

ERIT-FFAG and Beta Beams

F. Benedetto

September 15, 2011

Abstract

In this note we discuss the possibility to use ERIT-FFAG as a Production Ring for Beta Beam isotopes and/or to perform i-cooling experiments of interest for EuroNu.

Contents

1	Introduction	2
1.1	Beta Beams needs	2
2	ERIT (Emittance Recovery Internal Target) FFAG	2
3	Cooling rates, emittances,...	2
4	Proof of principle experiment proposal	4
5	Conclusion	5
6	Acknowledgments	5
A	appendix	6

1 Introduction

BetaBeams: radioactive ions for nu-oscillation experiments

storage ring with internal target for secondary particle production

Rubbia and Mori came with idea.

Main diff is that there is no horizontal-longitudinal coupling in ERIT-FFAG, but the very large horizontal acceptance can be a plus.

1.1 Beta Beams needs

${}^7\text{Li}(\text{D},?){}^8\text{Li}$

${}^6\text{Li}(\text{3He},?){}^8\text{B}$

Inverse kinematics, i.e. Lithium ions stored and gas jet target.

Requirements in term of target thickness to produce enough secondary ions are extremely challenging, therefore liquid Lithium target and direct kinematics.

R&D is going-on on the feasibility of a liquid lithium target

Solid target not an option because of several 100kW power deposited.

2 ERIT (Emittance Recovery Internal Target) FFAG

ERIT description, proof of principle, ref. to NIM article and Okabe-san papers

Vertical acceptance is limiting number of surviving turns, therefore studies for insertions Ref. JB, FFAG10.

3 Cooling rates, emittances,...

The blow-up due to Multiple Coulomb Scattering is evaluated using the Moliere rms angle equation:

$$\theta_c = \sqrt{\langle\theta^2\rangle} = \frac{14.1\text{MeV}}{\beta_r c p} z \sqrt{\frac{t}{\chi_0}} \left[1 + 0.038 \ln \frac{t}{\chi_0} \right] \quad (1)$$

where β_r, c, γ_r, p and z are the velocity, relativistic mass factor, momentum and charge of the incident ion and χ_0 is the radiation length.

The mean energy lost at the target is estimated via the Bethe-Bloch formula [1]:

$$\Delta E_{BB} = \left\langle \frac{dE}{dx} \right\rangle t = K z^2 \frac{Z}{A} \frac{1}{\beta_r^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta_r^2 \gamma_r^2 T_{\max}}{I^2} - \beta_r^2 - \frac{1}{2} \delta(\beta_r \gamma_r) \right] t \quad (2)$$

where A, Z and I are the target atomic mass, charge and mean excitation energy. The quantity

$$T_{\max} = \frac{2m_e c^2 \beta_r^2 \gamma_r^2}{1 + 2\gamma_r m_e / M + (m_e / M)^2} \quad (3)$$

is the maximum kinetic energy which can be imparted to a free electron in a single collision, with m_e the electron mass and M the mass of the incident particle, and $K = 4\pi N_A r_e^2 m_e c^2$ is a constant, being r_e the classical electron radius and N_A the Avogadro's number.

Following the derivation of [2], the equations for the normalized horizontal, vertical and longitudinal emittances are:

$$\frac{d\varepsilon_{N,x}}{ds} = -J_x \frac{1}{p} \frac{dp}{ds} \varepsilon_{N,x} + \beta_r \gamma_r \frac{\beta_x}{2} \frac{d\langle\theta_c^2\rangle}{ds} \quad (4)$$

$$\frac{d\varepsilon_{N,y}}{ds} = -J_y \frac{1}{p} \frac{dp}{ds} \varepsilon_{N,y} + \beta_r \gamma_r \frac{\beta_y}{2} \frac{d\langle\theta_c^2\rangle}{ds} \quad (5)$$

$$\frac{d\varepsilon_{N,l}}{ds} = -J_l \frac{1}{p} \frac{dp}{ds} \varepsilon_{N,l} + \beta_r \gamma_r \frac{\beta_l}{2} \frac{d\langle\delta_{rms}^2\rangle}{ds} \quad (6)$$

