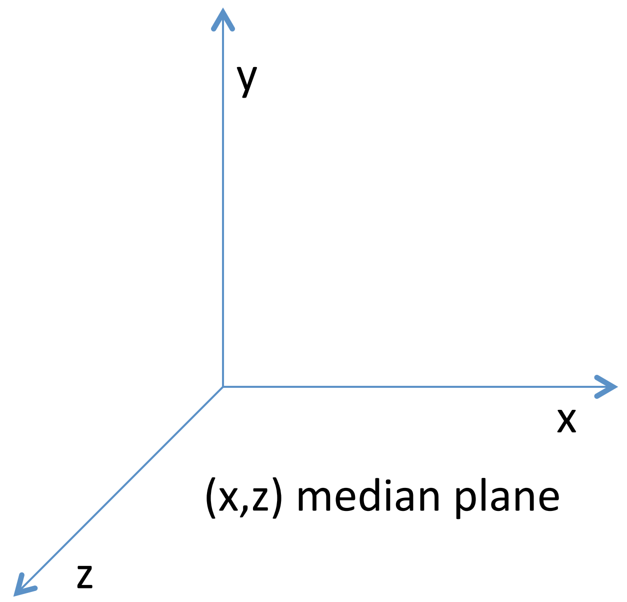
Derive **B**(x,y,z) fields from Median Plane **B**(x,0,z) field maps



Expand **B**(x,y,z) in Maclaurin series in y

$$B_x(x,y,z) = B_x(x,0,z) + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + \ldots$$

$$a_n(x,z) = \frac{1}{n!}\frac{\partial^n B_x(x,y,z)}{\partial y^n}\bigg|_{y=0}$$

$$B_y(x,y,z) = B_y(x,0,z) + b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4 + \ldots$$

$$b_n(x,z) = \frac{1}{n!}\frac{\partial^n B_y(x,y,z)}{\partial y^n}\bigg|_{y=0}$$

$$B_z(x,y,z) = B_z(x,0,z) + c_1 y + c_2 y^2 + c_3 y^3 + c_4 y^4 + \ldots.$$

$$c_n(x,z) = \frac{1}{n!}\frac{\partial^n B_z(x,y,z)}{\partial y^n}\bigg|_{y=0}$$

The task is to express the coefficients of each of the three expansions in terms of the derivatives of the function **B**(x,0,z)

We use the Maxwell equations.

$$\nabla \cdot B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\nabla \cdot B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} =$$

$$\frac{\partial B_x(x,0,z)}{\partial x} + \frac{\partial a_1(x,z)}{\partial x} y + \frac{\partial a_2(x,z)}{\partial x} y^2 + \frac{\partial a_3(x,z)}{\partial x} y^3 + \frac{\partial a_4(x,z)}{\partial x} y^4 +$$

$$\cancelto{0}{\frac{\partial B_y(x,0,z)}{\partial y}} + b_1 + 2b_2 y + 3b_3 y^2 + 4b_4 y^3 +$$

$$\frac{\partial B_z(x,0,z)}{\partial z} + \frac{\partial c_1(x,z)}{\partial z} y + \frac{\partial c_2(x,z)}{\partial z} y^2 + \frac{\partial c_3(x,z)}{\partial z} y^3 + \frac{\partial c_4(x,z)}{\partial z} y^4 = 0$$

$$\frac{\partial B_x(x,0,z)}{\partial x} + \cancelto{0}{\frac{\partial B_y(x,0,z)}{\partial y}} + b_1 + \frac{\partial B_z(x,0,z)}{\partial z} = 0 \quad (4)$$

$$\frac{\partial a_1(x,z)}{\partial x} + 2b_2 + \frac{\partial c_1(x,z)}{\partial z} = 0 \quad (5)$$

$$\frac{\partial a_2(x,z)}{\partial x} + 3b_3 + \frac{\partial c_2(x,z)}{\partial z} = 0 \quad (6)$$

$$\frac{\partial a_3(x,z)}{\partial x} + 4b_4 + \frac{\partial c_3(x,z)}{\partial z} = 0 \quad (7)$$

$$\frac{\partial a_4(x,z)}{\partial x} + \frac{\partial c_4(x,z)}{\partial z} = 0 \quad (8)$$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right)\hat{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right)\hat{j} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)\hat{k} = 0$$

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

$$\frac{\partial B_y(x,0,z)}{\partial x} + \frac{\partial b_1}{\partial x} y + \frac{\partial b_2}{\partial x} y^2 + \frac{\partial b_3}{\partial x} y^3 + \frac{\partial b_4}{\partial x} y^4 = \underbrace{\frac{\partial B_x(x,0,z)}{\partial y}}_{0} + a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3$$

$$a_1 = \frac{\partial B_y(x,0,y)}{\partial x} \quad (11)$$

$$2a_2 = \frac{\partial b_1}{\partial x} \quad (12)$$

$$3a_3 = \frac{\partial b_2}{\partial x} \quad (13)$$

$$4a_4 = \frac{\partial b_3}{\partial x} \quad (14)$$

$$\frac{\partial b_4}{\partial x} = 0$$

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x}$$

$$\frac{\partial B_x(x,0,z)}{\partial z} + \frac{\partial a_1}{\partial z} y + \frac{\partial a_2}{\partial z} y^2 + \frac{\partial a_3}{\partial z} y^3 + \frac{\partial a_4}{\partial z} y^4 = \frac{\partial B_z(x,0,z)}{\partial x} + \frac{\partial c_1}{\partial x} y + \frac{\partial c_2}{\partial x} y^2 + \frac{\partial c_3}{\partial x} y^3 + \frac{\partial c_4}{\partial x} y^4$$

$$\frac{\partial B_x(x,0,z)}{\partial z} = \frac{\partial B_z(x,0,z)}{\partial x}$$

$$\frac{\partial a_1}{\partial z} = \frac{\partial c_1}{\partial x} \qquad \frac{\partial a_2}{\partial z} = \frac{\partial c_2}{\partial x} \qquad \frac{\partial a_3}{\partial z} = \frac{\partial c_3}{\partial x} \qquad \frac{\partial a_4}{\partial z} = \frac{\partial c_4}{\partial x}$$

