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KURRI FFAG simulation update
- evaluation of TOSCA field quality -

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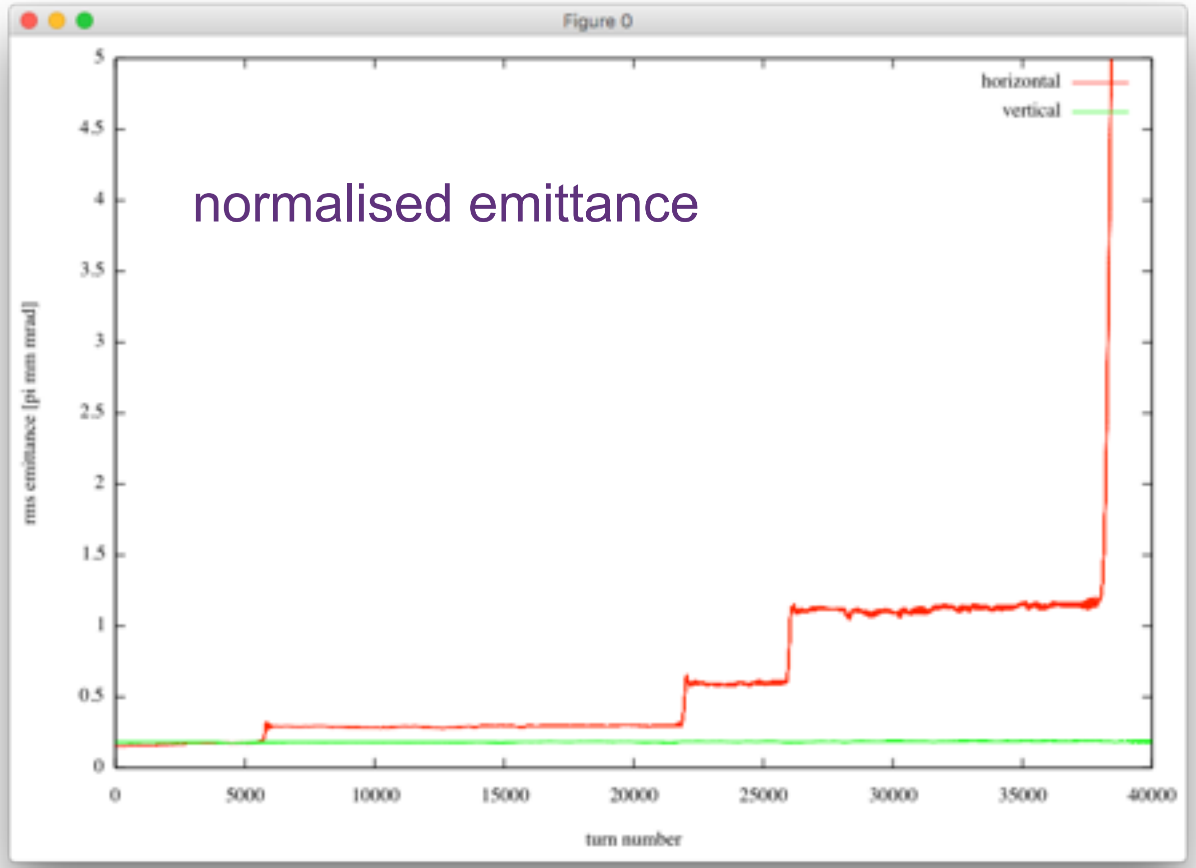
28 April 2016

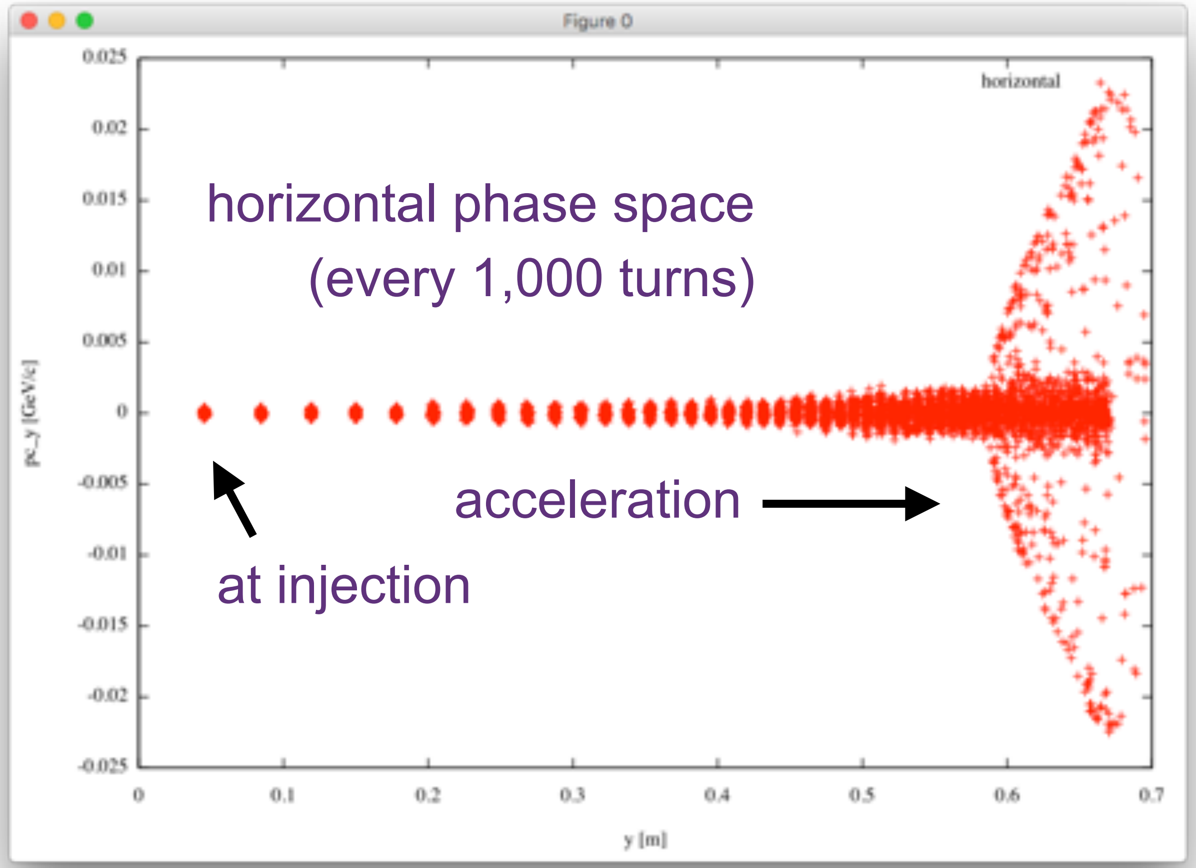
Emittance evolution

Zgoubi and Scode show similar emittance jump at some turns (energy).

