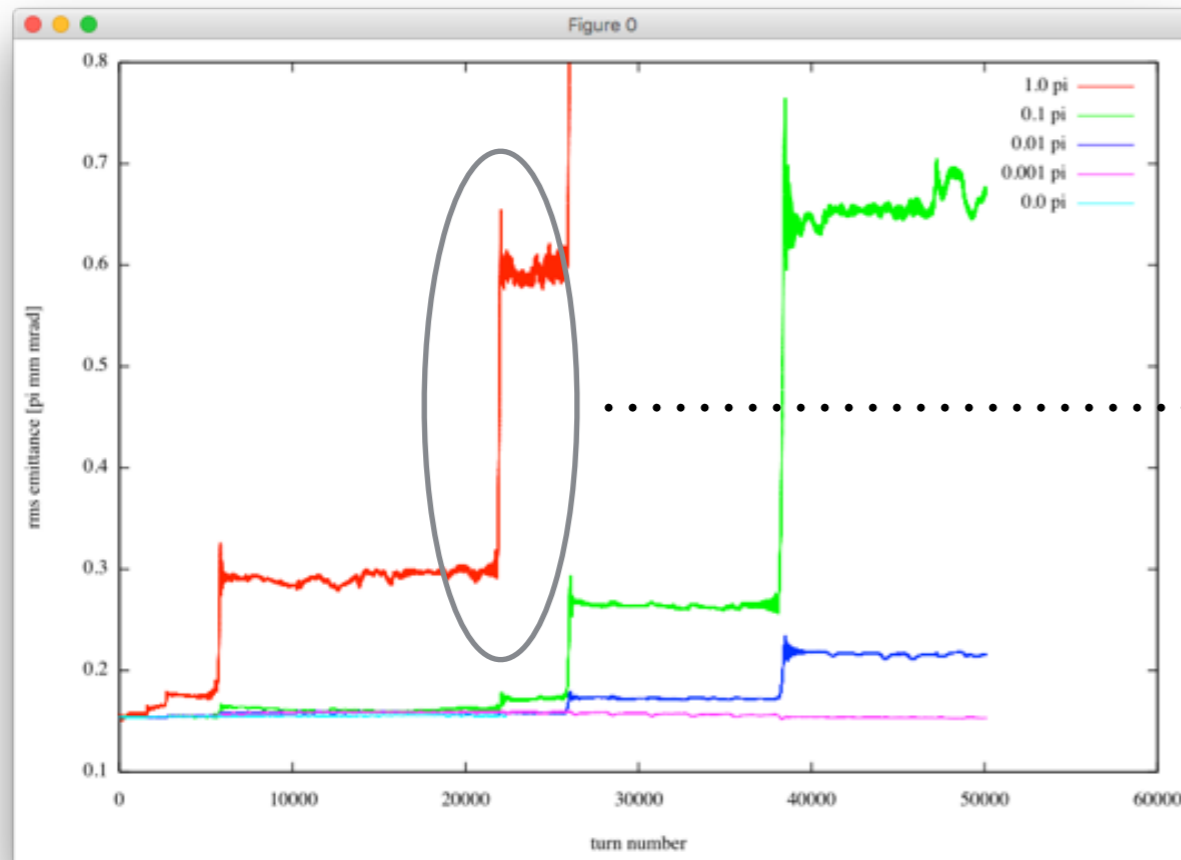
In fact, $Q_v \sim 1.37$ when jump appears.

$$Q_h + 6Q_v = 12 \quad \text{7th order coupling}$$

Since the order of vertical is much higher than horizontal, there is no jump in vertical?

Phase space topology at fixed momentum (1)

Look at phase space around the second jump occurs.