$$\frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z}$$

$$\cancelto{0}{\frac{\partial B_z(x,0,z)}{\partial y}} + c_1 + 2c_2 y + 3c_3 y^2 + 4c_4 y^3 = \frac{\partial B_y(x,0,z)}{\partial z} + \frac{\partial b_1}{\partial z} y + \frac{\partial b_2}{\partial z} y^2 + \frac{\partial b_3}{\partial z} y^3 + \frac{\partial b_4}{\partial z} y^4$$

$$c_1 = \frac{\partial B_y(x,0,z)}{\partial z} \quad (31)$$

$$2c_2 = \frac{\partial b_1}{\partial z} \quad (32) \qquad 3c_3 = \frac{\partial b_2}{\partial z} \quad (33) \qquad 4c_4 = \frac{\partial b_3}{\partial z} \quad (34) \qquad \frac{\partial b_4}{\partial z} = 0$$

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = 0$$

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} =$$

$$\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 a_1(x,z)}{\partial x^2} y + \frac{\partial^2 a_2(x,z)}{\partial x^2} y^2 + \frac{\partial^2 a_3(x,z)}{\partial x^2} y^3 + \frac{\partial^2 a_4(x,z)}{\partial x^2} y^4 +$$

$$\frac{\partial^2 B_x(x,0,z)}{\partial y^2} \to 0 \quad + 2a_2 + 6a_3 y + 12 a_4 y^2 +$$

$$\frac{\partial^2 B_x(x,0,z)}{\partial z^2} + \frac{\partial^2 a_1}{\partial z^2} y + \frac{\partial^2 a_2}{\partial z^2} y^2 + \frac{\partial^2 a_3}{\partial z^2} y^3 + \frac{\partial^2 a_4}{\partial z^2} y^4 = 0$$

$$2a_2 = -\left(\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 B_x(x,0,z)}{\partial z^2} \right) \quad (51)$$

$$6a_3 = -\left(\frac{\partial^2 a_1}{\partial x^2} + \frac{\partial^2 a_1}{\partial z^2} \right) \quad (52)$$

$$12a_4 = -\left(\frac{\partial^2 a_2}{\partial x^2} + \frac{\partial^2 a_2}{\partial z^2} \right) \quad (53)$$

$$\left(\frac{\partial^2 a_3}{\partial x^2} + \frac{\partial^2 a_3}{\partial z^2} \right) = 0$$

$$\left(\frac{\partial^2 a_4}{\partial x^2} + \frac{\partial^2 a_4}{\partial z^2} \right) = 0$$

$$\nabla^2 B_y = \frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_y}{\partial z^2} = 0$$

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} =$$

$$\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 b_1(x,z)}{\partial x^2} y + \frac{\partial^2 b_2(x,z)}{\partial x^2} y^2 + \frac{\partial^2 b_3(x,z)}{\partial x^2} y^3 + \frac{\partial^2 b_4(x,z)}{\partial x^2} y^4 +$$

$$\cancelto{0}{\frac{\partial^2 B_y(x,0,z)}{\partial y^2}} + 2b_2 + 6b_3 y + 12 b_4 y^2 +$$

$$\frac{\partial^2 B_y(x,0,z)}{\partial z^2} + \frac{\partial^2 b_1}{\partial z^2} y + \frac{\partial^2 b_2}{\partial z^2} y^2 + \frac{\partial^2 b_3}{\partial z^2} y^3 + \frac{\partial^2 b_4}{\partial z^2} y^4 = 0$$

$$2b_2 = -\left(\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 B_y(x,0,z)}{\partial z^2} \right) \quad (61)$$

$$6b_3 = -\left(\frac{\partial^2 b_1}{\partial x^2} + \frac{\partial^2 b_1}{\partial z^2} \right)$$

$$12b_4 = -\left(\frac{\partial^2 b_2}{\partial x^2} + \frac{\partial^2 b_2}{\partial z^2} \right) \quad (63)$$

$$\left(\frac{\partial^2 b_3}{\partial x^2} + \frac{\partial^2 b_3}{\partial z^2} \right) = 0$$

$$\left(\frac{\partial^2 b_4}{\partial x^2} + \frac{\partial^2 b_4}{\partial z^2} \right) = 0$$

$$\nabla^2 B_y = \frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_y}{\partial z^2} = 0$$

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} =$$

$$\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 b_1(x,z)}{\partial x^2} y + \frac{\partial^2 b_2(x,z)}{\partial x^2} y^2 + \frac{\partial^2 b_3(x,z)}{\partial x^2} y^3 + \frac{\partial^2 b_4(x,z)}{\partial x^2} y^4 +$$

$$\cancelto{0}{\frac{\partial^2 B_y(x,0,z)}{\partial y^2}} + 2b_2 + 6b_3 y + 12 b_4 y^2 +$$

$$\frac{\partial^2 B_y(x,0,z)}{\partial z^2} + \frac{\partial^2 b_1}{\partial z^2} y + \frac{\partial^2 b_2}{\partial z^2} y^2 + \frac{\partial^2 b_3}{\partial z^2} y^3 + \frac{\partial^2 b_4}{\partial z^2} y^4 = 0$$

$$2b_2 = -\left(\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 B_y(x,0,z)}{\partial z^2} \right) \quad (61)$$

$$6b_3 = -\left(\frac{\partial^2 b_1}{\partial x^2} + \frac{\partial^2 b_1}{\partial z^2} \right) \quad (62)$$

$$12b_4 = -\left(\frac{\partial^2 b_2}{\partial x^2} + \frac{\partial^2 b_2}{\partial z^2} \right)$$

$$\left(\frac{\partial^2 b_3}{\partial x^2} + \frac{\partial^2 b_3}{\partial z^2} \right) = 0$$

$$\left(\frac{\partial^2 b_4}{\partial x^2} + \frac{\partial^2 b_4}{\partial z^2} \right) = 0$$

$$\nabla^2 B_z = \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2} = 0$$

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} =$$

$$\frac{\partial^2 B_z(x,0,z)}{\partial x^2} + \frac{\partial^2 c_1(x,z)}{\partial x^2} y + \frac{\partial^2 c_2(x,z)}{\partial x^2} y^2 + \frac{\partial^2 c_3(x,z)}{\partial x^2} y^3 + \frac{\partial^2 c_4(x,z)}{\partial x^2} y^4 +$$