No space charge. No error in the lattice.

No jump in vertical plane.

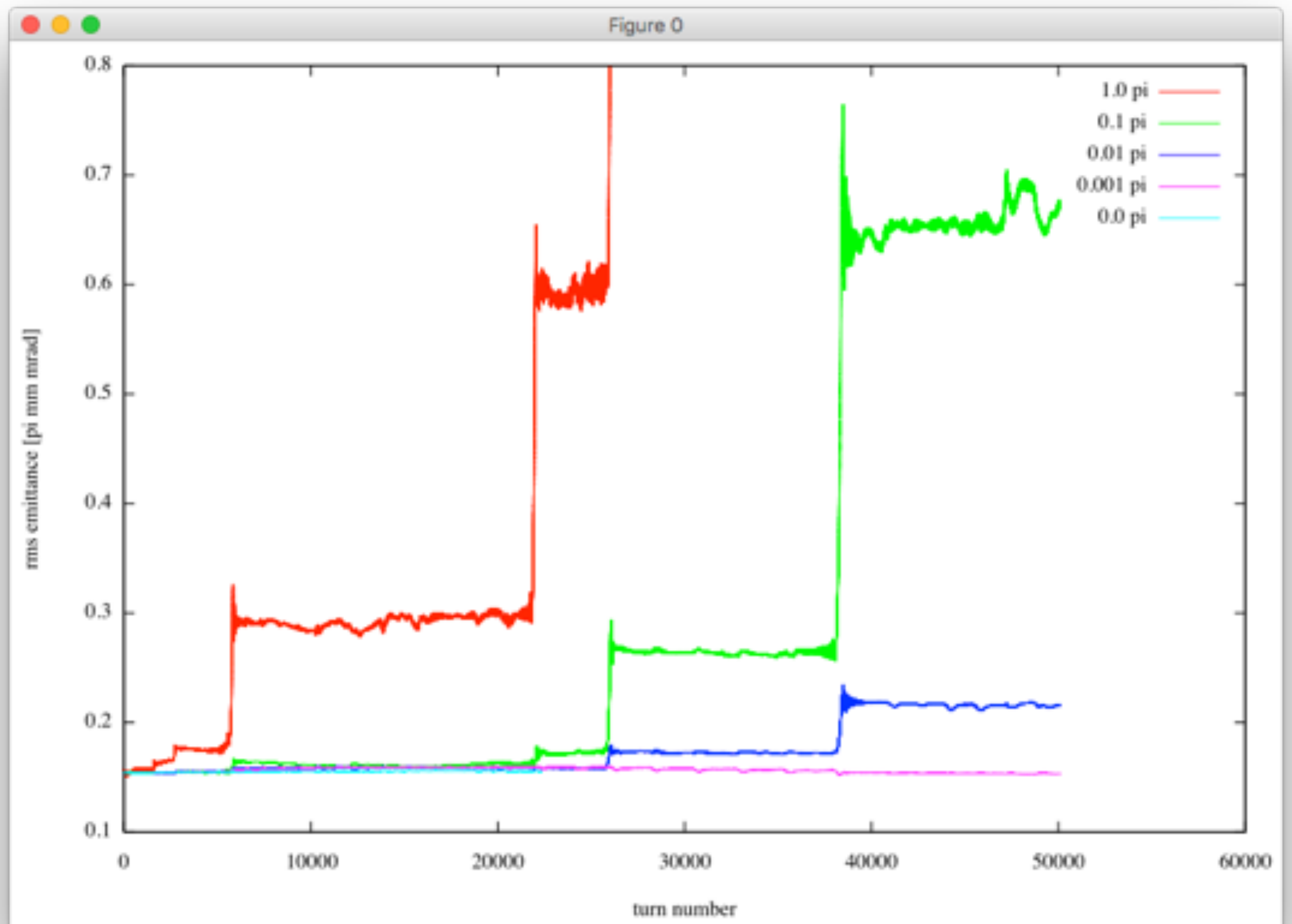




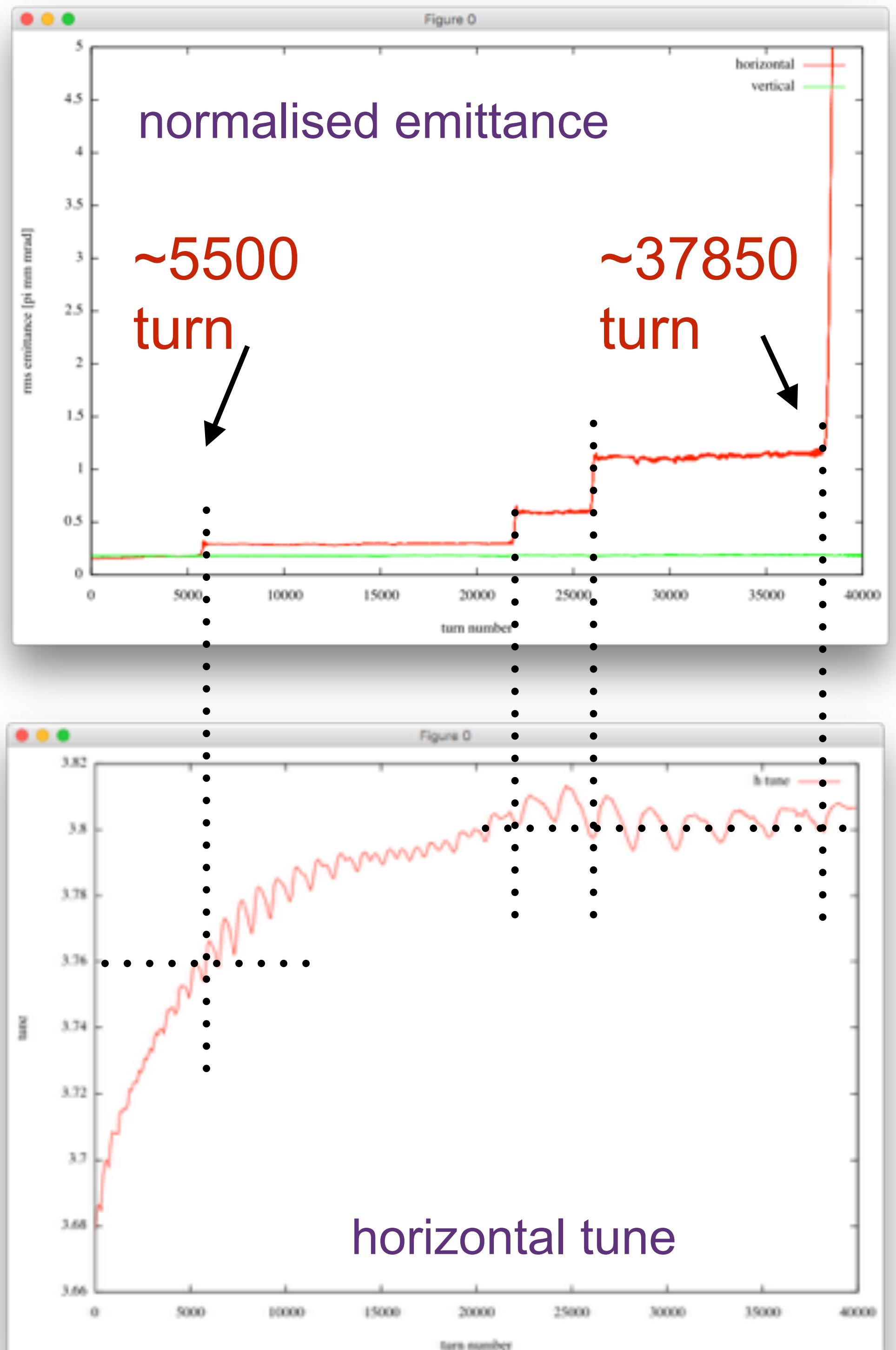
Kinetic energy [MeV]
= 11.0 + 0.002 x (turn number)

