KURRI scaling FFAG studies:

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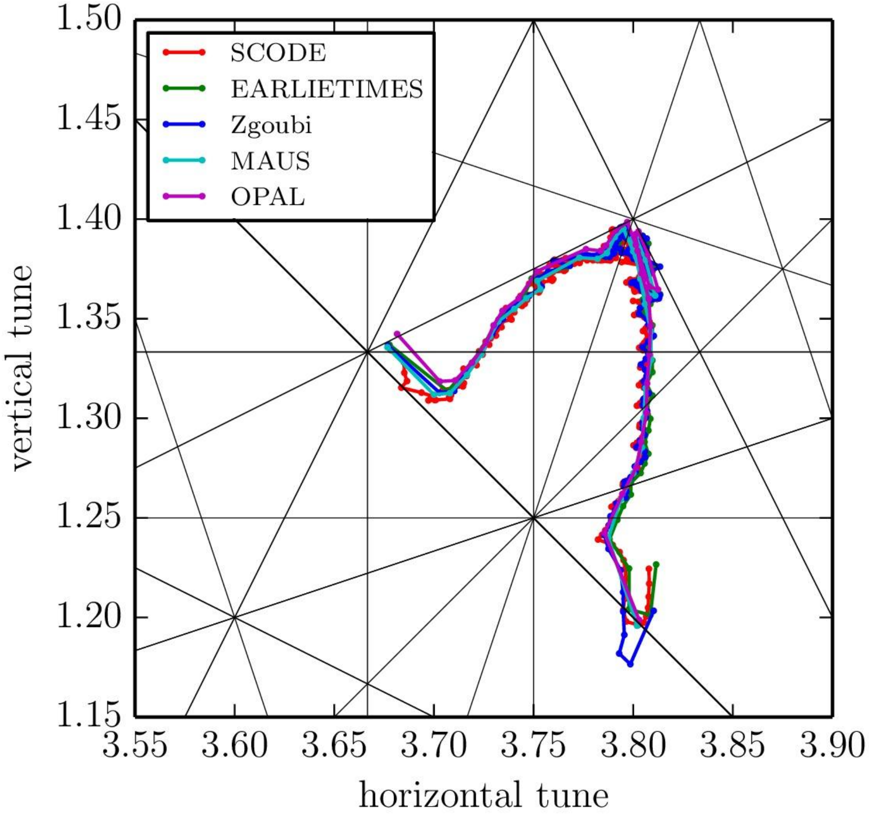
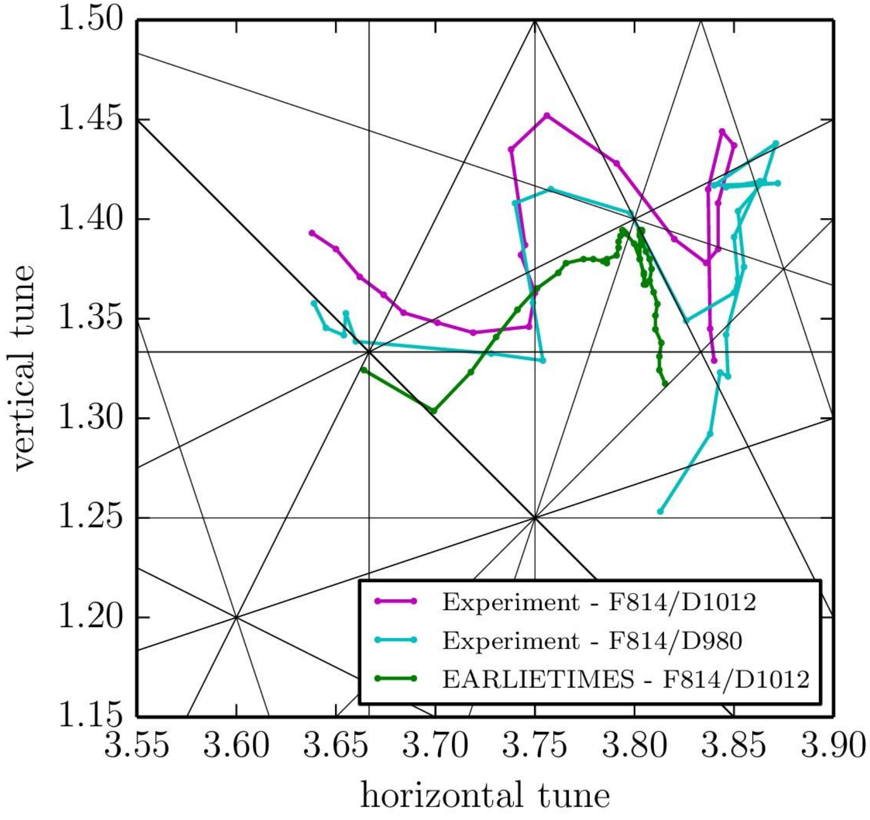
Motivation:



Benchmarking of the codes



Comparison with experiment

Origin of the problem?
How to remediate to it?

$$\frac{\Delta\nu_x}{\nu_x} = 5.78\ \% \quad \text{and} \quad \frac{\Delta\nu_y}{\nu_y} = 9.02\ \%$$

Scaling FFAG model (reminder):

- The magnetic field of a radial sector type has the form:

$$B(R,\theta) = B_0 \times \left(\frac{R}{R_0}\right)^k \times F(\theta)$$

where $F(\theta)$ is a periodic function of the azimuthal angle θ ⟹ **Fixed Field**

- **Alternating Gradient** is obtained by alternance of:

- Positive curvature field, focusing $\frac{\rho}{B}\frac{dB}{d\rho} > 0$
- Negative curvature field, defocusing $\frac{\rho}{B}\frac{dB}{d\rho} < 0$

Correct/Original form of the scaling factor:

- The original definition of the average scaling factor is: $k = \frac{\frac{dB}{B}}{\frac{dR}{R}}$

  where B is the field averaged in azimuth.

- Apply this form to compute the average scaling factor:

$$k(R_i) = \frac{\frac{B_{i+1} - B_i}{B_i}}{\frac{R_{i+1} - R_i}{R_i}}$$

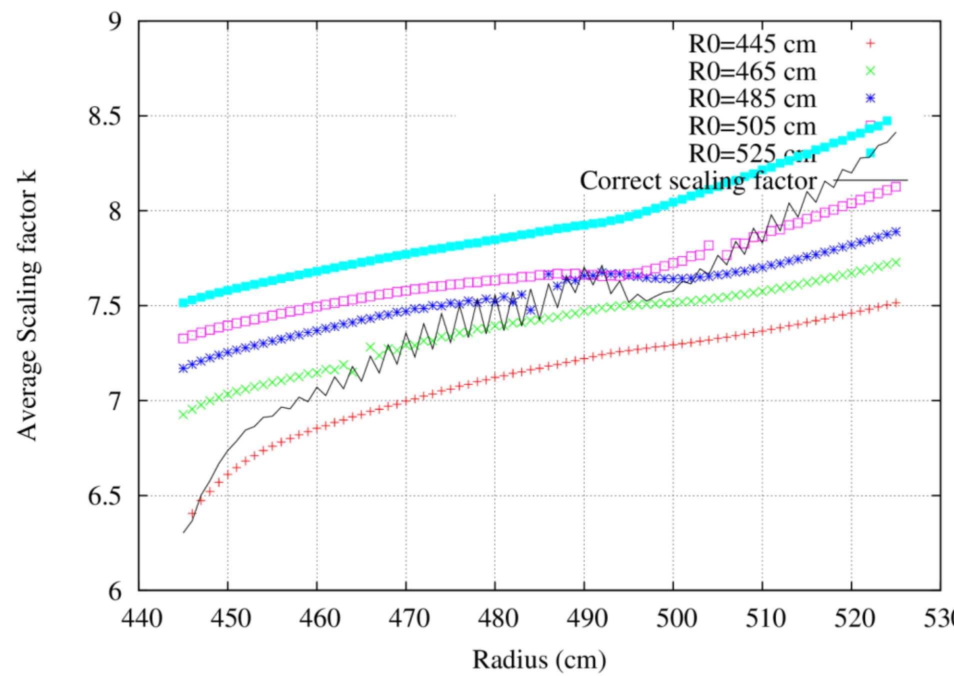
where R_i and R_{i+1} are limited by the mesh size (here $R_{i+1} - R_i = 1cm$)

Correct Scaling law:

• Strong variations that can be improved if we have higher mesh size.

• The correct scaling factor (curve in black) was obtained from the original definition (see previous slide) where no assumption is made to the form of the scaling factor.

• It can be seen that each one of the other curves which was obtained from a specific reference radius R0 is valid at the proximity of this reference radius where the variations of k are still not too big.