0

$$\frac{\partial^2 B_z(x,0,z)}{\partial y^2} + 2c_2 + 6c_3 y + 12c_4 y^2 +$$

$$\frac{\partial^2 B_z(x,0,z)}{\partial z^2} + \frac{\partial^2 c_1}{\partial z^2} y + \frac{\partial^2 c_2}{\partial z^2} y^2 + \frac{\partial^2 c_3}{\partial z^2} y^3 + \frac{\partial^2 c_4}{\partial z^2} y^4 = 0$$

$$2c_2 = -\left(\frac{\partial^2 B_z(x,0,z)}{\partial x^2} + \frac{\partial^2 B_z(x,0,z)}{\partial z^2} \right) \quad (71) \qquad 6c_3 = -\left(\frac{\partial^2 c_1}{\partial x^2} + \frac{\partial^2 c_1}{\partial z^2} \right) \quad (72)$$

$$12c_4 = -\left(\frac{\partial^2 c_2}{\partial x^2} + \frac{\partial^2 c_2}{\partial z^2} \right) \quad (73) \qquad \left(\frac{\partial^2 c_3}{\partial x^2} + \frac{\partial^2 c_3}{\partial z^2} \right) = 0 \qquad \left(\frac{\partial^2 c_4}{\partial x^2} + \frac{\partial^2 c_4}{\partial z^2} \right) = 0$$

$$b_1 = -\left(\frac{\partial B_x(x,0,z)}{\partial x} + \frac{\partial B_z(x,0,z)}{\partial z} \right) \quad (100)$$ From (4)

$$a_1 = \frac{\partial B_y(x,0,z)}{\partial x} \quad (101)$$ From (11)

$$c_1 = \frac{\partial B_y(x,0,z)}{\partial z} \quad (102)$$ From (31)

$$a_2 = \frac{1}{2}\frac{\partial b_1}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}\left(-\left(\frac{\partial B_x(x,0,z)}{\partial x} + \frac{\partial B_z(x,0,z)}{\partial z} \right) \right) \quad (103)$$ From (12)

$$a_2 = -\frac{1}{2}\left(\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 B_z(x,0,z)}{\partial x \partial z} \right) \quad (104)$$

$$2a_2 = -\left(\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 B_x(x,0,z)}{\partial z^2} \right) \quad (51)$$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right)\hat{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right)\hat{j} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)\hat{k} = 0$$

$$\frac{\partial a_1(x,z)}{\partial x} + 2b_2 + \frac{\partial c_1(x,z)}{\partial z} = 0 \qquad (5)$$

$$b_2 = -\frac{1}{2}\left(\frac{\partial a_1(x,z)}{\partial x} + \frac{\partial c_1(x,z)}{\partial z}\right) = -\frac{1}{2}\left(\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 B_y(x,0,z)}{\partial z^2}\right) \qquad (105)$$

$$2b_2 = -\left(\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 B_y(x,0,z)}{\partial z^2}\right) \qquad \text{see also (61)}$$

$$2c_2 = \frac{\partial b_1}{\partial z} \quad (32) \qquad\qquad c_2 = \frac{1}{2}\frac{\partial b_1}{\partial z} = -\frac{1}{2}\left(\frac{\partial^2 B_x(x,0,z)}{\partial x \partial z} + \frac{\partial^2 B_z(x,0,z)}{\partial z^2}\right) \quad (106)$$

$$2c_2 = -\left(\frac{\partial^2 B_z(x,0,z)}{\partial x^2} + \frac{\partial^2 B_z(x,0,z)}{\partial z^2}\right) \quad (71)$$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\hat{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\hat{j} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{k} = 0$$

$$3a_3 = \frac{\partial b_2}{\partial x} \quad (13)$$

$$a_3 = \frac{1}{3}\frac{\partial b_2}{\partial x} = -\frac{1}{6}\left(\frac{\partial^3 B_y(x,0,z)}{\partial x^3} + \frac{\partial^3 B_y(x,0,z)}{\partial x \partial z^2}\right) \quad (107)$$

$$6a_3 = -\left(\frac{\partial^2 a_1}{\partial x^2} + \frac{\partial^2 a_1}{\partial z^2}\right) \quad (52) \qquad a_1 = \frac{\partial B_y(x,0,z)}{\partial x} \quad (101)$$

$$\frac{\partial a_2(x,z)}{\partial x} + 3b_3 + \frac{\partial c_2(x,z)}{\partial z} = 0 \quad (6)$$

$$b_3 = -\frac{1}{3}\left(\frac{\partial a_2(x,z)}{\partial x} + \frac{\partial c_2(x,z)}{\partial z}\right) = -\frac{1}{3}\left(-\frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 B_z(x,0,z)}{\partial x \partial z}\right) - \frac{1}{2}\frac{\partial}{\partial z}\left(\frac{\partial^2 B_x(x,0,z)}{\partial x \partial z} + \frac{\partial^2 B_z(x,0,z)}{\partial z^2}\right)\right)$$

$$-\frac{1}{6}\left(\left(\frac{\partial^3 B_x(x,0,z)}{\partial x^3} + \frac{\partial^3 B_z(x,0,z)}{\partial x^2 \partial z}\right) + \left(\frac{\partial^3 B_x(x,0,z)}{\partial x \partial z^2} + \frac{\partial^3 B_z(x,0,z)}{\partial z^3}\right)\right) (108)$$

$$6b_3 = -\left(\frac{\partial^2 b_1}{\partial x^2} + \frac{\partial^2 b_1}{\partial z^2}\right) \quad (62)$$