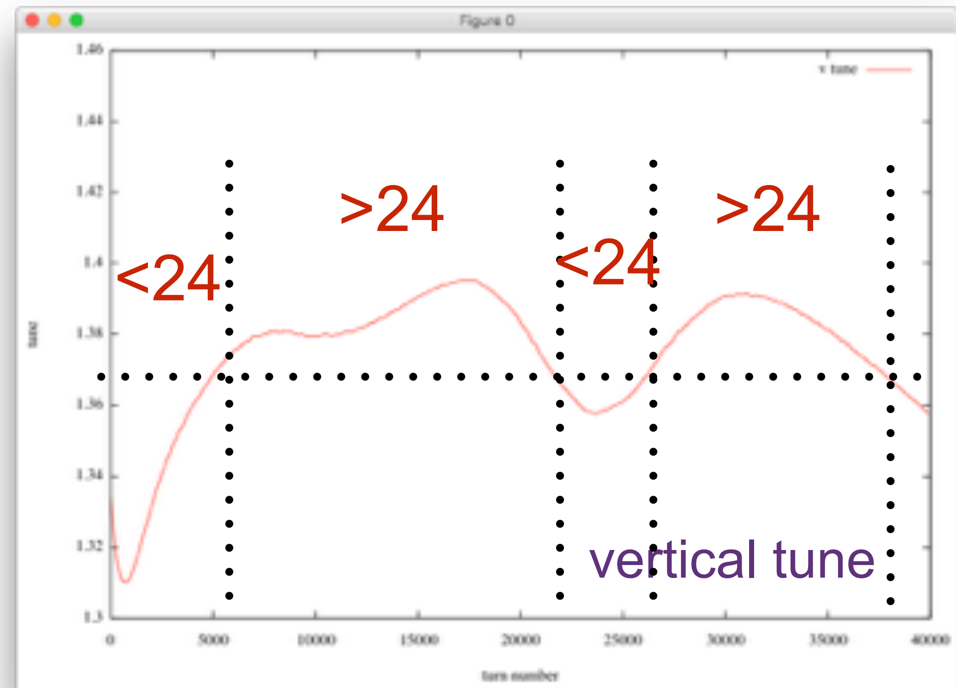
Parameter dependence
vertical emittance

When vertical emittance is reduced (1 pi to 0 pi), the jump disappears.



Tune evolution





6Qh+Qv=2*12   7th order coupling

Much higher order in horizontal plane is consistent with much higher increase of horizontal emittance.

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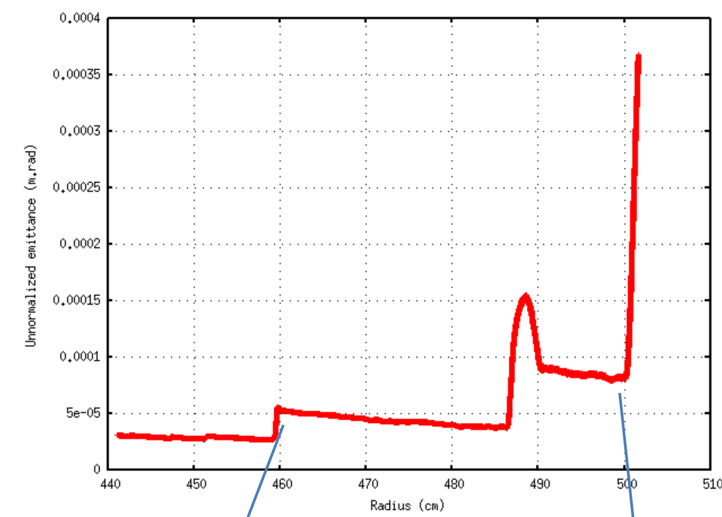
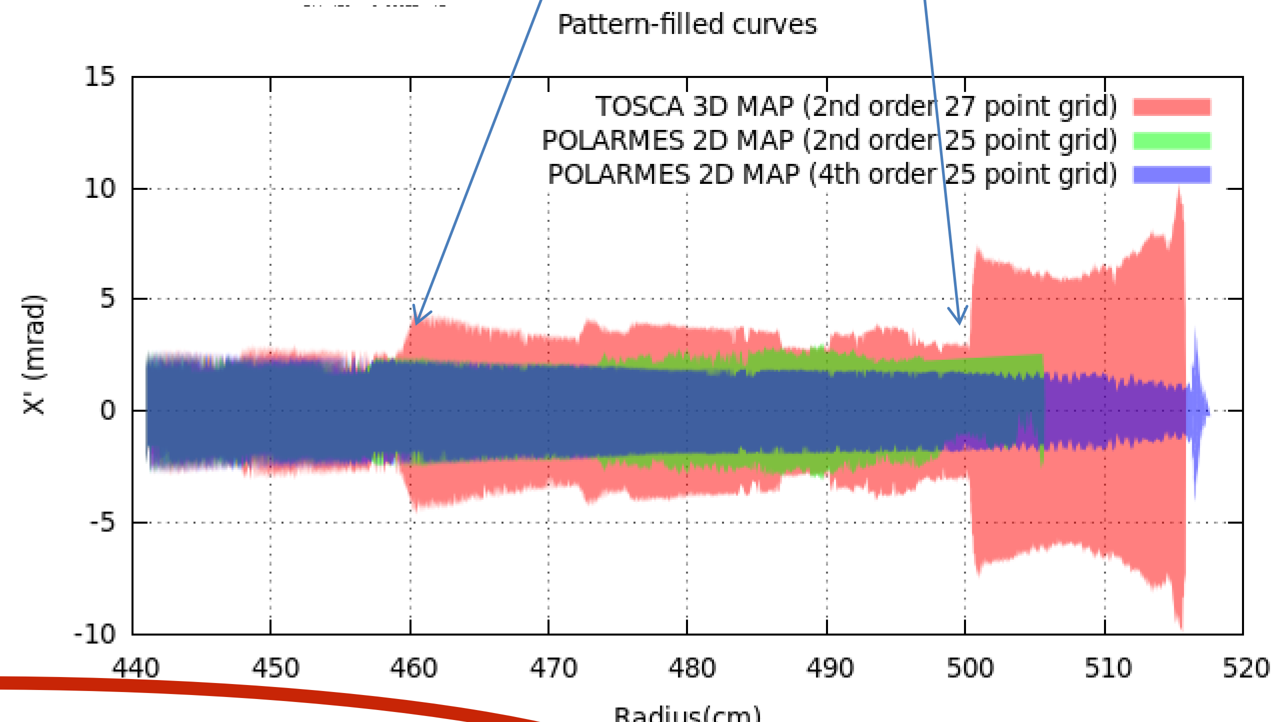
2D vs 3D field map