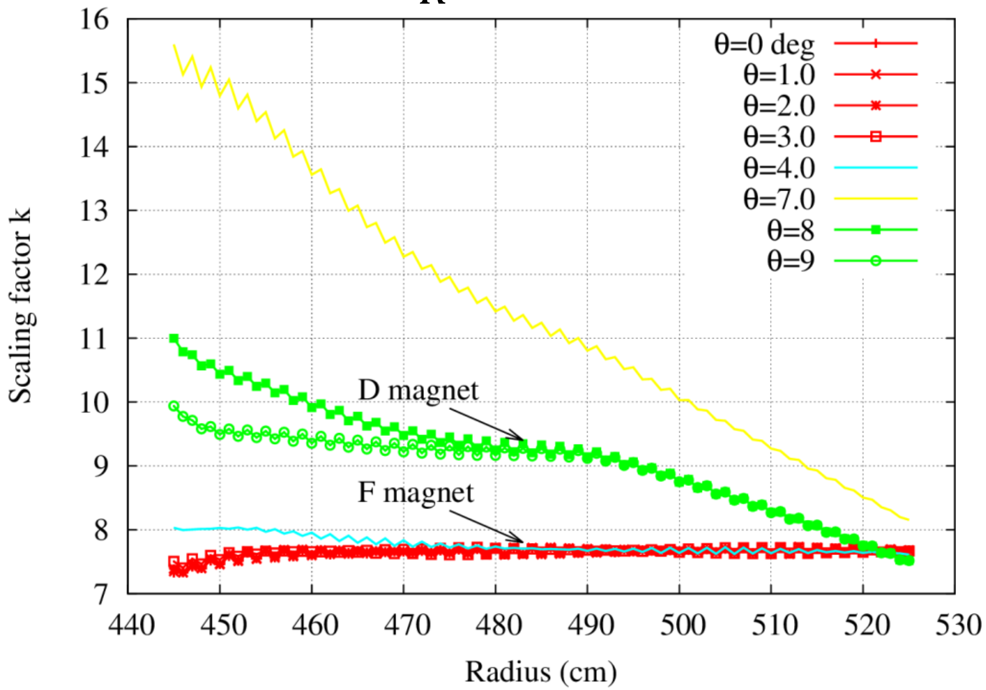
The scaling law $B_{ave}(R) = B_0 \times \left(\frac{R}{R_0}\right)^{k(R)}$ **is only valid in the vicinity of the reference radius** R_0, **simply because it is not a correct general solution of the equation** $k = \frac{\frac{dB}{B}}{\frac{dR}{R}}$.

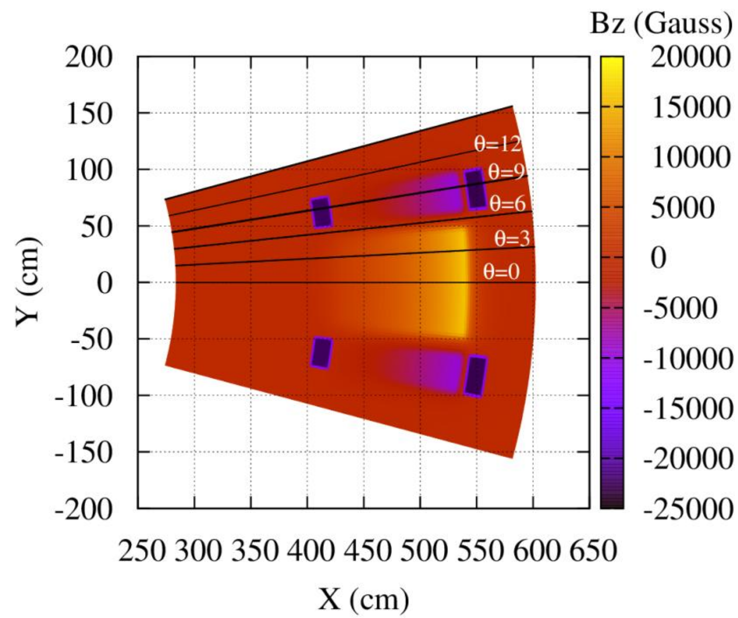
Thus when defining this equation, one has to choose R_0 **very carefully.**

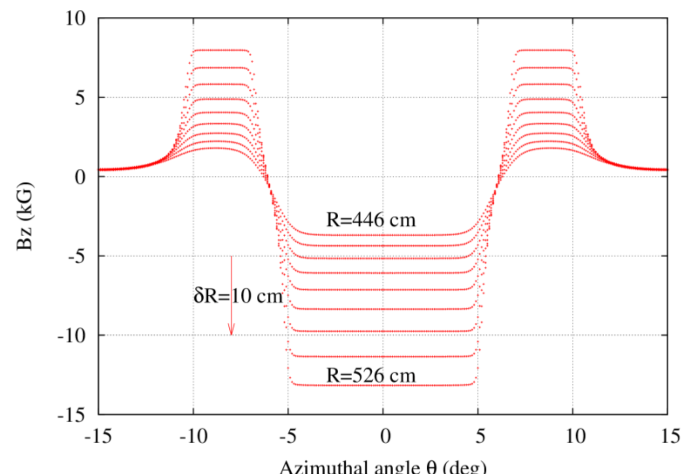
k F-magnet vs k D-magnet:

- The scaling factor of the F and D magnets are different:

$$k = \frac{\frac{dB}{B}}{\frac{dR}{R}}$$







k F-magnet vs k D-magnet:

- The average scaling factor of the F and D magnets are different:

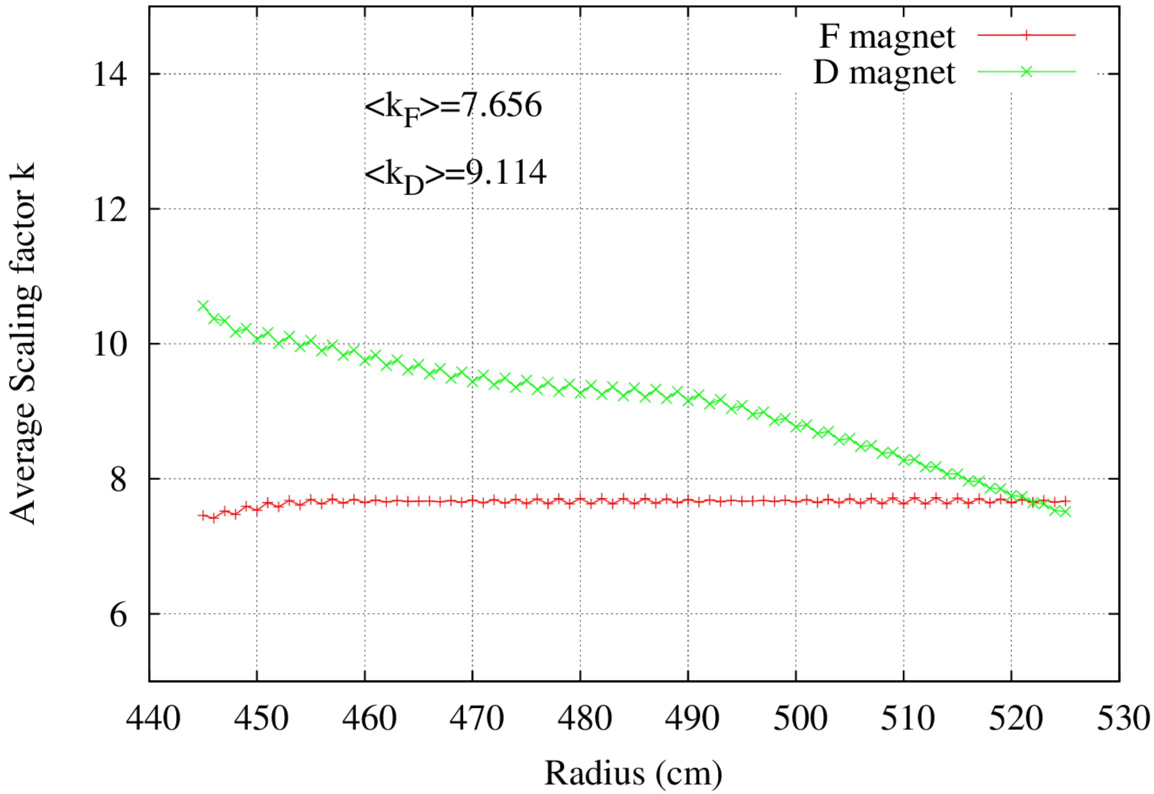
$$k = \frac{\frac{dB}{B}}{\frac{dR}{R}}$$ where,

For the F magnet

$$B = \langle B_F \rangle (R) = \frac{\int_{-4}^{4} B_z(R,\theta)d\theta}{\int_{-4}^{4} d\theta}$$

For the D magnet

$$B = \langle B_D \rangle (R) = \frac{\int_{7.5}^{9.5} B_z(R,\theta)d\theta}{\int_{7.5}^{9.5} d\theta}$$

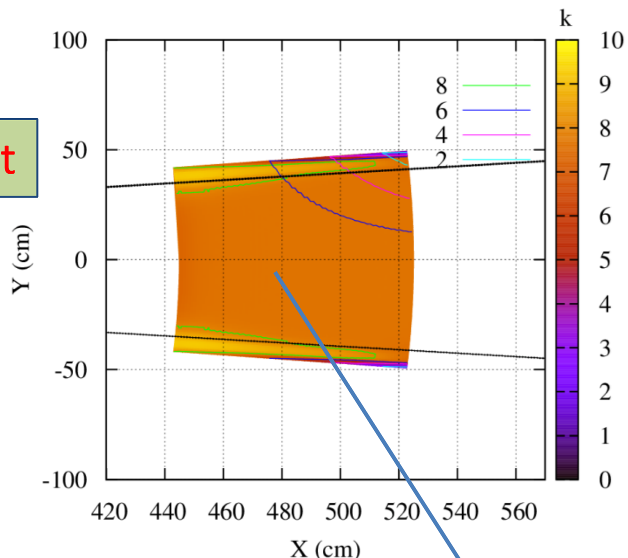


The scaling factor of the F-magnet is constant ≈ 7.656
Yet, for the D-magnet, the variations are non negligible and $< k > \approx 9.114$