$$b_3 = -\frac{1}{6}\left(\left(\frac{\partial^3 B_x(x,0,z)}{\partial x^3} + \frac{\partial^3 B_z(x,0,z)}{\partial x^2 \partial z}\right) + \left(\frac{\partial^3 B_x(x,0,z)}{\partial x \partial z^2} + \frac{\partial^3 B_z(x,0,z)}{\partial z^3}\right)\right) (108)$$

$$3c_3 = \frac{\partial b_2}{\partial z} \quad (33)$$

$$c_3 = -\frac{1}{6}\left(\frac{\partial^3 B_y(x,0,z)}{\partial x^2 \partial z} + \frac{\partial^3 B_y(x,0,z)}{\partial z^3}\right) \quad (109)$$

$$6c_3 = -\left(\frac{\partial^2 c_1}{\partial x^2} + \frac{\partial^2 c_1}{\partial z^2}\right) \quad (72)$$

$$c_1 = \frac{\partial B_y(x,0,z)}{\partial z} \quad (31)$$

$$c_3 = -\frac{1}{6}\left(\frac{\partial^3 B_y(x,0,z)}{\partial x^2 \partial z} + \frac{\partial^3 B_y(x,0,z)}{\partial z^3}\right) \quad (109)$$

$$4a_4 = \frac{\partial b_3}{\partial x} \quad (14)$$

$$b_3 = -\frac{1}{6}\left(\left(\frac{\partial^3 B_x(x,0,z)}{\partial x^3} + \frac{\partial^3 B_z(x,0,z)}{\partial x^2 \partial z}\right) + \left(\frac{\partial^3 B_x(x,0,z)}{\partial x \partial z^2} + \frac{\partial^3 B_z(x,0,z)}{\partial z^3}\right)\right) \quad (108)$$

$$a_4 = -\frac{1}{24}\left(\left(\frac{\partial^4 B_x(x,0,z)}{\partial x^4} + \frac{\partial^4 B_z(x,0,z)}{\partial x^3 \partial z}\right) + \left(\frac{\partial^4 B_x(x,0,z)}{\partial x^2 \partial z^2} + \frac{\partial^4 B_z(x,0,z)}{\partial x \partial z^3}\right)\right) \quad (110)$$

$$a_2 = -\frac{1}{2}\left(\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 B_z(x,0,z)}{\partial x \partial z}\right) \quad (104)$$

$$12a_4 = -\left(\frac{\partial^2 a_2}{\partial x^2} + \frac{\partial^2 a_2}{\partial z^2}\right) \quad (53)$$

$$2a_2 = -\left(\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2 B_x(x,0,z)}{\partial z^2}\right) \quad (51)$$

$$\frac{\partial a_3(x,z)}{\partial x} + 4b_4 + \frac{\partial c_3(x,z)}{\partial z} = 0 \quad (7)$$

$$c_3 = -\frac{1}{6}\left(\frac{\partial^3 B_y(x,0,z)}{\partial x^2 \partial z} + \frac{\partial^3 B_y(x,0,z)}{\partial z^3}\right) \quad (109)$$

$$a_3 = \frac{1}{3}\frac{\partial b_2}{\partial x} = -\frac{1}{6}\left(\frac{\partial^3 B_y(x,0,z)}{\partial x^3} + \frac{\partial^3 B_y(x,0,z)}{\partial x \partial z^2}\right) \quad (107)$$

$$12b_4 = -\left(\frac{\partial^2 b_2}{\partial x^2} + \frac{\partial^2 b_2}{\partial z^2}\right) \quad (63)$$

$$2b_2 = -\left(\frac{\partial^2 B_y(x,0,z)}{\partial x^2} + \frac{\partial^2 B_y(x,0,z)}{\partial z^2}\right) \quad \text{see also (61)}$$

$$4c_4 = \frac{\partial b_3}{\partial z} \quad (34) \qquad b_3 = -\frac{1}{6}\left(\left(\frac{\partial^3 B_x(x,0,z)}{\partial x^3} + \frac{\partial^3 B_z(x,0,z)}{\partial x^2 \partial z}\right) + \left(\frac{\partial^3 B_x(x,0,z)}{\partial x \partial z^2} + \frac{\partial^3 B_z(x,0,z)}{\partial z^3}\right)\right) (108)$$

$$12c_4 = -\left(\frac{\partial^2 c_2}{\partial x^2} + \frac{\partial^2 c_2}{\partial z^2}\right) \quad (73) \qquad c_2 = \frac{1}{2}\frac{\partial b_1}{\partial z} = -\frac{1}{2}\left(\frac{\partial^2 B_x(x,0,z)}{\partial x \partial z} + \frac{\partial^2 B_z(x,0,z)}{\partial z^2}\right) \quad (106)$$

EXPAND FIELDS IN TERMS OF

$$B_x(x,y,z) = B_x(x,0,z) + a_1 y + a_2$$

$$B_y(x,y,z) = B_y(x,0,z) + b_1 y + b_2$$

Use of $\nabla \cdot \vec{B} = 0$

$$\frac{\partial B_x}{\partial x} = \frac{\partial B_x(0)}{\partial x} + \frac{\partial a_1(x,z)}{\partial x} \cdot y + \frac{\partial a_2}{\partial x}$$

$$\frac{\partial B_y}{} \quad \partial B_y(0) \to 0$$

Adding 1 + 2 + 3 and setting
y^n_i

$$\frac{\partial B_{x(0)}}{\partial x} + \frac{\partial B_{y(0)}}{\partial y} \rightarrow 0 + \frac{\partial B_{z(0)}}{\partial z} +$$

Use of $\nabla \times \vec{B} = 0$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y} - \right.$$

B) $\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x}$

$$\frac{\partial B_{x(0)}}{\partial z} + \frac{\partial a_1}{\partial z} y + \frac{\partial a_2}{\partial z} y^2 + \frac{\partial a_3}{\partial z} y^3 + \frac{\partial a_4}{\partial z} y^4$$

$$\frac{\partial B_{x(0)}}{} \quad \frac{\partial B_{z(0)}}{}$$

2) $\frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z}$

$\frac{\partial B_{z(0)}}{\partial y} \to 0 = c_1 + 2c_2 y + 3c_3 y^2 + 4c_4 y^3 =$