Malek's findings:

The horizontal emittance increase at certain locations is observed from single particle tracking using the 3D TOSCA field map.

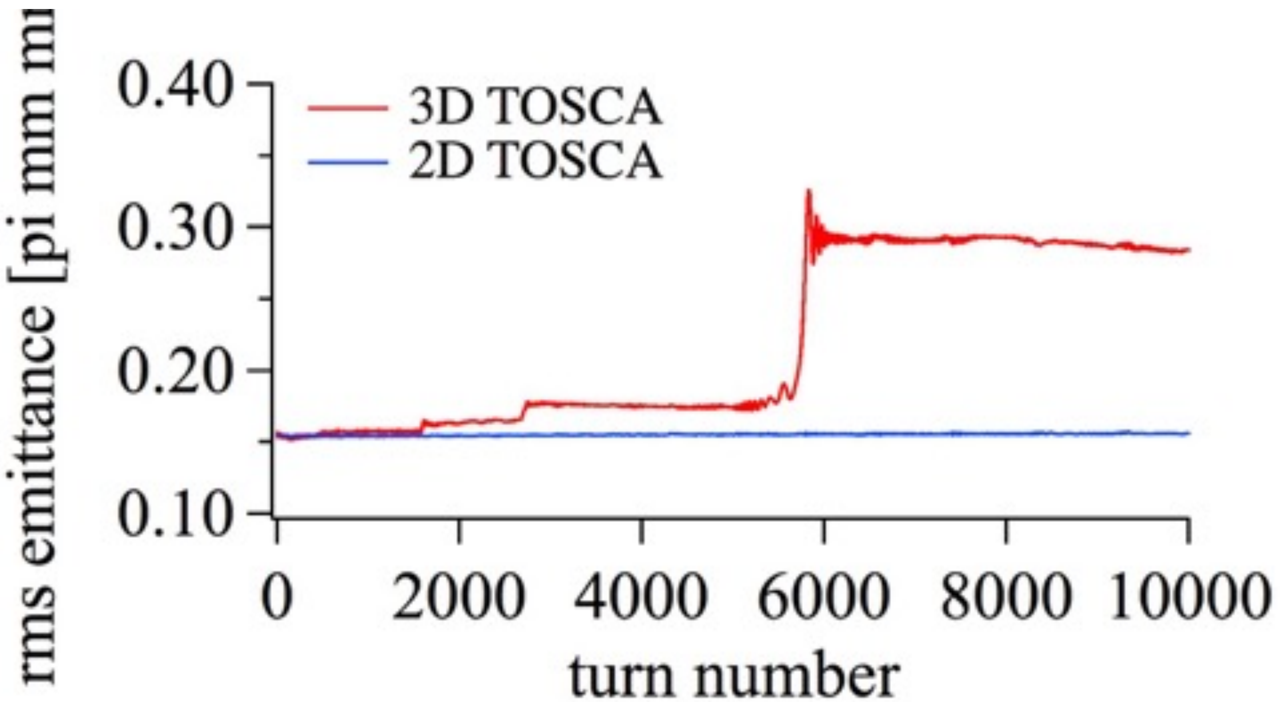


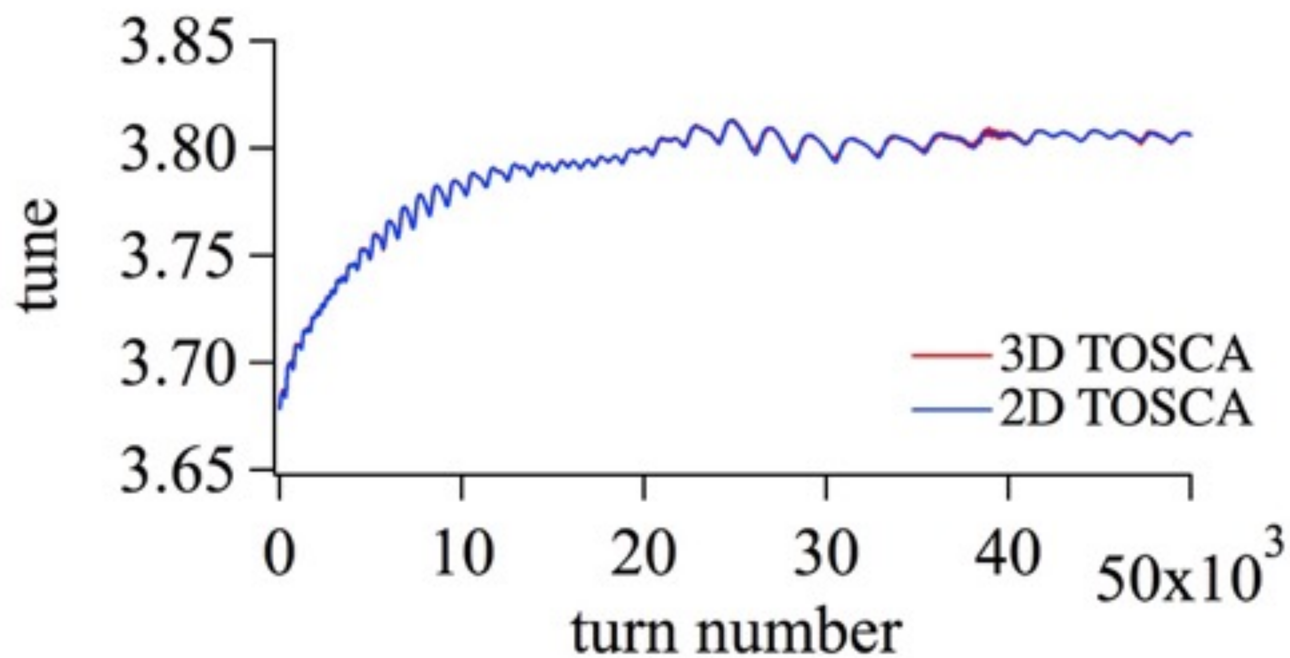
Single particle acceleration with 3D and 2D field maps and using different interpolation methods.



The problem seems to disappear with the 2D field map

Confirmed by s-code





Quality of field map

Can we estimate the quality of field map quantitatively?

> "2D field map on the mid-plane and extrapolation to off planes seems more accurate than 3D field map."

Is it obvious and correct statement?

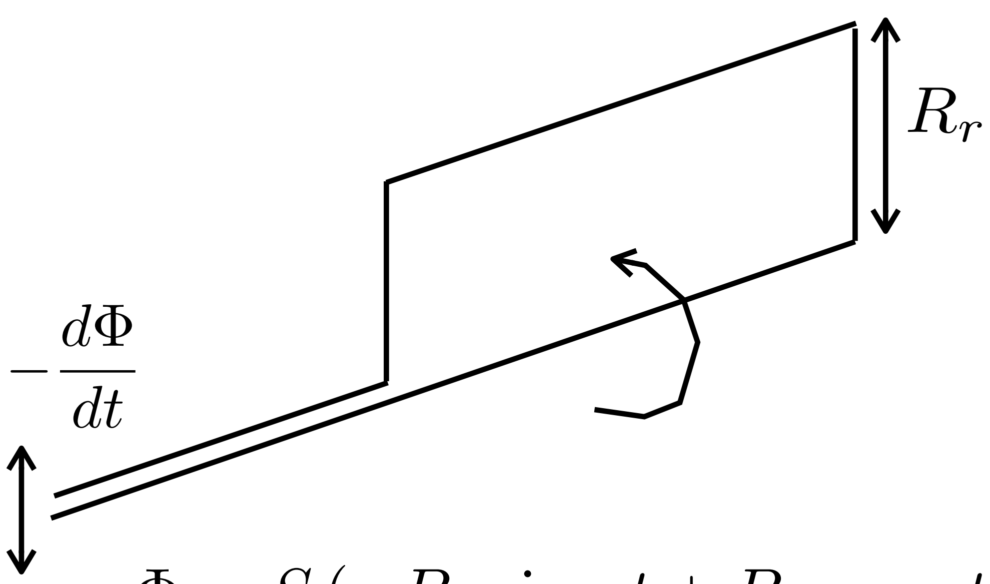
How do we know 2D field map is more accurate than 3D one?

Measurement of multipole fields

When we make a magnet, we measure magnetic fields whether it satisfies specifications.

We probably can do the same “magnetic field measurement” in simulation.

For example, measurement of multipole fields with rotating coil.



$$V = -\frac{d\Phi}{dt}$$

$$\Phi = S\left(-B_x \sin\omega t + B_y \cos\omega t\right)$$

Use $B_x(x,y,z)$ and $B_y(x,y,z)$ of field map.

Coefficient of Fourier expansion of V gives the relative strength of multipoles.