The first term represents the energy-loss cooling effect and is characterized by the partition numbers J_x , J_y , J_l which, in case there is no coupling between the planes, are:

$$J_x = 1, \quad J_y = 1, \quad J_l = \frac{\frac{\partial}{\partial E} \left(\frac{dE_L}{ds} \right)}{\frac{1}{p} \frac{dp}{ds}} \quad (7)$$

The second term in the equations is the blow-up term due to Multiple Coulomb Scattering ($\langle\theta_c^2\rangle$ is the rms multiple scattering angle) or Energy Straggling ($\langle\delta_{rms}^2\rangle$ is the rms induced relative momentum spread). γ_r is the relativistic mass factor, $\beta_{x,y}$ are the horizontal and vertical betatron functions at the target location and β_l is a longitudinal focusing function defined as:

$$\beta_l^2 \equiv \frac{\langle z^2 \rangle}{\langle \delta^2 \rangle} = \frac{\beta_r^2 p C \lambda_{RF} \alpha_c}{2\pi z e V_{RF} \sin \phi_s} \quad (8)$$

where C is the ring circumference, α_c the momentum compaction, ze the particle charge, λ_{RF} and V_{RF} the RF wavelength and RF voltage and ϕ_s the synchronous RF phase. (dp/ds) is the momentum loss rate at the target, which corresponds, in terms of kinetic energy E_c and energy losses $\left(\frac{dE_L}{ds}\right)$, evaluated via the Bethe-Bloch formula [1], to:

$$\frac{1}{p} \frac{dp}{ds} = \frac{\gamma_r}{\gamma_r + 1} \frac{1}{E_c} \left(\frac{dE_L}{ds} \right) \quad (9)$$

The damping time in the transverse plane is:

$$\tau = \left(\int_0^t \frac{1}{p} \frac{dp}{ds} ds \right)^{-1} \quad (10)$$

and the equilibrium emittance is:

$$\varepsilon_{\perp} = \frac{1}{\beta_r \gamma_r} \varepsilon_N = \frac{\int_0^t \frac{\beta_{\perp}}{2} \frac{d\langle\theta_c^2\rangle}{ds} ds}{\int_0^t \frac{1}{p} \frac{dp}{ds} ds} \quad (11)$$

Concerning the longitudinal plane, one should note that the rate of change of the energy lost at the target with respect to the change of

energy, thus the slope of the Bethe-Bloch curve, is strongly negative for the energies of interest and therefore in this plane there is no ionization cooling, but heating:

$$J_l = \frac{\frac{\partial}{\partial E} \left(\frac{dE_L}{ds} \right)}{\frac{1}{p} \frac{dp}{ds}} \approx -2.2 \quad (12)$$

The following Table shows the results for the original ERIT experiment (11 MeV protons on a Be target) and the results for a Beta Beam experiment (5.5 MeV Deuterium on Lithium7), keeping the same $B\rho$, for two different target thickness:

ERIT parameters	11 MeV p on Be	5.5 MeV D on ${}^7\text{Li}$	5.5 MeV D on $10\mu\text{m } {}^7\text{Li}$
Beam Kinetic Energy	11 MeV	5.5 MeV	5.5 MeV
Target thickness	5 μm	5 μm	10 μm
Energy lost at the target	43 keV	53 keV	105 keV
Relative momentum lost	0.2%	0.48%	1%
Cooling time τ	506	209	104
Multiple Coulomb Scattering $\langle\theta_c^2\rangle$	$1.92 \cdot 10^{-6}$	$1.40 \cdot 10^{-6}$	$3.09 \cdot 10^{-6}$
Equilibrium horizontal rms emittance	663 $\mu\text{m mrad}$	199 $\mu\text{m mrad}$	220 $\mu\text{m mrad}$
Equilibrium vertical rms emittance	385 $\mu\text{m mrad}$	116 $\mu\text{m mrad}$	128 $\mu\text{m mrad}$
Vertical rms beam size	22.5 μm	12.4 μm	13.0 μm