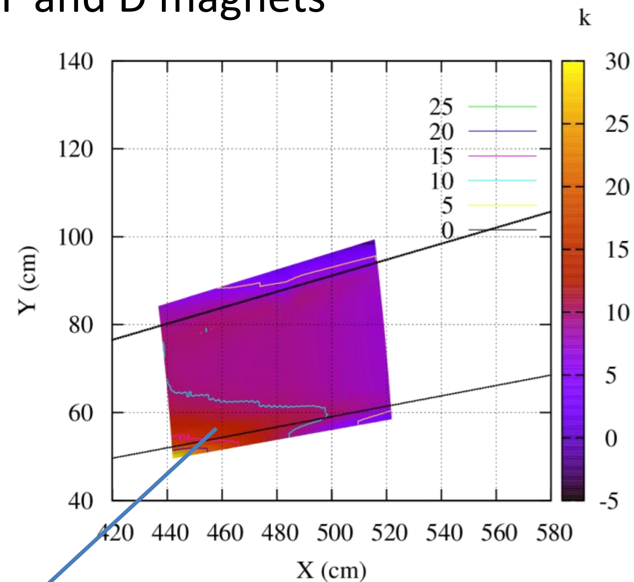
k F-magnet vs k D-magnet:

Scaling factor map for the F and D magnets

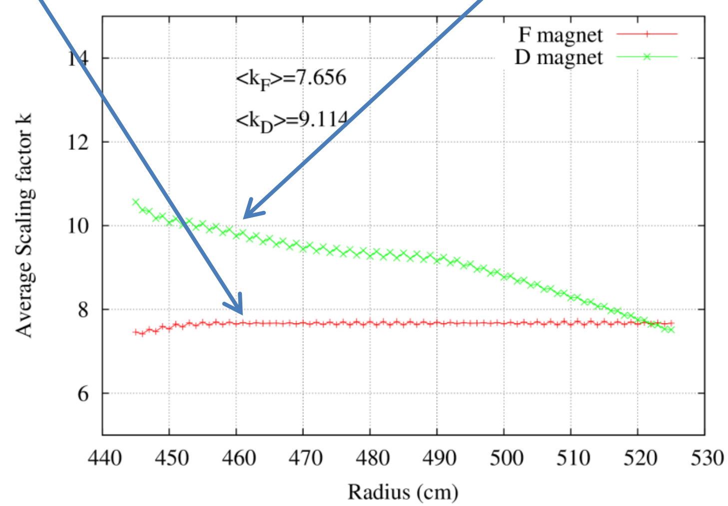
F-magnet



D-magnet



Most of the important variations of the scaling factor for the D-magnet come from the interaction region with the F-magnet: cross-talk between the F-D magnet.



Objective

- To carry out parametric studies of the space charge effects, the depressed phase advance is the natural parametrization.
- Therefore if one finds a relationship between the tune of the DFD triplet and the scaling factors k_F and k_D of the F and D magnets respectively, one can investigate space charge effects and also understand certain sources of imperfections of the KURRI FFAG.
- For that, the hard edge model of the cell was implemented in Mathematica.

Hard edge model

$\rho_F = O_F A;\;\; R_F = OA$

$\rho_D = O_D C;\;\; R_D = OC$

$L_B = BC$

$L_{drift} = DE$

$(BC) \perp (O_F B)$

$(BC) \perp (O_D C)$

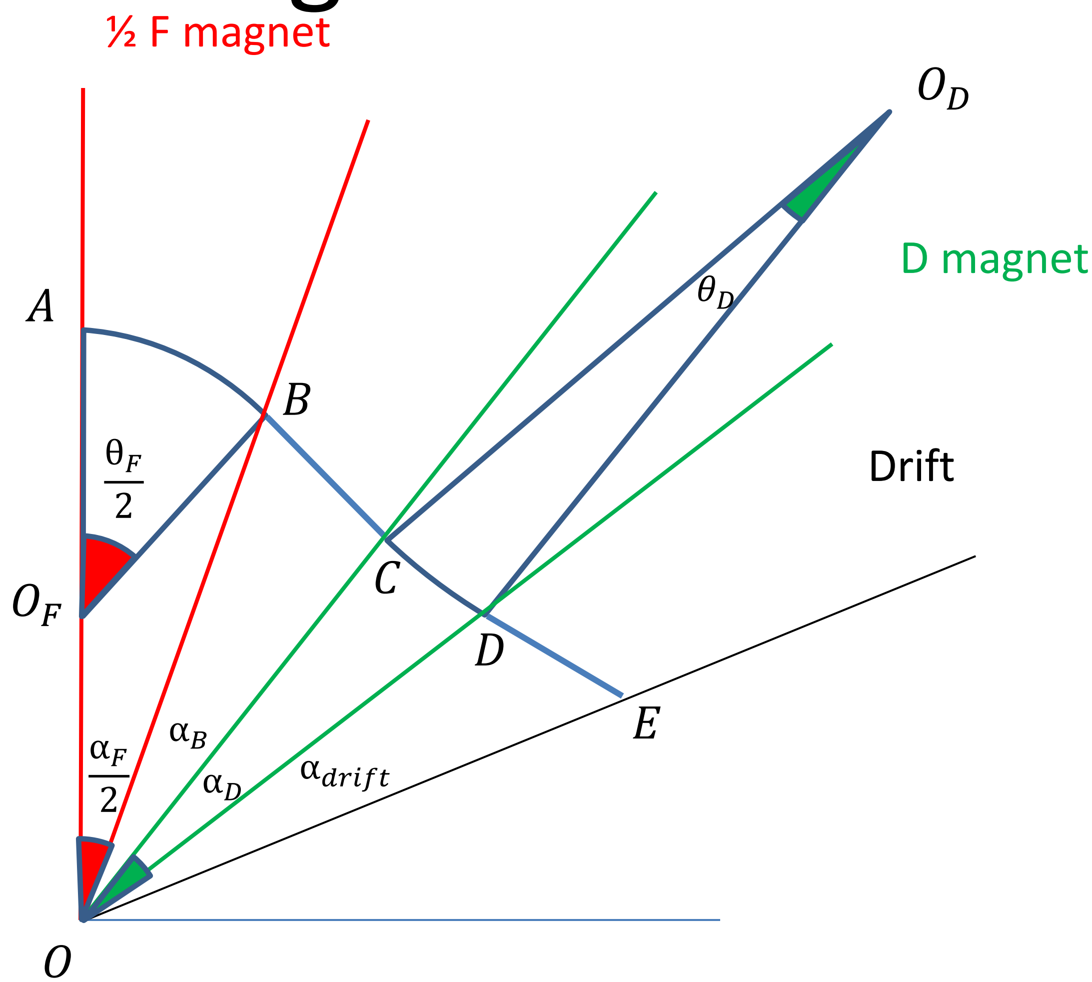
$O_F B = O_F A$

$O_D C = O_D D$

$\boldsymbol{OC \neq OD}$

$(DE) \perp (O_D D)$

$$\frac{\alpha_F}{2} + \alpha_B + \alpha_D + \alpha_{drift} = \frac{\pi}{N}$$



Conjecture:

We make the following conjecture based on the results of the hard edge model:

The Vertical tune of a DFD cell is given by:

$$\nu_z^2 = x_1 k_F + x_2 k_D + x_3$$

Analogy with $\nu_z^2 = -k + \frac{f^2}{2}$

The horizontal tune of a DFD cell is given by:

$$\nu_x^2 = x_1 k_F + x_2 k_D + (x_3 k_F + x_4 k_D + x_5)^2$$

Analogy with $\nu_x^2 = k + 1 + A.(k+1)^2$

What the Hard edge model is missing

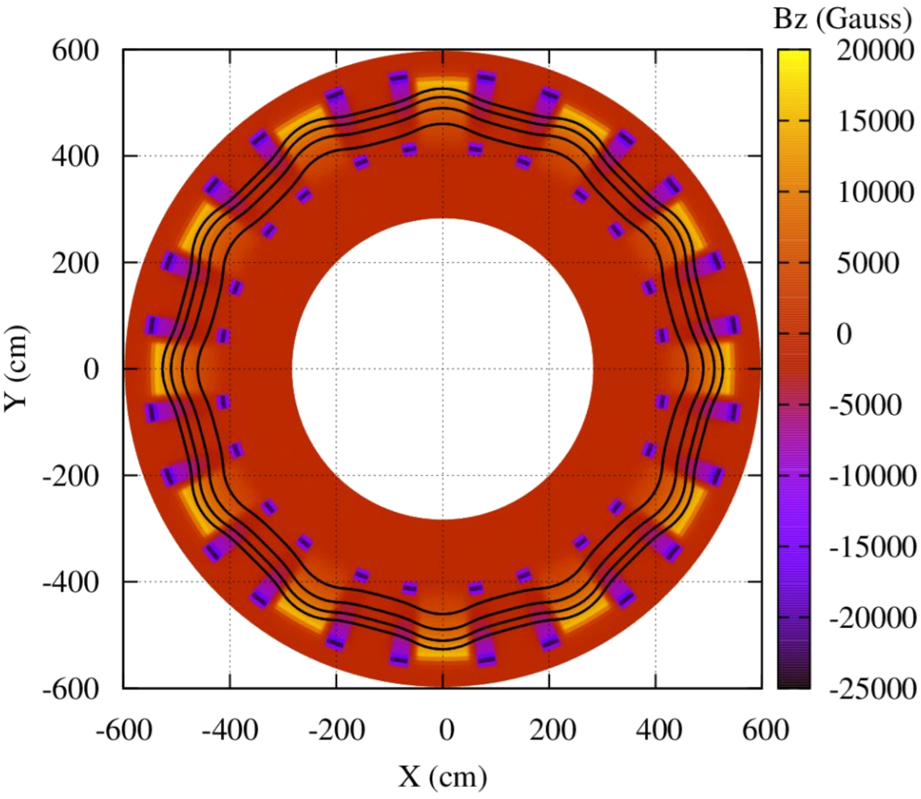
- THE BASIC ASSUMPTION OF THE HARD EDGE MODEL IS THAT ALL ORBITS ARE HOMOTHETIC TO EACH OTHER.
- YET, WITH

$$B_F(R) = B_{F0} \times \left(\frac{R}{R_0}\right)^{k_F}$$

$$B_D(R) = B_{D0} \times \left(\frac{R}{R_0}\right)^{k_D}$$

$$1 = \frac{(B\rho)_F}{(B\rho)_D} = \frac{B_F}{B_D} * \frac{\rho_F}{\rho_D}$$

$$\Rightarrow \quad \frac{\rho_F}{\rho_D} = \frac{B_{D0}}{B_{F0}} * \left(\frac{R}{R_0}\right)^{k_D - k_F}$$



See Annex for more complete form if kD and kF are R-dependent.