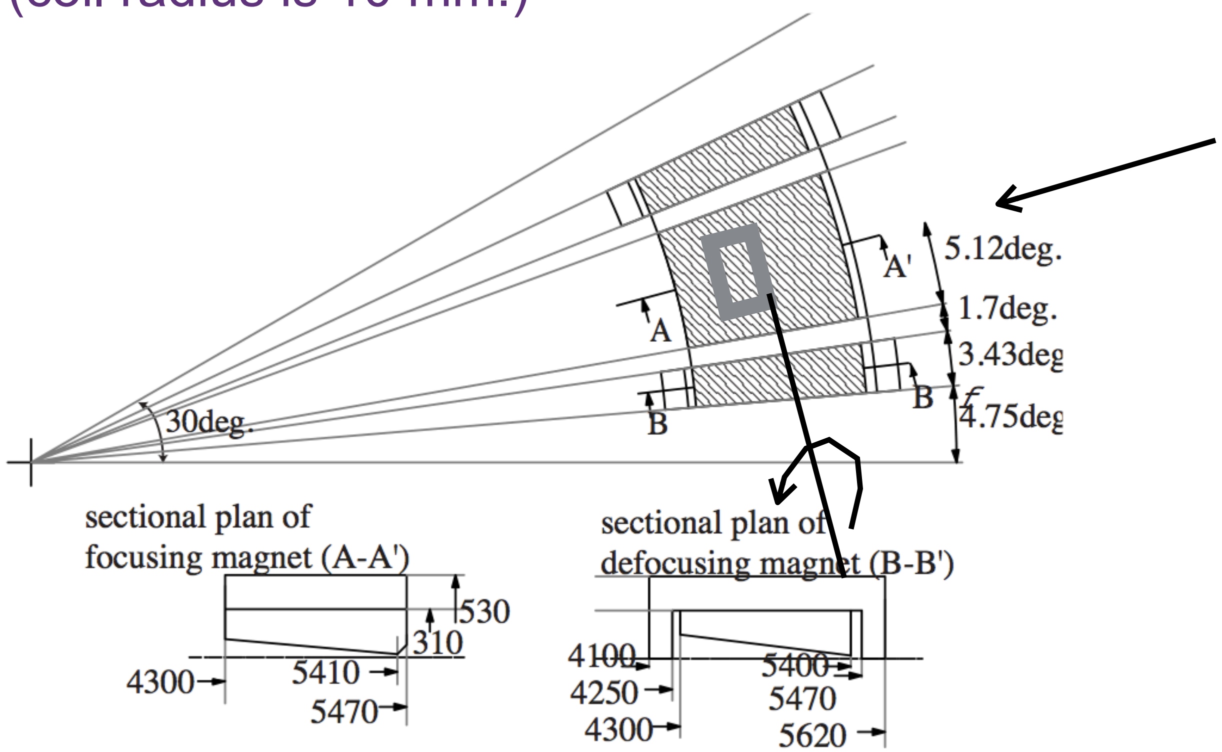
$$f_n = \sqrt{c_n^2 + s_n^2}$$

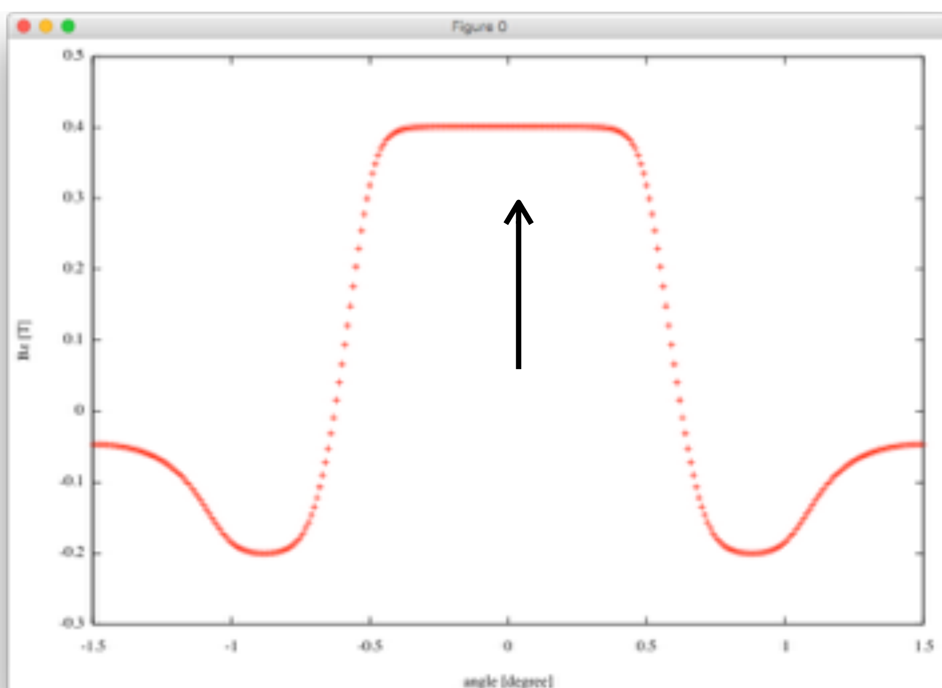
$$c_n = \int_0^{2\pi} \frac{V(\theta)}{R_r^n}\cos(n\theta)d\theta$$

$$s_n = \int_0^{2\pi} \frac{V(\theta)}{R_r^n}\sin(n\theta)d\theta$$

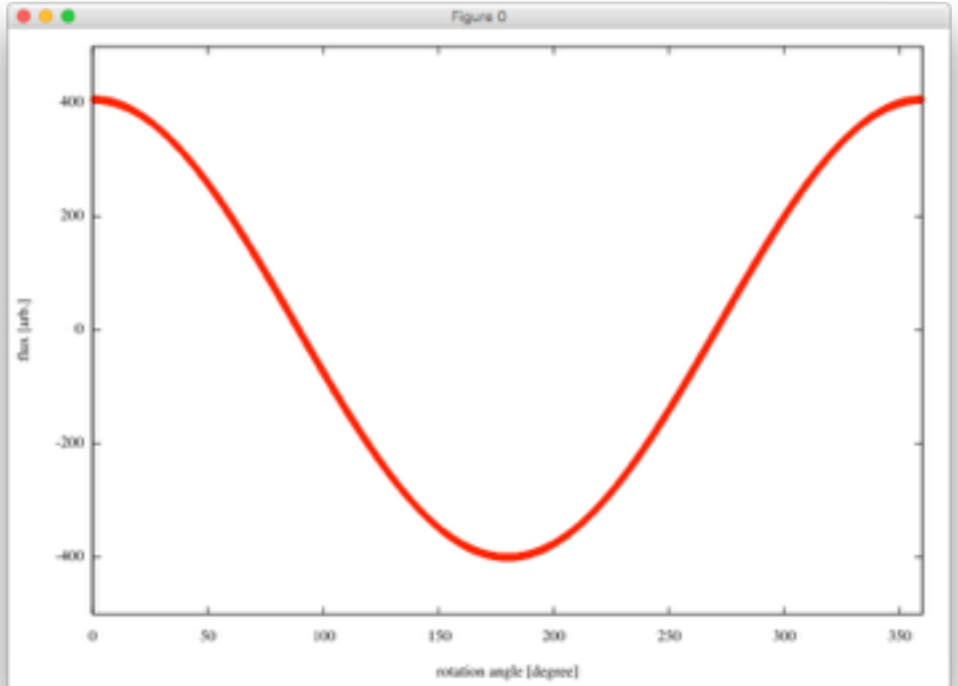
"Measurement"

Rotation coil is inserted at the centre of F magnet.
(coil radius is 10 mm.)