ERIT-FFAG has the following optics functions:

β_x at the target	1.36 m
β_y at the target	0.79 m
β_x maximum	1.56 m
β_y maximum	1.32 m
D_x	0.8 m

Such a large difference in terms of equilibrium emittance comes mainly from the higher relative momentum lost at the target by the 5.5 MeV deuterium beam, which corresponds to a more efficient ionization cooling. Multiple Coulomb Scattering is also $\sim 20\%$ smaller in the case of Deuterium traversing a Lithium foil.

In the Table, $5\mu\text{m}$ Lithium target is used in order to have a direct comparison with the Beryllium case. However, we are aware of the fact of the problem of manufacturing a lithium foil. The $10\mu\text{m}$ thickness is also reported as it represents a more realistic value, in case a liquid lithium target is built instead, as foreseen for the Beta Beam production ring. The RF cavity which is needed to compensate for the energy lost at the target needs in this case to provide twice as much voltage.

4 Proof of principle experiment proposal

As shown in the previous section, Deuterium beam on a target is better than Protons (for the same target thickness) in terms of equilibrium

emittances.

The deuterium beam could therefore survive longer in the machine than in the case of protons and therefore one could expect conclusive proof of transverse i-cooling.

We will need a Deuterium source and unfortunately another RFQ. DTL linac can work in 2π mode.

Diagnostic to count the produced 8Li (scintillators for the betas produced)

Main problem is Lithium target: fragile, difficult to handle, cannot get in contact with air or moisture(?)

For the purpose of the experiment it would probably be better to use another target (carbon?) with an appropriate thickness to get a similar blow-up and energy losses than Lithium.

5 Other studies

Following figures report the dependence of emittance,

6 Conclusion

ERIT-FFAG may be a possible candidate as a Beta-Beam Production ring.

Main issue could be the size of the internal target (which serves also as a charge-exchange stripper) to be placed in this compact lattice and the production rates. Calculations indeed seem to indicate that a kinetic energy higher than 5.5 MeV (i.e. around 25 MeV) would be better in terms of production rates [?], as a compromise between reduction of the production cross section and increase in the revolution frequency and in the possible target thickness.

A proof of principle experiment of secondary particle production with emittance recovery has been successfully performed in year???? Ref.Okabe??? with proton beam on Beryllium target. However, due to the relatively small vertical acceptance of the machine, the number of survival turns in the machine is limited and comparable with the i-cooling time.

Using deuterium as a stored beam, as we plan to do for the Beta Beam isotope 8Li production, goes in the right direction. The i-cooling is more efficient for the same target thickness and similar energy losses, and as a result the vertical equilibrium beam size would be almost a factor 2 smaller. The beam will therefore survive longer in the machine and may lead to a conclusive i-cooling experiment in a storage ring.

An experiment with 5.5 MeV circulating Deuterium requires no modification to the ERIT-FFAG, the DTL and the injection, but requires a D-source and a new RFQ. Due to the issue of handling Lithium target (and its fragility) a first experiment can be done with another material with an appropriate thickness to get similar energy losses and multiple coulomb scattering.

7 Acknowledgments

This work started during my visit at KURRI in Nov.'10 and would have not been possible without the encouragement, ideas, and financial support of Y. Mori. I would also like to thank K. Okabe, J.B. Lagrange and T. Planche for useful discussions at KURRI and E. Wildner for her continuous support.

EURONU, blabla lba

References

- [1] C. Amsler et al. (Particle Data Group). *Physics Letters B*, 1(667), 2008.
- [2] D. Neuffer. Muon cooling. *Nucl. Instrum. Methods Phys. Res. A*, 532:26, 2004.

A appendix