Use of $\nabla^2 B_x = 0$

$$\frac{\partial^2}{\partial x^2} B_x + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial B_x}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{B_x}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial B_x}{\partial z}\right) = 0$$

Use of $\nabla^2 B_y = 0$

$$\frac{\partial^2}{\partial x}\left(\frac{\partial B_y}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial B_y}{\partial y}\right) + \frac{\partial^2}{\partial z}\left(\frac{\partial B_y}{\partial z}\right)$$

$$\frac{\partial^2 B_{y(0)}}{\partial x^2} + \frac{\partial^2 b_1}{\partial x^2} y + \frac{\partial^2 b_2}{\partial x^2} y^2 + \frac{\partial^2 b_3}{\partial x^2} y^3 + \frac{\partial^2 b_4}{\partial x^2}$$

Use of $\nabla^2 B_z = 0$

$$\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial B_z}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial B_z}{\partial z}\right.$$

$$\frac{\partial^2 B_z(0)}{\partial x^2} + \frac{\partial^2 c_1}{\partial x^2} y + \frac{\partial^2 c_2}{\partial x^2} y^2 + \frac{\partial^2 c_3}{\partial x^2} y^3 -$$

From (4)

$$b_1 = -\left[\frac{\partial B_x(x,0,z)}{\partial x} + \frac{\partial B_z(x,0,z)}{\partial z}\right]$$

From (11)

FROM (12)

$$a_2 = \frac{1}{2}\frac{\partial b_1}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}\left\{-\left[\frac{\partial B_x}{\partial}\right.\right.$$

$$= -\frac{1}{2}\left[\frac{\partial^2 B_x(x,0,z)}{\partial x^2} + \frac{\partial^2}{\partial}\right.$$

From (5)

$$b_2 = -\frac{1}{2}\left[\frac{\partial a_1}{\partial x} + \frac{\partial c_1}{\partial z}\right] = -\frac{1}{2}$$

From (32)

$$C_2 = \frac{1}{2} \frac{\partial b_1}{\partial z} = -\frac{1}{2} \left[\frac{\partial B_{x(0)}}{\partial x \partial z} \right.$$

From (13)

$$a_3 = \frac{1}{3}\frac{\partial b_2}{\partial x} = -\frac{1}{6}\left[\frac{\partial^3}{\partial \ldots}\right.$$

From (6)

$$b_3 = -\frac{1}{3}\left[\frac{\partial a_2}{\partial x} + \frac{\partial c_2}{\partial z}\right] z -$$

From 10G

$$+\frac{\partial}{\partial}\left[-\frac{1}{}\left[\frac{\partial^2 B_x}{} + \frac{\partial^2 B_z}{}\right]\right]$$

From (33)

$$C_z = \frac{1}{3}\frac{\partial b_2}{\partial z} = -\frac{1}{6}\left[\frac{\partial^3 B_y}{\partial z \partial x^2} + \right.$$

$$C_z = -\frac{1}{6}\left[\frac{\partial^3 B_y}{\partial \ \partial x^2} + \frac{\partial^3}{\ }\right.$$

From (14)

$$a_4 = \frac{1}{4}\frac{\partial b_3}{\partial x} = \frac{1}{24}\left[\frac{\partial^4 B_x}{\partial x^4} + \right.$$

From (7)

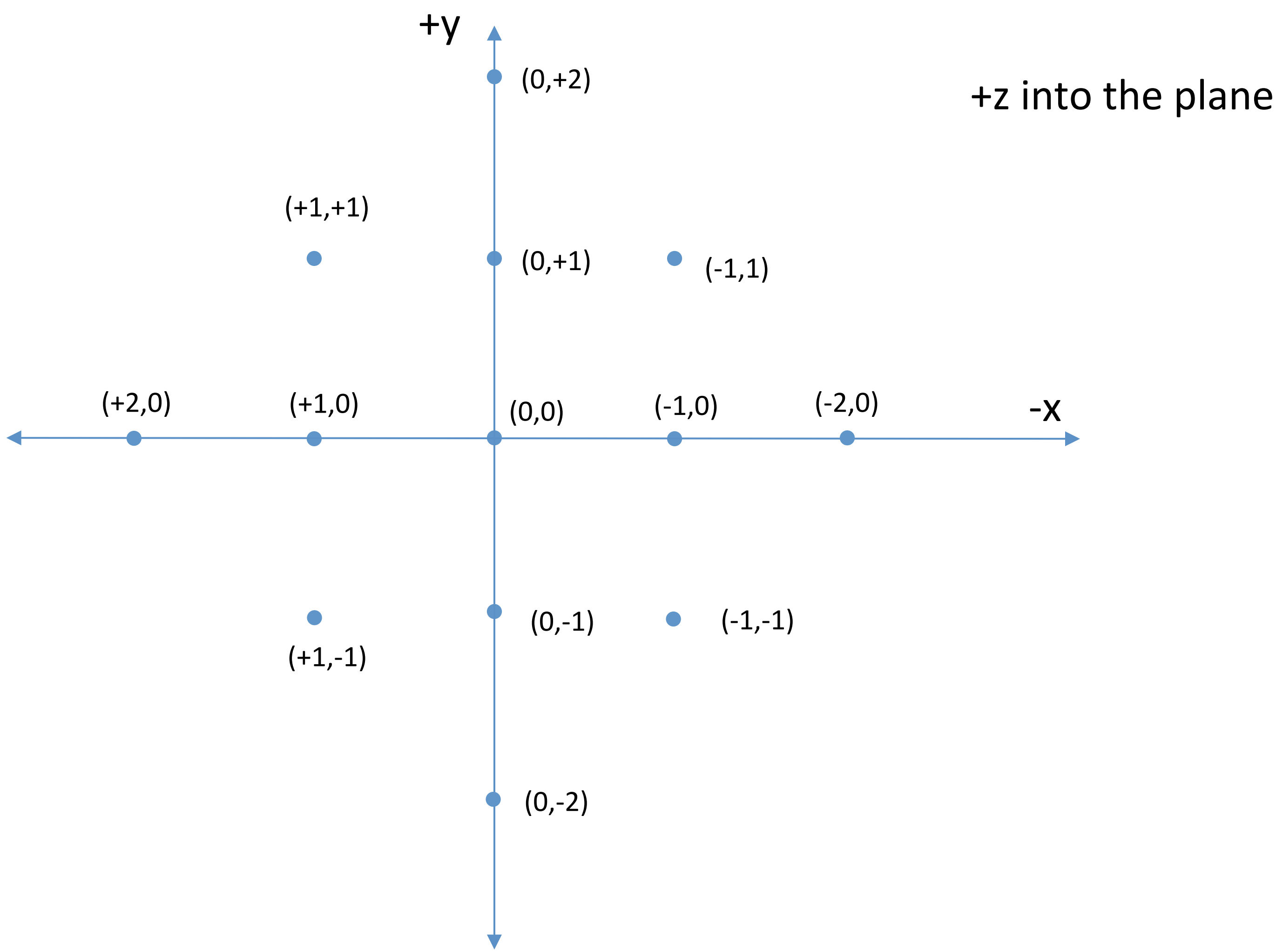
$$b_4 = -\frac{1}{4}\left[\frac{\partial a_3}{\partial x} + \frac{\partial c_3}{\partial z}\right] = +$$

