Multibunch acceleration

Exact solutions

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Harmonic acceleration

- The most common acceleration mode in a circular accelerator is harmonic
- The bunch passes repeatedly through an RF cavity
- The frequency of the RF cavity is chosen such that the bunch always see and accelerating voltage

$$C = hB \tag{1}$$

- C is the frequency of the Cavity
- h is harmonic number, integer
- B is revolution frequency of the bunch

Multiple solutions

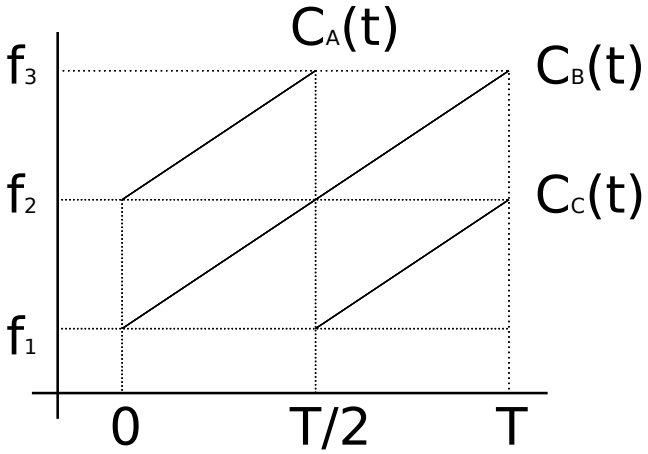
$$C = hB$$

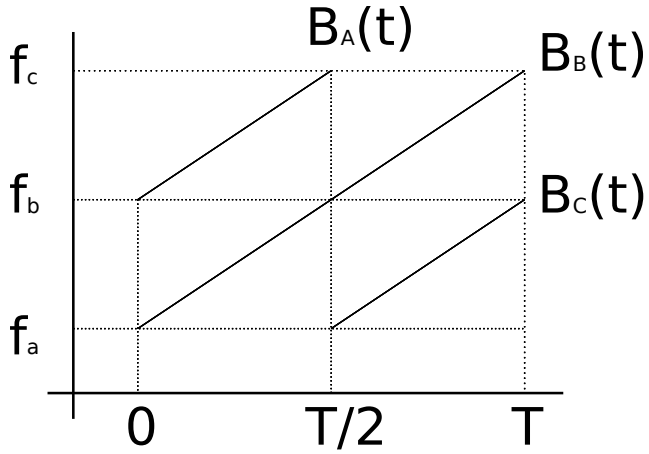
- Has multiple solutions
  - For a given cavity frequency, there are an infinite number of matched revolution frequencies
    - If C = 12 MHz, B can be 12, 6, 4, 3, 2.4 ... MHz
  - For a given revolution frequency, there are an infinite number of matched cavity frequencies
    - If B = 1 MHz, C can be 1, 2, 3, 4, 5 ... MHz

Multiple solutions

- With the bunch always seeing an accelerating gradient:
  - it is possible to have multiple bunches at different velocities, accelerated by 1 cavity frequency
  - and it is possible to have multiple cavities at different RF frequencies, accelerating the same bunch
    - (or multiple frequencies in a single high bandwidth cavity)
- So is it possible to accelerate multiple bunches, with multiple frequencies, such that they all always see an accelerating gradient?

Conditions





- Need functions that meet harmonic conditions $C_A(t) = hB_A(t)$
- Need repetition, $C_A(t) = C_B(t + T/n)$ (n is number of bunches)
- Need to sweep from f1 to f2, $C_A(t) = f2/f1 C_A(t + T)$

Results

- To ensure that the frequencies remain separated by a constant fraction, we find that C(t) and B(t) must be exponentials in t
- Also we find strict conditions on the harmonics used

- For 2 bunch acceleration, with a frequency sweep of 2, harmonic numbers of 100 or bigger are required.
- For 3 bunch harmonic numbers need to be 1296 or bigger.
- This continues to rise rapidly

- (The full working is in my thesis)

Practicality

- For a small accelerator a harmonic number of 1000 is big.
- It is not possible accelerator more than 3 bunches, and for each bunch to always see an accelerating gradient.
- However Takeichiro has shown that acceleration can work if there is relaxation in these conditions