The rms tune variations are not accounted for in the hard edge model.

Fringe field model in Zgoubi

- We carry out parametric studies of the tune variations as a function of the scaling factor kF and kD of the F and D-magnet respectively.

- We generate several field maps by varying kF and kD so that the median plane field is written in the form:

$$B_Z(R) = B_{F0} \times \left(\frac{R}{R_0}\right)^{k_F} \times F_F(\theta) + B_{D0} \times \left(\frac{R}{R_0}\right)^{k_D} \times F_D(\theta)$$

Only kF and kD are allowed to change.

Procedure

1) Generate a field map for a given (kF,kD).

2) A set of 30 closed orbits are generated between injection (@ 11MeV) and extraction (@100 MeV).

3) For each lattice, the average value of the tune is calculated: $< \nu_{x,y} > = \frac{1}{N} \sum_{i=1}^{N} \nu_{x,y,i}$; N = 30

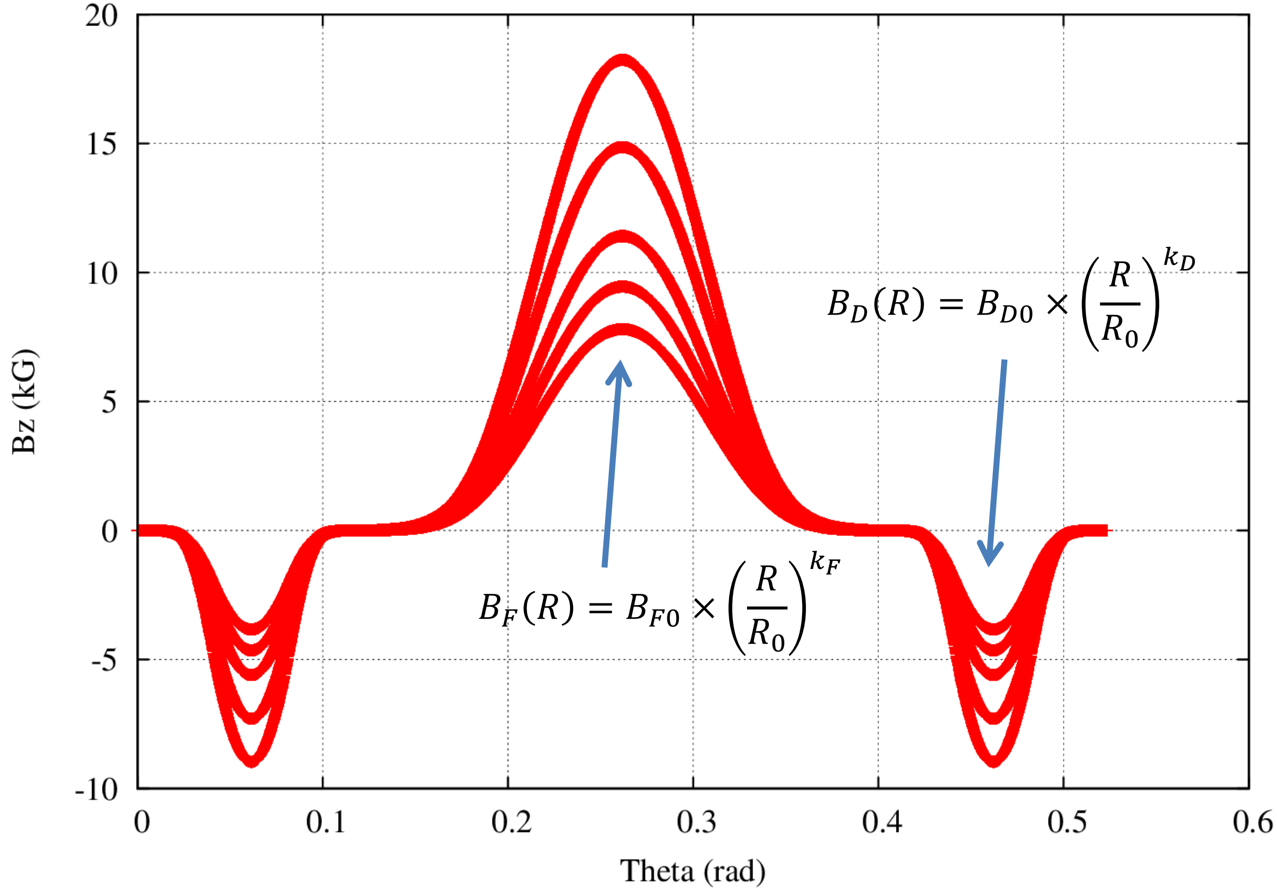
4) For each lattice, the rms value of the tune is calculated

$$< \nu_{x,y}^{rms} > = \frac{1}{N} \sum_{i=1}^{N} (\nu_{x,y,i} - < \nu_{x,y} >)^2$$

= a measure of the tune spread

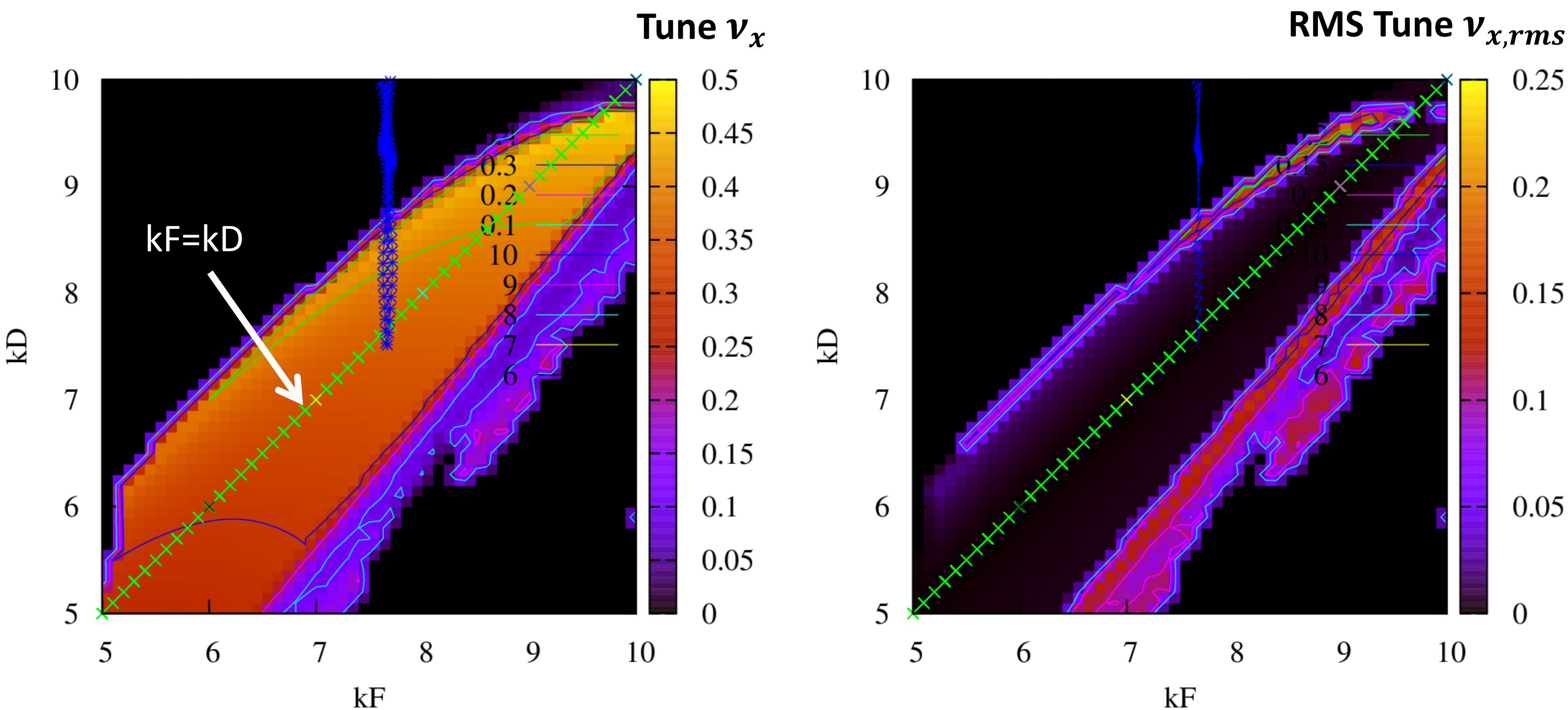
For a perfectly scaling FFAG, $< \nu_{x,y}^{rms} >$ = 0