From (34)

$$C_4 = \frac{1}{4}\frac{\partial b_3}{\partial z} = \frac{1}{24}\left[\frac{\partial^4 B_x}{\partial x^3 \partial z} + \cdots\right.$$

$$C_4 = \frac{1}{\cdots}\left[\frac{\partial^4 B_x}{\cdots} + \frac{\partial^4 B_x}{\cdots}\right.$$

How we calculate the derivatives



$$B_y(x,z) = B_y(0,0) + \frac{1}{1!}\left.\frac{\partial B_y(x,z)}{\partial x}\right|_{\substack{x=0\\z=0}} \Delta x + \frac{1}{1!}\left.\frac{\partial B_y(x,z)}{\partial z}\right|_{\substack{x=0\\z=0}} \Delta z$$

$$\frac{1}{2!}\left(\left.\frac{\partial^2 B_y(x,z)}{\partial x^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 + 2\left.\frac{\partial^2 B_y(x,z)}{\partial x \partial z}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z + \left.\frac{\partial^2 B_y(z,z)}{\partial z^2}\right|_{\substack{x=0\\z=0}} \Delta z^2 \right) +$$

$$\frac{1}{3!}\left(\left.\frac{\partial^3 B_y(x,z)}{\partial x^3}\right|_{\substack{x=0\\z=0}} \Delta x^3 + 3\left.\frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\right|_{\substack{x=0\\y=0}} \Delta x^2 \Delta z + 3\left.\frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x \Delta z^2 + \left.\frac{\partial^3 B_y(x,z)}{\partial z^3}\right|_{\substack{x=0\\z=0}} \Delta z^3 \right) +$$

$$\frac{1}{4!}\left(\left.\frac{\partial^4 B_y(x,z)}{\partial x^4}\right|_{\substack{x=0\\z=0}} \Delta x^4 + 4\left.\frac{\partial^4 B_y(x,z)}{\partial x^3 \partial z}\right|_{\substack{x=0\\y=0}} \Delta x^3 \Delta z + 6\left.\frac{\partial^4 B_y(x,z)}{\partial x^2 \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z^2 + 4\left.\frac{\partial^4 B_y(x,z)}{\partial x \partial z^3}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z^3 + \left.\frac{\partial^4 B_y(x,z)}{\partial z^4}\right|_{\substack{x=0\\z=0}} \Delta z^4 \right)$$

$$B_y(1,0) = B_y(0,0) + \frac{1}{1!}\frac{\partial B_y(x,z)}{\partial x}\bigg|_{\substack{x=0\\z=0}}\Delta x + \frac{1}{2!}\left(\frac{\partial^2 B_y(x,z)}{\partial x^2}\bigg|_{\substack{x=0\\z=0}}\Delta x^2\right) + \frac{1}{3!}\left(\frac{\partial^3 B_y(x,z)}{\partial x^3}\bigg|_{\substack{x=0\\z=0}}\Delta x^3\right) + \frac{1}{4!}\left(\frac{\partial^4 B_y(x,z)}{\partial x^4}\bigg|_{\substack{x=0\\z=0}}\Delta x^4\right) \quad (1)$$

$$B_y(-1,0) = B_y(0,0) - \frac{1}{1!}\frac{\partial B_y(x,z)}{\partial x}\bigg|_{\substack{x=0\\z=0}}\Delta x + \frac{1}{2!}\left(\frac{\partial^2 B_y(x,z)}{\partial x^2}\bigg|_{\substack{x=0\\z=0}}\Delta x^2\right) - \frac{1}{3!}\left(\frac{\partial^3 B_y(x,z)}{\partial x^3}\bigg|_{\substack{x=0\\z=0}}\Delta x^3\right) + \frac{1}{4!}\left(\frac{\partial^4 B_y(x,z)}{\partial x^4}\bigg|_{\substack{x=0\\z=0}}\Delta x^4\right) \quad (2)$$

$$B_y(1,0) + B_y(-1,0) = 2B_y(0,0) + \frac{2}{2!}\left(\frac{\partial^2 B_y(x,z)}{\partial x^2}\bigg|_{\substack{x=0\\z=0}}\Delta x^2\right) + \frac{2}{4!}\left(\frac{\partial^4 B_y(x,z)}{\partial x^4}\bigg|_{\substack{x=0\\z=0}}\Delta x^4\right) \quad (1)+(2) => (3)$$

$$B_y(1,0) - B_y(-1,0) = \frac{2}{1!}\frac{\partial B_y(x,z)}{\partial x}\bigg|_{\substack{x=0\\z=0}}\Delta x + \frac{2}{3!}\left(\frac{\partial^3 B_y(x,z)}{\partial x^3}\bigg|_{\substack{x=0\\z=0}}\Delta x^3\right) \quad (1)\text{-}(3) => (4)$$