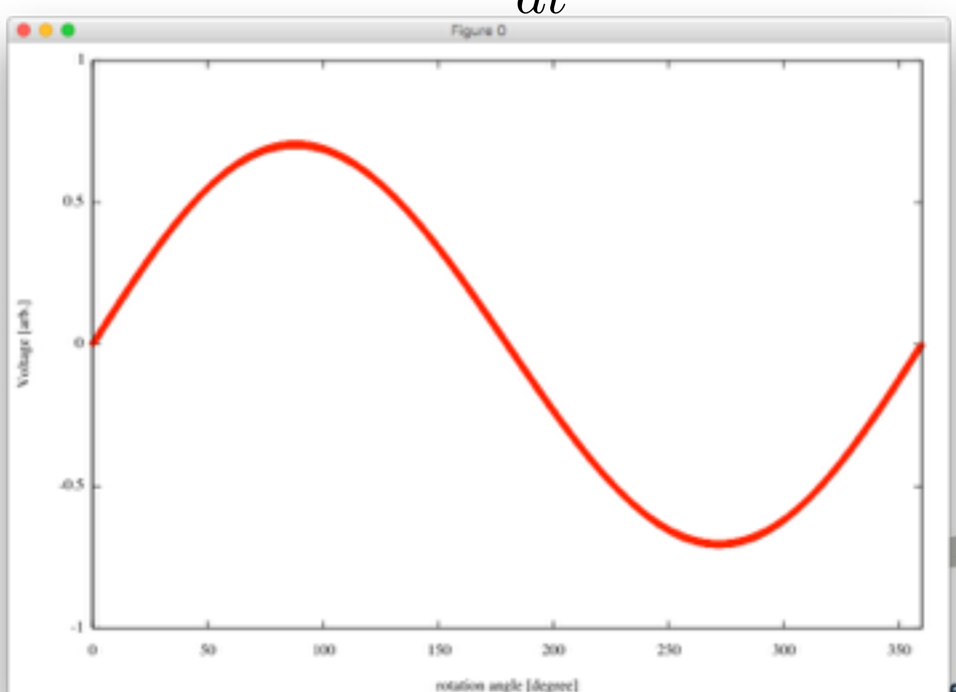




flux: $\Phi = S\left(-B_x \sin\omega t + B_y \cos\omega t\right)$



voltage: $V = -\frac{d\Phi}{dt}$ (sign maybe wrong)



First test

almost ideal field map

Almost ideal field map (B) vs analytical multipole coefficient (A)

B. On the mid-plane, the field is $\left(\frac{r_0+x}{r_0}\right)^k$ and calculate fields off plane using Maxwell equations up to z^6 order.

"Measure" multipoles with rotating coil.

A. Analytical FFAG field can be expanded as

$$\left(\frac{r_0+x}{r_0}\right)^k = 1 + \frac{k}{1!r_0}x + \frac{k(k-1)}{2!R_0^2}x^2 + \frac{k(k-1)(k-2)}{3!r_0^3}x^2 + \quad = 1 + \sum_{n=1}^{\infty}\frac{k!/(k-n)!}{n!r_0^n}x^n$$

mltipoleor der	1	2	3	4	5	6	7	8
A	1	1.688	1.285	0.552	0.147	2.46E-02	2.46E-03	2.41E-02
B	1	1.689	1.239	0.514	0.131	2.1E-02	2.02E-03	1.03E-04

Second test

2D mid-plane based field map

2D mid-plane based field map (C) vs analytical multipole coefficient (A)