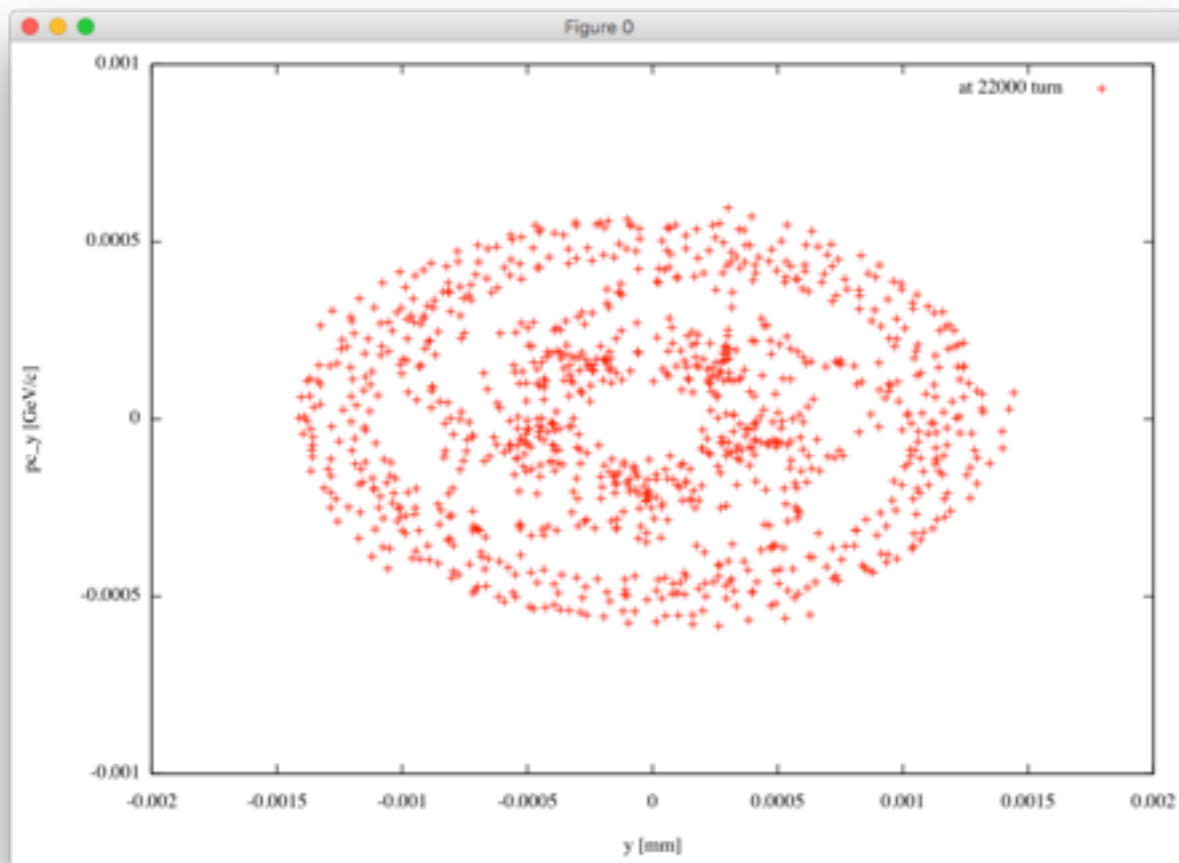


Phase space topology at fixed momentum (2)

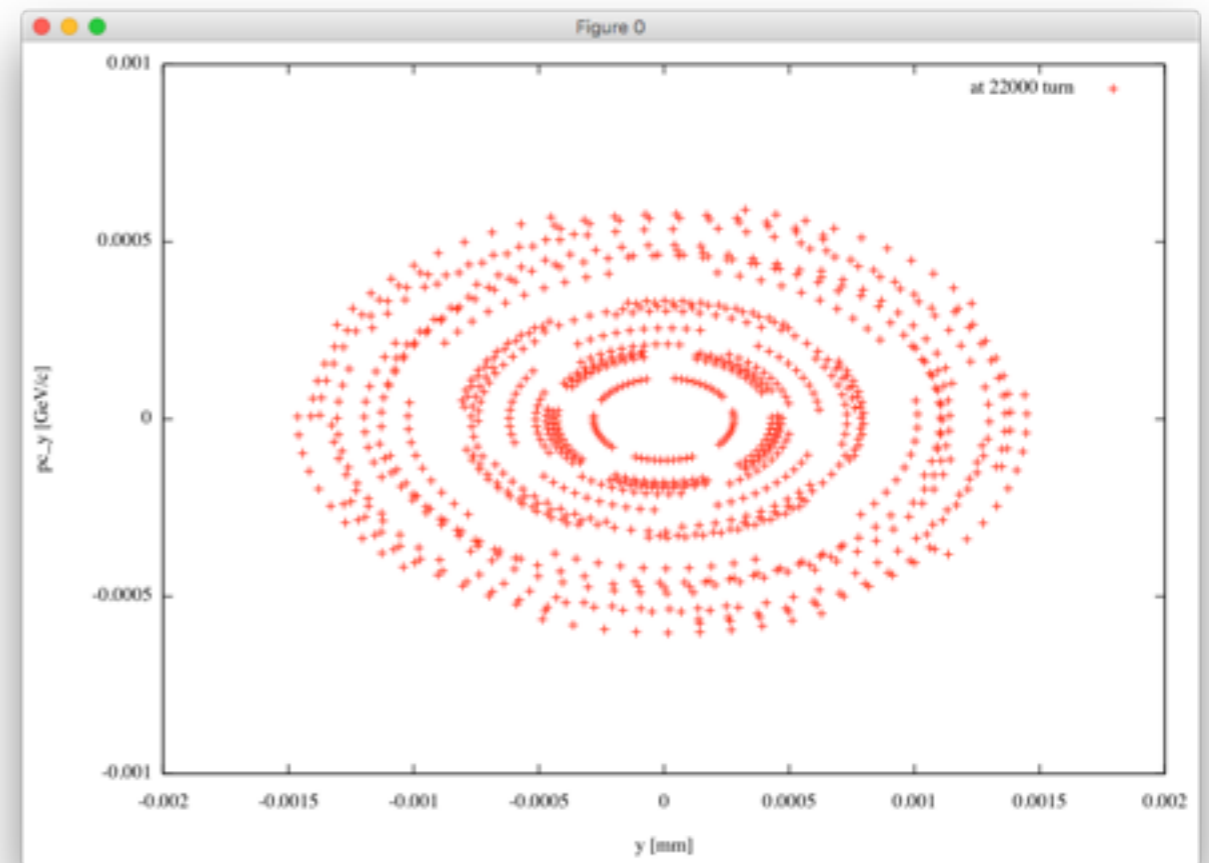
No acceleration. Momentum is fixed at 55.0 MeV (at 22,000 turn).

With finite vertical oscillation, fixed points appear as expected.

with vertical emittance of 1 pi mm mrad



without vertical oscillation



Summary

If there is a coupling between horizontal and vertical, 12 fold symmetry cannot be applied to horizontal direction only.

FFAG have all order of nonlinear coupling.

In principle, the same should be observed in a synchrotron if nonlinear elements exist.

Why it is so clear in KURRI FFAG, and only in horizontal?

Crossing resonances slowly?

Large asymmetry in horizontal and vertical tune?

In principle, we can avoid resonance crossing for single particle optics. However, some tune spread by space charge is inevitable. Does it becomes more serious in FFAG?