$$B_y(2,0) + B_y(-2,0) = 2B_y(0,0) + \frac{8}{2!}\left(\frac{\partial^2 B_y(x,z)}{\partial x^2}\bigg|_{\substack{x=0\\z=0}}\Delta x^2\right) + \frac{32}{4!}\left(\frac{\partial^4 B_y(x,z)}{\partial x^4}\bigg|_{\substack{x=0\\z=0}}\Delta x^4\right) \quad (5)$$

$$B_y(2,0) - B_y(-2,0) = +\frac{4}{1!}\frac{\partial B_y(x,z)}{\partial x}\bigg|_{\substack{x=0\\z=0}}\Delta x + \frac{16}{3!}\left(\frac{\partial^3 B_y(x,z)}{\partial x^3}\bigg|_{\substack{x=0\\z=0}}\Delta x^3\right) \quad (6)$$

$$8\Big(B_y(1,0) - B_y(-1,0)\Big) - \Big(B_y(2,0) - B_y(-2,0)\Big) = \frac{12}{1!}\left.\frac{\partial B_y(x,z)}{\partial x}\right|_{\substack{x=0\\z=0}} \Delta x \qquad 8*(4)-(6) => (7)$$

$$\Big(B_y(2,0) - B_y(-2,0)\Big) - 2\Big(B_y(1,0) - B_y(-1,0)\Big) - = \frac{12}{3!}\left.\frac{\partial^3 B_y(x,z)}{\partial x^3}\right|_{\substack{x=0\\z=0}} \Delta x^3 \qquad (6)-2*(4) => (8)$$

$$\Big(B_y(2,0) + B_y(-2,0)\Big) - 4\Big(B_y(1,0) + B_y(-1,0)\Big) = -6B(0,0) + \frac{24}{4!}\left.\frac{\partial^4 B_y(x,z)}{\partial x^4}\right|_{\substack{x=0\\z=0}} \Delta x^4 \qquad (5)-4*(3) => (9)$$

$$16\Big(B_y(1,0) + B_y(-1,0)\Big) - \Big(B_y(2,0) + B_y(-2,0)\Big) = 30B(0,0) + \frac{24}{2!}\left.\frac{\partial^2 B_y(x,z)}{\partial x^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \qquad 16*(3)-(5) => (10)$$

$$\frac{\partial B}{\partial x}(\Delta x) = \frac{1}{12}\left[8(B_{10}-B_{-10})-(B_{20}-B_{-20})\right]$$

$$\frac{\partial^2 B}{\partial x^2}(\Delta x)^2 = \frac{1}{12}\left[16(B_{10}+B_{-10})-(B_{20}+B_{-20})-30B_{00}\right]$$

$$\frac{\partial^3 B}{\partial x^3}(\Delta x)^3 = \frac{1}{2}\left[(B_{20}-B_{-20})-2(B_{10}-B_{-10})\right]$$

$$\frac{\partial^4 B}{\partial x^4}(\Delta x)^4 = \left[(B_{20}+B_{-20})-4(B_{10}+B_{-10})+6B_{00}\right]$$

$$\frac{\partial B}{\partial z}(\Delta z) = \frac{1}{12}\left[8(B_{01}-B_{0-1})-(B_{02}-B_{0-2})\right]$$

$$\frac{\partial^2 B}{\partial z^2}(\Delta z)^2 = \frac{1}{12}\left[16(B_{01}+B_{0-1})-B_{02}+B_{0-2})-30B_{00}\right]$$

$$\frac{\partial^3 B}{\partial z^3}(\Delta z)^3 = \frac{1}{2}\left[(B_{02}-B_{0-2})-2(B_{01}-B_{0-1})\right]$$

$$\frac{\partial^4 B}{\partial z^4}(\Delta z)^4 = \left[(B_{02}+B_{0-2})-4(B_{01}+B_{0-1})+6B_{00}\right].$$

$$B_y(1,-1) - B_y(-1,1) = \frac{2}{1!}\frac{\partial B_y(x,z)}{\partial x}\bigg|_{\substack{x=0\\z=0}}\Delta x - \frac{2}{1!}\frac{\partial B_y(x,z)}{\partial z}\bigg|_{\substack{x=0\\z=0}}\Delta z +$$

$$\frac{1}{3!}\left(2\frac{\partial^3 B_y(x,z)}{\partial x^3}\bigg|_{\substack{x=0\\z=0}}\Delta x^3 - 6\frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\bigg|_{\substack{x=0\\y=0}}\Delta x^2 \Delta z + 6\frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\bigg|_{\substack{x=0\\z=0}}\Delta x \Delta z^2 + \frac{\partial^3 B_y(x,z)}{\partial z^3}\bigg|_{\substack{x=0\\z=0}}\Delta z^3\right) \quad (11)$$

$$B_y(1,-1) - B_y(-1,1) = \big(B_y(1,0) - B_y(-1,0)\big) - \big(B_y(0,1) - B_y(0,-1)\big) + \left(\frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\bigg|_{\substack{x=0\\z=0}}\Delta x \Delta z^2 - \frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\bigg|_{\substack{x=0\\y=0}}\Delta x^2 \Delta z\right) \quad (11)$$

$$B_y(1,1) - B_y(-1,-1) = \frac{2}{1!}\frac{\partial B_y(x,z)}{\partial x}\bigg|_{\substack{x=0\\z=0}}\Delta x + \frac{2}{1!}\frac{\partial B_y(x,z)}{\partial z}\bigg|_{\substack{x=0\\z=0}}\Delta z +$$

$$\frac{2}{3!}\left(2\frac{\partial^3 B_y(x,z)}{\partial x^3}\bigg|_{\substack{x=0\\z=0}}\Delta x^3 + 3\frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\bigg|_{\substack{x=0\\y=0}}\Delta x^2 \Delta z + 3\frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\bigg|_{\substack{x=0\\z=0}}\Delta x \Delta z^2 + \frac{\partial^3 B_y(x,z)}{\partial z^3}\bigg|_{\substack{x=0\\z=0}}\Delta z^3\right) \quad (12)$$