Example of a DFD triplet



Magnetic field along several closed orbits

Stability diagram for a scaling FFAG Horizontal plane

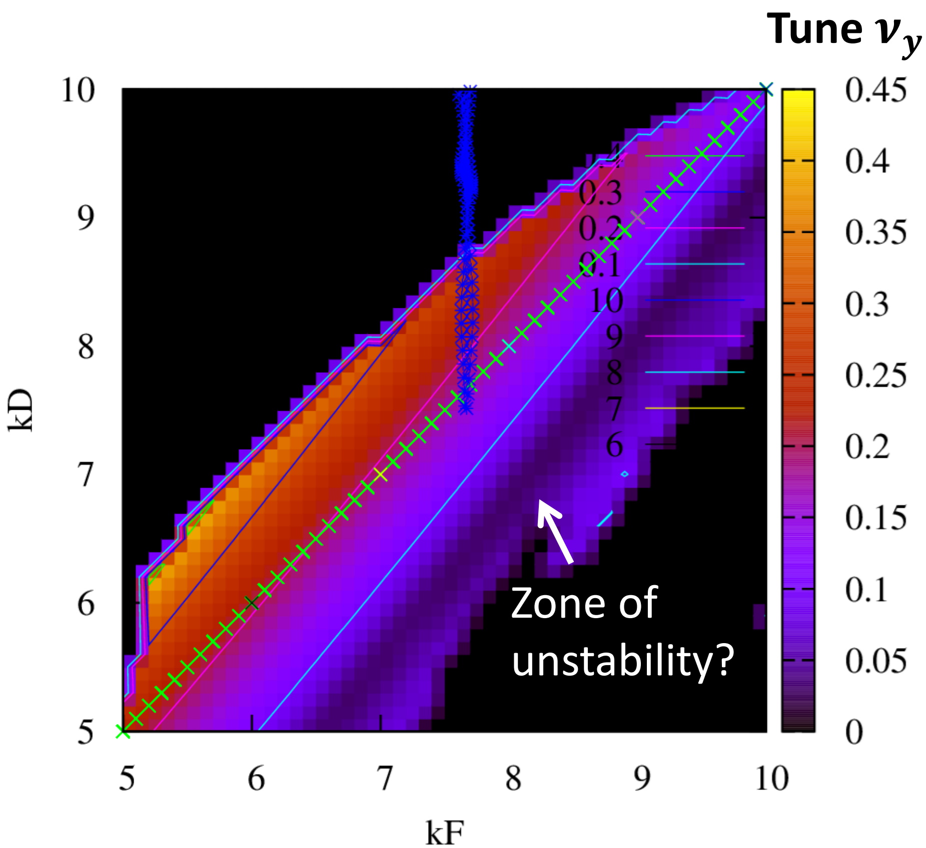


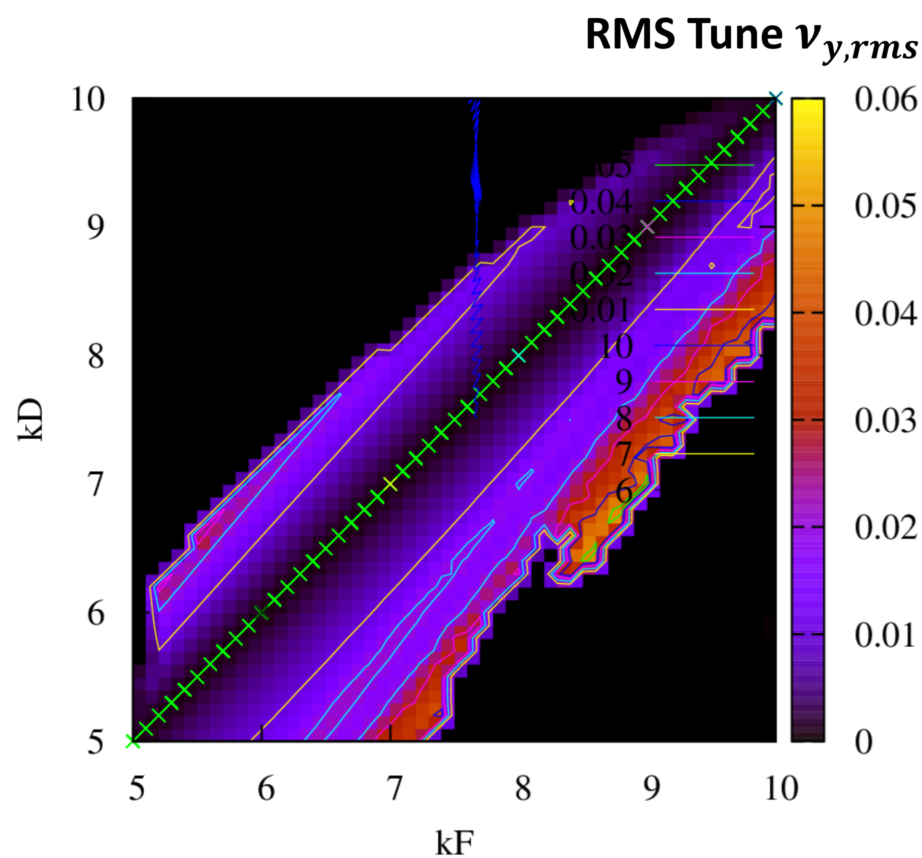
In the horizontal plane, the tune of a scaling FFAG can be approximated by:

$$\nu_x^2 = k + 1 + A.(k+1)^2$$

Less tolerance to imperfections when k becomes large.

Stability diagram for a scaling FFAG
Vertical plane



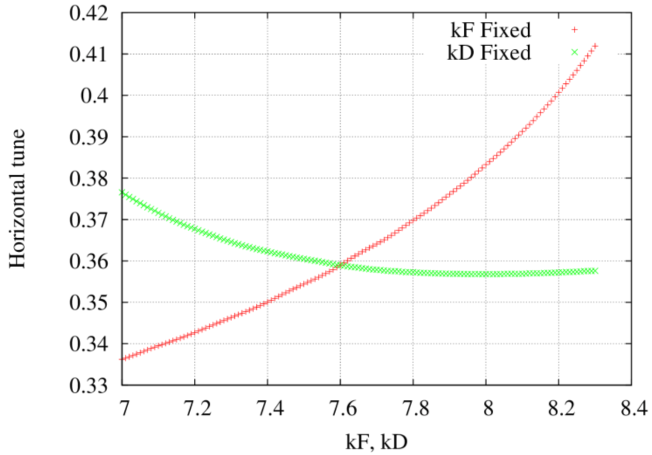


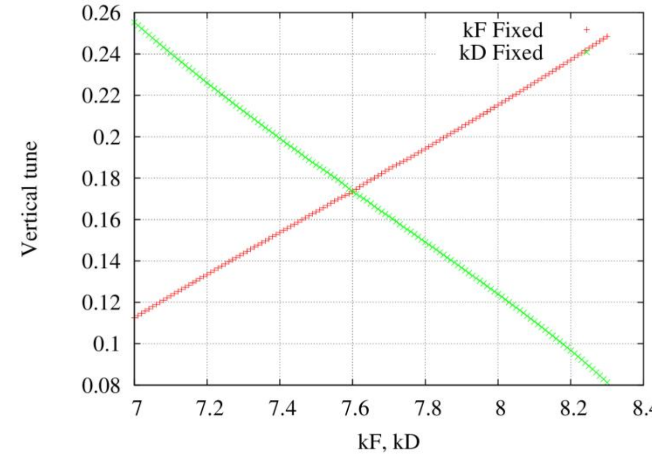
In the vertical plane, the tune of a scaling FFAG can be approximated by:

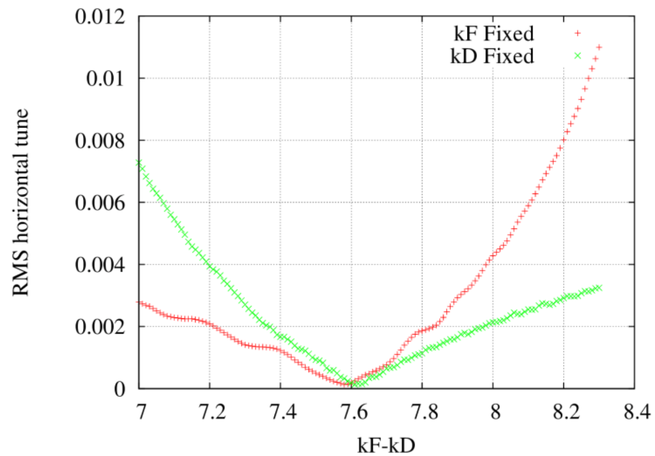
$$\nu_y^2 = -k + \frac{f^2}{2}$$

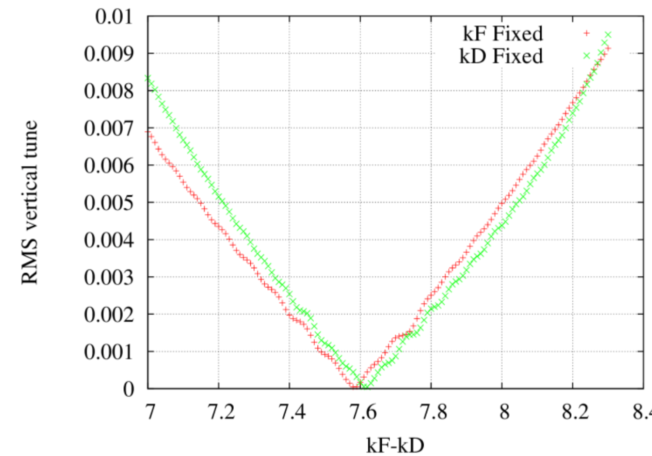
Conclusion: due to imperfect scaling law (kF ≠ kD), the rms variations of the tune are more important in the vertical plane. This is consistent with the measured tune variations (twice as important in the vertical plane than in the horizontal, see 1st slide).

TUNE variations due to imperfections









In the vicinity of the line $k_F = k_D$, the rms tune variations are proportional to $|k_F - k_D|$

Linear depdencen of the tune in the vertical plane, and parabolic dependence in the horizontal plane

The rms quantities yet to be included in the hard edge model.

Tune diagram from multi-particle tracking

In order to investigate the tune variations:

Same exercise as before with multi-particle : launch a KV beam distribution on several closed orbits with initial conditions /

$$\begin{cases} r_x(s+L) = r_x(s) = 2 < x^2 >^{1/2} = \sqrt{\epsilon_x \beta_x} \\ r_y(s+L) = r_y(s) = 2 < y^2 >^{1/2} = \sqrt{\epsilon_y \beta_y} \\ \epsilon_{x,y} = \dfrac{\varepsilon_{norm}}{\beta\gamma} \end{cases}$$

Each distribution (with the same energy, no dispersion included) contains 900 particles for which the average value of the tune is computed as well as the rms tune variations.