C. On the mid-plane, use TOSCA 2D field map and calculate fields off plane using Maxwell equations up to z^1 order.

"Measure" multipoles with rotating coil.

A. Analytical FFAG field can be expanded as

$$\left(\frac{r_0+x}{r_0}\right)^k = 1+\frac{k}{1!r_0}x+\frac{k(k-1)}{2!R_0^2}x^2+\frac{k(k-1)(k-2)}{3!r_0^3}x^2+ \quad = 1+\sum_{n=1}^{\infty}\frac{k!/(k-n)!}{n!r_0^n}x^n$$

mltipoleorder	1	2	3	4	5	6	7	8
A	1	1.688	1.285	0.552	0.147	2.46E-02	2.46E-03	2.41E-02
C	1	1.687	1.389	15.61	852.1	9.7E+05	2.1E+06	5.1E+08

Third test

3D field map

3D field map (D) vs analytical multipole coefficient (A)

D. TOSCA 3D field map everywhere.

“Measure” multipoles with rotating coil.

A. Analytical FFAG field can be expanded as

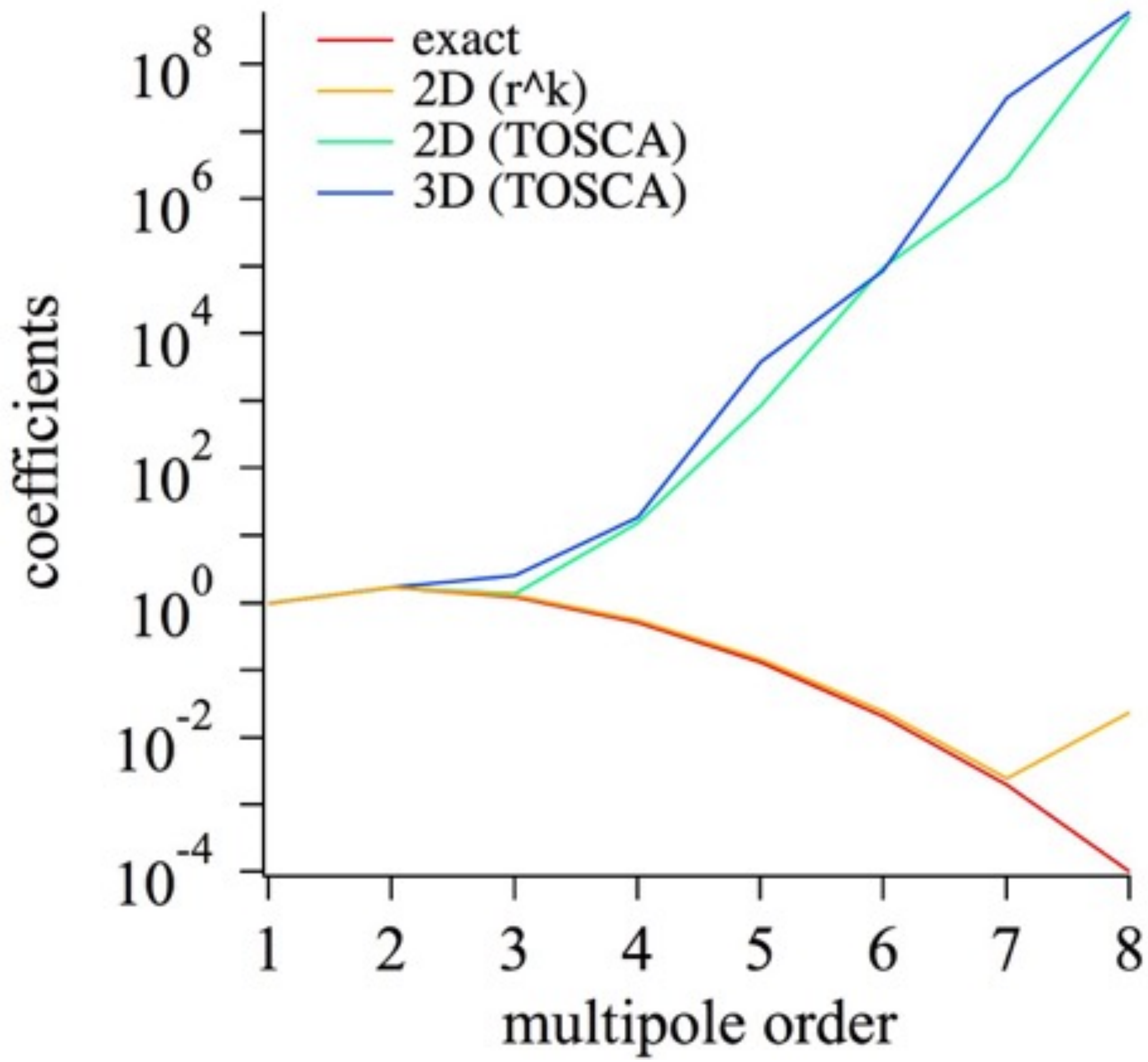
$$\left(\frac{r_0+x}{r_0}\right)^k = 1+\frac{k}{1!r_0}x+\frac{k(k-1)}{2!R_0^2}x^2+\frac{k(k-1)(k-2)}{3!r_0^3}x^2+ \quad = 1+\sum_{n=1}^{\infty}\frac{k!/(k-n)!}{n!r_0^n}x^n$$