$$B_y(1,1) - B_y(-1,-1) = \big(B_y(1,0) - B_y(-1,0)\big) + \big(B_y(0,1) - B_y(0,-1)\big) + \left(\frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\bigg|_{\substack{x=0\\y=0}}\Delta x^2 \Delta z + \frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\bigg|_{\substack{x=0\\z=0}}\Delta x \Delta z^2\right) \quad (12)$$

$$B_y(1,1) + B_y(-1,-1) = 2B_y(0,0)$$

$$\frac{2}{2!}\left(\left.\frac{\partial^2 B_y(x,z)}{\partial x^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 + 2\left.\frac{\partial^2 B_y(x,z)}{\partial x \partial z}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z + \left.\frac{\partial^2 B_y(z,z)}{\partial z^2}\right|_{\substack{x=0\\z=0}} \Delta z^2 \right) +$$

$$\frac{2}{4!}\left(\left.\frac{\partial^4 B_y(x,z)}{\partial x^4}\right|_{\substack{x=0\\z=0}} \Delta x^4 + 4\left.\frac{\partial^4 B_y(x,z)}{\partial x^3 \partial z}\right|_{\substack{x=0\\y=0}} \Delta x^3 \Delta z + 6\left.\frac{\partial^4 B_y(x,z)}{\partial x^2 \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z^2 + 4\left.\frac{\partial^4 B_y(x,z)}{\partial x \partial z^3}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z^3 + \left.\frac{\partial^4 B_y(x,z)}{\partial z^4}\right|_{\substack{x=0\\z=0}} \Delta z^4 \right) +$$

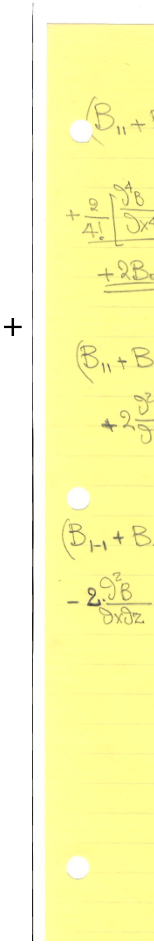
$$2B(0,0) - 2B(0,0) \quad (13)$$

$$B_y(1,1) + B_y(-1,-1) = \left(B_y(1,0) - B_y(-1,0)\right) - 2B(0,0)$$

$$\left(2\left.\frac{\partial^2 B_y(x,z)}{\partial x \partial z}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z + \frac{1}{3}\left.\frac{\partial^4 B_y(x,z)}{\partial x^3 \partial z}\right|_{\substack{x=0\\y=0}} \Delta x^3 \Delta z + \frac{1}{2}\left.\frac{\partial^4 B_y(x,z)}{\partial x^2 \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z^2 + \frac{1}{3}\left.\frac{\partial^4 B_y(x,z)}{\partial x \partial z^3}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z^3 \right) \quad (13)$$

$$B_y(1,-1) + B_y(-1,1) = \left(B_y(1,0) + B_y(-1,0)\right) + \left(B_y(0,1) + B_y(0,-1)\right) - 2B(0,0) +$$

$$\left(-2\left.\frac{\partial^2 B_y(x,z)}{\partial x \partial z}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z - \frac{1}{3}\left.\frac{\partial^4 B_y(x,z)}{\partial x^3 \partial z}\right|_{\substack{x=0\\y=0}} \Delta x^3 \Delta z + \frac{1}{2}\left.\frac{\partial^4 B_y(x,z)}{\partial x^2 \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z^2 - \frac{1}{3}\left.\frac{\partial^4 B_y(x,z)}{\partial x \partial z^3}\right|_{\substack{x=0\\y=0}} \Delta x \Delta z^3 \right) \quad (14)$$



$$\left(B_y(1,-1)-B_y(-1,1)\right)+\left(B_y(1,1)-B_y(-1,-1)\right)=2\left(B_y(1,0)-B_y(-1,0)\right)+2\left(\left.\frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x \Delta z^2\right) \quad (11)+(12) => (15)$$

$$\left(\left.\frac{\partial^3 B_y(x,z)}{\partial x \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x \Delta z^2\right) = \frac{1}{2}\left(\left(B_y(1,1)-B_y(-1,-1)\right)+\left(B_y(1,-1)-B_y(-1,1)\right)\right)-\left(B_y(1,0)-B_y(-1,0)\right)+2 \quad (11)+(12) => (15)$$

$$\left(B_y(1,1)-B_y(-1,-1)\right)-\left(B_y(1,-1)-B_y(-1,1)\right)+=2\left(B_y(0,1)-B_y(0,-1)\right)+2\left(\left.\frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z\right) \quad (11)-(12) => (16)$$

$$\left(\left.\frac{\partial^3 B_y(x,z)}{\partial x^2 \partial z}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z\right) = \frac{1}{2}\left(\left(B_y(1,1)-B_y(-1,-1)\right)-\left(B_y(1,-1)-B_y(-1,1)\right)\right)-\left(B_y(0,1)-B_y(0,-1)\right)+2 \quad (11)-(12) => (16)$$

$$\left(B_y(1,1)+B_y(-1,-1)\right)+\left(B_y(1,-1)-B_y(-1,1)\right)=2\left(B_y(1,0)+B_y(-1,0)\right)+2\left(B_y(0,1)+B_y(0,-1)\right)+4B(0,0)+\left(\left.\frac{\partial^4 B_y(x,z)}{\partial x^2 \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z^2\right) \quad (13)+(14) \Rightarrow (17)$$

$$\left(\left.\frac{\partial^4 B_y(x,z)}{\partial x^2 \partial z^2}\right|_{\substack{x=0\\z=0}} \Delta x^2 \Delta z^2\right)=\left(B_y(1,1)+B_y(-1,-1)\right)+\left(B_y(1,-1)-B_y(-1,1)\right)-2\left(B_y(1,0)+B_y(-1,0)\right)-2\left(B_y(0,1)+B_y(0,-1)\right)+4B(0,0)+ \quad (13)+(14) \Rightarrow (17)$$