Tune diagram from multi-particle tracking

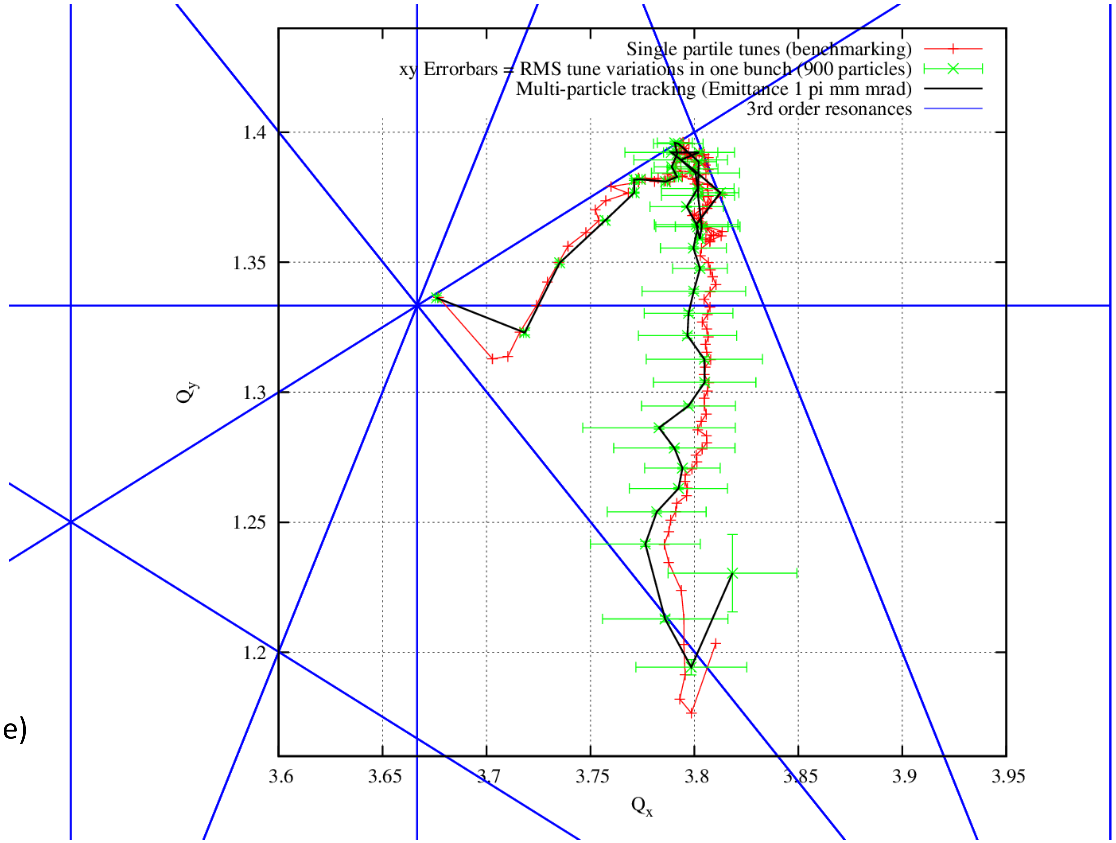
The errorbars represent the rms tune variations for a bunch of particles (one closed orbit).

$$errorbars_x = 1/4 < \nu_x^2 >^{1/2}$$

$$errorbars_y = 1/4 < \nu_y^2 >^{1/2}$$

$$\frac{\delta\nu_x}{\nu_x} \approx \frac{< \nu_x^2 >^{\frac{1}{2}}}{\nu_x} \approx \frac{6.10^{-3}}{0.3168} = 1.9\%$$

(see next slide)



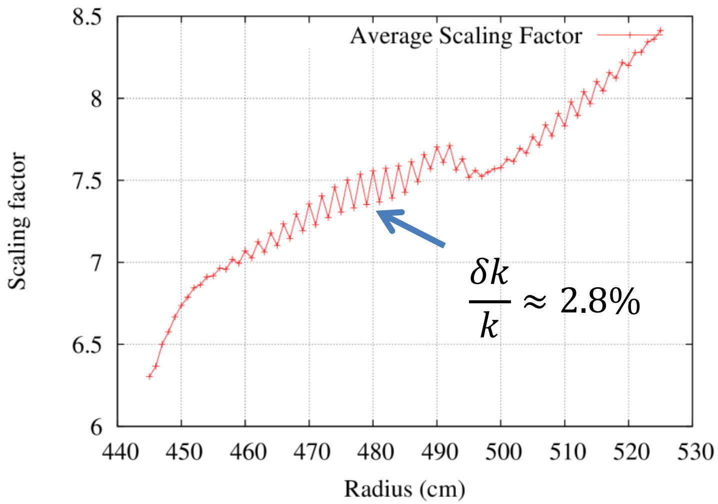
Taking the tune averages help reduce the oscillatiory behavior obtained from single particle calculation. Yet, the oscillation is contained in the rms calculation.

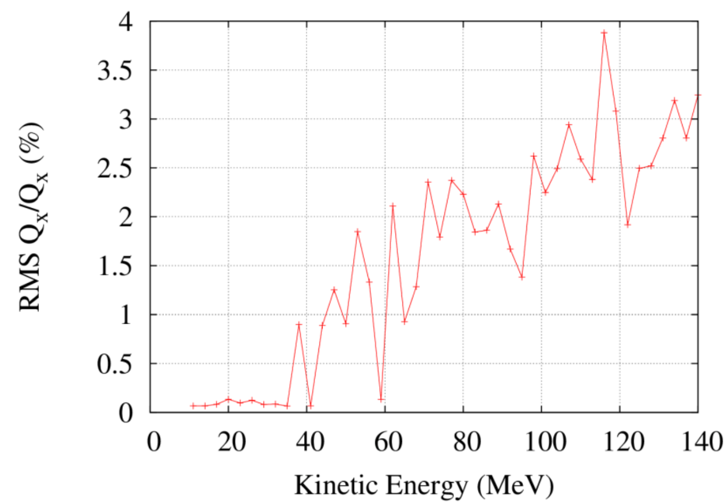
NB: in the above notation, $< \nu_x^2 >^{1/2} = < (\nu_x - < \nu_x >)^2 >^{1/2}$

Tune diagram from multi-particle tracking

Observations: The average value of the tunes from multi-particle tracking is comparable to what we obtain from single particle tracking.
Although, the rms tune variations in the vertical plane are small (see error bars), the same does not apply in the horizontal plane. Source of the discrepancy?

$$\nu_x^2 \approx k + 1 \quad \Rightarrow \quad \frac{\delta k}{k} = 2\frac{\delta \nu_x}{\nu_x}$$



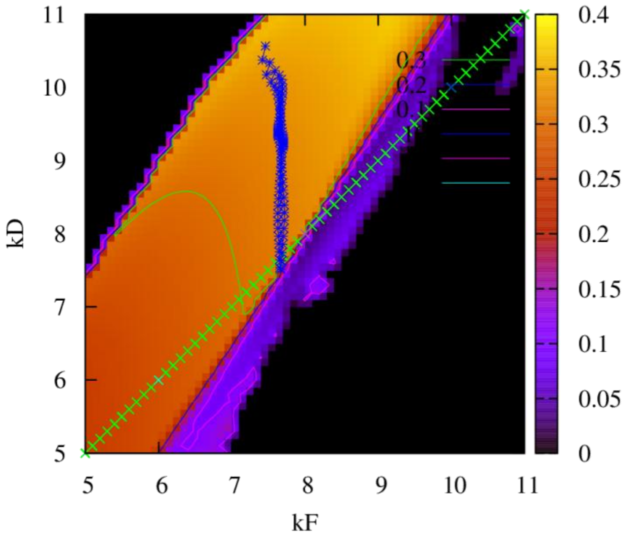


As observed, the rms tune variations are obtained at higher radii and are consistent with the observation that the scaling factor oscillates at higher radii. Need for a higher mesh size to conclude that the rms variations of the tune are only related to the granularity of the map.

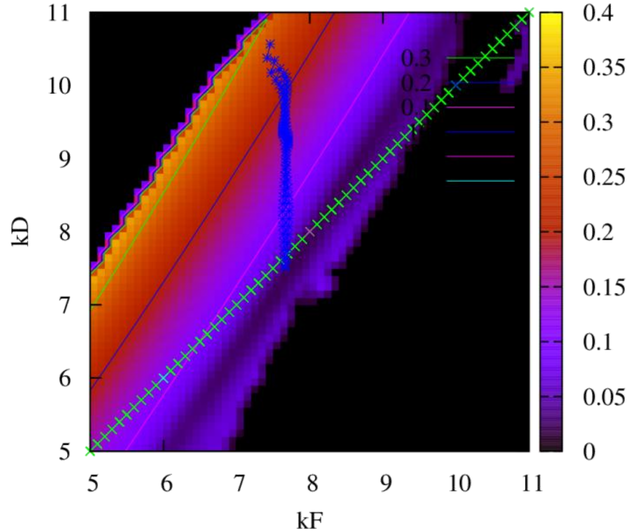
NB: you're very welcome to check!