mltipoleorder	1	2	3	4	5	6	7	8
A	1	1.688	1.285	0.552	0.147	2.46E-02	2.46E-03	2.41E-02
D	1	1.686	2.542	18.82	3772	8.7E+05	3.2E+07	6.0E+08

All compared

Multipoles obtained from 2D and 3D TOSCA field maps are significantly different from ideal at n=4, 5 and higher.

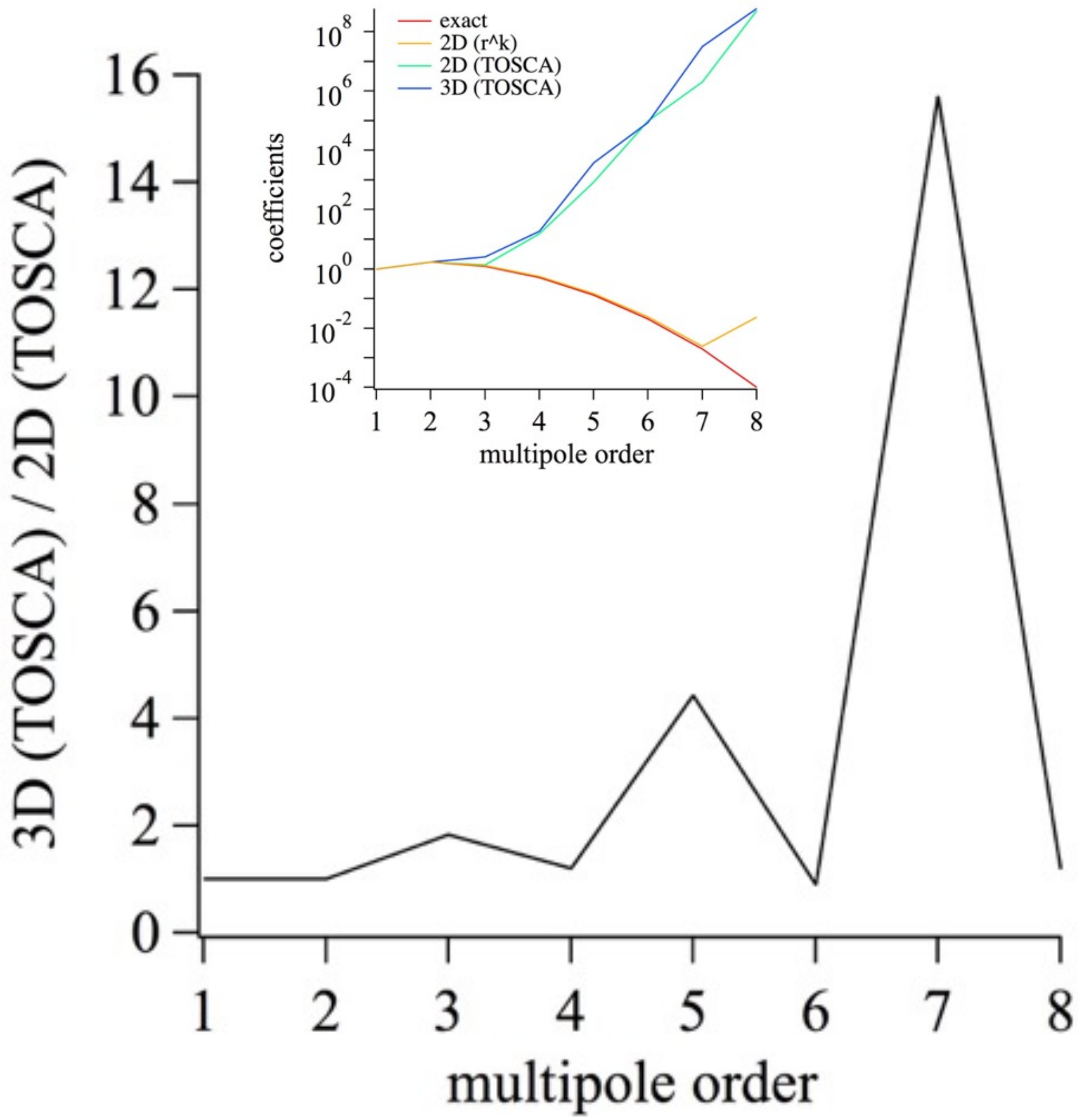
It depends on
1. the order of vertical coordinate z^m when extrapolating from 2D.
2. the location of “rotating coil”.



Difference between 2D and 3D

If you compared coefficients from 2D TOSCA and 3D TOSCA, odd order (sext, deca, 14 pole, …) has large difference.

This may explain the reason why the emittance jump (6Qh +Qv=2*12, 7th order coupling?) appears in 3D TOSCA field, but not in 2D TOSCA field.



Questions

Do we have to improve the quality of TOSCA field map when we track particles?

Higher multipoles (4, 5 or higher) does not make any contribution?

> Emittance jump appears in the simulation gives some indication to this question.

How can the quality of TOSCA field map be improved.

> Fine mesh is obviously one solution. But there must be some limits.

I suspect this is the first time those questions are asked, because tracking with field map before (e.g. cyclotron) did NOT look at higher multipoles.