Stability diagram for the KURRI 150 MeV FFAG

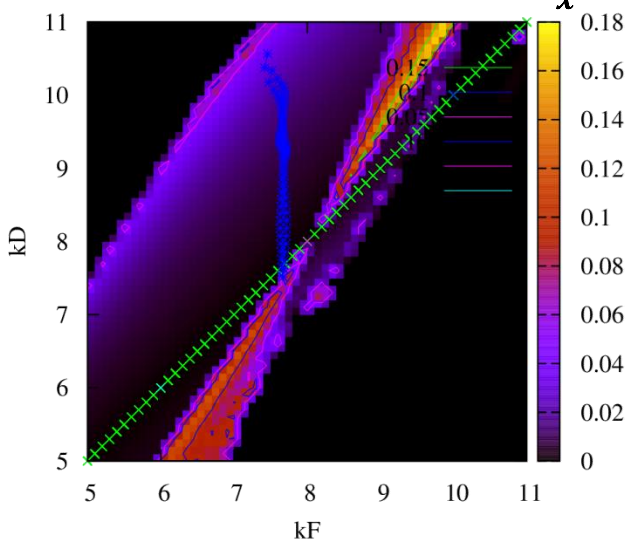
Tune ν_x



Tune ν_y

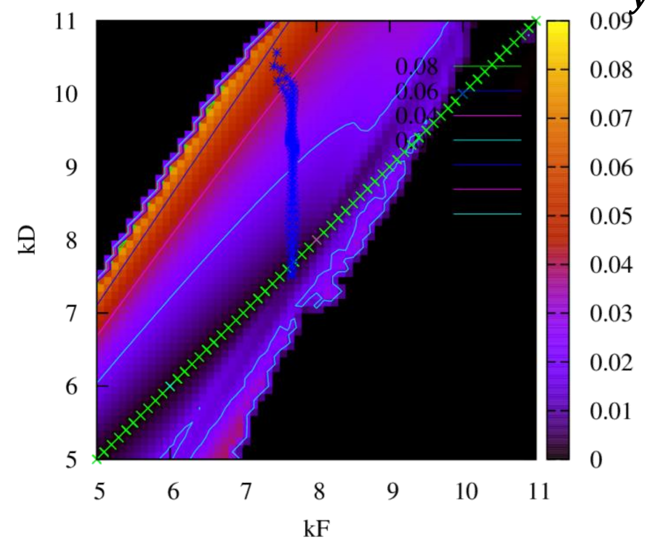


RMS Tune ν_x



This is only an approximation, given that the flutter is R-dependent.

RMS Tune ν_y



Proposal

- Now, how to remediate to the problem of the tune dispersion?

- The scaling factors are fixed by the gap size of the magnet. But the FD ratio (current in the F-D magnet) can be adjusted.

Therefore, a parametric study on the FD ratio aiming at finding the minimum of $< \nu_x^{rms} > \times < \nu_y^{rms} >$ can minimize the area scanned by the tune.

FD ratio

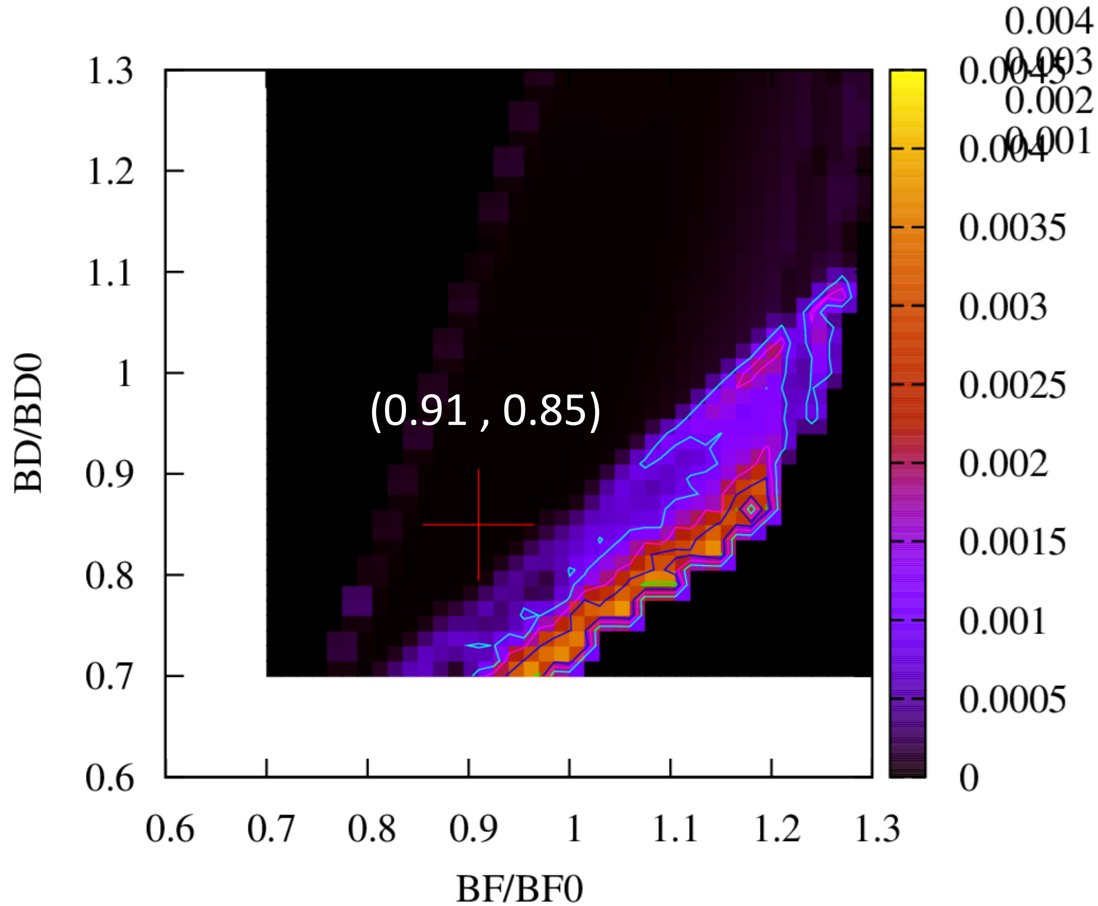
The minimum of

$$P = < \nu_x^{rms} > \times < \nu_y^{rms} >$$

obtained from the scan of BD and BF

reduces P by 17.3 %.

Yet, this is insufficient to justify any changes to the current design.
(Also the current scan is only based on tweaking the field map, and is not as accurate as a 3D TOSCA calculation.



However, one would expect that using some trim coils that can be manipulated independently along the radius (as in cyclotrons) could help reduce the discrepancy?

Needs to be answered:

- To what extent, a KV beam can be utilized to model space charge effects in FFAG (non self consistent distribution).
- Find a matching condition for FFAG in presence of space charge (general assumption is that there exists one unique matching condition!). Linear perturbation is one way to find an approximate solution ..
- Add space charge to the previous to see how it affects the tune and therefore the dependence of kF and kD.

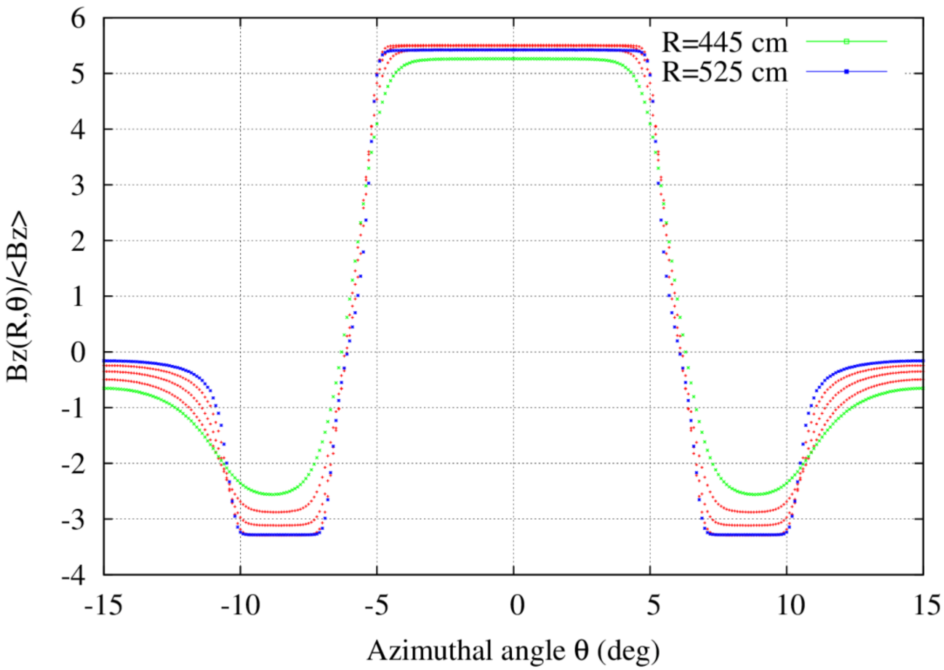
Backup Slides

Flutter

- If $B(R,\theta) = \langle B(R) \rangle \times F(\theta)$ were an exact solution, then, F is independent of R.
- Yet, the plot shows that F is dependent of R as well.

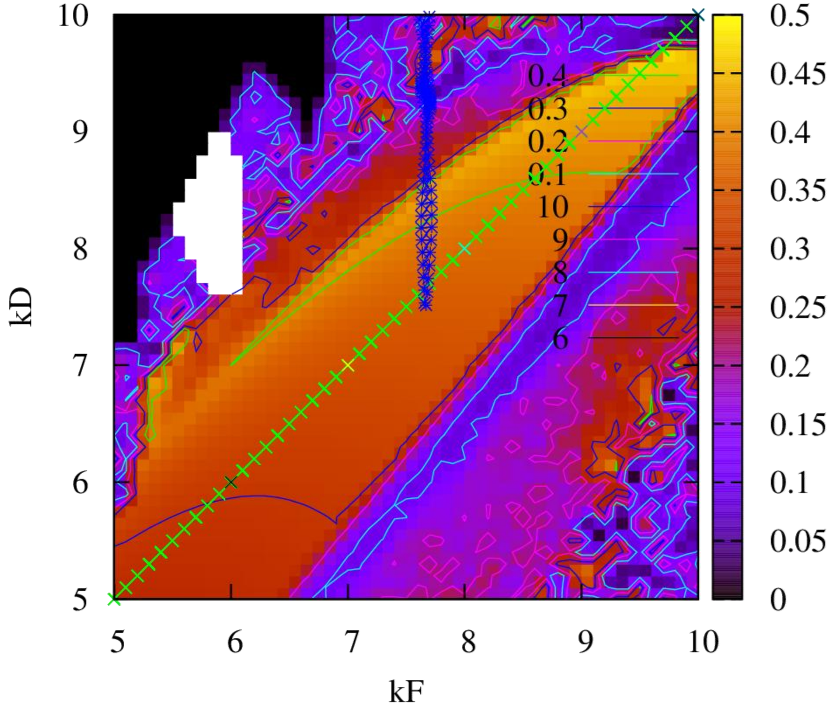
$$B(R,\theta) = \langle B(R) \rangle \times F(R,\theta)$$

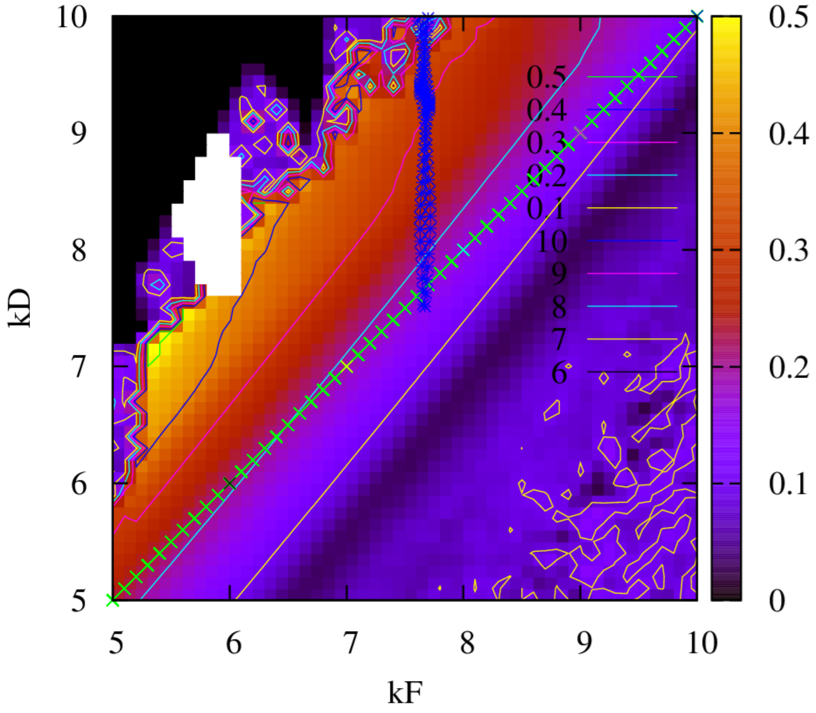
$$\int_0^{\frac{2\pi}{N}} F(R,\theta)d\theta = \frac{2\pi}{N}$$



Flutter F(R,θ)

Tune stability diagram in the large





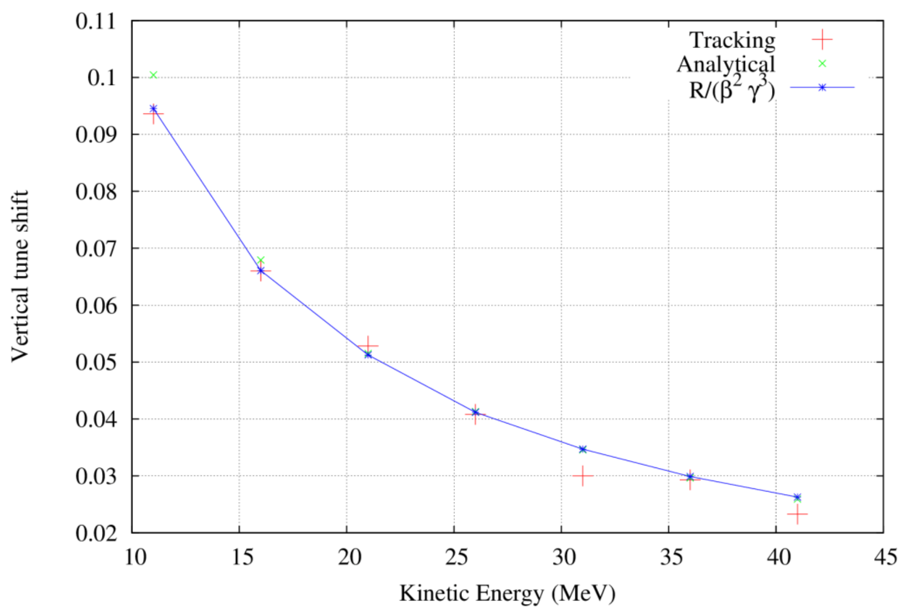
Laslett tune shift

- Investigate the change in betatron oscillation frequency due to space charge forces. The linear Laslett tune shift is given by:

$$\Delta Q_x = \frac{1}{4\pi}\int_0^C \beta_x(s)\frac{2Q}{r_x(r_x+r_y)}ds \quad \Rightarrow \quad \Delta Q_x \propto \frac{R}{\beta^2\gamma^3}$$

If the emittance is kept the same for all energies

⇨Tracking results are consistent with the Scaling law of the Laslett tune shift (dispersion is neglected here)



$Q \approx 6.4 \times 10^{-8}$ at injection.

Modeling of imperfect scaling:

- In order to determine the correct solution of the magnetic field based on the knowledge of k(R), one assumed that k(R) can be fitted with a n-degree polynomial:

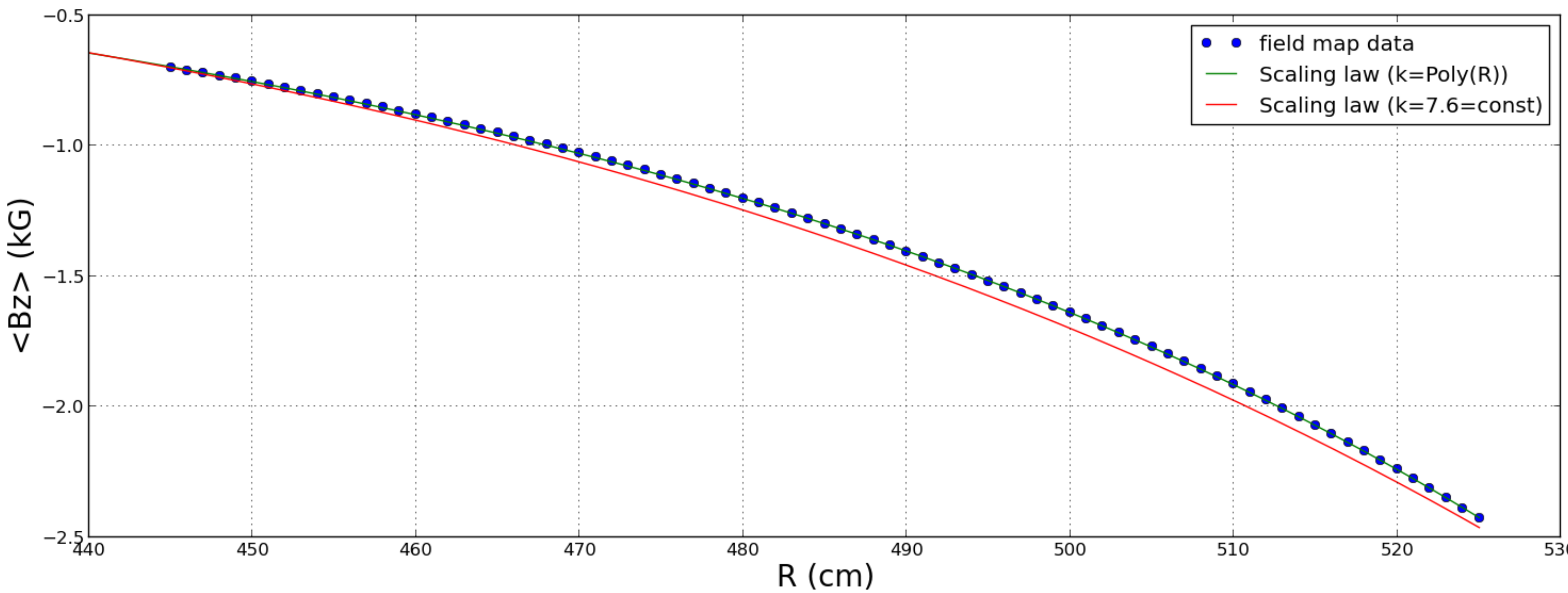
$$k(R) = \sum_{i=1}^{n} a_i R^i$$

- After some math … the new solution is:

$$B(R) = B_0(\frac{R}{R_0})^{a_0} \times \exp(\sum_{i=1}^{n} a_i \frac{(R^i - R_0^i)}{i})$$

B(R) is the median plane field averaged in azimuth

Modeling of imperfect scaling:



The new scaling law (shown in green) was obtained from the equation

$$B(R) = B_0\left(\frac{R}{R_0}\right)^{a_0} \times \exp\left(\sum_{i=1}^{n} a_i \frac{(R^i - R_0^i)}{i}\right)$$

where n=3, $a_0 = 7.6$ and

$a_1 = 4.74e-03$; $a_2 = -5.39e-05$; $a_3 = 9.04e-08$

The perturbed scaling law takes into account the R-dependence of the scaling factor and thus is more accurate to describe the magnet.